THE DEVELOPMENT OF SPECIALIZED CONTENT KNOWLEDGE IN BEGINNING ALGEBRA AMONG SECONDARY MATHEMATICS PRE-SERVICE TEACHERS

by

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The goal of teacher preparation programs is to equip future secondary mathematics teachers for the broad spectrum of mathematical learners they will encounter. Part of that process includes developing their Specialized Content Knowledge, defined by Ball, Hill, and Bass (2005) as a deeper understanding of mathematics that allows teachers to explain new ideas, work problems in multiple ways, and analyze student solutions. This study examined the development of Specialized Content Knowledge among secondary mathematics pre-service teachers. Forty-seven pre-service teachers completed a demographic survey and an assessment measuring Specialized Content Knowledge. Twenty-three of those pre-service teachers were then interviewed to further illuminate experiences that influenced their responses on the assessment, and to elaborate on experiences that they identified as helping them develop Specialized Content Knowledge. Qualitative analysis revealed two broad categories aiding in the development of Specialized Content Knowledge: coursework and interactions with learners. Within the category of coursework, pre-service teachers highlighted course components that were also related to interactions with learners, often in a hypothetical way (e.g., analyzing fictional student work). Findings indicate that the development of Specialized Content Knowledge is strongly influenced by interactions with learners, both face to face and hypothetical. These results are discussed along with recommendations for practice and future research.
All of the work was complete. Certifications, licenses, and exams were in the past. A feeling of relief rushed over the new teacher who had done everything it takes to land her first real job, as an algebra teacher at the local high school. She had been dreaming of the day she'd have her own classroom and she knew she was prepared to teach algebra. After all, she had taken every mathematics course that her university had to offer. Alongside aspiring mathematicians, engineers, and scientists, she had been the only aspiring teacher, and she found herself helping classmates who struggled with challenging material. Her education professors had taught her everything she needed to know about teaching, from designing Powerpoint presentations to managing a disruptive classroom. Finally, the bell rang and students filed into the classroom.

The day started out just as she had imagined. Her lesson plans helped guide her throughout the day, and she was pleased with her decision to become a teacher and all of the hard work she had put in. In the afternoon class, things took a turn. Some of her students struggled to grasp the strategy she was using to simplify a rational expression. This method had worked with all of the students in the morning classes, and this was the way she remembered learning it in high school. The new teacher could not believe how quickly she became flustered, and on the first day of teaching beginning algebra! She knew that topic like the back of her hand!

After the students left for the day, the stress of the situation was too much for her to hold inside any longer. Her eyes welled up with tears as she made a mad dash for her car. On the way, she bumped into an experienced mathematics teacher, one of her new
co-workers that she had befriended in meetings the week prior. They talked about what had happened with the student who could not understand her teaching strategy for simplifying rational expressions. Her experienced friend calmed her down and showed her some other ways of explaining how to simplify rational expressions. The experienced teacher said she had learned these alternative strategies in her mathematics education courses. The new teacher sighed with relief knowing that she had survived the first day, but anxiously worried if maybe her teacher preparation program had not taught her everything she needed after all.

**What do Aspiring Teachers "Need to Know"?**

The *Mathematical Education of Teachers II* (2012) is an attempt by the mathematics community to answer that question. Published by the Conference Board of the Mathematical Sciences with a writing team representing expert mathematicians and mathematics educators across the United States, this report provides recommendations for teacher preparation programs in K-12 Mathematics. Two major themes throughout the *MET II* (2012) are that "[p]roficiency with school mathematics is necessary but not sufficient mathematical knowledge for a teacher" (p. xii), and "the mathematical knowledge needed for teaching differs from that of other professions" (p. xii). As a means to develop this specialized knowledge at the secondary level of mathematics, the *MET II* (2012) "recommends that the mathematics courses taken by prospective high school teachers include at least...9 semester-hours explicitly focused on high school mathematics from an advanced standpoint" (p. 55), while also acknowledging that
recommends a total of 9 semester-hours of coursework designed for prospective teachers are ambitious and will take years to achieve and institutions that serve only a few prospective teachers per year may be unable to offer many courses with prospective teachers as the sole audience (p. 55). This is one reason there are many teachers facing the same dilemma as the hypothetical new teacher described in the introduction.

The *MET II* (2012) describes a special type of mathematical knowledge needed for teaching, that is usually not needed in other mathematical professions. This type of knowledge, sometimes referred to as Specialized Content Knowledge (SCK), is the main focus of this study. By definition, Specialized Content Knowledge is not typically acquired in mathematics courses for audiences other than teachers. The writers of *MET II* (2012) imply that it is difficult, if not impossible, to incorporate mathematics courses solely for future teachers into many teacher preparation programs. Such programs may benefit from the findings of this study, which explores alternative ways that Specialized Content Knowledge can be developed. The following section describes how SCK fits into the larger framework of research on teaching knowledge.

Knowledge for Teaching

**Pedagogical Content Knowledge**

During the nineteenth century and the majority of the twentieth century, it was believed that an understanding of basic content knowledge was sufficient for a teacher of mathematics. This is exemplified in the teacher examinations administered in the 1980s, which only tested basic skills in reading, writing, and arithmetic (Shulman, 1986).
However, in 1986, Lee S. Shulman published "Those Who Understand: Knowledge Growth in Teaching." This article was the first introduction of pedagogical content knowledge, and it is still regularly cited today. Shulman (1986) suggested that a teacher's content knowledge could be subdivided into three categories. Up until this point, researchers had been focusing on subject matter knowledge. Shulman (1986) suggested that researchers should also include pedagogical content knowledge, which he defined as "subject matter knowledge for teaching" (p. 9) and curricular knowledge, which he defined as "the knowledge of alternative curriculum materials for a given subject or topic within a grade...the curriculum materials under study by his or her students in other subjects they are studying at the same time, [and] familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school." (p. 10)

Mathematical Knowledge for Teaching

More recently, building on Shulman’s (1986) Pedagogical Content Knowledge (PCK), "Mathematical Knowledge for Teaching" (MKT) has been introduced as a type of knowledge that includes PCK (Hill, Ball, & Schilling, 2008). Ball, Thames, and Phelps (2008) defined MKT as "the mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). Mathematical Knowledge for Teaching is a necessary component of a teacher's knowledge. Hill, Rowan, and Ball (2005) found that "teachers' content knowledge for teaching mathematics was a significant predictor of student gains" (p. 396) in a study of first and third grade teachers. Figure 1: Diagram of MKT Components shows a representation of MKT, often informally referred to as "the egg,"
which differentiates Pedagogical Content Knowledge from Subject Matter Knowledge (Hill et al., 2008).

As shown in the Diagram of MKT Components (refer to Figure 1), Subject Matter Knowledge can be further broken down into Common Content Knowledge, Knowledge at the Mathematical Horizon, and Specialized Content Knowledge. Common Content Knowledge can be described as the typical mathematics that a student studying mathematics will learn. Common Content Knowledge has no specific connection to teaching, but teachers need to learn Common Content Knowledge so that they have their own understanding of the mathematics they will teach. Knowledge at the Mathematical Horizon is the mathematics that follows what is currently being taught. It is important for teachers to have an understanding of what students will be learning next to ensure a better understanding of the current material and to be able to properly answer student questions (Hill et al., 2008). Common Content Knowledge and Knowledge at the Mathematical Horizon are not the focus of this study.
Specialized Content Knowledge. Ball, Hill, and Bass (2005) defined Specialized Content Knowledge as a deeper understanding of the material that allows teachers to explain new ideas, work problems in multiple ways, and analyze student solutions (as cited in Hill et al., 2008). The researchers explained that none of the three types of Subject Matter Knowledge (Common Content Knowledge, Knowledge at the Mathematical Horizon, and Specialized Content Knowledge) require specific knowledge of students or of teaching, and that a typical mathematician would have that knowledge (Hill et al., 2008). Additionally, Specialized Content Knowledge (SCK) can further be defined as "mathematics that is useful in teaching, but is not typically taught in conventional mathematics classes either at the high school or postsecondary levels" (McCrorry, Floden, Ferrini-Mundy, Reckase, & Senk, 2012, p. 598). McCrorry et al. (2012) go on to say that "for most teachers, [Specialized Content Knowledge] is not included in their formal mathematical education" (p. 598).

Statement of the Problem

Mathematics education researchers have recommended that "it could be useful to study whether and how different approaches to teacher development have different effects on particular aspects of teachers' pedagogical content knowledge" (Ball, Thames, & Phelps, 2008, p. 405). Investigating the knowledge needed for teaching continues to be a primary area of mathematics education research. This study explores two topics within that field of research that have not received much attention: Mathematical
Knowledge for Teaching at the high school level, and Specialized Content Knowledge as a subset of MKT.

In the past decade, an impressive number of studies of MKT and/or SCK have focused on Elementary or K-8 Teachers (e.g. Bair & Rich, 2011a; Baumert et al., 2010; Campbell et al., 2014; Copur-Gencturk, 2015; Hill, Rowan, & Ball, 2005; McCrory et al., 2012). In contrast, there is a dearth of research on Mathematical Knowledge for Teaching (MKT), and especially Specialized Content Knowledge (SCK), at the secondary level. Researchers studying SCK have tended to concentrate on elementary teachers. Why would this be so? Researchers may place greater priority on studying elementary teachers, who receive limited mathematics preparation, compared to prospective high school teachers who study mathematics throughout their entire program. It may be assumed that SCK is naturally developed in a typical mathematics class. Perhaps the lack of courses designed specifically for secondary mathematics teachers simply limits opportunities for researchers to study the knowledge of high school mathematics teachers. Whatever the reason, researchers studying Specialized Content Knowledge have tended to concentrate on elementary mathematics.

Research at the elementary level has resulted in changes in the way elementary teachers are prepared. At the program level, some programs have increased the mathematics course requirements for K-8 teacher preparation based on MET II (2012) recommendations. However, findings from Hill et al. (2005) in a study of first and third grade teachers suggest "neither ensuring teacher certification nor increasing teachers' subject-matter or methods coursework (two common approaches to improving teacher
quality) ensures a supply of teachers with strong content knowledge for teaching mathematics" (p. 393). At the classroom level, some textbooks and instructors aim to "teach" SCK directly in mathematics courses for K-8 teachers that are specifically designed for teachers. For example, Sybilla Beckmann's *Mathematics for Elementary Teachers with Activity Manual* (2010) features activities that walk pre-service teachers through important concepts. The activities ask pre-service teachers to solve problems in multiple ways and explain their reasoning, as well as look at fictional student work to determine whether the students' methods were valid.

Similar changes do not easily translate into coursework for secondary teachers. In many secondary mathematics teacher preparation programs, only the methods course is designed specifically for mathematics pre-service teachers. The additional mathematics courses required of secondary pre-service teachers are designed for a diverse audience, and are often taught by mathematics professors who do not specialize in education. For the mathematics education departments that are able to offer courses specifically designed for secondary mathematics pre-service teachers, the textbook options are limited.

Furthermore, textbooks that could be used for a secondary mathematics course are not designed to support the exploratory activities that are commonly found in elementary mathematics courses. For example, *Mathematics for High School Teachers: An Advanced Perspective* (Usiskin, 2003) is a text specifically designed for pre-service and in-service high school mathematics teachers, focusing on the connections between upper level mathematics courses (e.g. Calculus, Abstract Algebra, etc.) and high school
mathematics topics. While the content of the book is ideal for a prospective secondary mathematics teacher, it has a typical textbook layout of examples and explanations followed by mathematics problems for the students to practice, unlike the activity-based texts available for elementary mathematics courses. Also, more than ten years after its release, only one edition of the textbook has been published. A search for additional textbooks appropriate for a mathematics course for secondary teachers only revealed three other options: *Mathematical Connections: A Companion for Teachers and Others* (Cuoco, 2005), *Mathematics for Secondary School Teachers* (Bremigan, Bremigan, & Lorch, 2011), and *Mathematics Methods and Modeling for Today’s Mathematics Classroom: A Contemporary Approach to Teaching Grades 7-12* (Dossey, 2002), each of which had a similar layout. Additionally, with a wide variety of high school mathematics topics to be included in the curriculum, there is very little time for exploratory activities like those commonly undertaken in an elementary mathematics course.

It is imperative to study how SCK develops and is acquired among secondary pre-service teachers. All K-12 teachers need SCK to be able to explain mathematics in different ways so that struggling students can understand it, make connections between topics, assess student work for solid reasoning, and determine whether one method will apply to solving other mathematics problems. A unique challenge in the high school setting is meeting the needs of students who enter high school from different backgrounds and knowledge levels. Specialized knowledge of mathematics allows secondary teachers to adapt mathematics instruction for this diverse student audience.
Research Questions

The purpose of this study is to investigate, identify, and catalog experiences that contribute to the development of Specialized Content Knowledge (SCK) among a population of secondary mathematics pre-service teachers (PST) in the domain of beginning algebra. A research design based in grounded theory will allow these experiences to emerge from the perspective of the pre-service teachers. A combination of surveys, interviews, and a written assessment will generate data related to teacher preparation programs, but also extend into participants' experiences prior to and outside of formal teacher preparation. One overarching question guides this study, supported by clarifying sub-questions:

What kinds of experiences support the development of Specialized Content Knowledge (SCK) for beginning algebra among pre-service secondary mathematics teachers?

a. In what ways do secondary mathematics pre-service teachers exhibit Specialized Content Knowledge on an assessment measuring SCK in beginning algebra?

b. What teacher preparation experiences do secondary mathematics pre-service teachers identify as aiding in the development of Specialized Content Knowledge as indicated on the SCK assessment?

c. What experiences outside of teacher preparation do secondary mathematics pre-service teachers identify as aiding in the development of SCK?
Significance of Study

The *MET II* (2012) guidelines set forth an ambitious program of study for the preparation of secondary mathematics teachers that recommends nine credit hours of mathematics content coursework designed for mathematics teachers. These guidelines are based on an analysis of the content of secondary school mathematics and the authors’ ideas about how to organize undergraduate mathematics content for teachers. For some teacher preparation programs, the limitations imposed by low student enrollments make it difficult, if not impossible, to meet these goals. Only large teacher preparation programs are likely to have the demand for the recommended courses. Furthermore, while the ideas proposed in *MET II* (2012) seem reasonable, the research has not been done to show that courses designed for secondary mathematics teachers are more effective than traditional coursework. This study may reveal alternative ways to embed SCK in more traditional program requirements and other learning opportunities.

This study investigates relatively unexplored terrain and has the potential to make a difference to researchers and practitioners in three ways. First, it extends what we know about SCK in mathematics by investigating how it is acquired through secondary mathematics teacher preparation. Second, findings that result from this study will provide a new framework for organizing ideas about how secondary mathematics pre-service teachers develop SCK, which is helpful to future research. Third, the results generate suggestions for emphasizing the development of SCK in secondary mathematics teacher preparation programs, which is beneficial to practitioners.
2. REVIEW OF LITERATURE

This chapter begins by defining and clarifying Mathematical Knowledge for Teaching and its various components as they relate to this study. Following that are descriptions of assessment instruments used to measure components of Mathematical Knowledge for Teaching (MKT) at both the elementary and secondary levels, and research that has used or examined these instruments. To provide context for the study, the next section presents research and discusses issues relevant to teacher preparation and retention. The final sections provide justification for the use of qualitative methods to investigate the research questions, including a theoretical framework for the study.

Mathematical Knowledge for Teaching

Lee Shulman (1986) first introduced Pedagogical Content Knowledge (PCK) to educational researchers as

"a second kind of content knowledge, which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching. [This is] still...content knowledge, but...the particular form of content knowledge that embodies the aspects of content most germane to its teachability." (p. 9)

Shulman (1986) goes on to describe skills requiring pedagogical content knowledge, such as identifying the most useful representation of a concept, understanding what makes the learning of specific topics easy or difficult, and recognizing common student misconceptions and how to rectify those misconceptions. As described in Chapter 1, Ball, Thames and Phelps (2008) and Hill, Ball, and Schilling (2008) further defined Mathematical Knowledge for Teaching (MKT) as having two distinct components:
Subject Matter Knowledge and Pedagogical Content Knowledge. Those researchers further argued that Subject Matter Knowledge and Pedagogical Content Knowledge can each be broken down into three types of knowledge, as shown in their Diagram of MKT Components (see Figure 1 in Chapter 1).

Subject Matter Knowledge as a Subset of MKT

Ball et al. (2008) suggest that Subject Matter Knowledge consists of Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and Specialized Content Knowledge (SCK). Common Content Knowledge is defined as "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008, p. 399). Examples of CCK include using terms and notations correctly, and recognizing when a student is giving an incorrect answer. Horizon Content Knowledge (HCK) is defined as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p. 403). Ball et al. (2008) give an example of a teacher who is introducing a number line with only the integers labeled. This teacher would apply HCK when introducing the number line by ensuring students realize that the number line contains other numbers between the labeled integers, even though integers may be the only type of numbers used in that particular example.

The last type of Subject Matter Knowledge and the focus of this study is Specialized Content Knowledge (SCK). Ball et al. (2008) define SCK as "the mathematical knowledge and skill unique to teaching [and] not typically needed for purposes other than teaching" (Ball et al., 2008, p. 400). Some examples of SCK include
looking for patterns in student errors[,]... sizing up whether a nonstandard approach would work in general[,]... understanding different interpretations of the operations[,]... appreciating the difference between... models, talk[ing] explicitly about how mathematical language is used[,] how to choose, make, and use mathematical representations effectively, and how to explain and justify one's mathematical ideas. (Ball et al., 2008, p. 400)

SCK will be discussed in more detail after the three components of pedagogical content knowledge are defined in the following paragraphs.

Pedagogical Content Knowledge as a Subset of MKT

Pedagogical Content Knowledge can be subdivided into Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum. Ball et al. (2008) define Knowledge of Content and Students as, "knowledge that combines knowing about students and knowing about mathematics" (p. 401), and continue by explaining that "[t]eachers must anticipate what students are likely to think and what they will find confusing" (Ball et al., 2008, p. 401). In contrast, Knowledge of Content and Teaching "combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction" (Ball et al., 2008, p. 401). For a definition of Knowledge of Content and Curriculum, Ball et al. (2008) rely on what Shulman (1986) referred to as "curricular knowledge," or

"the knowledge of alternative curriculum materials for a given subject or topic within a grade... the curriculum materials under study by his or her students in other subjects they are studying at the same time...[and] familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school. (p. 10)
It is important to understand these other components of Mathematical Knowledge for Teaching, because they help to clarify what Specialized Content Knowledge is not. SCK is not an understanding of the basic content taught in a mathematics course (CCK); it is not about what is taught in sequence after a given topic (HCK); it is not about how students learn mathematics and knowing the topics in mathematics that students will likely struggle with (KCS); it is not being able to design a mathematics lesson, or select the best example for a struggling student (KCT); and lastly, it is not familiarity with the materials available for teaching mathematics or connecting mathematics to other topics (KCC). Instead, SCK is "subject matter knowledge needed by teachers for specific tasks of teaching...but still clearly subject matter knowledge" (Ball et al., 2008, p. 402). The tasks of teaching they refer to include, but are not limited to,

- recognizing what is involved in using a particular representation, linking representations to underlying ideas and to other representations, evaluating the plausibility of students' claims (often quickly), giving or evaluating mathematical explanations, and using mathematical notation and language and critiquing its use. (Ball et al., 2008, p. 400)

Ball et al. (2008) continue,

> These tasks of teaching depend on mathematical knowledge, and, significantly, they have aspects that do not depend on knowledge of students or of teaching. These tasks require knowing how knowledge is generated and structured in the discipline and how such considerations matter in teaching, such as extending procedures and concepts of whole-number computation to the context of rational numbers in ways that preserve properties and meaning. These tasks also require a host of other mathematical knowledge and skills-knowledge and skills not typically taught to teachers in the course of their formal mathematics preparation. (p. 402)

Recalling this explanation of SCK, and the fact that it is "not intertwined with knowledge of pedagogy, students, curriculum, or other noncontent domains" (Ball et al., 2008, p.
will help distinguish the knowledge that is of interest to this research from other types of Mathematical Knowledge for Teaching.

Relationships between Components of MKT

While the Diagram of MKT Components (refer to Figure 1) represents the six types of MKT as distinct, with no overlaps, some researchers claim that the relationships between these components are actually more complicated. For example, McCrory et al. (2012) examined a variety of individual assessment items, most from different sources, that measure SCK in algebra, and found that "both knowing through experience and figuring out the answer logically or mathematically" (p. 600) could produce good results on the items. In other words, when measuring SCK, test-takers may rely on Knowledge of Content and Students to answer the items if they have experience with students. Bair and Rich (2011) explain that others have referred to the Diagram of MKT Components (refer to Figure 1 in Chapter 1) as an omelet, as opposed to the familiar "egg," with inseparable categories. They conducted a study with students in Algebraic Reasoning and Number Theory courses intended for K-8 teachers, most of whom were in a middle school mathematics specialization program, and also included mathematics education graduate students. Based on their results, Bair and Rich (2011) propose a new perspective on the subdivision of MKT:

We see that at the lower levels of SCK development, there is a distinct separation of the content knowledge from the PCK, but as [pre-service teachers] grow in SCK you begin to see more connections to the PCK. Thus, we see the dividing line between subject matter knowledge and pedagogical content knowledge as a permeable membrane in the egg, and as [pre-service teachers] develop deeper SCK, they begin to move across that membrane seamlessly. (p. 315)
In addition to describing the way pre-service teachers move between subject matter knowledge and pedagogical knowledge, the researchers also presented conclusions about how a pre-service teacher's Common Content Knowledge (CCK) is related to his or her SCK:

[Pre-service teachers] who had insufficient CCK to successfully solve mathematical problems could exhibit characteristics of growth in SCK involving the same content. Our findings indicate that [pre-service teachers] can work at simultaneously developing lower-level components of SCK as they developed their CCK. However, they would not be able to move to the higher levels of SCK without CCK. (Bair & Rich, 2011b, p. 315)

Frameworks for Research on MKT

Silverman and Thompson (2008) introduced a framework for studying MKT that relies on the claim that a high level of Common Content Knowledge (CCK) for a certain topic does not necessarily indicate that a pre-service teacher has a pedagogical understanding of that topic. It is suggested that once a pre-service or in-service teacher has acquired a key developmental understanding, or the ability to conceptualize that topic and relate it to other topics, a reflective process relating that topic to teaching is needed in order to develop a pedagogical understanding of that topic (Silverman & Thompson, 2008).

Kahan, Cooper, and Bethea (2003) also introduced a framework for studying what they call the Mathematical Content Knowledge of teachers. They suggested that teachers should be observed with a focus on the elements of teaching (p. 228), the processes of teaching (p. 231), and an intersection of those two. Kahan et al. (2003) consider the elements of teaching to be goals and objectives, selection of tasks and representations,
motivation of content, development: connectivity and sequencing, allocation of time, points and emphasis, and discourse. They identify the processes of teaching to include preparation, instruction, assessment, and reflection. They intended for this framework to "guide research on the relationship between mathematics teachers' knowledge of content and their teaching" (p. 224). They report on a study conducted with pre-service secondary mathematics teachers during student teaching. The researchers created and administered an assessment that measured the mathematical content knowledge of pre-service teachers in the first week of a mathematics methods course; the participants were then observed during student teaching.

Kahan et al. (2003) did not employ their framework during those observations; however, the observation notes were later analyzed using the framework. One interesting case from the study involved a participant who had achieved the highest score among the participants on the mathematical content knowledge test and had taken many upper level mathematics courses. During the observation of student teaching, the observer noticed that this participant was unable to explain why one is not a prime number and had difficulty explaining why, when finding composite numbers less than 100, she could have stopped finding multiples of prime numbers once she reached ten (Kahan, et al. 2003). This participant exemplifies a pre-service teacher who appears to have developed a high level of mathematical content knowledge, but still lacks the SCK needed to provide meaningful explanations about mathematics to students.

McCrory et al. (2012) also introduced a framework for studying MKT, specifically in algebra. They describe three types of knowledge in algebra: knowledge of
advanced mathematics, knowledge of school algebra (the algebra that high school students are expected to learn), and mathematics-for-teaching knowledge, which they equate with SCK. They also identify three processes that are useful for teachers of high school algebra: trimming ("scaling up or down, intentionally omitting or adding detail, or modifying levels of rigor, and it also involves recognizing mathematics that has been trimmed too much, namely, instances in which important details or special cases are missing" (McCrory et al., 2012, p. 604)); bridging ("efforts to connect and link mathematics across topics, courses, concepts, and goals, including connecting the ideas of school algebra to those of abstract algebra and real analysis, and linking one area of school mathematics to another" (McCrory et al., 2012, p. 606)); and decompressing ("being able to separate advanced concepts from their precursors, decompressing involves attaching fundamental meaning to symbols and algorithms that are typically employed by sophisticated mathematics users in automatic unconscious ways" (McCrory et al., 2012, p. 603)). Based on their framework, McCrory et al. (2012) recommend that measures of algebra teachers' Mathematical Knowledge for Teaching include combinations of all three types of knowledge with the three types of teaching processes. The following section discusses attempts to develop measures of MKT and SCK.

Measures of Mathematical Knowledge for Teaching

Although other assessments may be in development, a thorough review of literature revealed only two measurement instruments currently used to assess Mathematical Knowledge for Teaching at the secondary level. These will be discussed
first, followed by a review of research on instrument development and validation at the elementary level. Many of the theories and processes that have been used in creating assessments of elementary MKT apply to the secondary level as well.

Measures of MKT for Secondary Mathematics Teachers

The first assessment identified for secondary teachers is the *Knowledge of Algebra for Teaching* instrument, which remains unfinished and only has four released items (McCrorry et al., 2012). The second measure of MKT for secondary mathematics teachers, the one used in this study, is called the *Content Knowledge for Teaching: Algebra 1 Assessment* (2012). This instrument was developed for use in research by a team of mathematicians and mathematics educators led by Geoffrey Phelps and Drew Gitomer, as a part of the Measures of Effective Teaching Project sponsored by the Bill and Melinda Gates Foundation. Gitomer and Phelps (2012) explain

> [Content Knowledge for Teaching] refers to more than explicit content knowledge that an educated individual needs; it is the content knowledge that is used in the day-to-day work of teaching. Examples of test questions include those that require test-takers to interpret student work, to represent content in ways that are accessible to students, and to select appropriate instructional examples. (p. iii)

In addition to the assessment for *Algebra 1*, the project has also created assessments for Mathematics Grades 6-8, Mathematics Grades 4-5, and for English/Language Arts Grades 4-6 and Grades 7-9 (Phelps & Gitomer, 2012).

Items for the *Content Knowledge for Teaching: Algebra 1 Assessment* (2012) were developed based on selected tasks of teaching: *anticipating student challenges, eliciting and evaluating student work, ideas, and interactions, explaining and using...*
concepts and procedures, and using examples, models, and representations (p. iv). "The assessment was first piloted with approximately 300 teachers to assess basic question-level measurement characteristics. A small number of cognitive interviews were conducted and questions were subsequently revised" (Phelps & Gitomer, 2012, p. v). In its final form, the assessment consists of 22 items. Eighteen of those items are multiple-choice with four options. The remaining four questions are tables of items with three to five items per question. Phelps and Gitomer (2012) point out that "[o]ne way to use the Algebra 1 assessment is to create a score for each participant. This can be done by simply counting the total number of correct responses. The maximum possible score is 35" (p. v).

Measures of MKT for Elementary Teachers

Although resources for measuring MKT at the secondary level are limited, much can be learned from similar research and development at the elementary level. Hill, Schilling, and Ball (2004) describe the process of writing an assessment that measures the MKT of elementary teachers. They started by examining current literature about the domains of knowledge that researchers identify as important types of knowledge for teachers of mathematics to possess (e.g. Mathematical Knowledge for Teaching, Specialized Content Knowledge, Common Content Knowledge, Knowledge of Content and Students). Then, they looked at curriculum materials and student work, and identified tasks of teaching (e.g. "choosing representations, explaining, interpreting student responses, assessing student understanding, analyzing student difficulties, evaluating the correctness and adequacy of curriculum materials" (Hill, Schilling, & Ball,
2004, p. 16). Using this information, the research team drafted items related to those tasks and knowledge types and piloted an assessment. Results were analyzed with factor analyses and scaling techniques, which helped to delineate the different types of MKT. Hill et al. (2004) concluded that the aforementioned analyses "show evidence of multidimensionality in [the] measures [of MKT], suggesting that teachers' knowledge of mathematics for teaching is at least partly domain specific rather than simply related to a general factor such as overall intelligence, mathematical ability, or teaching ability" (p. 26).

In a later article, Ball, Hill and Bass (2005) describe, in more detail the steps taken to develop the MKT assessment for elementary teachers. First, they "set out a 'domain map,' or a description of the topics and knowledge to be measured" (p.22). Then, experts were invited to write items for the assessment that related to the daily work of teachers; experts included "mathematics educators, professional developers, project staff, and classroom teachers" (Ball et al., 2005, p. 22). Over 250 total multiple-choice items included measures of both Common Content Knowledge and Specialized Content Knowledge (Ball et al., 2005).

Another study of the elementary MKT assessment focused on items that were intended to measure Knowledge of Content and Students (KCS). Teachers involved in professional development were given the MKT assessment as a pre- and post-test, and participated in cognitive interviews about the assessment after the post-test (Hill, Ball, & Schilling, 2008). By examining the strategies employed by the teachers to respond to KCS items, Hill et al. (2008) found that teachers not only relied on their Knowledge of
Content and Students, but also used mathematical reasoning, and sometimes even test-taking strategies, to arrive at an answer. It is important to consider this when administering a multiple choice assessment; the researcher will not know what knowledge base a teacher is relying on to answer assessment items.

Schilling and Hill (2007) lined out the ways in which the elementary MKT assessment could be validated. The results reporting the validity of the assessment are not included in the article; instead, it focuses on three assumptions the researchers planned to test including a structural assumption, an elemental assumption, and an ecological assumption. Structural validity relies on whether items assessing one domain of MKT will have a strong correlation with other items assessing that same domain (Schilling & Hill, 2007). The elemental assumption addresses whether each test item actually assesses MKT, as opposed to participants using test-taking strategies (Schilling & Hill, 2007). Based on the findings of Hill et al. (2008), it seems that teachers can sometimes use test-taking strategies to determine a correct answer, rather than considering the elements of the task. The third assumption that supports validity of the assessment is ecological, testing whether a higher score on the MKT assessment implies that a teacher will be more effective and increase student achievement (Schilling & Hill, 2007). This was shown to be the case by Hill et al. (2005).

Buschang, Chung, Delacruz, and Baker (2012) took a different approach to validating the elementary MKT assessment. Along with administering the MKT assessment, they asked participants to analyze both student responses and concept maps. Participants included subject matter experts (mathematicians), pedagogical content
knowledge experts (mathematics educators), novice teachers, and experienced teachers. Findings indicate that analyzing student responses required a different type of knowledge than taking the assessment or examining concept maps. The authors conclude by saying that "teacher knowledge is a complex construct to measure...no one assessment can accurately measure it in its entirety. Instead a battery of well-developed tests is needed to measure pedagogical content knowledge of teachers" (Buschang, et al. 2012, p. 24)

Teacher Preparation and Retention

The Mathematical Education of Teachers II (2012) makes recommendations for teacher preparation in mathematics at the elementary, middle grades, and high school levels. This unique document was created by a writing team of mathematicians and mathematics educators, and included mathematicians and teachers in the editing process. In the chapter on recommendations for training high school mathematics teachers, the introduction explains that high school mathematics teachers often have a "double discontinuity" experience. That is, when entering college from high school, the college mathematics is presented at a level that is not easily related to the mathematics learned in high school. This disruptive change is experienced a second time when the teacher goes back to high school to teach mathematics, and the content is not easily relatable to what was learned in college. To address these discontinuities, the MET II (2012) suggests that teacher preparation focus on the connections between high school mathematics with middle grades mathematics and college level mathematics.
The MET II (2012) also makes specific course recommendations for preparing high school teachers. They suggest that a teacher preparation program should "include at least a three-course calculus sequence, an introductory statistics course, an introductory linear algebra course, and 18 additional semester-hours of advanced mathematics, including 9 semester-hours explicitly focused on high school mathematics from an advanced standpoint" (Conference Board of the Mathematical Sciences, 2012, p. 55). They also recommend a set of education courses typically required of most programs, but do not go into detail except to explain that the methods course should focus on mathematics-specific instructional methods.

Recognizing the fact that programs may be unable to offer courses specifically for secondary mathematics pre-service teachers, the MET II (2012) makes additional recommendations that conventional mathematics courses should emphasize experiences with reasoning, proof, and technology. They suggest that theorems proved in coursework are well motivated and related to high school mathematics when feasible, so they are seen as helpful.

Alternatives and Adaptations

Depending on their size and capacity, it may be difficult for teacher preparation programs to provide the recommended courses designed specifically for mathematics education majors. The National Task Force on Teacher Education in Physics: Report Synopsis (2010) notes ideas that could be applicable to teacher preparation programs that may not attract enough students to offer the recommended mathematics education specific courses. For example, they recommend that typical mathematics courses could
"have a reflective component connecting the course material to the demands of the precollege classroom" (Physics, 2010, p. 10). Additionally, they recommend courses for pre-service teachers that could also benefit in-service teachers in the form of professional development, to increase the number of students in those courses.

*American Journal of Physics* (2010, p. 10)

A Professional Program for Preparing Future High School Mathematics Teachers (2015), a follow up report to the *MET II* (2012), agrees with those recommendations and makes suggestions for how to incorporate high school mathematics concepts as well as reasoning and proof into existing teacher preparation programs. The report includes specific recommendations for each advanced mathematics course and how it should be modified to better align with the needs of pre-service teachers. While based on the recommendations in the *MET II* (2012), the new report elaborates on that information and includes details that would be of benefit to any faculty member who is designing or teaching those courses (Tucker, Burroughs, & Hodge, 2015).

In his article “On the Education of Mathematics Majors” (1999), published prior to even the first *Mathematical Education of Teachers* (2001), Wu provides some helpful suggestions for modifying typical mathematics courses to prepare mathematics teachers. In fact, Wu (1999) recommends that any mathematics major who does not plan to attend graduate school should enroll in these courses with a different perspective. Typical college mathematics courses are taught on the concept of delayed gratification- the idea that material taught in undergraduate mathematics courses provides foundational knowledge preparing students to learn the really interesting mathematics when they reach graduate school. Wu (1999) recommends six "desirable characteristics" that should be
included in mathematics courses intended for undergraduate mathematics or mathematics education majors who do not plan to attend graduate school:

(1) Only proofs of truly basic theorems are given, but whatever proofs are given should be complete and rigorous...(2) In contrast with the normal courses which are relentlessly 'forward-looking'...considerable time should be devoted to 'looking back'...(3) Keep the course on as concrete a level as possible, and introduce abstractions only when absolutely necessary...(4) Ample historical background should be provided...(5) Provide students with some perspective on each subject, including the presentation of surveys of advanced topics...(6) Give motivation at every opportunity. (p. 13-14)

Wu (1999) implemented these ideas into some of his own undergraduate courses and discovered two problems. First, it was difficult to find a textbook aligned to the content he wanted to include. Second, he was not able to cover all of the material that the courses typically cover. He does not recommend that courses of this nature should replace all undergraduate mathematics courses, but that they should be offered as additional options for students not intending to pursue graduate studies in mathematics.

**Influences of Teacher Preparation on Retention**

The aforementioned documents and studies make well-researched recommendations for teacher preparation programs. However, they are mostly based on anticipating the types of knowledge that high school mathematics teachers will need without considering the teachers themselves. Another area of concern for teacher preparation programs is the attrition rates of new teachers. Regardless of the quality of his or her preparation, a teacher is not contributing the education of students if that teacher leaves the classroom within the first few years.
Ingersoll, Merrill, and May (2012) looked at the attrition rates of beginning teachers and their pedagogical preparation. To measure beginning teachers’ pedagogical preparation, the researchers asked about the number of courses teachers had taken in teaching methods and strategies, learning theory, child psychology, and materials selection, as well as the amount of time spent practice teaching and observing other teachers and the amount of feedback they had received on their own teaching. "Those receiving little or no pedagogical preparation] were more than twice as likely to leave after one year as those who received a comprehensive pedagogy package" (Ingersoll et al., 2012, p. 33).

Pedagogical preparation is certainly not the only factor influencing teacher attrition. Content preparation may also play a role, as do competing job opportunities. Darling-Hammond (2000) believes teacher shortages are systemic. She writes, "[I]n some fields like mathematics...real shortfalls do exist, largely because the knowledge and skills required by teachers command much greater compensation in fields outside of teaching" (Darling-Hammond, 2000, p. 8).

In response to the shortage of mathematics teachers nationwide, a variety of alternative teacher certification programs have arisen. Preparing Teachers: Building Evidence for Sound Policy (2010) examined the demographics of students enrolled as pre-service teachers, and found that between 20 and 30 percent of all the pre-service teachers in America (including all subjects and all grade levels) are enrolled in non-traditional teacher preparation programs. These alternative programs gain popularity and support due to their ability to quickly prepare teachers, but critics warn that teachers may
be underprepared, and in turn, leave the teaching profession at higher rates than teachers completing a traditional teacher preparation program (Brantlinger & Smith, 2013). Such concerns align with Darling-Hammond's (2000) claim that teachers with lower pedagogical preparation are more likely to leave the teaching profession.

Supporters of alternative teacher preparation programs argue that teacher candidates in those programs "have greater subject matter expertise, subject-relevant work experience, linguistic competence, and prior academic achievement" (Brantlinger & Smith, 2013, p. 2). These arguments are based on the claim that alternative programs "recruit 'the best and the brightest' professional career changers and recent college graduates who hold college degrees in something other than education" (Brantlinger & Smith, 2013, p. 2). However, Brantlinger and Smith (2013) point out that alternative teacher preparation programs, particularly those using an accelerated program, often take a "technicist view of the role of teachers" (p. 2), which exacerbates what some are calling "the new professionalism." From this perspective, a teacher is viewed as someone who delivers information to students, but not as someone who makes decisions about what to teach or how to teach it (Brantlinger & Smith, 2013). The next section describes the theoretical framework around which this study was designed.

**Theoretical Framework**

The theoretical framework that guides this study is founded on prior research in Mathematical Knowledge for Teaching (specifically Specialized Content Knowledge). In particular, this study hypothesizes that Specialized Content Knowledge may be
developed throughout a secondary mathematics pre-service teacher preparation program, and that the experiences aiding in the development of SCK will be identifiable.

Experiences play a major role in shaping how teachers at all stages perceive and practice mathematics teaching. The unique circumstances and dispositions of pre-service teachers make them especially receptive to gaining knowledge through those experiences. Chapter 1 outlines issues surrounding the preparation of pre-service secondary mathematics teachers and suggestions (Conference Board of the Mathematical Sciences, 2012) for how they might acquire Specialized Content Knowledge. Chapter 2 presents prior research results and background information that inform the investigation of Specialized Content Knowledge within this population. This knowledge can be synthesized into a theoretical framework as described below.

Mathematical Knowledge for Teaching, defined as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395) has been identified as a distinct type of knowledge that is separate from content knowledge. For example, the pre-service teacher described in a study by Kahan et al. (2003) exhibits a high level of mathematical content knowledge on an assessment instrument, but struggles with the underlying mathematics of that subject when teaching it.

Furthermore, MKT has been shown to be an influential component of a teacher's knowledge; for example, Hill et al. (2005) found that teachers with higher levels of MKT produced greater achievement gains in their first and third grade students over a year. Those findings brought MKT to the forefront as a topic of interest for ongoing studies in mathematics education. Through further dissection, MKT has been deconstructed into
six different types of mathematical knowledge needed for teaching, one of which is Specialized Content Knowledge. However, SCK has been largely ignored in the research, especially at the secondary level and in regard to pre-service teachers.

The goal of teacher preparation programs is to educate future secondary mathematics teachers, and part of that process includes developing their Specialized Content Knowledge. While pre-service teachers enter preparation programs with more than twelve years of experience as a student, only in rare cases have they experienced classrooms from a teaching perspective. Feiman-Nemser (2001) cautions that “[a] focus on teachers as learners begins with a recognition that preservice [teachers] come with images and beliefs that must be extended or transformed” (p. 1025). Similarly, their knowledge of mathematics may be based on a narrow set of strategies and understandings. Darling-Hammond (2006) notes that it is “important to organize prospective teachers’ experiences so that they can integrate and use their knowledge in skillful ways in the classroom” (p. 305). Pre-service teachers’ malleable dispositions regarding the knowledge needed for teaching mathematics render them a plausible population for studying the development of SCK.

Becoming a teacher is a multifaceted process that involves coursework, field placements, and other experiences outside of formal teacher preparation. These experiences vary greatly across different teacher preparation programs, and even pre-service teachers with the same experiences are still likely to internalize them differently. Furthermore, SCK is a subtle component of the mathematical knowledge needed for teaching, which is not always directly addressed in coursework, making it difficult to
observe or measure. Finally, there is a lack of prior research on Specialized Content Knowledge among pre-service teachers at the secondary level. Given all these considerations, a grounded theory approach is the most appropriate methodology for exploring the development of SCK (Corbin & Strauss, 2008). Creswell (2013) explains that “the intent of a grounded theory study is to move beyond description and to generate or discover a theory…for a process or an action” (p. 83). Grounded theory grew out of the philosophy of Pragmatism, a belief system that “knowledge arises through (note the verbs) acting and interacting of self-reflective beings” (Corbin & Strauss, 2008, p. 2).

Pragmatists also believe that actions are influenced by perceived possible outcomes, that “new knowledge [is] provisional until checked out empirically by peers” (Corbin & Strauss, 2008, p. 3), and that “knowledge can be useful for practice or practical affairs” (Corbin & Strauss, 2008, p. 4).

Pragmatism and grounded theory guide the research questions posed in this study, and the methodology used to explore them by examining the actions and interactions of pre-service teachers. Corbin and Strauss (2008) eloquently describe the implications of pragmatism in relation to the grounded theory methodology:

The world is very complex. There are no simple explanations for things. Rather, events are the result of multiple factors coming together and interacting in complex and often unanticipated ways. Therefore any methodology that attempts to understand experience and explain situations will have to be complex. We believe that it is important to capture as much of this complexity in our research as possible, at the same time knowing that capturing it all is virtually impossible. We try to obtain multiple perspectives on events and build variation into our analytic schemes. (p. 8)

The complexity surrounding how pre-service teachers develop Specialized Content Knowledge drove the methodological decisions and processes used in this study.
Interview questions, participant selection processes, and methods of data analysis informed each other and evolved as the study progressed. This clearly exemplifies grounded theory, where “[p]rocess is integral to…studies because we know that experience, and therefore any action/interaction that follows, is likely to be formed and transformed as a response to consequence and contingency” (Corbin & Strauss, 2008, p. 8).

Research Methodology and Grounded Theory

When grounded theory first emerged as a formal method for conducting qualitative research, two researchers were best known for conducting grounded theory studies: Anselm Strauss and Barney Glaser. Essentially, all grounded theory can trace its roots back to the work of either Glaser or Strauss. However, their techniques were different from one another (Morse et al., 2009). Anselm Strauss worked closely alongside Juliet Corbin in developing their grounded theory techniques (Morse et al., 2009). Grounded theorists who emerged later were typically colleagues or students of Anselm Strauss, Juliet Corbin, or Barney Glaser. One exception to this rule is constructivist grounded theory, developed by Kathy Charmaz, although she still incorporated the ideas of both Anselm Strauss and Barney Glaser (Morse et al., 2009). This study was conducted using the qualitative perspective and methods suggested by Juliet Corbin and Anselm Strauss in Basics of Qualitative Research, Third Edition (2008).

Corbin and Strauss (2008) define grounded theory as “[a] specific methodology developed by Glaser and Strauss (1967) for the purpose of building theory from data. In
this book the term grounded theory is used in a more generic sense to denote theoretical constructs derived from qualitative analysis of data" (p. 1). This research study of pre-service teachers' Specialized Content Knowledge in mathematics fits that definition, as it depends on qualitative analysis of data to achieve results, but does not solely generate original theory. For example, the researcher has made an assumption that experiences both within and outside teacher preparation contribute to the development of Specialized Content Knowledge and seeks to catalog and describe those experiences.

Like Kathy Charmaz, Corbin and Strauss (2008) support a constructivist perspective: "concepts and theories are constructed by researchers out of stories that are constructed by research participants who are trying to explain and make sense out of their experiences and/or lives, both to the researcher and themselves" (p. 10). In this study, an autobiographical survey and the Content Knowledge for Teaching: Algebra 1 Assessment (Phelps & Gitomer, 2012) will be used as a springboard for construction of the participants' stories. These stories will be collected through semi-structured interviews, the method recommended by Corbin and Strauss (2008). Through a structured process of data collection and analysis, the researcher will construct accurate concepts and theories arising from those stories. Data analysis will be cyclical and continuous, following recommendations from Glaser and Strauss (1967) and Strauss (1987) to "begin the analysis after completing the first interview or observation" (as cited in Corbin & Strauss, 2008, p. 57). Methods of data collection and analysis are described in detail in Chapter 3.
3. DESIGN AND METHODS

It goes without saying that a secondary mathematics teacher should have a reasonable amount of Specialized Content Knowledge in beginning algebra. However, very little is known about how SCK is developed in pre-service teachers, especially secondary mathematics pre-service teachers. A grounded theory approach was used in this study to develop a theory out of the data for how SCK is developed among secondary mathematics pre-service teachers. This chapter describes the methods used for carrying out the research, as well as reasons for choosing those methods. More detail on the use of grounded theory will be provided throughout this chapter.

Research Questions

The goal of this study was to explore the experiences of pre-service secondary mathematics teachers to learn more about the development of Specialized Content Knowledge (SCK) in beginning algebra. The research questions are repeated below.

What kinds of experiences support the development of Specialized Content Knowledge (SCK) for beginning algebra among pre-service secondary mathematics teachers?

a. In what ways do secondary mathematics pre-service teachers exhibit specialized content knowledge on an assessment measuring SCK in beginning algebra?
b. What teacher preparation experiences do secondary mathematics pre-service teachers identify as aiding in the development of Specialized Content Knowledge as indicated on the SCK assessment?

c. What experiences outside of teacher preparation do secondary mathematics pre-service teachers identify as aiding in the development of SCK?

**Research Design**

It is common in mathematics education studies of pre-service teachers for the researcher to use a case study approach. A case study allows the researcher to gain a lot of information from different sources about a few cases. Cases might be defined as individual pre-service teachers, but they may also be identified as classrooms, schools, or teacher preparation programs. However, a case study is not the most suitable approach for this research context. For the research questions posed here, a case study of individuals might reveal more in-depth information about how SCK is developed in those specific individuals, but it would not uncover the wide variety of experiences that different pre-service teachers have during, and even before, completion of a teacher preparation program. One may also suggest that a case study of teacher preparation programs could be appropriate. However, the numerous and diverse experiences that make up a teacher preparation program would limit the number of programs feasible to include in the study. Furthermore, focusing on a study of programs would neglect the experiences that pre-service teachers have outside of formal teacher preparation.
In his text on qualitative research methods, Creswell (2013) explained how grounded theory is different from other approaches:

While narrative research focuses on individual stories told by participants, and phenomenology emphasizes the common experiences for a number of individuals, the intent of a grounded theory study is to move beyond description and to generate or discover a theory, a 'unified theoretical explanation' (Corbin & Strauss, 2007, p. 107) for a process or an action. Participants in the study would all have experienced the process, and the development of the theory might help explain practice or provide a framework for further research. [Italics added for emphasis.] (p. 83)

For this study, the process being explored is the development of SCK in beginning algebra for secondary pre-service mathematics teachers. Creswell (2013) continued with a brief definition of grounded theory and its purpose:

A key idea is that this theory development does not come 'off the shelf,' but rather is generated or 'grounded' in data from participants who have experienced the process (Strauss & Corbin, 1998). Thus, grounded theory is a qualitative research design in which the inquirer generates a general explanation (a theory) or a process, an action, or an interaction shaped by the views of a large number of participants. (p. 83)

Finally, Creswell (2013) confirmed that a grounded theory approach is appropriate for a topic with little research surrounding it:

The researcher needs to begin by determining if grounded theory is best suited to study his or her research problem. Grounded theory is a good design to use when a theory is not available to explain or understand a process. [Italics added for emphasis.] The literature may have models available, but they were developed and tested on samples and populations other than those of interest to the qualitative researcher. (p. 88)

While there are some published studies examining the development of SCK in elementary pre-service teachers, there is a dearth of research surrounding SCK at the secondary level. In other words, there is not "a theory available to explain or understand" (Creswell, 2013, p. 83) how SCK is developed in pre-service secondary mathematics
teachers; therefore, a researcher would not know what aspect of teacher preparation programs would need to be studied to learn more about SCK. Since there is not a theory grounded in research that describes the factors contributing to the development of SCK in secondary mathematics teachers, a grounded theory approach was appropriate for exploring the research questions.

**Background**

Pre-service teacher preparation programs are the ideal setting for studying the development of SCK. A major goal of secondary teacher preparation programs is to train teachers who are well qualified and equipped to teach in their content area. As a well-established component of the Mathematical Knowledge for Teaching identified by Ball, Hill, and Bass (2005), developing SCK is a desirable outcome of any teacher preparation program. Furthermore, the *MET II* (2012) recommends that "[t]eachers need to have more than a student's understanding of the mathematics in [grades K-12]." (Conference Board of the Mathematical Sciences, 2012, p. 1) SCK is one type of knowledge that will satisfy this recommendation regarding the mathematical education of teachers. Finally, studying pre-service teachers who are still enrolled in teacher preparation programs will maximize the opportunity to identify essential experiences that generate SCK in its early stages. In contrast, data collected from practicing teachers might be contaminated by experiences gained during professional development, curriculum training, mentoring, and practice in the field. It would be difficult for both teacher and researcher to untangle these overlapping experiences to find the source of SCK development.
Research Setting

The research was conducted across a variety of secondary mathematics teacher preparation programs in a Rocky Mountain state. The following section includes an overview of the types of teacher preparation programs found throughout the United States, followed by descriptions of the preparation programs included in the study.

Types of Teacher Preparation Programs

Several different types of colleges and universities offer teacher preparation programs in the United States. Many teacher preparation programs are housed in public universities, which rely on state and other funding and may be required to meet guidelines imposed by the state. Private colleges and universities, on the other hand, typically do not get funding from the government, and are often affiliated with a religious entity. They are not necessarily required to maintain the same standards and requirements as state-sponsored public colleges and universities.

In addition to being classified as a public or private university or college, a school often can be categorized by purpose or mission. For example, a college or university may be a Liberal Arts, Land Grant, and/or Research Institution. A Liberal Arts institution often encourages a broad education and exposure to a variety of disciplines. Land Grant institutions were given land by the government to build a college or university, and typically emphasize agriculture and engineering. Apart from being categorized as a certain type of institution, many universities emphasize research across a
variety of disciplines. Some institutions do not categorize themselves as any particular type of school.

Another way that one institution may differ from another is by its size. The number of students attending a particular institution changes the atmosphere of a program in many ways. Schools with fewer students often have smaller classes, and a smaller variety of instructors teaching undergraduate courses. On the other hand, students in smaller institutions typically receive more individual attention. The variation in program sizes is particularly applicable to this study because in small schools, it is common for students in a given academic program to have limited options in course offerings, and even to have the same instructor for multiple courses. In small secondary teacher preparation programs, the options narrow even further for receiving specialized teacher training in the subject area. Therefore, pre-service teachers enrolled in a small school are likely to have very different experiences than pre-service teachers who are enrolled in a larger institution.

In addition to the typical four year programs for teacher education training, there are alternative or add-on programs available. These programs are designed for those interested in teaching who have earned a degree in a field outside of education, and who typically have some experience working with children. For example, a college student may complete an undergraduate degree in mathematics, and then go to an add-on program to attain his or her mathematics education degree and/or license. Or an engineer, after working in his or her field for several years, may decide to earn a teaching license through additional coursework in mathematics and education. Other alternative
programs provide a route to licensure for people who are currently teaching without a license. Teachers in this category usually have a degree in another field and may take additional online or evening courses, participate in formal internships, and complete other requirements to gain a teaching license and keep their jobs. These alternative routes to obtain a teaching license are a relatively new option, intended to aid in repairing the shortage of teachers in high needs subjects, such as mathematics, and in high needs schools.

Online teacher education programs have also gained popularity, but the curriculum and methods of lesson delivery in these programs vary greatly. Online programs are not typical in the Rocky Mountain state of this study, and findings about one online program would not likely extend to others. Some alternative teacher preparation programs, including online programs, remain unaffiliated with any institution of higher education. While online programs and alternative programs are expected to uphold the same requirements as teacher preparation programs, they sometimes gain permission to alter certain guidelines (Brantlinger & Smith, 2013).

Teacher Preparation Programs in the Study

A total of eight traditional secondary mathematics teacher preparation programs are offered by institutions in the Rocky Mountain State chosen for this study. The eight institutions represent the various types of programs described above, with the exception of online programs and alternative programs. It is reasonable to exclude online programs and alternative programs in this study because they are so vastly different from the others, and have a great amount of individual variation among them. A ninth program,
the newest secondary mathematics teacher preparation program in the state, is located at a Native American tribal college. This program is in its earliest stages, so does not contain any juniors or seniors. Therefore, this program was not included in the study.

Table 1 describes the eight institutions that are included in the study, each of which has been assigned a pseudonym. Information for each institution includes the approximate size of the full student body, organizational affiliation, type of institution, accreditation (either universal or specific to teacher education), and the type of degree awarded to secondary mathematics teachers. Accreditations include the Northwest Commission on Colleges and Universities (universal accreditation) and the National Council for the Accreditation of Teacher Education (teacher education accreditation).

The original goal was to include participants from each of the eight programs; however, in the end, no pre-service teachers from the programs at Ellis and Farley participated. A mathematics professor at Ellis indicated that she was unsure of whether there were any current mathematics education majors in the program. The names and e-mail addresses of two secondary mathematics education majors were provided by Farley, but neither person responded to e-mails requesting their participation in the study. Further attempts to contact them through a retired mathematics professor at Farley had the same result.

Ellis and Farley represent smaller universities, with Farley additionally being a private school. Despite their absence in the data, these types of schools were still represented, with Anderson being a small private university, and Davis being a small public university. Descriptions of these programs and their requirements for secondary
Table 1. Institutions in the Study

<table>
<thead>
<tr>
<th>Institution</th>
<th>Enrollment</th>
<th>Affiliation</th>
<th>Type</th>
<th>Accreditation</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>1,500</td>
<td>Catholic</td>
<td>Liberal Arts</td>
<td>NCCU</td>
<td>B.A. Secondary Mathematics Education</td>
</tr>
<tr>
<td>Bailey</td>
<td>15,000</td>
<td>Public</td>
<td>Land Grant</td>
<td>NCATE</td>
<td>B.S. Mathematics w/Teaching Option</td>
</tr>
<tr>
<td>Connor</td>
<td>5,000</td>
<td>Public</td>
<td>N/A</td>
<td>NCATE</td>
<td>B.S. Mathematics w/Teaching Option, Mathematics Teaching Minor</td>
</tr>
<tr>
<td>Davis</td>
<td>3,000</td>
<td>Public</td>
<td>N/A</td>
<td>NCCU</td>
<td>B.S. Mathematics</td>
</tr>
<tr>
<td>Ellis</td>
<td>1,000</td>
<td>Private</td>
<td>Liberal Arts</td>
<td>NCCU</td>
<td>Mathematics Education Major, Mathematics Education Minor</td>
</tr>
<tr>
<td>Farley</td>
<td>1,000</td>
<td>Roman Catholic</td>
<td>Liberal Arts</td>
<td>NCCU</td>
<td>B.S. Mathematics Education, Mathematics Education Minor</td>
</tr>
<tr>
<td>Gaines</td>
<td>15,000</td>
<td>Public</td>
<td>N/A</td>
<td>NCATE</td>
<td>B.A. Mathematics w/Teaching Option</td>
</tr>
<tr>
<td>Hamilton</td>
<td>1,500</td>
<td>Public</td>
<td>N/A</td>
<td>NCCU</td>
<td>B.S. Mathematics and Secondary Education Double Major, Education Major with a Minor in Mathematics</td>
</tr>
</tbody>
</table>

Pre-service teachers at Anderson take most of their mathematics coursework alongside mathematics majors. The students at this private Catholic college take the same coursework as a mathematics major, except they are additionally required to take a *History of Mathematics* course and one mathematics methods course. After the calculus sequence, mathematics coursework for content majors includes *Real Analysis*,
Probability and Statistics, Discrete Mathematics, Advanced Differential Equations & Linear Algebra, Applied Numerical Methods & Analysis, Abstract Algebra & Modern Geometry, Complex Analysis, and Mathematical Optimization, Applications, & Analysis. Mathematics education majors in this program also participate in extensive field experience. They begin in-school observations their first semester, and continue to visit classrooms with increasing interaction with students, throughout their teacher preparation. There were two secondary mathematics education majors enrolled at Anderson at the time of this study.

Bailey. Mathematics education majors at Bailey begin a path of coursework separate from mathematics majors upon completion of the calculus sequence and Linear Algebra. This land-grant university offers several mathematics content courses designed specifically for pre-service secondary mathematics teachers: Modern Geometry, Higher Mathematics for Secondary Teachers, Mathematical Modeling for Teachers, Algebraic Thinking and Number Sense in the Middle Grades (not required), and Geometry, Measurement, and Data in the Middle Grades (not required). One offering of Methods of Proof was also reported as being tailored towards teaching. In addition to the courses above, students at this university are required to complete nine credits of upper level mathematics courses and two methods courses, taught in the mathematics department: Methods: 5-8 Mathematics and Methods: 9-12 Mathematics. There were approximately 40 secondary mathematics education majors enrolled at Bailey at the time of this study.
**Connor.** The coursework required of mathematics education majors at Connor includes the calculus sequence, *Methods of Proof, Discrete Structures, Modern Geometry, Linear Algebra, Numerical Computing, Abstract Algebra, Mathematical Analysis, Introduction to Complex Analysis,* and *Introduction to Probability and Statistics.* The students also complete a methods course for mathematics grades 5-8, and methods for teaching mathematics in grades 9-12 is not addressed in the program. There were just under twenty students enrolled in the mathematics education program at Connor at the time of this study.

**Davis.** The mathematics department at Davis, a traditional school of mining and technology, recently added a mathematics education option. The university still does not offer a full mathematics education major; instead, mathematics majors who intend to be teachers first complete a mathematics degree at Davis, and then attend Hamilton (see below) to acquire the necessary coursework for a teaching license. A faculty member at Davis reported that in the past eight years, only one out of roughly fifteen mathematics majors has opted to complete the teacher licensure program at Hamilton after attending Davis. At the time of this study, there were two mathematics majors at Davis who planned to gain teacher licensure.

**Gaines.** At this institution, secondary mathematics education majors are required to complete *Calculus I* and *Calculus II, Linear Algebra, Introduction to Abstract Mathematics* (described as an introduction to proofs course), *Number Theory, History of Mathematics, Abstract Algebra, Geometry,* and *Introduction to Probability and Statistics.*
Mathematics education majors also take a course intended specifically for teachers: 
*Mathematics Technology for Teaching*, and choose one additional mathematics content 
course from a list of upper level courses. They take one methods course that addresses 
the full range of grades 5-12 mathematics. There were approximately 40 students 
enrolled in secondary mathematics education at Gaines at the time of this study.

**Hamilton.** All coursework at Hamilton is offered on a unique block schedule. 
Each course is either a morning or afternoon class meeting for three hours, Monday 
through Friday, for 18 days. Students take only one or two courses at a time. Secondary 
mathematics education majors are required to complete the calculus sequence, in addition 
to *Math Software, Linear Algebra, Modern Geometry, Foundations of Mathematics*, a 
statistics course (there are three to choose from), and three additional courses from a list 
of upper level mathematics content options. They must also complete a course for 
methods of teaching grades 5-12 mathematics. There were approximately fifteen 
mathematics education majors at Hamilton at the time of this study.

Table 2 summarizes the course requirements of each program in the study. Note 
that course titles are not consistent across programs, so the most common course names 
are displayed in Table 2. The actual content of these courses is not easily described; 
courses with the same titles may include different content or may be taught using 
different teaching approaches, depending on the program. “R” indicates that the course is 
required, and “E” indicates that the course is an elective. Later in this chapter, courses 
that pre-service teachers have indicated as aiding in the development of SCK will be 
described in more detail.
In addition to the coursework in mathematics described above, field experience required of mathematics education majors is of particular interest to developing SCK.

However, the quantity and quality of field experiences in each program is difficult to quantify. In some cases, field experiences are a stand-alone, credit-bearing requirement. At other times they are linked to classes, where pre-service teachers enter classrooms to practice what they have been learning in their own coursework. Field experience may also be embedded in classroom activities within coursework. Data collected about field experiences will be described later in this chapter.
Role and Experience of the Researcher

The researcher is not involved in designing any aspect of secondary mathematics teacher preparation programs. However, I am a graduate of a teacher preparation program in Mathematics (Grades 5-Adult) at a very small land-grant institution in another state. Upon program completion in December 2010, there were only two other students in the secondary mathematics education program at that institution, so the program primarily included most of the mathematics courses typically required for a B.S. in Mathematics, along with education courses taken alongside elementary education majors. The program did include one methods course titled "Teaching Mathematics in Elementary/Middle School," which contained three students. The last semester of teacher preparation included a student teaching assignment in a suburban high school, where I was responsible for teaching Algebra 2 and College Preparation Mathematics, under the supervision of two coordinating teachers.

I believe I was adequately prepared for secondary teaching in this program, but felt some required courses, such as Abstract Algebra, included material well beyond what is needed to be a successful high school mathematics teacher. In my opinion, courses with a focus on high school mathematics topics would have been more beneficial. Graduating from such a small teacher preparation program influenced the focus of this study, which seeks to reveal experiences that develop SCK in pre-service teachers; perhaps these experiences can be incorporated into a program that does not offer courses designed specifically for mathematics education majors.
During my undergraduate program, I decided that I wanted to continue studying mathematics education and entered graduate school the following year. Therefore, I have not had the experience of teaching secondary mathematics without the help of supervising teachers. My current graduate program is housed in a large university where I have assisted teaching a geometry course for secondary teachers. Experiences with both large and small teacher preparation programs are a possible source of bias in this study. As the researcher, I will be conscious of these conflicts when collecting, analyzing, interpreting, and reporting data.

During graduate studies, I have taught three different mathematics content courses for prospective K-8 teachers over seven semesters. The first of those two courses have a focus on numbers and operations, the second emphasizes geometry, and the third course examines number theory, ratio and proportions, algebra, and statistics. These courses are designed around Sybilla Beckmann's *Mathematics for Elementary Teachers with Activity Manual* (2010). The courses are taught using an inquiry-based approach, in which the students work through activities with limited guidance from their instructor. The courses in general, and the Beckmann book in particular, are intended to teach elementary mathematics at a level deeper than what a typical elementary student would need to know. For example, one activity requires the elementary pre-service teachers to explain how to order a list of fractions without converting them to decimals or finding a common denominator. This type of activity, like many others in the course, represents an effort to intentionally build Specialized Content Knowledge in pre-service elementary teachers.
Methods of Data Collection

Initial data were collected using an SCK Assessment and a Prospective Teacher Survey (see Appendix A). These data were then used to select interview participants, who were asked follow-up questions about their performance on the SCK Assessment and what experiences helped them develop SCK. Each of these instruments and participant selection are described in the following sections.

SCK Assessment

A key source of data for the study was an assessment tool designed to measure knowledge specific to mathematics teaching, the Content Knowledge for Teaching Algebra 1 Assessment developed by Geoffrey Phelps and Drew Gitomer (2012) as part of the Measures of Effective Teaching Project, sponsored by the Bill and Melinda Gates Foundation. Individual items on the assessment were written with the help of several mathematics educators, Mark Hoover Thames and Hyman Bass among them. For the purposes of this study, the instrument will be referred to as the "SCK Assessment."

Close examination of the items in this instrument reveal that they can be assumed to target Specialized Content Knowledge. The assessment developers wrote:

[Content Knowledge for Teaching] refers to more than explicit content knowledge that an educated individual needs; it is the content knowledge that is used in the day-to-day work of teaching. Examples of test questions include those that require test-takers to interpret student work, to represent content in ways that are accessible to students, and to select appropriate examples. (Phelps & Gitomer, 2012, p. iii)
This explanation aligns with the accepted definition of Specialized Content Knowledge as a deeper understanding of the material that allows teachers to explain new ideas, work problems in multiple ways, and analyze student solutions (as cited in Hill et al., 2008).

The SCK Assessment tool was designed for research purposes and is not intended to evaluate or inform decisions about teachers. Phelps and Gitomer (2012) note:

[The] assessment is not to be used as a selection or evaluation instrument for individual teachers.... The assessment is also not designed with a criterion reference. There is no score on the assessment that indicates a particular proficiency level, such as readiness to teach or a direct connection to a level of teaching quality. (p. v)

The developers also indicate that the assessment may be used for research purposes without further permission. The SCK Assessment was discovered through e-mail correspondence with Mark Hoover Thames, who contributed to writing the items on the assessment instrument.

The SCK Assessment consists of 22 items, each of which are either multiple choice questions or a table combining three to five items. See Figure 2: SCK Assessment Item 1 for an example of a multiple choice item and Figure 3: SCK Assessment Item 3 for an example of a table with four items. Multiple choice items are each worth one point. The tables are worth three to five points, depending on how many items are in the table. The overall possible score on the assessment is 35.
Figure 2. SCK Assessment Item 1

1. Mr. Wright asked his students to solve the equation \( 6 - 3(x - 5) = 24 \). After reviewing his students’ work, he found two interesting methods for solving the equation and asked those two students to present their methods on the board.

<table>
<thead>
<tr>
<th>Brenda’s method</th>
<th>Daniel’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6 - 3(x - 5) = 24 )</td>
<td>( 6 - 3(x - 5) = 24 )</td>
</tr>
<tr>
<td>( 6 - 3x + 15 = 24 )</td>
<td>( -3(x - 5) = 18 )</td>
</tr>
<tr>
<td>( -3x + 21 = 24 )</td>
<td>( x - 5 = -6 )</td>
</tr>
<tr>
<td>( -3x = 3 )</td>
<td>( x = -1 )</td>
</tr>
<tr>
<td>( x = -1 )</td>
<td></td>
</tr>
</tbody>
</table>

After Brenda and Daniel presented their methods, Mr. Wright’s class discussed these two methods. One student, Steve, compared them and then said, “I like Daniel’s method because there are less steps. However, if it is a harder division, Brenda’s method would be easier.” For which of the following equations would Steve be most likely to use Brenda’s method?

A) \( 5 - 2(x - 3) = 21 \)

B) \( 10 - 7(x + 5) = 6 \)

C) \( 6 - 3(x + 1) = 9 \)

D) \( 8 + 4(x - 3) = 14 \)
During a lesson on solving multistep equations, Ms. Kane asked her students to solve the equation $-5x + 8 = 13x - 10$. While walking around the classroom looking at what the students were writing, she noticed several different strategies. For each of the following student solutions, indicate whether or not the work provides evidence that the student is reasoning correctly about this problem.

Each item on the SCK Assessment was examined using the SCK framework developed by Bair and Rich (2011). That framework measures SCK on five levels across four different components: (1) Ability to correctly solve a task, explain their work, justify

<table>
<thead>
<tr>
<th></th>
<th>Provides Evidence of Correct Student Reasoning</th>
<th>Does Not Provide Evidence of Correct Student Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>$-5x + 8 = 13x - 10$&lt;br&gt;$8 = 18x - 10$&lt;br&gt;$18x = 18$&lt;br&gt;$x = 1$</td>
<td></td>
</tr>
<tr>
<td>B)</td>
<td>$-5x + 8 - 13x + 10 = 15x - 10 - 15x + 10$&lt;br&gt;$-18x + 18 = 0$&lt;br&gt;$x = 1$</td>
<td></td>
</tr>
<tr>
<td>C)</td>
<td>$-5x + 8 = 13x - 10$&lt;br&gt;$-5x + 8 = 3x$&lt;br&gt;$5x = 8$&lt;br&gt;$x = 1$</td>
<td></td>
</tr>
<tr>
<td>D)</td>
<td>$-5x + 8 = 13x - 10$&lt;br&gt;$-13x - 8 = -5x - 10$&lt;br&gt;$-18x = -12$&lt;br&gt;$x = 1$</td>
<td></td>
</tr>
</tbody>
</table>
their reasoning, and make connections; (2) Ability to use multiple representations; (3) Ability to recognize, use, and generalize relationships among conceptually similar problems; and (4) Ability to pose problems. After classifying the items in accordance with that framework, e-mail correspondence with Dr. Bair verified the correct use of her framework. All but Item 5 and Item 21 assess some aspect of SCK. Fifteen of the items are aligned with component one, one item is aligned with component three, and five of the items are aligned with the fourth component. Note that items were allowed to measure more than one component of SCK. Dr. Bair (personal communication, August 19, 2015) also cautioned that given its use of multiple choice items, the instrument "rarely provides a way of assigning the level of development of a teacher...[and] the vast majority of the tasks address category one."

Prospective Teacher Survey

Participants were asked to complete a brief Prospective Teacher Survey asking about the mathematics content and methods courses they had completed (as part of teacher preparation and for other reasons) and the courses they were currently taking. The survey was designed uniquely for each teacher preparation program, so that the listed mathematics courses aligned with the courses that a given college or university offered. In addition to the mathematics courses, middle school mathematics methods courses, high school mathematics methods courses, and statistics courses that were required for each program were included in the list on each survey.

Pre-service teachers were also asked to respond to questions about their pathway in deciding to become a teacher, such as, "What events and experiences influenced your
decision to become a high school mathematics teacher?" The Prospective Teacher Survey ended with collecting demographic data on academic class, gender, and whether participants graduated from a high school in the same state as their teacher preparation program. The participants were also asked for the semester they planned to student teach. This prompt provided more useful information than class standing, because students who changed their major could be considered seniors in the first year of a teacher preparation program while their experiences were more appropriately aligned with those of a sophomore. This demographic information was used for the quantitative portion of analysis, and to select interview participants. For example, survey responses were examined to be sure at least one non-traditional student was included in the interviews. More information on this process is found in the section titled "Participant Selection."

**Administering the SCK Assessment and Prospective Teacher Survey**

The SCK Assessment and Prospective Teacher Survey were administered to every willing secondary mathematics pre-service teacher in the state. The survey was completed after the assessment. It was considered that completing the survey first might affect responses on the SCK Assessment, because of a possible self-stereotype activation. If a participant answered a survey question that reminded him or her that a certain stereotype was applicable, he or she could succumb to that stereotype (Wheeler & Petty, 2001). The survey asked for respondents' names so they could be contacted later for an interview or to gather additional information. More detail about both of these scenarios is provided in the sections titled "Participant Selection" and "Analysis of Data."
The researcher communicated with an established faculty contact at each campus to arrange the administration of the SCK Assessment and Prospective Teacher Survey. At two of the eight institutions, pre-service teachers did not participate in the study (described in Chapter 4). The researcher asked permission to visit a class such as high school mathematics methods for forty-five minutes, so that participants would have time to complete the survey and assessment during the visit. It was expected that the SCK Assessment would take less than thirty minutes, and that the survey could be completed in less than fifteen minutes. The researcher traveled to each campus and when granted enough time, proceeded to explain the study, administer the SCK Assessment, and administer the Prospective Teacher Survey in as many classes as possible. When access to a class was granted, but a full forty-five minutes was not available, the researcher briefly described the study and distributed a handout with instructions for accessing digital online versions of the SCK Assessment and Survey. When the researcher was not granted access to a course, the instructor was asked to distribute a handout stating the purpose of the study and instructions for accessing the digital online versions of the assessment and survey. Pre-service teachers who were not present during classroom visits received e-mail invitations asking them to attend a meeting in the early evening where they could complete the SCK Assessment and Prospective Teacher Survey on site. Any remaining pre-service teachers were contacted via e-mail asking them to complete the digital online versions of the SCK Assessment and Prospective Teacher Survey.

It would appear to be a difficult task to contact all of the pre-service mathematics teachers in teacher preparation programs across an entire state. However, at five out of
the eight schools in the study, there were less than ten pre-service teachers enrolled in the program. It was relatively easy to arrange meetings with those pre-service teachers, where they were introduced to the study and asked to complete the assessment and survey. In addition, the researcher is a graduate student at one of the larger programs; pre-service teachers in that program were contacted over a longer period of time, since the researcher was regularly on campus. The strongest efforts were made to include pre-service teachers in their third or final year of a preparation program, as they take more courses together, making them easier to contact. Furthermore, they have had the most experience in teacher preparation, so the data they contributed was richer than that coming from underclass participants.

**Interview Protocol**

Interviews were semi-structured using an interview guide (see Appendix B), and lasted between 40 and 100 minutes. Two hours were allotted when reserving an interview location, so that space would be available if the participant was willing to continue the interview. The interviews were all conducted face to face, which increased the comfort level of participants and made it easier for the researcher to notice body language throughout the interview. Body language can enhance the meaning of a statement made by a participant or indicate that an interviewee is becoming bored with the questions or topic at hand. At the end of each face to face interview, participants were asked if they were willing to be contacted via video chat for clarifications, to complete an unfinished interview, or for a brief follow up interview later in the study. Having a virtual alternative for wrapping up the interview was necessary for two reasons:
to minimize travel to distant locations and to make data collection as convenient and noninvasive as possible for prospective teachers. Each interview was audio recorded, transcribed, coded, and organized into memos (discussed in Chapter 4).

The interview sessions opened with a reminder about the purpose of the study, the purpose of the interview, and a request for official consent to complete and audio record the interview. The interview continued with questions about mathematics courses the participant was currently taking, and questions about his or her survey responses regarding the decision to become a teacher. This reminded the participants of their circumstances as pre-service teachers, so they would be more likely to reflect on their experiences as a pre-service teacher when they were asked questions in the next part of the interview.

Five SCK Assessment items were chosen as the basis for the main body of the interview. Participants were questioned about their responses to each item in the following way. They were shown a clean copy of an item from the assessment and asked to select an answer again. Prompts led them to explain how they decided on that answer. If their answer in the interview was different than their test response, they were notified of their answer change and asked if they could recall what they were thinking before, and how or why their thinking changed. Then, they were asked to recall experiences that helped them answer the question, regardless of whether the solution was right or wrong.

In the third and final part of the interview, participants were invited to review the entire SCK Assessment to help them recall what was on the instrument. Then, they were asked to make connections between what was seen on the assessment and their previous
experiences. One interview prompt asked, "If you were developing a teacher preparation program for high school mathematics teachers, what experiences would you include to help prepare students for an assessment like this?"

The SCK Assessment items were analyzed based on the type of Mathematical Knowledge for Teaching (i.e. components of MKT) needed to answer each item. Then, the items that were coded as measuring SCK were examined closer. Preference was given to five items that required evaluating a student's thinking or method for solving a problem. For many of the items, Common Content Knowledge could be used to select the correct choice. For example, on Item 2 (see Figure 4), applying the distributive property on each of the four choices will reveal which of the problems becomes simpler using the distributive property. Item 3 (see Figure 3) was also eliminated because Common Content Knowledge easily reveals which student methods are correct. In Item 3C, a basic understanding of combining like terms reveals that the student incorrectly combined $13x$ and $-10$, getting a result of $3x$. Items 3A, 3B, and 3D can easily be checked for accuracy using similar Common Content Knowledge in beginning algebra. It is assumed that pre-service teachers have a solid grasp of basic algebra manipulation skills upon entering the teacher preparation program, so responses to an interview question about this item, and others like it, would rely on Common Content Knowledge as opposed to SCK.

On the other hand, Item 9 (see Figure 5) asked participants to evaluate a student's approach to a problem. The student's words are written out, and the pre-service teacher needs to analyze the student's thinking based on what she said. This encourages the pre-
service teachers to think non-procedurally about answering the problem, and they put themselves in the position of teacher when thinking about the problem. Each of the five problems selected for the interview is similar to Item 9 in that it surrounds student reasoning that is not typical, and the pre-service teacher is asked to evaluate that reasoning.

Figure 4. SCK Assessment Item 2

2. A lesson in Ms. Hagerman’s textbook defines the distributive property, but the exercises merely ask for its definition. To motivate her students to learn the definition, Ms. Hagerman tells them that the distributive property can often be used to simplify the evaluation of expressions.

She wants to give her students an example that will focus their attention on how the distributive property can be useful in evaluating expressions. Of the following expressions, which would best serve her purpose?

A) $12 \times \left( \frac{3}{4} + \frac{1}{4} \right)$
B) $18 \times \left( \frac{3}{5} - \frac{1}{10} \right)$
C) $36 \times \left( \frac{5}{12} + \frac{2}{9} \right)$
D) Each of these expressions would serve her purpose equally well.

Ultimately, Item 9, Item 10, Item 12, Item 15, and Item 22 on the SCK Assessment were chosen because they seemed to have the most potential to reveal experiences that develop SCK. In early interviews, the participants were not necessarily interviewed about these items in the order they occurred on the SCK Assessment. It was important to first ask interview participants about two items that they answered correctly, because correct responses were the most likely indicators that they had developed some
degree of SCK for that topic. It was valuable to learn through follow up questions what experience(s) helped them answer that item. One incorrect item was discussed next because reflecting on incorrect responses and thinking through how to correct errors could also reveal SCK. In addition, the researcher hoped that participants would be able to identify knowledge they acquired between the time of the SCK Assessment and the interview that could be identifiable as SCK. After completing the first three interviews, it was clear that an hour allowed enough time for discussing all five SCK Assessment items with each participant, so the items were discussed in the order they appeared on the assessment for the remainder of the interviews.

Figure 5. SCK Assessment Item 9

9. Having taught her students to factor quadratics with integer coefficients, integer roots, and a leading coefficient of 1, Ms. Quezada explained that she was going to give them a harder problem. She then asked them to solve the following.

\[3x^2 - 3x - 6 = 0\]

After a few minutes of work, the class discussed their solutions. Letitia said that \(x\) was \(-1\) or \(2\) and explained, “I added \(3x\) to both sides and divided by \(3\).”

\[3x^2 - 6 = 3x\]
\[x^2 - 2 = x\]

She then continued, “The parabola’s just down a little and the line’s at 45 degrees, so it’s just below zero and about 2 to the right. \(x\) can be \(-1\) and \(2\), and those are the only possible ones.”

Of the following, which best characterizes Letitia’s approach to this problem?

A) Letitia’s method is wrong because she should have first divided by 3 and then factored the left side of the equation.

B) Letitia’s method is wrong because this is a parabola and you could graph it, but you would have to graph the original equation and look for the roots.

C) Letitia’s reasoning is correct, but her method often leads to points of intersection that might be hard to determine visually.

D) Letitia’s reasoning is correct, but her method requires knowledge of calculus.
As is typical in a grounded theory study, the interview guide was modified as incoming data informed topics of interest to the research questions. Data analysis, in some ways, drove data collection, which is a hallmark of grounded theory’s iterative process of data collection and data analysis. This is discussed more fully in the section on theoretical sampling.

Prospective Teacher Survey and SCK Assessment Participants

All students enrolled in secondary mathematics teaching programs in the state were asked to complete the SCK Assessment and the Prospective Teacher Survey. This provided the broadest spectrum of assessment and survey responses, which served a number of purposes. Most importantly, later stages in the study called for interview participants to be chosen based on their SCK Assessment scores and survey responses. Maximizing the pool of participants who completed the data collection instruments increased the likelihood that those who were willing to participate in an interview would satisfy the needs of the study. The assessment scores and survey responses were also used to complete a separate analysis. Lastly, the data collected from pre-service teachers who completed the SCK Assessment were shared with the original developers of the instrument to further validate the instrument and expand its use.

A total of forty-seven participants provided demographic and background information on the Prospective Teacher Survey. Some of these data are summarized in Table 3. Six participants were student teaching during the academic year of the study (2015-2016). Twenty-three participants planned to student teach the following year
(2016-2017); they are referred to as “juniors.” Fourteen participants planned to student teach during the 2017-2018 academic year, and are considered “sophomores.” A remaining four participants planned to student teach after Spring 2018, classifying them as “freshmen.” Only fourteen of the forty-seven participants did not attend a high school in the same state as their teacher preparation program. Their ages ranged from 18 to 43 years old, with seven of the participants age 30 or older. Only twenty-three of the participants reported that they originally declared mathematics education as their major, with more than half of the participants transferring into mathematics education from other majors. Four of those participants completed degrees in other fields before studying mathematics education.

Table 3. Summary Demographics of Prospective Teacher Survey Participants

<table>
<thead>
<tr>
<th></th>
<th>Anderson</th>
<th>Bailey</th>
<th>Connor</th>
<th>Davis</th>
<th>Gaines</th>
<th>Hamilton</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Participants</td>
<td>2</td>
<td>20</td>
<td>4</td>
<td>2</td>
<td>13</td>
<td>6</td>
<td>47</td>
</tr>
<tr>
<td>Completing the Survey</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Changed Major to Mathematics</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>23</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Age ≤ 25</td>
<td>2</td>
<td>18</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>6</td>
<td>39</td>
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<tr>
<td>Age 26-30</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Age ≥ 31</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Graduated from a High School</td>
<td>1</td>
<td>14</td>
<td>4</td>
<td>1</td>
<td>9</td>
<td>4</td>
<td>33</td>
</tr>
<tr>
<td>in the State</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agreed to an Interview</td>
<td>2</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>12</td>
<td>4</td>
<td>32</td>
</tr>
</tbody>
</table>
Participant Selection for Interviews

Twenty-three pre-service teachers participated in follow-up interviews to further explore selected SCK Assessment items and the experiences that helped them answer those items. The selection process for interviewees began with the identification of a pool of volunteers willing to participate in an interview. The last question on the Prospective Teacher Survey asked participants if they would be willing to participate in a follow-up interview about their experiences. Thirty-two responded, "yes," and all but ten of them were interviewed. One additional interviewee indicated that he would not be interested in a follow-up interview, but agreed to participate when asked directly, bringing the total number of interviews to 23. They included three freshmen, five sophomores, twelve juniors, and three who were at various stages of student teaching. Table 4 provides more information about the interviewees.

Of the ten willing participants who were not interviewed, four did not reply to e-mail requests to meet for an interview. Three others scheduled an interview, but had to cancel at the last minute for various personal conflicts. Two other participants were out of state; one had moved and the other was studying abroad. The remaining participant was student teaching at the time she completed the Prospective Teacher Survey and SCK Assessment, and had graduated by the time she was contacted for an interview.

Interview participants, each selected from the pool of SCK Assessment and Prospective Teacher Survey participants, were interviewed in four stages. The first stage was based on convenience sampling, which took place during campus visits when administering the SCK Assessment and Prospective Teacher Survey. The second group
of interview participants was chosen based on maximum variation sampling, to ensure that the data gathered from the convenience sample was supplemented with data from a wide variety of pre-service teachers. The third round of interview participants were selected over time based on theoretical sampling. Finally, convenience sampling rounded out the participant pool during the last month of the Spring semester. All of these sampling techniques are described in detail below. The interview sampling timeline in Table 5 shows when each of these phases of interview sampling were carried out.

Table 4. Summary Demographics of Interview Participants

<table>
<thead>
<tr>
<th>Teacher Preparation Program</th>
<th>Intended Student Teaching</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anderson</td>
<td>Spring 2016</td>
<td>21</td>
</tr>
<tr>
<td>Anderson</td>
<td>Fall 2016</td>
<td>21</td>
</tr>
<tr>
<td>Bailey</td>
<td>Fall 2016</td>
<td>22</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2017</td>
<td>20</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2017</td>
<td>20</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2017</td>
<td>21</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2017</td>
<td>21</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2017</td>
<td>30</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2018</td>
<td>19</td>
</tr>
<tr>
<td>Bailey</td>
<td>Spring 2018</td>
<td>19</td>
</tr>
<tr>
<td>Connor</td>
<td>Spring 2017</td>
<td>19</td>
</tr>
<tr>
<td>Connor</td>
<td>Fall 2019</td>
<td>39</td>
</tr>
<tr>
<td>Davis</td>
<td>Fall 2017</td>
<td>24</td>
</tr>
<tr>
<td>Gaines</td>
<td>Spring 2016</td>
<td>36</td>
</tr>
<tr>
<td>Gaines</td>
<td>Spring 2017</td>
<td>20</td>
</tr>
<tr>
<td>Gaines</td>
<td>Spring 2017</td>
<td>20</td>
</tr>
<tr>
<td>Gaines</td>
<td>Spring 2017</td>
<td>23</td>
</tr>
<tr>
<td>Gaines</td>
<td>Spring 2017</td>
<td>43</td>
</tr>
<tr>
<td>Gaines</td>
<td>Fall 2017</td>
<td>26</td>
</tr>
<tr>
<td>Hamilton</td>
<td>Spring 2016</td>
<td>21</td>
</tr>
<tr>
<td>Hamilton</td>
<td>Fall 2017</td>
<td>20</td>
</tr>
<tr>
<td>Hamilton</td>
<td>Fall 2018</td>
<td>18</td>
</tr>
<tr>
<td>Hamilton</td>
<td>Spring 2019</td>
<td>18</td>
</tr>
</tbody>
</table>
### Table 5. Interview Sampling Timeline

<table>
<thead>
<tr>
<th>Month</th>
<th>Description</th>
</tr>
</thead>
</table>
| October–December 2015 | Campus visits:  
|                 | • 47 PSTs completed Prospective Teacher Survey and SCK Assessment  
|                 | • First 5 PSTs interviewed (convenience sampling); transcribing and analysis |
| January 2016    | 2 PSTs interviewed (1 convenience sampling; 1 maximum variation sampling); transcribing and analysis |
| February–April 2016 | 7 PSTs interviewed (theoretical sampling); transcribing and analysis |
| April 2016      | 9 PSTs interviewed (return to convenience sampling) |

#### Convenience Sampling.** The selection of interview participants began with “sampling on the basis of convenience (whoever walks through or whoever agrees to participate). This is a more practical way to gather data…Often, differences in data emerge naturally because of the natural variations in situations” (Corbin & Strauss, 2008, p. 153). Since the SCK Assessment and Prospective Teacher Survey were administered during site visits, convenience sampling was used to interview one participant while visiting each of the six programs. It was assumed that the mathematics education major who was most successful on the assessment would be ablest to talk about the experiences that led to that success. Therefore, the pre-service teacher from each program who earned the highest score on the SCK Assessment was contacted for an interview. In some cases, that pre-service teacher was unwilling or unable to meet for an interview, in which case the second highest scoring pre-service teacher was selected. At Gaines, a larger program where six mathematics education majors completed the survey and assessment on site in one day, one pre-service teacher was able to meet for an interview later in the evening. At the remaining programs, circumstances did not allow for interviewing a mathematics education major during the visit, as outlined below.
At Hamilton, four pre-service teachers dropped in to a room in the library to complete the survey and assessment throughout the day. Others stopped in briefly and agreed to complete the survey and assessment online at a later time. None of the four pre-service teachers who completed the survey and assessment were able to meet for an interview that same day. At Connor, the SCK Assessment and Prospective Teacher Survey were introduced in two mathematics courses, but no one in those classes had time to complete the survey and assessment on site. The two participants at Anderson completed the Prospective Teacher Survey and SCK Assessment online, with no site visit. At Bailey, three classrooms were visited throughout a week; participants then completed the survey and assessment on their own time in the mathematics department’s testing center, over a period of two weeks. Finally, the two mathematics education majors at Davis were able to complete the survey and assessment on site during their geometry course, but neither was able to meet later in the day for an interview.

All of the program sites are located within a three-hour radius of my home institution. As described above, some campuses did not allow for an interview to be completed during the first visit. Rather than immediately taking time to pursue one interview at each campus to fulfill the original plan, the decision was made to first focus on contacting more mathematics education majors at each program to complete the survey and assessment. This would allow for a larger, more diverse, selection pool of interview participants in the long run.

In the grounded theory approach, findings from earlier interviews are used to select later interview participants. Having interview data from participants at each of the
programs would provide essential variety, allowing for interview selections that would help build on the data from earlier interviews in a productive manner. Therefore, it was important to have a wide variety of experiences reported early in the study. By the time site visits and personal contacts had been completed for each program, only two interviews had taken place: one at Davis on the way to the Hamilton site visit, and one at Gaines the day of the initial visit, as planned. However, it was still possible to interview one mathematics education major from each of the six programs, before interviewing duplicates from the same program. During the final month of the Fall semester, participants at three of the remaining four programs were contacted and interviewed. A participant in the sixth and final program was interviewed at the start of the Spring semester.

**Maximum Variation Sampling.** After interview data were collected from the first six participants, one from each program, maximum variation sampling began. Maximum variation is a sampling technique that has the goal to “adequately capture the heterogeneity in the population. The purpose [of maximum variation sampling] is to ensure that the conclusions adequately represent the entire range of variation, rather than only the typical members or some ‘average’ subset of this range” (Maxwell, 2013, p. 98). To select the next group of interviewees, survey data for the first six interviewees were analyzed against the participants who agreed to participate in a follow-up interview. This analysis is described below.

On the Prospective Teacher Survey, participants were asked which teacher preparation program they attended, which mathematics and mathematics methods courses
they had taken, when they planned to student teach, whether they had originally declared mathematics education as their major or which major they transferred from, at what age they made the decision to become a teacher, what influenced them to become a teacher, their experiences working with learners, their age, and whether they attended a high school in the same state. For the purpose of maximum variation sampling, the following dimensions were examined for each of the first six interview participants: teacher preparation program, when they planned to student teach, when they made the decision to become a teacher, their experiences with learners, and their age. The other dimensions were used later in the sampling process, during theoretical sampling, to be described later.

In the first group of six interview participants, the majority were planning to student teach in the next few semesters, had decided to become a teacher when they were in high school, had mathematics tutoring experience, and the oldest was 24 years old. Also, one person from each teacher preparation program had been interviewed. To increase variation, the next interviewee was characterized as a nontraditional student, planning to student teach in Fall 2018 or later, who made the decision to become a teacher before or after high school. Six participants satisfied two or more these requirements. One participant was a nontraditional student who was just starting in the secondary mathematics program, and had decided after high school that he wanted to become a mathematics teacher. Additionally, his only reported mathematics tutoring experience was helping his young children with their homework. He satisfied the most
criteria for providing maximum variation, and was therefore selected for the next interview.

The demographic analysis was reexamined for the seventh interview, and showed a variety in the interviewed population that would expose a broad range of experiences potentially aiding in the development of SCK. This variety was also evident in the memos that were developing. The process of creating the memos is described below.

With seven interviews completed, further interviews were suspended to allow time for transcribing the interviews, analyzing the interview data, and organizing the data into memos. A memo was initiated each time a new theme arose out of the data, and expanded each time there was another occurrence or extension of that theme in an interview. The entries in each memo were labeled according to which participant provided that piece of data, and included quotes from the interviewees as well as summaries of what participants reported.

Theoretical Sampling. Once maximum variation sampling was completed and each of those interviews was analyzed, memos based on the coded data began to reveal both common and unique themes, leading to a period of theoretical sampling.

Theoretical sampling has been defined as:

A method of data collection based on concepts/themes derived from data. The purpose of theoretical sampling is to collect data from places, people, and events that will maximize opportunities to develop concepts in terms of their properties and dimensions, uncover variations, and identify relationships between concepts. (Corbin & Strauss, 2008, p. 143)

Each new participant was theoretically selected based on the data provided in his or her survey and SCK Assessment, with the goal of potentially filling in gaps in the developing
theory. The goal of a grounded theory study is saturation of the developing theory, the point when the participants are repeatedly giving similar responses (Corbin & Strauss, 2008). Once saturation has been reached in one area, it is necessary to shift the focus of the study to concepts that are lacking information. Theoretical sampling allowed for new interview participants to be selected in such a way to reveal new data. The following paragraphs include detail about how theoretical sampling was used to select interview participants during this study.

To begin theoretical sampling, existing memos were organized into folders that categorized similar themes. The categorized folders were titled: Experiences, Misconceptions, Participant Profile, Role Models, Specialized Content Knowledge, Ways of Approaching SCK Assessment Items and Change in Viewpoint of What Teachers Need to Know. The titles of those folders are not as important as their contents, but served as an organizational tool to aid in the process of incorporating new data. A summary was written for each memo to reflect key ideas and synthesize conflicting and agreeing data found in each memo at that time. The summaries were compiled in one document in an outline format, with folder titles as headings and memo titles as subheadings, to get a big picture idea of the data that had been collected so far. One memo, titled Change in Viewpoint of What Teachers Need to Know, did not fit into any of the folders, so it remained as a stand-alone memo throughout analysis.

Analysis of this outline of summaries showed that the memos with the most data, and also most relevant to the research, were Coursework that Develops SCK in the Experiences folder and Approaching SCK Assessment Items like Proofs in the Ways of
Approaching SCK Assessment Items folder. Two participants said that they approached the items on the SCK Assessment the same way they approach a proof, so that concept needed to be examined further. Additionally, a pre-service teacher said that his experiences in Real Analysis and Abstract Algebra helped him develop a deeper understanding of high school mathematics, so that would also be a focus of further interviews. To further investigate both how other mathematics education majors viewed upper level mathematics courses and how they related proof writing to answering SCK Assessment items, ideally one participant with both of those experiences would need to be interviewed.

Abstract Algebra and Real Analysis typically involve writing proofs, creating a connection between upper level mathematics courses and proof writing. Only one participant who had not been interviewed, had taken a Real Analysis course. Only two participants had completed an Abstract Algebra course, both pre-service teachers at Gaines, and they were selected for the next two interviews. These two participants were the first interviewees chosen using theoretical sampling.

Theoretical sampling often results in modifying the interview questions to focus on a particular aspect of the theory to be investigated. Corbin and Strauss (2008) elaborate:

Once data collection begins, the initial interview or observational guides (used to satisfy committees) give way to concepts derived from analysis. Adhering rigidly to initial questions throughout a study hinders discovery because it limits the amount and type of data that can be gathered. (p. 152)

Previous interviewees did not talk much about the actual content of their Abstract Algebra courses, but more about the proof writing aspect of the course. Some
participants had described how writing proofs was helpful to them, but others found it unhelpful for high school teachers to be able to write proofs, while the discrepancy was not explained. The data did not show saturation, meaning that there was more to be learned about writing proofs, upper level mathematics courses, and how they might aid in developing SCK. When interviewing the two aforementioned pre-service teachers, questions specific to how *Abstract Algebra* helped them understand high school level mathematics and how proof writing is related to answering SCK Assessment items were incorporated into the interviews.

While Bailey does not require *Abstract Algebra* of mathematics education majors, they do require a proof writing class, which is typically taught at the sophomore level. It was expected that participants at Bailey would have had different experiences with proof writing than those already interviewed at Gaines. Furthermore, Bailey enrolled the most secondary mathematics education majors of any program in the study, but so far only one participant in that program had been interviewed. Therefore, the next interview participants were selected from that program. Four of the five participants who reported taking the *Introduction to Proofs* course on the Prospective Teacher Survey responded to e-mails, agreed to meet, and were interviewed.

At this point, the decision was made to begin investigating a different topic that had emerged relating to developing SCK. Although the four students from Bailey were selected for reasons specific to proof writing, in the earlier parts of their interviews, each of the participants talked about their lack of field experience. They expressed that they did not have enough experience with students, and that interactions with students would
have helped them develop SCK. Pursuing this theme using theoretical sampling called for additional interviews with participants who reported having more field experiences than the pre-service teachers at Bailey. This led to selection of the second participant from Anderson, who had reported on the Prospective Teacher Survey that she had more field experience than other participants. This interview was modified by incorporating more questions about whether her field experiences helped her develop SCK. The findings based on the interviews described above will be described later in this chapter.

**Returning to Convenience Sampling.** At this point, there were less than four weeks left in the spring semester. Most college students travel during the summer, so it was important to complete all of the desired interviews before the programs finished their academic year. It was likely that participants would be less agreeable to an interview during the last week of classes or during final exams, which further limited the amount of time available for completing interviews.

With the end of the semester approaching quickly, convenience sampling was used once again in an attempt to interview as many pre-service teachers as possible. Another analysis of survey data showed that sixteen mathematics education majors remained who had agreed to be interviewed: one at Davis, three at Hamilton, three at Bailey, and nine at Gaines. All four pre-service teachers at Davis and Hamilton responded to e-mail requests, and interviews were scheduled. Interviews were also scheduled for five mathematics education majors at Gaines. On the day of the interviews, the participant at Davis and two of the participants at Gaines were unable to make their scheduled interviews for personal reasons. The other six interviews were successfully
completed. Three more interviews were conducted at Bailey during the final week in the semester. At this point, all participants who were willing and able to participate in an interview had been interviewed, and they provided a great deal of data about the development of SCK.

**Participant Incentives**

Participants were presented with an incentive in hopes that the offer would help increase the number of pre-service teachers willing to participate in the study. Participants who completed the Prospective Teacher Survey and the SCK Assessment were entered into a drawing. Those participants who also indicated on the survey that they were willing to participate in a future interview were entered into the drawing twice. Once all surveys and SCK Assessments were collected, five names were chosen from the drawing pool. The five winning participants each had the choice between a lift ticket to a local ski resort (valid for the 2015/2016 ski season) and $50 cash.

**Other Supporting Data**

In addition to the SCK Assessment, the Prospective Teacher Survey, and the series of interviews, a variety of other data sources supported the study. Archival data including online course catalogs and department web pages were accessed to flesh out program requirements, such as courses needed for degree completion, before the beginning of the study. The document archive continued to be referenced throughout the study, and its contents increased as information was collected from each teacher preparation program during campus visits.
Program coordinators and mathematics education instructors from all eight programs were contacted during the study preparation phase. Face to face and telephone conversations allowed the researcher to gather background information about the logistics of each program and to establish a point of contact for administering surveys and assessments. These individuals were an ongoing resource for program and student data.

After analyzing data collected through interviews, some questions remained concerning the details of specific coursework and program requirements. Participants reported that some courses helped develop SCK, so more information was gathered about the nature of those courses through interviews with three course instructors. The goal of the interviews was to share details that may have been responsible for the success of the courses. Another instructor was asked for more information about a class activity that participants referred to. The instructor was able to provide more information about that task, so that it could be described in more detail. All four of these interviews were conducted face to face or via video chat. The interviews were recorded, but they were not transcribed. The instructors’ descriptions were summarized and added to the information about those courses and class activities, so deep analysis was not needed.

Another university professor explained the details of an undergraduate program that allows pre-service teachers to work alongside instructors of undergraduate courses. Lastly, one program coordinator was contacted to verify program requirements. Both of these exchanges took place via e-mail.
Analysis of Data

SCK Assessment and Prospective Teacher Survey

The SCK Assessment consisted of 22 items, each of which was a multiple choice item or a table of items, with each item in the table having one correct answer. Each multiple choice item was worth one point, but the tables of items were worth three to five points total, depending on the number of items in the table. SCK scores were calculated out of a total of 35 points. Each participant's score on each item was entered into a spreadsheet matrix, so that individual participant's scores on each item could be examined, as well as the performance of the entire sample on any particular item. Prospective Teacher Survey data was also organized in a spreadsheet so that data could be viewed by looking at one participant's responses to all questions and also by looking at all participants' responses to a single question. This enabled comparisons between participants on a given question, and also made it possible to compose short profiles for each participant. Spreadsheets were used to help with theoretical sampling in the later stages of the study.

At the most basic level, descriptive statistics were calculated to report overall results of the SCK Assessment, which were informative in their own right. Aggregated demographic data was also reported to provide a thorough description of the study population. When interviewees indicated experiences that develop SCK, the researcher intended to analyze survey and SCK Assessment scores based on that finding, but that did not prove useful because of the small sample size and the variety among them. Very few participants reported the same experiences and those experiences were not organized
chronologically, meaning it was difficult to make fair comparisons between pre-service teachers who have had a particular experience and those who have not. Other summary statistics from quantitative analysis are included in Chapter 4.

Interviews

Interview data were transcribed and coded using suggested strategies from Corbin and Strauss (2008). First, interviews were transcribed word for word. Then, the entire interview was read in one sitting to help recollect the main focus of that particular interview. The coding process was based on memoing (Corbin & Strauss, 2008). As each interview was read a second time, memos were created for each new topic that arose. When possible, previously created memos were expanded. At significant points in data analysis, the growing collection of memos was re-examined for similar topics that could be grouped together into one memo and renamed to reflect that collection of codes.

The method of constant comparison was used to group occurrences that were conceptually similar. While coding, each new concept encountered was constantly compared to previous findings, both within that particular interview and across previous interviews. When multiple occurrences of a conceptually similar incident were found in the data, they were grouped together to become a concept. In contrast to looking for similar events and ideas through constant comparison, negative case analysis involves looking for contradictions or exceptions in the data. Negative cases often emerge naturally during constant comparison, but deserve special attention because of the importance of exceptions to the typical pattern in fully explaining a concept.
Corbin and Strauss (2008) also recommend the use of questioning during analysis. When coding data, the researcher maintained a constant thought process to question who, what, when, where, how, and with what consequences. Additionally, analysis was enhanced by considering issues of frequency, duration, and timing. For example, if an interview participant said, "Tutoring really helped me learn how to explain simplifying a rational expression," the researcher might ask herself the following questions about this short excerpt: "Who was this respondent tutoring?," "What topic was he or she tutoring?," "When did this tutoring take place? Was it recently?," "Where did this respondent tutor? Was it in a formal tutoring setting?," "How did the respondent start tutoring?," "Did the respondent have to give up other experiences that might help develop SCK in order to be able to tutor (e.g. taking less college courses so he or she has time to tutor),," and "How often does this respondent tutor? And for how many hours?" This strategy of thinking about the data helped analysis move past the words on the transcript to more meaningful findings.

Another strategy that Corbin and Strauss (2008) recommend is considering various meanings of a word. For example, "nice," "challenge," "easy," "difficult," and "a long time ago" could mean very different things across participants or between the participant and the researcher. When words like this are found in a transcribed interview, there is the "problem of accepting one's own interpretation of what is being said; that is, assigning meaning without careful exploration of all possible meanings" (Corbin & Strauss, 2008, p. 78). During analysis, close attention was paid to words from an interview that could have multiple meanings.
Researcher memos were the product of the coding process described above. Memos helped keep track of codes, themes, and categories as data collection and analysis continued. Memos allowed, and actually enforced, the organization of concepts, as opposed to working with raw data (Corbin & Strauss, 2008). Ongoing memos were kept in folders with titles that represented their content (see Appendix C for a list of the folder titles and memo titles within each folder). This way, similar codes could be recognized and grouped into one memo. The memos also included excerpts of relevant raw data to help recall details of the interviews, and could conveniently be used for quotes in the final write up (Corbin & Strauss, 2008). Early memos were like the margin notes others sometimes use when coding data, except they were kept separately from transcripts for organizational purposes.

A thorough analysis was completed on the initial six interviews before data collection continued. The codes and themes that emerged from these first six interviews were the basis for the earliest memos. When possible, each interview was coded and memos were written before beginning the next interview, although multiple interviews were sometimes conducted without intervening analysis when traveling long distances to conduct the interviews. As analysis continued, many concepts aligned with previously coded data, so earlier memos were updated and expanded to reflect new findings (Corbin & Strauss, 2008).

Data captured in memos were used to modify the developing theory as interviews continued, and interviews continued until saturation was reached for the developing theory. Saturation occurs when all themes, categories, and concepts have been fleshed
out, and new themes are no longer emerging from the interviews (Corbin & Strauss, 2008).

**Issues of Validity and Reliability**

**Prospective Teacher Survey**

The Prospective Teacher Survey used in this study collected factual demographic information and asked for open-ended responses to questions about individuals' experiences related to mathematics teaching. This is qualitative and individualized information that does not raise concerns about validity. The online version of the survey used identical language and resembled the paper version as closely as possible.

**SCK Assessment**

Phelps and Gitomer (2012) were able to achieve a scale reliability score of 0.77 on the *Content Knowledge for Teaching: Algebra 1 Assessment*. They explain the process of validating the instrument as follows:

The assessment was first piloted with approximately 300 teachers to assess basic question-level measurement characteristics. A small number of cognitive interviews were conducted and questions were subsequently revised. A final assessment was administered to [143] ninth grade teachers of Algebra 1. (p. 5)

This resulted in a scale reliability score of 0.77.

When SCK Assessments were administered in a campus classroom, the researcher was present during the assessment to explain its structure and purpose and to discourage cheating. Students taking the assessment online read a stern warning about the
importance of taking the test using their own abilities. All students were reminded that results would not be seen by anyone except the researcher.

The SCK Assessment was administered on paper in a classroom setting to as many participants as possible. Where a convenient classroom setting was not available, students were asked to complete the assessment in an online format. There are two concerns regarding the difference in results between the two formats. First, pre-service teachers who completed the assessment in a campus classroom may have felt limited regarding time to complete it. Participants were allotted time after class to finish the survey and assessment, but in one case as indicated during an interview, that a pre-service teacher was unaware that she could have had extra time to complete the assessment and survey. Second, students taking the assessment in person were encouraged to use scratch paper while completing the assessment. Students completing the online assessment might be less likely to use paper to calculate, compare, and reason through possible answers. The difficulties presented by each assessment format were unavoidable, but caution was taken to limit the effects as much as possible. For example, students who were unable to finish the assessment during a class setting had the opportunity to meet the researcher later in the day to complete the remainder of the assessment on paper.

Interviews

Qualitative analysis and findings based on interview data are judged on the notion of trustworthiness. Criteria supporting trustworthiness include credibility, confirmability, transferability, and dependability (Trochim & Donnelly, 2006).
Credibility can be defined as "establishing that the results of qualitative research are credible or believable from the perspective of the participant in the research" (Trochim & Donnelly, 2006). Member-checking helped with the credibility of interview findings. Four interview participants were contacted via e-mail to review a summary of the researcher's analysis of their interviews. They had the opportunity to correct errors, approve quotes, confirm that they agreed with the researcher's interpretations, and to clarify any obscurities in the interview.

Confirmability can be defined as "the degree to which the results could be confirmed or corroborated by others" (Trochim & Donnelly, 2006). Interview transcripts and corresponding memos were reviewed by the researcher's willing committee members to verify that they were aligned. Direct quotes from the interviews were included in the findings to help with confirmability of the findings. An audit trail consisting of all written memos, interview transcripts, notes on how the interview guide changed over time, and the selection process for interview participants further advanced the confirmability of the study.

Transferability can be defined as "the degree to which the results of qualitative research can be generalized or transferred to other contexts or settings" (Trochim & Donnelly, 2006). Some degree of transferability from this study to other settings is assured, since the variety of teacher education programs included in the study are representative of many teacher preparation programs across the United States. Rich descriptions of the programs and participants in this study, the process of data collection,
and the results of analysis enable readers to decide whether the findings apply to specific teacher preparation programs or those with different characteristics.

Dependability can be defined as "the need for the researcher to account for the ever-changing context within which research occurs" (Trochim & Donnelly, 2006). One can think of dependability in qualitative research as being analogous to reliability in quantitative research. Reliability ensures that if an experiment were repeated, the second researcher would find the same results. In qualitative research, it is impossible to ensure that a second researcher would reach the same conclusions. However, continuing the research until saturation has been reached is one way to ensure dependability. In this study, if only one interview participant reported a unique experience, it was important to follow through on that experience to learn more about it, and to find out if other participants had that experience. In-depth explanations detailing how saturation was reached are included in the final write up. Additionally, descriptions of procedures that would allow another researcher to repeat a similar study are included as a way of supporting dependability.

Limitations and Delimitations

Since most data were collected via interviews, claims about gaining SCK were self-reported by the participants and not directly observable. Furthermore, the teacher preparation experiences were not randomly assigned to participants, so any quantitative findings do not indicate a causal inference about how SCK is developed.
The researcher chose to limit theoretical sampling and the interview cycles to several months. As time went on participants would have new experiences not captured in the Prospective Teacher Survey. Some would even complete teacher preparation programs and begin working in fulltime classrooms. Furthermore, after the academic year ends, participants are difficult, if not impossible, to contact and interview. In addition to time constraints, the number of participants willing to be interviewed were limited. Despite the fact that by the end of the study, every willing participant had been contacted for an interview, the number of participants with particular experiences (e.g., advanced content coursework) were still limited. Therefore, saturation was not fully reached in all conceptual themes. However, efforts were made to reach saturation in the conceptual themes most strongly related to the research questions.

Fewer participants in the interview pool were in the earliest stages of teacher preparation (e.g., entering freshman who immediately enroll in a secondary mathematics program.) It is common for students to change from another major to focus on education in their first or second year of college, often after completing several mathematics courses. It is impossible to know who these students are when they are just beginning the program.

No programs outside of one Rocky Mountain state were studied. This decision was made in part due to travel costs and accessibility issues. In order to access participants, it was important to have a familiar faculty contact in each participating program. The faculty contacts helped in successfully accessing pre-service teachers to administer the survey and assessment. Faculty contacts were only available in one Rocky
Mountain state. Staying within the state was justifiable because the available programs to be studied include such a wide variety of teacher preparation programs.

Finally, the design of this study may be oversimplifying SCK as a well-defined, distinct category of knowledge. Some researchers suggest that SCK has four different components that can each be measured at five different levels (Bair & Rich, 2011). This study did not compartmentalize SCK in this way. Furthermore, the Content Knowledge for Teaching: Algebra 1 Assessment (Phelps & Gitomer, 2012) used in this study does not claim to measure SCK. However, the researcher has attempted to carefully define the concept of SCK and explain the parameters and limitations of the instrument. The results of this study help to further clarify and differentiate the nature of SCK, how it is acquired, and how it might be assessed.

**Conclusion**

Data collection for this study consisted of an SCK Assessment and a Prospective Teacher Survey, which were administered to all willing secondary mathematics pre-service teachers in the state, and interviews with select pre-service teachers. The interview participants were selected based on convenience sampling, maximum variation sampling, and theoretical sampling. Recall that theoretical sampling is a way to select interview participants who have the most potential to reveal data that fills gaps in current findings. Qualitative analysis primarily consisted of transcribing and coding interview data, organizing the coded concepts into memos and developed them into theoretical findings. Quantitative analysis examined SCK Assessment scores overall and across
different groups, based on likely predictors of SCK development and interview findings. The results of these analyses are discussed in Chapter 4.
4. ANALYSIS AND FINDINGS

The goal of this study is to identify and describe experiences of secondary mathematics pre-service teachers that aid in the development of Specialized Content Knowledge (SCK). Data collected from 47 participants using methods described in Chapter 3 brought to light a variety of experiences that may contribute to SCK among secondary mathematics pre-service teachers. In this chapter, findings will be presented based on analysis of data collected from the Prospective Teacher Survey, the SCK Assessment, and individual interviews. The discussion will focus on answering the research questions restated below:

What kinds of experiences support the development of SCK for beginning algebra among pre-service secondary mathematics teachers?

   a. In what ways do secondary mathematics pre-service teachers exhibit specialized content knowledge on an assessment measuring SCK in beginning algebra?

   b. What teacher preparation experiences do secondary mathematics pre-service teachers identify as aiding in the development of specialized content knowledge as indicated on the SCK Assessment?

   c. What experiences outside of teacher preparation do secondary mathematics pre-service teachers identify as aiding in the development of SCK?
The SCK Assessment

Depending on the situation, either four or five SCK Assessment items were referenced in the interview protocol. These provided examples of specialized content knowledge for the participants, and motivated them to reflect on experiences that supported their development of SCK. The participants’ performance on those items was not the focus of interviews. Instead, data were collected on the reasoning and prior experiences that supported pre-service teachers’ responses to the assessment items. This section offers an in-depth analysis of specific items and participant responses, and concludes with a quantitative analysis of the SCK Assessment data collected during this study.

Perceived Challenges in the SCK Assessment

The Prospective Teacher Survey included a question asking how confident participants were about their answers on the SCK Assessment. The responses indicate that very few of the 47 who completed the survey and assessment felt confident about their performance on the assessment. The findings revealed during interviews are closely aligned with the beliefs expressed on the survey. Eleven participants referenced the difficulty of the SCK Assessment during the interview. Ten found the assessment difficult, while one was confident in her answers.

Four interviewees found the questions challenging because the “students” referenced in the items were not present. One explained, "It was hard because it wasn't like I had this person here [to ask], 'And what do you mean by this?' Because I might not
understand it right off the piece of paper." Another participant said, "This is kind of difficult never having dealt with another student's work before."

One pre-service teacher believed that reviewing the basic content would help her perform better on the assessment: “It's not like these are hard questions content wise, but...you don't know what to analyze or where to start or what your rules are. So maybe if you were prepping for that kind of thing, I would definitely...basic knowledge, kind of refresh stuff.” A respondent on the Prospective Teacher Survey also mentioned needing to "brush up on some algebra fundamentals."

One pre-service teacher felt the SCK Assessment items "can be tricky if you're not paying attention to what they're asking." This was also reflected in the responses of eight participants on the Prospective Teacher Survey, who reported having difficulty interpreting the assessment items. Others mentioned that some of the problems seemed to have more than one correct answer. Two participants found the wording of the items confusing, while another simply said the items were confusing. Similarly, two participants had trouble following the student work in the items, while another said it was difficult to understand some of the student responses. Another participant got "lost in longer question setups."

Two participants expressed interest in studying questions like the SCK Assessment items in their programs. One pre-service teacher who was in her final semester of coursework said, "I would like more assessment items like this...maybe the methods class addressed things like this more...I don't feel like I was very prepared. Everything was kind of a guess. But I liked answering the questions, so I don't know."
The most frequently identified misconceptions and misunderstandings occurred with Item 9, Item 12, and Item 22, and are discussed below.

**SCK Assessment Misconceptions**

As participants worked through assessment items during twenty-three interviews, some misconceptions were apparent. Some misconceptions were purely of a mathematical nature. Other participants reflected an understanding of the knowledge needed to correctly answer an item, but were not able to interpret what the item was asking, so they selected the incorrect answer choice. There were also cases where misunderstanding the mathematics and the item were intertwined. Each of the misconceptions are described below.

**Item 9.** Item 9 is shown in Figure 6. One participant thought that Letitia, the fictional student in Item 9, was the teacher. Others were unsure what Letitia meant when she described a “line at 45 degrees.” Three interviewed participants stated that graphing a parabola and a line to find their intersection is not a valid method of solving the equation, and one participant explained why she thought so: “The standard [way] to graph a line would [be] \( y = mx + b \); there’s no \( y \) here.” These participants mostly demonstrated mathematical misconceptions as the reason for misunderstanding the item, because they did not understand that the intersection would provide a solution to the equation.
Figure 6. SCK Assessment Item 9

9. Having taught her students to factor quadratics with integer coefficients, integer roots, and a leading coefficient of 1, Ms. Quezada explained that she was going to give them a harder problem. She then asked them to solve the following.

\[3x^2 - 3x - 6 = 0\]

After a few minutes of work, the class discussed their solutions. Letitia said that \(x\) was \(-1\) or \(2\) and explained, “I added 3x to both sides and divided by 3.”

\[3x^2 - 6 = 3x\]
\[x^2 - 2 = x\]

She then continued, “The parabola’s just down a little and the line’s at 45 degrees, so it’s just below zero and about 2 to the right. \(x\) can be \(-1\) and \(2\), and those are the only possible ones.”

Of the following, which best characterizes Letitia’s approach to this problem?

A) Letitia’s method is wrong because she should have first divided by 3 and then factored the left side of the equation.

B) Letitia’s method is wrong because this is a parabola and you could graph it, but you would have to graph the original equation and look for the roots.

C) Letitia’s reasoning is correct, but her method often leads to points of intersection that might be hard to determine visually.

D) Letitia’s reasoning is correct, but her method requires knowledge of calculus.

**Item 12.** Item 12 is shown in Figure 7. In Item 12, one participant had correctly selected answer choice “B” during the assessment, but changed her answer to “D” during the interview. When asked why, she explained, “‘B’ is saying the graph changes all the time but cannot have sudden changes at some of the points. I was thinking of other graphs…like sometimes it’s a piecewise function…But I guess if they’re talking about just this graph, [this answer] makes sense for this function.” Another participant also eliminated answer choice “B” because it only applies to “this specific graph.” One participant thought answer choice “A” was not accurate, explaining, “It’s not changing at
a constant rate, but it’s changing all the time, depending on $x$. However, she still selected “A” as her answer, even after considering the other answer choices. These participants appear to have selected the incorrect answers because of a misunderstanding.
of the problem. They are seeing the answer choices as general statements, as opposed to seeing them in the context of the problem. Another participant confused “continuous” with “differentiable.” He said, “When you have corners, you’re not going to have continuity.”

**Item 22.** Regarding Item 22 (shown in Figure 8), five participants explained that they did not choose “C,” the correct answer choice, because the statement taken out of context of the answer choice is too universal. One participant explained “You can’t make a general rule” that assumes $x$ is positive. Another participant, while exploring other answer choices, substituted values for $x$ and realized the inequality only held true for positive numbers. However, after rereading the question and even emphasizing “identify what is problematic,” she still chose answer “A” because she disagreed with the wording in answer choice “C.” Similar to the participants described above who were confused about Item 12, these participants were analyzing answer choice “C” outside the context of the problem. One of the five participants was able to correctly interpret what the question was asking, and changed his answer to “C.” Another participant thought $x$ must represent a positive number: “Normally they would put a negative sign if $x$ was less than zero.” Another participant interpreted the condition $x > 0$ to mean $x \neq 0$. Others did not understand how to interpret the conclusion $0 < 2$. One said, “With the way that she solved it, I don’t know anything about the variable; it just disappeared…When you eliminate the variable you’re looking for…you’re not solving the problem. You’re just stating a fact that zero is less than two.”
Some participants also struggled with the idea of eliminating all variables in the inequality. One who chose answer “A” explained, “Because when you eliminate the variable you’re looking for…you’re not solving the problem. You’re just stating a fact that zero is less than two.” Eighteen of the forty-seven participants chose answer “A,” while only seven participants chose answer “C,” the correct answer. The participants who incorrectly answered this item may have done so for two different reasons: misunderstanding the item or mathematical misconceptions.

Figure 8. Assessment Item 22

22. Mr. Baas’ students were solving inequalities. Cheryl wrote the following solution on the board.

\[ \frac{x - 2}{x} < 1 \]
\[ x - 2 < x \]
\[ 0 < 2 \]

She concluded that because this is always true, every \( x \) would work. After some discussion, students decided that Cheryl’s solution was not correct, but they were unsure why. Of the following explanations, which best identifies what is problematic about Cheryl’s work on the problem?

A) It’s true that 2 is always greater than 0, but because you have eliminated all \( x \), we cannot say what \( x \) is.

B) We can see that the numerator always is two less than the denominator, so the fraction will always be less than 1 for all \( x \). However, we have to require the denominator \( x \neq 0 \).

C) You don’t know what \( x \) is, so when you multiply with \( x \) like this, you must assume \( x > 0 \).

D) She should have simplified the left-hand side of the inequality to \( 1 - \frac{2}{x} \) and then subtracted 1 from both sides and added \( \frac{2}{x} \) to both sides. This would yield \( 0 < \frac{2}{x} \), which is true for \( x > 0 \).

Ways of Approaching SCK Assessment Items

As participants revisited SCK Assessment items during the interviews, they took different approaches to selecting an answer. Seven participants examined the items
through the lens of a teacher, considering what grade level the fictional students might be in or what methods students might find easier for attempting the problem. One said, “I could sit down and read these and feel like a teacher. Like, ‘Okay, I see what you did…That’s right, but there’s a better way to do this.’” She added, “It’s really important to, even if you…know the right way to do something, you can’t just disregard other people’s thought process. That’s not fair. That’s not [going to] help them learn.”

Fourteen interviewees worked through the mathematics in the items first, and then compared the students’ work to their own, to decide if the student was correct. While discussing Item 9, one participant explained, “I solved it out myself first, just to see what I would get solving it in the method that I knew how.” The participant was unable to understand the student’s work, but still relied on the final answer from her own work to conclude, “Initially I wanted to put that her method was wrong, but…her method’s not necessarily wrong, like the answers are right.”

Eight participants considered student thinking when answering the items. They either put themselves in the “mindset of the students” in the problems, or tried to figure out what the students might be thinking as they examined the student work. One said,

You try to get in the mindset that these kids might be in…I think you need to also think about the process that the students are going through when they answer these problems…When you find something that has an incorrect…solution, being able to diagnose why they may have thought of that problem in that regard is pretty important.

Several participants, when asked how they answered the SCK Assessment items, described general test taking strategies that they use for any multiple choice assessment. They referenced strategies such as reading the answer choices first, reading the problem...
multiple times, eliminating answer choices that did not make sense, and using guess and check.

Early in the study, interviewees indicated that they approached the SCK Assessment items in a way that aligned with how they approach proofs. This was a common theme that in some ways contradicted statements made by other participants who did not see the value in studying proofs. This approach to answering the SCK Assessment items became a focus of many interviews and is discussed more fully below.

**Approaching SCK Assessment Items like Proofs.** Some participants indicated that they used the same process to answer assessment items that they used to read, understand, and write proofs. When asked to explain, one participant said, “First, [you] have to decide what’s true and what’s not true…Then you have to go through and [decide why].” She compared this to her experience of writing proofs in *Introduction to Abstract Mathematics*: “First, I have to convince myself that it’s true or not true, and then, if it’s not true, I need a counter example.” If interviewees who had taken a proofs course did not mention proofs on their own, they were asked whether reasoning through the items was similar to reasoning through proofs. Each agreed that being able to write proofs was helpful in answering assessment items. Some explained that being able to read, write, and understand proofs helped them understand the mathematics underlying the assessment items. One participant elaborated, “One thing that proofs does for me is make me try to understand why something is, instead of just taking it at face value.” Another said that learning to write proofs in *Introduction to Abstract Mathematics* “gives a deeper
knowledge of the mechanics behind a lot of the stuff [that we] just take for granted and do.”

Exploratory Quantitative Analysis

The developers of the SCK Assessment used in this study administered it to 143 ninth grade teachers and found the assessment to have a scale reliability score of 0.77 (Phelps & Gitomer, 2012). While Phelps and Gitomer (2012) caution that “no score on the assessment…indicates a particular proficiency level” (p. v), the quantitative results of this study are still worth examining. Although the data produced from the SCK Assessment must be viewed as incomplete and for the most part inconclusive, it is informative to examine some comparisons between certain groups of participants. Several comparative analyses are discussed in the following sections.

SCK by Program. One might be interested in comparing the scores of participants enrolled in different programs in the study. This is best done by examining the scores of seniors (student teaching during the 2015-2016 academic year) at each of the six programs in the study, since they are most representative of their respective programs. However, there were only six seniors in the study, with no more than two seniors enrolled in any program. To compare programs, we are left to examine the scores of all participants enrolled in each program; however, creating subsets of participants from the same program and class reveals that the number of pre-service teachers in each class is not proportional across programs. For example, both Bailey and Gaines had five participants who were juniors, while there were twelve sophomores at Bailey and only
seven sophomores at Gaines. Under the assumption that pre-service teachers develop SCK as they move through teacher preparation, Gaines would be more likely to have a higher overall score, based on this unfair proportion of juniors to sophomores. The number of participants at each program, and the number in each class (defined by when the participants planned to student teach) are shown in Table 6. A plot of the SCK Assessment scores for each program is shown in Figure 9 with medians for each group indicated by a black dash. The reader is cautioned not to draw unsupported conclusions.

Table 6. Number of Participants in each Program, by Class

<table>
<thead>
<tr>
<th>Class</th>
<th>Anderson</th>
<th>Bailey</th>
<th>Connor</th>
<th>Davis</th>
<th>Gaines</th>
<th>Hamilton</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshmen</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Sophomores</td>
<td>1</td>
<td>12</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>Juniors</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>Seniors</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>20</td>
<td>4</td>
<td>2</td>
<td>13</td>
<td>6</td>
<td>47</td>
</tr>
</tbody>
</table>
SCK by Participant Class Standing. It is also of interest to compare the scores of participants organized by class (again defined by when the participant planned to student teach), but some considerations need to be made. Course requirements are not strictly aligned with pre-service teachers’ class status in a given program; under the assumption that coursework aids in the development of SCK, caution is needed when grouping participants by class. For example, qualitative analysis revealed that methods coursework aids in the development of SCK. The majority of pre-service teachers complete methods coursework during their junior year, but one participant at Anderson completed a methods course during her sophomore year. The same is true of student
interactions, with different programs assigning field experience at different levels of the program. Even so, observations gleaned from a graphical analysis of these data (see Figure 10 for a plot of SCK Assessment scores by class with medians indicated by a black dash) does lead to some findings. While there are only five freshmen in the study, their scores range from 11 points (one point above the lowest scoring participant) to thirty points (the highest score of any participant). This is an indication that pre-service teachers may enter teacher preparation with different levels of SCK. Sophomores, juniors, and seniors have a higher median score on the SCK Assessment than freshman by at least five points.

Figure 10. SCK Assessment Scores by Class
SCK by Coursework. Qualitative analysis of interview data revealed that some coursework may aid in the development of SCK. These qualitative analyses and findings are described in the sections that follow. It is worth examining the SCK Assessment scores of pre-service teachers who have completed such courses. For comparison, recall that the mean score on the SCK Assessment (out of 35) for the forty-seven participants who completed it was 21.7 points, with a median of 22 points and a standard deviation of 4.67 points. Of the twelve survey participants who had taken the course Higher Mathematics for Secondary Teachers, the mean score on the SCK Assessment was 24.25, with a median of 25 and standard deviation of 4.15. Twenty-seven survey participants had completed a proof writing course, or were currently enrolled in one. Their mean score on the SCK Assessment was 23.48, with a median of 23 and a standard deviation of 3.57. Of the three survey participants who had taken a Number Theory course, their mean score on the SCK Assessment was 23.33 with a median of 24 and standard deviation of 1.70. Finally, of the seven survey participants who had completed Abstract Algebra or were enrolled in the course, their mean score on the SCK Assessment was 21.28, with a median of 23 and standard deviation of 3.57. These data are summarized in Table 7.

Table 7. SCK Assessment Scores by Coursework

<table>
<thead>
<tr>
<th>Course</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher Mathematics for Secondary Teachers</td>
<td>24.25</td>
<td>25</td>
<td>4.15</td>
<td>12</td>
</tr>
<tr>
<td>Proof Writing</td>
<td>23.48</td>
<td>23</td>
<td>3.57</td>
<td>27</td>
</tr>
<tr>
<td>Number Theory</td>
<td>23.33</td>
<td>24</td>
<td>1.70</td>
<td>3</td>
</tr>
<tr>
<td>Abstract Algebra</td>
<td>21.28</td>
<td>23</td>
<td>3.57</td>
<td>7</td>
</tr>
<tr>
<td><strong>Overall</strong></td>
<td><strong>21.7</strong></td>
<td><strong>22</strong></td>
<td><strong>4.67</strong></td>
<td><strong>47</strong></td>
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</tbody>
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Coursework Reported as Aiding in the Development of SCK

The six teacher preparation programs in the study have different course requirements, but many mathematics content courses are required at all programs. This section describes the courses, and experiences in those courses, that pre-service teachers reported as aiding in the development of SCK. On the Prospective Teacher Survey, participants were asked what mathematics content and methods coursework they had completed, but there were no questions about the nature or content of those courses. All data reported in this section was collected from the twenty-three interviewees, called "participants."

Mathematics Content Courses Designed for Teachers

At some colleges and universities, secondary mathematics teacher preparation programs design and offer mathematics courses that examine high school level mathematics in the context of teaching, and sometimes connect that content to the upper level mathematics that is common in typical college level mathematics courses. The content of such courses, as well as the teaching techniques used in those courses, differ between schools, and even sometimes between courses within the same program, so each of the courses discussed in this study include richer descriptions later in this chapter. The mathematics courses designed for teachers that are discussed in this study were taught by someone who considers himself or herself a mathematics teacher educator. Bailey and Gaines each offer at least one mathematics content course designed for teachers, and the other programs in the study have at least one mathematics methods course that is
designed for mathematics education majors. These are discussed in the sections that follow.

Higher Mathematics for Secondary Teachers. The instructor of Higher Mathematics for Secondary Teachers explained that most of the course is taught with the text, Higher Mathematics for Secondary Teachers: An Advanced Perspective (Usiskin, 2003). They focus on number theory and abstract algebra, “but not number theory and abstract algebra as you would study them if you were preparing to go to grad school in math. Instead, it’s more looking for ways that problems that arise in high school algebra have…either their foundations [in] or lead to ideas in…higher level math courses.” She explains that one of her favorite topics in the course is the rules of equality, where pre-service teachers learn about the assumptions that underlie applying functions or operations to both sides of an equation. They build on this to explore when it is necessary to check for extraneous solutions in high school algebra. She says that her “biggest instructional challenge at the beginning of the course is convincing [the pre-service teachers] that you can dig deeper than [high school level mathematics]” when looking at a problem you would talk about in high school. One goal of the course is to connect ideas in high school mathematics to ideas throughout mathematics that typical mathematics majors would learn about.

Of the eight participants at Bailey who were interviewed, five had taken Higher Mathematics for Secondary Teachers. They recognized the relevancy of this course and its unique approach to mathematics, teaching them “how to think behind the problems.” One pre-service teacher said of the course, "We would...often have to...solve the
problems algebraically, then look at it graphically and think of it visually." Another said, "Higher Mathematics for Secondary Teachers is going really well. [The instructor] does a really good job with making sure we think of more of the reasoning behind how we can solve those problems instead of… ‘Just go solve this problem.' Which is, I agree, really important for us as teachers."

Another pre-service teacher explained that he would include a course like Higher Mathematics for Secondary Teachers in a high school teacher preparation program, saying, "That was [a] super helpful class. Honestly that one helped me more on [the SCK Assessment] probably than anything else." The five Bailey interviewees who had taken this course only had positive comments about the course.

Algebraic Thinking and Number Sense in the Middle Grades. Bailey also offers a course titled Algebraic Thinking and Number Sense in the Middle Grades, taken by three of the interviewees. All three participants mentioned that the course helped them view problems in different ways, but did not provide details about the course itself. One pre-service teacher who completed the course explained that it took an in-depth look at “why things work, and how we know that it works that way." This course was also commended more indirectly as participants listed courses with positive attributes. For example, one pre-service teacher said that Higher Mathematics for Secondary Teachers and Algebraic Thinking and Number Sense in the Middle Grades "kind of opened me up to a lot of different solution methods… I feel like knowing how to solve problems in as many ways as possible is very, very important."
Mathematics Technology for Teachers. The Gaines program features a course for prospective secondary mathematics teachers titled *Mathematics Technology for Teachers*. The instructor said the course focuses on one investigation per week on high school topics across statistics, probability, geometry, algebra, and functions, with attention paid to modeling throughout each of the investigations. In addition to examining a variety of content, the pre-service teachers also learn how to use technology to explore the topics, with a new technology, or a new use of previously used technology, introduced each week. The instructor said they do not work mathematics problems in the context of teaching, but the majority of the coursework is in an open-ended problem solving setting, and the pre-service teachers explore mathematical ideas that have the potential to develop SCK.

All six participants interviewed at Gaines had taken the course. One participant was enrolled in *Mathematics Technology for Teachers* at the time of the interview. She explained that the course allowed her to look back at content she had not seen since taking geometry in high school, and that it was also a good review of algebra. The course helped her with "seeing connections between algebra and geometry" and "with connections and reasoning through things...Making me think about proofs; that type of reasoning." She confirmed that this helped her answer items on the SCK Assessment.

Mathematics Methods Coursework. Every teacher preparation program in the study offers and requires at least one mathematics methods course. These courses are designed for future secondary teachers, but may target different grade levels within the secondary range. Gaines and Hamilton offer one all-inclusive methods course that spans
grades five through twelve. Since pre-service teachers at Davis attend Hamilton for their methods coursework, they also complete a single methods course for grades five through twelve. Bailey requires two methods courses, one focused on grades five through eight, and the other on grades nine through twelve. Anderson offers one methods course titled *Teaching Mathematics in the Secondary School*, which typically addresses grades nine through twelve, and is modified for pre-service teachers who intend to teach middle school. Connor requires one methods course that focuses on grades five through eight, but the two participants at Connor had not yet taken the course. Findings about pre-service teachers’ perceptions and experiences related to methods coursework in the other programs are reported below.

At Anderson, both participants completed the methods course the semester prior to their interview. They were the only students in the course; not a surprise, since they were the only two mathematics education majors in the program. These pre-service teachers had a unique experience in their methods course. The instructor was simultaneously teaching *Calculus I*, and the two methods students helped teach the *Calculus I* course. The pre-service teachers and the instructor alternated teaching *Calculus I* for one week at a time throughout the semester. The three also met weekly in the methods course, where they would work together on lesson plans for *Calculus I*, discuss issues such as time management in the course, and read articles about mathematics education. The pre-service teachers also informally met daily, usually without their methods instructor, to discuss teaching *Calculus I*.
The Bailey participants referenced the middle school methods course, formally titled *Methods: 5-8 Mathematics*, more than the high school methods course. Three pre-service teachers at Bailey said that the middle school methods course helped them develop specialized content knowledge. They emphasized the value of number talks that take place in that course daily. The instructor of the course described a number talk as a five to fifteen minute classroom activity usually based around a computational problem or a string of problems that are carefully crafted to elicit students’ mental strategies for finding a sum or a product, or counting the number of dots in a particular pattern...So by engaging in a number talk...children are practicing communicating their thinking, listening to each other, justifying their thinking, and...working on mental strategies for...computations.

The structure of the number talks remained the same in the middle school method course, but pre-service teachers were assigned to take turns leading the number talks, while the rest of the class participated as though they were middle school students. The pre-service teacher leading the number talk also completed a “planning document” before the number talk to “anticipate multiple solutions and anticipate ways they could connect potential solutions.” The pre-service teacher also recorded the number talk that he or she led, transcribed that recording, and wrote a reflection on the number talk.

One pre-service teacher noted,

> Probably the most [SCK] that I've developed is in this semester in my [middle school] methods class...She has us do these little number talks and they're fantastic for getting me to think...We were...trained to just go through the motions...There's not a lot of logic there, and not a lot of real world connection...She has taught us how to think in different ways to get solutions to different problems, and it's been very useful.

The high school methods course at Bailey helped another pre-service teacher develop SCK when they looked at actual student work and how students took different
approaches to solving a problem. This opportunity allowed them to analyze student solutions and study multiple approaches, both of which are ways that SCK is applied in the practice of teaching.

In the methods course at Gaines, mathematics education majors also explored different ways to solve problems. The one interviewee who had taken the methods course at Gaines said that the instructor would present a problem, and ask pre-service teachers to come up with their own way to solve it. After all of the different solution methods were presented, the pre-service teachers would attempt to come up with additional ways to solve the problem. The problems covered topics including infinite series, geometric representations, recognizing patterns, and various word problems.

One participant completed the methods course at Hamilton. When asked if she had ever looked at different approaches to problems, she explained that in her methods course, they examined problems through two lenses: as concrete sequential learning and as abstract random learning. Later in the semester, methods students taught two mathematics lessons on a topic they had chosen: one lesson as if it were being taught to concrete sequential learners and the other lesson to abstract random learners. She said the methods course "was really helpful; that's probably one of my most beneficial classes."

Mathematics Courses Beyond Calculus

After completing two to four semesters of calculus, mathematics education majors must meet different course requirements depending on which teacher preparation program they attend. Courses beyond calculus require a shift in mathematical thinking
from procedural to proof-based reasoning. During the calculus sequence through *Differential Equations*, students are mostly asked to master the concepts and procedures their instructors demonstrate in class. However, after calculus, students are expected to use formal reasoning and abstraction, distinguishing these courses from the ones that come before them.

**Proofs Courses.** All programs in the study require pre-service teachers to complete a course designed to introduce methods of proof and proof writing. In most cases, like at Bailey, the course is titled *Methods of Proofs*. Gaines lists the course as *Introduction to Abstract Mathematics*, but the main goal is still for students to be able to write proofs. Mathematics education majors have typically seen proofs in other coursework before taking this course. They will be expected to write and analyze proofs more formally in courses that follow this introductory course. Proofs courses at each campus are also required of non-teaching mathematics majors, so it is common to have both mathematics education majors and mathematics majors in the same course. Proofs courses are typically taught purely from a mathematical perspective with no focus on mathematics education.

In one case, during the period of data collection at Bailey, the *Methods of Proofs* course was taught by a mathematics educator who tailored the course towards an educator audience. One participant described this course as a "discussion community." One participant described exam problems asking them to find the error in a proof that was already written. She concluded, "And that made us...not only know how to be able
to write the proof correctly, but be able to analyze somebody who didn't and fix it for
them. So that was cool."

**Number Theory.** Another course required of most secondary mathematics
education majors, *Number Theory*, helped two pre-service teachers at Gaines answer Item
10 on the SCK Assessment. This item suggests an infinitely extended geoboard, with
classroom students wondering if they would be able to make an infinite amount of slopes
using the pegs on the geoboard and discussing how "spread out" the points on the
geoboard are. The item asks participants which answer choice is most closely related to
the underlying mathematics behind the students' discussion. The correct answer is
"Density of numbers on the real line- the rational numbers are dense, but not every real
number is rational." The Gaines participants studied rational and real numbers in their
*Number Theory* class, which helped them select the correct answer. In contrast, a pre-
service teacher at Hamilton had completed *Advanced Number Theory*, but had a difficult
time answering Item 10. When asked whether she learned about classifying the real
numbers into rational numbers, irrational numbers, integers, etc. in that course, she said,
"We did a little bit of that, but…it wasn't very in depth. It was more of the theory behind
mathematics, not exactly the numbers. It was more of how some of the mathematics
equations came about…like a history of mathematics class." Despite the assumed
outcomes of an *Advanced Number Theory* course, she did not seem to have a grasp of the
real number system and its various sub-classifications.
Abstract Algebra. Many programs require secondary mathematics education majors to complete a course in *Abstract Algebra*, but participants had differing opinions about whether the course is necessary for secondary mathematics teachers. While some pre-service teachers struggled with finding the value in taking *Abstract Algebra*, others explained how it deepened their knowledge of the underlying mathematics they will be teaching. One pre-service teacher at Anderson said, "You get into classes like *Real Analysis*, or *Abstract Algebra*, where you start seeing the underlying [reasons] behind why the things that you're teaching your students works." Another participant at Gaines had conflicting feelings about the *Abstract Algebra* course. She studied integers in *Abstract Algebra*, and that experience helped her answer Item 10, but despite that acknowledgement, she later proclaimed, "I think most of us felt our time was wasted in classes like *Abstract Algebra*, which is supposed to make us well-rounded mathematicians, but not practical for what we felt we needed." She saw a loose connection between *Abstract Algebra* and the mathematics she will be teaching in high school, but continued, "It was really hard to find value in that class…Here's the thing, if *Abstract Algebra* was being taught by a math ed perspective mathematician, it would be a lot different experience, but that's not what happens here." Another participant at Gaines was able to find the value in *Abstract Algebra*: "Abstract, the number theory stuff, the proof writing classes, help you find those little holes or fill in those blanks where a student did something different that you don't recognize."

Geometry. Geometry courses are typically required of mathematics education majors, and offered as an elective course for mathematics majors. Since geometry is a
common subject in high school mathematics, it might be assumed that it would be emphasized in discussions around developing SCK. Somewhat surprisingly, geometry courses were rarely mentioned during the interviews, other than being listed among courses the interviewees were currently taking. This may be because the interviews focused on an SCK Assessment that only examines SCK in beginning algebra, and the items discussed during the interviews were not geometry based. Those who mentioned the course said it was a good review of material they had not seen in a long time.

**Contributions of Calculus Coursework**

All six teacher preparation programs require *Calculus I* and *Calculus II*. At the time of the study, only four of the 23 interviewees had not yet completed *Calculus II*, and the majority of them had completed *Calculus III*. Gaines is the only program in the study that currently does not require *Calculus III*, but it is offered as an elective, and many of the participants reported taking the course. Participants explained that calculus helped them answer items on the SCK Assessment, gain a deeper understanding of mathematics, and see different approaches to problems. These outcomes are addressed in the following sections.

**How Calculus Contributed to Answering SCK Assessment Items.** Content from their calculus courses helped participants answer Item 10, Item 12, and Item 22 on the SCK Assessment. Three participants reported that calculus helped then answer Item 10 on the SCK Assessment about geoboards and the density of the real numbers. Two said that calculus helped them learn about slopes and derivatives, which they used to answer
this item. The third explained that calculus helped her understand the density of numbers on the real line. In calculus, she was told, "Not just the integers. Not just the fractions. There's all of the numbers [in an interval]."

Six participants said that calculus helped them answer Item 12, which shows a student's rendition of the graph of \( y = \frac{1}{x} \) with line segments connecting the points, instead of a smooth curve. The answer choices display student explanations of what is incorrect with the graph, and participants are asked to choose which of the student explanations "provides the best mathematical explanation of why drawing connected line segments is inappropriate for this graph" (Phelps & Gitomer, 2012, p. 13). Two pre-service teachers found that calculus helped them learn to graph functions, in the sense that they became more familiar with commonly graphed functions. Three other pre-service teachers said that calculus gave them a better understanding of slope, which helped them choose an answer. A pre-service teacher from Bailey explained in more detail, "They introduced the derivatives to us in Calc I by having us...graph a lot of points close to, to try to get a slope, so that's kind of where I got that from." At Bailey, Calculus I students are introduced to the concept of tangent lines by graphing secant lines and recognizing that the limit of the secant lines approach the tangent line. This appeared to have helped her understand why the graph of \( y = \frac{1}{x} \) does not have any sharp corners, because she could visualize the slope of the secant lines slowly approaching the slope of the tangent line.

Two participants believed calculus helped them answer Item 22, which shows the work of a student solving the inequality: \( \frac{x-2}{x} < 1 \). The student multiplies both sides by \( x \), and gets the end result: \( 0 < 2 \), concluding that the inequality is true for all \( x \). The item
asks participants to select the answer choice that "best identifies what is problematic" with the work shown. Neither participant described a strong connection to calculus, saying only that they used inequalities in their calculus classes. When faced with answering SCK Assessment items, participants found that calculus exposed them to a new understanding of slope, density of numbers on the real line, graphing functions, and inequalities.

**Deeper Understanding of Algebra.** Six participants mentioned contributions of calculus outside of helping them answer SCK Assessment items. In general, their comments implied that calculus helped them gain a deeper understanding of mathematics. One participant conveyed that she used algebra to solve many problems in her calculus courses, helping her solidify what she knew about algebra. Another participant entered calculus with a limited view of variables as missing values that needed to be found through solving equations. After completing calculus, she realized, "x could really be anything. You don't think about that on an algebra-based idea."

**Different Approaches to Problems.** In addition to gaining an understanding of algebra, a participant explained that calculus helped her develop SCK by encouraging her to take different approaches to a problem. One participant pointed out that *Calculus II*, which emphasizes integration, was the first time a problem had required her to decide which approach to use, and which was best: "If you get stuck…'Okay, let me try this.'… Compared to Algebra or Calc 1 where it's like, ‘Oh it's this type of derivative, so you're
going to do this type of method’ or ‘It's this type of equation, then you're going to do this step.’" She said this aspect of Calculus II helped prepare her for the SCK Assessment.

**Necessity of Calculus for Secondary Mathematics Pre-service Teachers.** One pre-service teacher claimed that calculus did not help prepare him as a teacher, except "Maybe Calc I was good because I could potentially teach that in high school." Another participant who was taking Calculus III at the time of the interview expressed an internal conflict with the necessity of completing Calculus III and Differential Equations, when she would only be teaching up through Calculus I in high school. However, she recognized that taking the courses helped put her in the "students' shoes" and helped her learn to reason mathematically. She concluded, "I think if I wanted to be a good teacher, I would [take up through Differential Equations], just so I would know the overall basics of math, because…math builds…so if I know the higher end, I can help my students if they have questions with Calc I."

**Interactions with Learners**

All pre-service teachers in this study were required to complete field experiences during their programs. This meant spending time in a K-12 setting with public school students, although the nature of this experience varied greatly. In addition to formal field experiences, a few also worked with college students by tutoring or assisting in lower level college mathematics courses. These experiences helped pre-service teachers develop SCK in various ways, described in more detail in the sections that follow.
Tutoring

Because they reported tutoring experience on the Prospective Teacher Survey, 13 of the 23 interviewees were asked directly to describe how tutoring helped them develop SCK. Only one pre-service teacher who referred to tutoring said it did not help him develop SCK, although some of his later comments contradicted that statement (see below). Nine others were not asked about tutoring during the interview because they had not reported tutoring experience on the Prospective Teacher Survey.

In general, participants found tutoring to be a rewarding experience. They described their sense of enjoyment when their students experience an "a-ha" moment, when tutoring has helped the students understand something that they were struggling with. One participant said of her tutoring experience, "It clears up everything for the students. It's great. I love that feeling. Watching them just like, 'Oh, I get it!' It's the best." Participants also said that tutoring helped them learn about different approaches to problems, practice their teaching, revisit old material, and answer SCK Assessment items. They viewed tutoring as a rewarding experience that drew them to teaching. Each of these concepts are described in more detail below.

**Different Approaches to Problems.** Pre-service teachers explained that tutoring is different than working with peers in a class, because it is more difficult to explain something to someone when you understand it, and they do not. Two pre-service teachers noted that mathematics came easy to them; before tutoring, they never had to think about it on the level needed to help others understand the material. One explained, "It's tricky when it was something that came easy to you and then taking those steps back
to see how you can get someone else to grasp it." Three pre-service teachers reported that when they first started tutoring, their explanations would go “right over their [students’] heads.” Tutoring forced them to learn different ways to explain the same concept, which helped them realize that people learn differently. One interviewee benefited from the practice of having students work through a problem, and then looking over that work to find out where the student went wrong. Participants explained that during tutoring, they often had to figure out what the student was actually doing, because the student had approached the problem differently than the pre-service teacher would have himself or herself. One of those interviewees said, "If you sat down to take this [SCK] Assessment not having any student interactions...you'll see it and be like, 'Well, I didn't think of it that way, so that's wrong.' But if you have that experience, you'll be like, 'Oh, I see students thinking like this all the time. Is that necessarily wrong? Is it necessarily right? Is it necessarily a bad way to think?'" One pre-service teacher had been exposed to unfamiliar approaches, when she tutored international students who were not taught the same methods as students in the United States.

**Practice Teaching.** For some participants, tutoring was a way to gain hands on experience with teaching, as opposed to studying theories about teaching in their education coursework. When asked what he would include in a teacher preparation program to prepare pre-service teachers for an assessment like the SCK Assessment, one interviewee said, "Experience tutoring or helping in a math lab or something [where] they actually get to practice teaching. I guess we need more [time] looking from the education
perspective... You can only sit in the classroom and learn about teaching so much before you have to actually just... go do it."

Revisiting High School Material. Tutoring also motivated pre-service teachers to revisit prior knowledge. They reported that tutoring helped them review material that they had not seen in a long time, and also learn different ways to explain those concepts. They cited tutoring as helping them acquire a deeper understanding of concepts they did not fully understand when they first learned them. One pre-service teacher said of the content she reviewed through tutoring, "It probably wouldn't even stick with me as much if I weren't tutoring it."

Answering SCK Assessment Items. Pre-service teachers identified tutoring experience as helping them answer Item 9, Item 10, and Item 22 on the SCK Assessment. Three pre-service teachers said that tutoring gave them a better understanding of the algebra and knowledge of parabolas they needed to answer Item 9. In Item 22, the hypothetical student gets $0 < 2$ as the solution to an inequality. Many pre-service teachers thought that this was an invalid solution, but not because of the mistakes made earlier in the problem; they could not interpret a solution where the variable had been eliminated. One pre-service teacher cited her experience with tutoring as helping her understand this solution. She had a lot of tutoring experience, starting in high school and continuing throughout college, and claimed that tutoring prepared her for the SCK Assessment: “My tutoring job, and then just all of the experience in high school... helping classmates, and grading and stuff like that, have probably helped me out a ton.”
**Unique Tutoring Experiences.** Pre-service teachers at Anderson and Davis reported unique tutoring experiences. At Davis, the multi-subject tutoring center provides training and certification for tutors. The pre-service teacher at Davis reported that she learned about different learning and teaching styles, and ways of interacting with students that "people who aren't part of tutoring wouldn't learn." The Anderson participants worked in the mathematics tutoring lab on campus, located near the offices of their mathematics professors. This allowed them to get help from their professors if they came across a problem they were unable to solve, or if they were not able to explain to a student in a way that the student could understand. Having the opportunity to be “able to go to my professors to…get that new knowledge…to then use that and then being able to test it out…through…tutoring…It just solidified it a lot more.”

**Tutoring Versus Field Experience.** In general, pre-service teachers viewed tutoring experiences as having a different quality than field experiences for pre-service teachers. Field experiences vary based on the cooperating teacher, the type and grade level of placement, and many other factors. On the survey, one pre-service teacher at Anderson reported having extensive field experience throughout her teacher preparation program. When asked whether that experience helped her develop SCK, she responded, "Not as well as tutoring, because a lot of times, you know, you're just observing." Often, pre-service teachers find themselves observing during early field experiences, with only occasional opportunities to teach in front of the class. Tutoring, on the other hand, primarily involves working hands on with students.
An Alternative Viewpoint. One pre-service teacher who had a difficult time with the SCK Assessment reported that tutoring did not help him develop SCK. However, some of his statements contradicted that viewpoint. When asked if his experiences tutoring friends in high school and college helped him develop SCK, he replied, "Not really," but went on to say, “All I do is repeat the stuff I've learned through my mistakes, and sometimes I'll be able to catch them before they make some of the mistakes I do." He also described tutoring as helping him "understand exactly how [the students] are going to think, and... which methods are going to stick with them better.”

Student Teaching

Three of the participants were student teaching at the time of their interview. The pre-service teacher at Anderson was interviewed after his first week in a high school classroom where the students were taking final exams, and he had not done any teaching yet. The pre-service teacher at Gaines who was student teaching had been responsible for some teaching, and was preparing to take over the full course load the week following the interview. She was co-student teaching in seventh grade, which meant another pre-service teacher from Gaines was placed in the same classroom with her, under the same cooperating teacher. At Hamilton, the participant with student teaching experience had just finished her student teaching in a middle school.

Since only two participants in this study had experience student teaching, the findings about how student teaching helps develop SCK are limited. The participant at Gaines who was co-student teaching in seventh grade added that working with another pre-service teacher in the classroom, in addition to the classroom teacher, was "kind of
challenging because you've got three different teaching styles in the classroom." She had the opportunity to practice giving clear mathematical explanations while student teaching with seventh graders: "Sometime[s] the things that we think are going to be simple to teach are the most complex because trying to find a way to explain...a simple concept is kind of a challenge."

The participant who had just finished student teaching did refer to this experience when answering two of the SCK Assessment items, stating it was "kind of nice" looking back at middle school level mathematics "from a teacher standpoint instead of a student." She felt she was just expected to know middle school mathematics content in college, but did not specifically study that content during her teacher preparation. This belief is visible in her responses to Item 22, which shows the work of a student solving the inequality: $\frac{x-2}{x} < 1$. That student multiplies both sides by $x$, and gets the end result: $0 < 2$, concluding that the inequality is true for all $x$. The item asks participants to select the answer choice that "best identifies what is problematic" (Phelps & Gitomer, 2012, p. 23) with the work shown. The Hamilton participant selected the incorrect answer choice for this item when she completed the assessment the previous semester, before student teaching. However, at the time of the interview, she was able to describe that when the student multiplied both sides of the equation by $x$, she assumed that $x$ was positive, because she did not flip the inequality sign when multiplying by $x$. When asked why she changed her answer, or if she knew why she selected the other answer choice the first time, she explained that student teaching helped her "because there's so many rules that
you don't remember until you do it. And I know that a huge one is multiplying or dividing by a negative number."

Learning Assistant

Gaines sponsors a *Learning Assistants Become Teachers* (LABT) scholarship, where pre-service teachers assist in a college mathematics course for up to three years before student teaching. The Gaines participants assisted in a variety of courses, ranging from a mathematics content course for elementary teachers to *Honors Calculus II*. Each pre-service teacher in the program is supported by a mentor, the instructor of a mathematics course, who determines how active the pre-service teacher will be in that course. In general, the pre-service teachers attend classes when they can, and take on various roles inside and outside of the classroom. One pre-service teacher said he had the opportunity to "jump in" and help during class, while another held review sessions for the students outside of class. In addition to being assigned to a course, pre-service teachers in the LABT program attend weekly two hour seminars where they discuss mathematical pedagogy, about half of which is discussing common student misconceptions.

Four of the six participants interviewed at Gaines commented on their experiences as a learning assistant. One pre-service teacher reported that she liked the "practice teaching" aspect of the LABT project. Two of the four talked about how being a learning assistant exposed them to different approaches to problems. The pre-service teacher who held review sessions outside of class explained that he and the students would work through a problem on their own, at the same time, on the board and compare the work. He described, "Working through these problems with [students], we all have different
sort[s] of ways going about certain things, and I think that might be...the most relatable experience to something like this [SCK Assessment]." He added that revisiting continuous functions as a learning assistant in *Calculus I* and *Calculus II* helped him select an answer for Item 12 on the SCK Assessment. Another pre-service teacher explained that being a learning assistant helped him develop SCK because "being in a classroom and not having to pay attention to learn everything, I can see how the instructor goes about explaining when someone is correct or incorrect." When answering Item 22 on the SCK Assessment, he reported that the problem was similar to a student misconception they had discussed in a LABT seminar the week prior to the interview.

**Teaching Assistant**

Teaching assistants at Bailey have similar experiences to the learning assistants at Gaines. The teaching assistant program is a way for undergraduate students to earn credits toward their degree, as well as a small monetary reward. Most undergraduate mathematics courses at Bailey are taught by graduate students, and undergraduate teaching assistants have the opportunity to assist in *College Algebra* and *Introduction to Statistics*. Both courses are taught in an active learning environment. In *College Algebra*, the course supervisor posts videos online that students are assigned to watch before class, and class time is spent working on problems in teams with other students. In *Introduction to Statistics*, students use technology during exploratory group activities to learn the content. In both courses, short lectures (five to ten minutes) are sometimes presented to introduce material at the beginning of class, wrap up material at the end of class, or to clarify a concept when several students in the class need help with the same
topic. Graduate students typically present the short lectures, while teaching assistants assigned to these courses are responsible for helping students as they work through problems. They also assist the graduate student instructors with grading student work for the course.

Two Bailey participants had been teaching assistants, one in College Algebra and the other in Introduction to Statistics. The pre-service teacher in College Algebra said this experience helped him develop SCK and elaborated, "[It] definitely has made me remember the algebra things…That just helped me look at it from a broader perspective again, where I can actually get experience helping students think through a problem."

The pre-service teacher who assisted in an Introduction to Statistics course said that being a teaching assistant helped her explain new ideas (a component of SCK) because she realized that the teacher's explanations do not always make sense to the students, and that "you have to explain it a couple different ways and hopefully one of them click[s]."

Field Experience

Several participants described field experience prior to student teaching as helpful in preparing them to become a teacher. Two participants from Anderson said they benefited from the hands-on experience of being in a classroom. They contrasted field experiences with their education coursework, saying that being in front of students is where they learn to teach. Another participant said her field experiences also helped her make professional connections. For one pre-service teacher, field experiences were helpful because it gave him the opportunity to “see a lot of different teachers.” While observing in a middle school geometry class, one pre-service teacher realized that the
students were approaching the problem in different ways, some so different that she could not tell they were working on the same problem as the rest of the class. This was an opportunity for her to identify multiple ways of solving a problem (a component of SCK).

Two participants reported that field experiences helped them answer items on the SCK Assessment. Referring to Item 22, one said that while assigned to a middle school classroom, he learned of the difficulties that students often have with inequalities. The other participant referred to his field experiences when discussing the SCK Assessment, saying he was able to "see how it's really beneficial for [the students] to think through problems without telling them how to do it."

Not all of the pre-service teachers spoke highly of their field experiences. One participant from Anderson found the hands on experience with students to be helpful, but when asked whether her field experiences helped her develop SCK, she responded, "Not as well as tutoring, because a lot of times, you know, you're just observing." She suggested that cooperating teachers should be aware that pre-service teachers need to be interacting with the students. Another said of his field experience, "It was kind of a mess, so it was really hard to stay organized to get much out of [the experience]."

When asked what components they would include in a teacher preparation program if they were designing it, one participant from Gaines and four participants at Bailey said they desired more field experience. These mathematics education majors spent time observing a class early in their program, but their next field experience was in conjunction with the methods class, often as much as two years later. One said, “Sitting
in a classroom and talking about teaching doesn’t teach you how to teach." Another pre-service teacher said that analyzing the student thinking in the SCK Assessment “would be a thousand and one times easier if I had the experience being out with kids, that perhaps thought these ways.” Another suggested that there should be "a little bit of instruction" for pre-service teachers going into the classroom and then they should "come back and talk about the issues they saw, because a lot of teachers learn more by [talking to other teachers]." She believed that field experience is a good way to see different teaching styles and "how the styles affect different learners."

Other Student Interactions

The two pre-service teachers at Anderson were the only students in their methods course during the period of the study. Their methods instructor was also teaching Calculus I, and invited the two pre-service teachers to alternate teaching the course with him for one week at a time. One of the pre-service teachers said this experience gave him a chance to be in front of a classroom, where he could figure out his own way of doing things. He said of the opportunity, “Until you’re actually in front of the classroom and you’re actually working with the students, it is completely different.” Another pre-service teacher had been a recitation leader for an economics class. When asked if that experience helped him develop SCK, he conveyed that it helped him think about problems in multiple ways “Economics and math [are] a lot different, but there are definitely some algebra applications in economics. Yeah, I think it probably helped me to see different ways of approaching the problem, and different ways to arrive at the same answer."
Two more pre-service teachers recalled unique experiences working with other students while they were in high school. One, during her senior year of high school, gained experience teaching with an eighth grade class. She visited the class on a regular basis throughout her senior year, helping the teacher with various tasks. She recalled that sometimes the teacher would split the eighth grade class, and she would be assigned to teach half of the students. She did not have time to prepare for these mini lessons, and said that having to figure out how to teach that material on the spot "was the biggest thing" that helped her develop SCK, as she practiced explaining new ideas. Another pre-service teacher was a tutor and helped with grading in an integrated mathematics class in high school. The teacher helped her write feedback for the students on their homework, and she said that experience helped her develop SCK; in particular, she had the opportunity to practice analyzing student solutions.

Other Contributing Experiences

Some of the experiences identified by participants as contributing to SCK were not related to required coursework or interactions with students. Pre-service teachers reported that being students themselves helped them learn from their own mistakes, brought them into contact with “good” and “bad” teachers, and exposed them to different teaching styles from a student's perspective. One participant attended a mathematics education conference during her teacher preparation, where she was influenced by a session about using students' incorrect answers as teaching tools. Another pre-service teacher reported that her mother was good at mental math, and would challenge her to
work arithmetic problems from a very young age. More data were collected on how experiences with learners outside of mathematics and opportunities to take courses alongside elementary education majors aided in the development of SCK. These two topics are discussed more fully below.

Experiences with Non-Mathematics Learners

Six interviewees described experiences with non-mathematics learners, five that involved coaching sports such as volleyball, basketball, marching band, and Tae Kwon Do. Those experiences helped them explain concepts relevant to their particular sport in different ways, made them more confident as leaders, taught them how to be fair and honest with children, and improved their reasoning abilities. The participant who taught marching band found that it gave her the opportunity to examine others’ thinking: "Not everyone thinks the same, and not everyone's going to respond to one explanation, so I guess in terms of...specialized content [marching band has] helped with my ability to...be like, 'Alright well, this is how people think of it. This is how I think of it. This is what I know is correct.'" For another participant, coaching girls' volleyball and basketball helped her see how students "work through things, mentally" and how to motivate students, which she sees as important for a mathematics teacher.

The remaining participant spent several years working in management at a bookstore before returning to college to become a mathematics teacher. She explained that time spent training employees was one factor that influenced her to become a teacher. During training, she "always tried to give extra background knowledge" that would explain "how it connects into something you're not seeing here, but it connects into
another department in the store, or it flows in a holistic fashion." She tried to apply that same concept to teaching mathematics. In the context of student teaching in a seventh grade mathematics class, she explained, "I like to make connections and try to draw something that's really a holistic view of the situation when I can." This is an example of linking representations to underlying ideas (a component of SCK).

Shared Courses with Elementary Education Majors

Some participants had opportunities to take mathematics education courses alongside elementary education majors. They found some aspects of the courses (e.g., classroom management) less applicable to themselves, since these issues would be different in a secondary classroom. However, they also reported valuable learning experiences from studying mathematics alongside elementary education majors. One Bailey participant explained that when he started the teacher preparation program, he thought there were only one or two valid solutions to a mathematics problem. However, after taking Middle School Methods and Algebraic Thinking and Number Sense in the Middle Grades, both of which serve secondary mathematics majors along with elementary education majors, he said, "I was working with a lot of different people [who] were going to teach different age ranges, and they had way different solutions than I did that would yield the exact same answers." Another participant had the opportunity to observe an elementary education course and said, "The teacher really emphasizes there are a million ways to approach a problem, and let them do it how it comes naturally and build off of that."
Additional Findings

The findings reported in this section do not directly address the research questions. However, their importance cannot be overlooked. There is much to learn from participants’ beliefs about what should be taught in a teacher preparation program, how their perspectives regarding the necessity for SCK changes throughout teacher preparation, the unique viewpoints and experiences of nontraditional students, and motivating factors in pre-service teachers' decisions to become teachers.

Coursework Focusing on Mathematics in the Context of Teaching

Some courses discussed in earlier sections examine high school mathematics content in the context of teaching and are assumed to have the potential to increase SCK among pre-service teachers. Participants in programs that do not offer such courses are not aware that they exist, but they expressed desire for those types of courses. Pre-service teachers reported that it would be helpful if they had coursework that directly “taught” them SCK in beginning algebra and other subjects they will teach. Four participants said they would like to be exposed to problems like those found in the SCK Assessment. Four interviewees also wished for coursework that would further examine high school level mathematics. Three participants thought it would be helpful to look at student work, and one elaborated, “true [student] work, and maybe even from real students if they’re able to get [it].” Two participants said that examining student thinking should be part of teacher preparation: “I remember the first time I started thinking about
things further than just, ‘Okay, I’m going to get the right answer.’ Thinking about it like, ‘Okay, what is this person thinking about?’ I think that’s huge for teachers, and before taking something like [the SCK Assessment] it’s important to be exposed to thinking about other people’s thinking.” One participant suggested that working open-ended problems that allow for seeing multiple solutions to a problem would help her develop SCK. Another conveyed her frustration about not learning high school level mathematics in the context of teaching when discussing the SCK Assessment: “The higher math is great and it's good to stretch your brain, but it would be great to have a class where we did just this kind of stuff.”

Change in Perception of SCK and its Importance

Mathematics education majors have the unique opportunity to observe practicing teachers throughout grades K-12, leaving many of them with the inclination that they know exactly what it takes to be a high school mathematics teacher. However, as they move through teacher preparation, mathematics education majors come to recognize that to be a successful teacher, they will need a broader knowledge of the mathematics they will teach. Before being given a definition of SCK, interviewees were asked, “In your view, what kind of specialized mathematics knowledge does a high school mathematics teacher need?” Subsequently, they were asked, “How, if at all, has your view of the specialized mathematics knowledge needed for high school mathematics teaching changed over time?” Most participants changed their perceptions during teacher preparation, and could usually identify the semester or academic year when that shift took place, but had difficulty elaborating on what specific experience(s) changed their
viewpoints. Areas of shift included: the level of knowledge and understanding needed by high school mathematics teachers, the nature of mathematics, and whether higher mathematics courses are necessary to be a successful secondary mathematics teacher.

**Understanding High School Mathematics.** Ten participants reported that through teacher preparation, they came to realize that teachers of high school mathematics need a deeper conceptual understanding of secondary content, stronger preparation to teach problem solving skills, an ability to explain "why" to students, and knowledge of applications of the mathematics they are teaching.

Three participants noted that teachers need a deeper conceptual understanding of the high school mathematics content they will be teaching. One said that his viewpoint changed starting in *Multivariable Calculus*, but solidified in *Real Analysis* where he realized, "Knowing this stuff can help you…have a deeper understanding of what it is that you're telling your kids." Another participant came to this realization during her high school methods course. Writing lesson plans and anticipating student questions during her practicum, in particular, made her realize, "It's not just simply like how to do stuff, but the mathematical concepts…develop math thinking, that kind of thing." The third participant recognized a need for higher order thinking as he completed his upper level mathematics courses, starting with the proof writing course *Introduction to Abstract Mathematics*. Finally, a nontraditional student said that the need for teachers to possess a deeper understanding of the content they will teach "intrigued" him to become a secondary mathematics teacher.
Two participants explained that over time, they have recognized the importance of problem solving skills in high school mathematics. One said, "I didn't really realize what a teacher really needed…Now taking some of the more difficult math classes, you kind of need the problem solving." A nontraditional pre-service teacher explained that she "place[s] a lot more value on...the problem solving aspect" than she did as a student. When she started the program, she had inclinations that mathematics was being taught differently, based on her awareness of mathematics education through the media. She confirmed she has seen some changes in the focus of mathematics during her teacher preparation, "In my [mathematics] ed classes, they seem to touch on this more, math is this problem solving, creative thing."

Four participants commented on the need to be able to explain mathematical concepts to students. A Bailey pre-service teacher changed her viewpoint during Higher Mathematics for Secondary Teachers. She said, "I can't think of specifically what we learned, but I know there have been times...I was like, 'Wow I was never taught why that works.'…It actually helped make solving that problem easier for me, knowing that reasoning behind ‘why.”" Two other participants appreciated the importance of explaining mathematical concepts to students through tutoring and helping friends with mathematics in college. One said, "For the longest time, I just thought that if I knew all the math in the world, I would be a good teacher, and that's definitely not the case." He continued, "What makes you a better teacher is knowing how to relate to what your students are thinking and transfer it to what they need to know." Another participant saw the importance of “more higher order thinking…more of like ‘why’” when he was in
upper level mathematics courses, specifically when he started learning "really hard proofs."

Finally, two participants noted the importance of showing their students applications of the mathematics they are learning. One said her *Calculus III* teacher emphasized applications, and she herself could make more sense of the mathematics in *Calculus III* and *Differential Equations* when provided with applications.

**The Nature of Mathematics.** Six participants described gaining a better understanding of the nature of mathematics through their teacher preparation. Participants appreciated the breadth of mathematics, learned that problems can have multiple valid solutions, and recognized connections between different content domains.

Two participants explained that through their upper level mathematics coursework, they began to realize the vastness of mathematics. One said, "I guess there's just a lot more to know than I realized…You feel like you've learned so much in math going along in school, and then you get to college and you're like, 'Yeah, what else am I really going to learn?' And you get into these upper level classes that are just going way beyond what you ever thought." Another participant discovered that there is a difference between applied mathematics and pure mathematics and that she preferred applied mathematics. She considered that an "interesting discovery" for her personally, but did not consider it necessary for her to know as a teacher.

After taking *Calculus II* and *Calculus III*, one participant saw that algebra connects much of the mathematics she has learned; she recognized "bridge points of seeing how specifically algebra kind of connects to everything." She clarified why this
information is important for a teacher: "You need to know what you're doing; you need to know how to explain it. And if a student asks, 'Why do you need this?' [It is] our job as a teacher, to kind of connect that." Overall, throughout teacher preparation, pre-service teachers gained a broader perspective on mathematics as an interconnected body of knowledge.

Higher Mathematics Courses. Two participants commented directly on the value of learning higher mathematics. One said, "In high school, I thought proofs were really dumb," adding that taking a proofs writing class and other proof-related courses in college made her start to realize the importance of proof. Her K-12 teachers could not explain to her why a negative times a negative was a positive: "It wasn't until college, that I finally saw a proof of why...For me, [that] was like, 'Okay, I guess to explain to students...is really helpful.'"

Another participant expressed his thoughts about the value of higher mathematics more generally. Through taking "more and more classes," he realized that having "this deeper background knowledge" would help him answer students' questions with confidence and better understand student thinking. He said, "As I get higher and higher up in the math classes, I'm using a lot of the stuff that I'll be teaching, but in totally different ways than I'll directly have to teach it." He referred to his Linear Algebra course, noting that he will not teach that content explicitly, but will be teaching systems of equations. "Being able to talk about different methods knowledgeably, for students to be able to solve problems in different ways, I think can be helpful."
Nontraditional Students

Six of the interviewees were nontraditional students, in the sense that they did not enter a teacher preparation program immediately after high school. Two were in their mid-twenties and did not discuss their nontraditional status during the interview. Another participant in his late thirties was in his first year of teacher preparation, while a thirty-year-old participant was further along in the process. Neither mentioned experiences unique to being a nontraditional student. The two remaining participants, one in her mid-thirties and the other in her early forties, did link their challenges in the program with their nontraditional status.

One participant explained,

For me, as a nontraditional student coming back, there wasn't a great support built in for everything I'd forgotten. [I've had to] relearn stuff I'm going to be teaching. So, that's kind of scary to me, just because it's been so long. When I knew that [mathematics], I knew it. But now, there's so many gaps, so I'll be relearning it with my students, which is mildly frightening.

She feared that she had forgotten many details of the mathematics she will be teaching and provided one example; during a field experience, she was unable to remember that a positive rotation in geometry goes in the counterclockwise direction. She summarized, "I needed a review of the actual K through 12 content, rather than a whole lot of Abstract Algebra I feel like I'm not going to use."

The other nontraditional student was challenged to adopt new approaches in mathematics. "When we went through learning math there was just one way to do it. At least when I learned it. Focus on the procedures, chug them out like a robot, and you're good. And that's not [how it is now]." To prepare for reentering college, she studied GRE
review materials, and also got a precalculus book and "basically did it cover to cover" by herself. Upon entering the teacher preparation program, she took only Calculus I the first semester and Calculus II the second semester. Although she was only taking one class per semester, it "totally felt like I was going to school full time because I had to back up so much."

Decision to Become a Teacher

All twenty-three interviewees were asked what influenced their decision to become a teacher. The participants cited experiences with their own K-12 teachers, tutoring mathematics, helping people with subjects outside of mathematics, and knowing family members who are teachers.

Ten participants remarked on positive experiences with their own K-12 teachers. Somewhat surprisingly, negative experiences with K-12 teachers compelled three participants toward teaching. One who had a bad experience explained, “I hopefully want to be able to give students a chance at not having a bad math teacher like that…[It’s] part of the reason why I want to be a good math teacher, so I don’t end up doing that to students.” One participant cited the combined influence of both a good and a bad teacher as helping him decide to become a teacher. He liked one teacher because, "Every now and then, she would give you some real world situations, and give you opportunity to think of mathematics in a different light." The negatively viewed teacher would "basically give you a set of problems...you never felt like you wanted to ask the teacher why this works or have a better understanding."
For six participants, formally tutoring mathematics or informally helping friends in mathematics influenced them to become teachers. One participant said, "Coming into college, I was pretty indecisive" until he reached “a point where I was tutoring other people and I was working as a recitation course leader at night, and just realized I enjoyed helping students…I knew if I was going in to help students, I felt like I could be excited about that, every day."

Similarly, an additional six participants described how helping people outside of mathematics had influenced them to become teachers. One helped a classmate learn to read during recess in elementary school. Another was a manager in a bookstore, and enjoyed training employees. Two participants had intentions of going into medicine so they could help people, but realized that teaching was a better fit for them. One said, "I always liked helping people, and I thought [medicine] would be a fun way to help people. And then I got into a cadaver lab and was like, 'Oh my god, this is not for me!'" She changed her major to mathematics education: "I still want to help people…I have a pretty good strength in math, so that's what led me to go towards that." The two remaining participants expressed a general desire to help people, and saw teaching as a way to accomplish that.

Five participants had family members who were teachers; four of those said that having family in the education field influenced them to become teachers. One emphasized, "Education has always been very important in our home." The fifth participant did not want to just follow in the footsteps of his parents who are teachers, so he started out majoring in economics. However, in an economics class, he read an article
about the importance of good teachers, and changed his major to mathematics education the next day.

When describing their journey into the teaching profession, six pre-service teachers mentioned that they first considered being engineers. They chose mathematics education over engineering for various reasons. One participant enjoyed the mathematical aspects of engineering, but was not interested in the physics she had to learn. Another did not like the professors in engineering. A third said that during *Shadow an Engineer* day, she "realized that it sounded awful because I didn't want to sit in an office…Then I realized that I do like math, and might as well use what I'm good at to help others. And I like socializing and being with people, so I felt like teaching would be a good option."

**Conclusion**

Throughout teacher preparation, mathematics education majors complete coursework, acquire hands-on experience with students who are learning mathematics, and have other unique opportunities to develop specialized content knowledge. Course requirements vary amongst teacher preparation programs, with some requiring more upper-level mathematics content courses and others offering courses specifically designed for mathematics teachers that study high school level mathematics content in the context of teaching. Both of these approaches appear to aid in the development of various aspects of specialized content knowledge. The implications of these findings are discussed in the next chapter.
5. CONCLUSIONS

The purpose of this study was to learn about experiences that aid in the development of Specialized Content Knowledge (SCK) among secondary mathematics pre-service teachers. Data were primarily collected through interviews with twenty-three secondary mathematics pre-service teachers at six campuses in a Rocky Mountain state; forty-seven pre-service teachers also completed a twenty-two item multiple choice assessment and a survey. Analysis of data revealed that specific experiences in coursework and various interactions with different types of learners contribute to the development of SCK. While this result is not unexpected, findings from this study offer additional insights regarding how pre-service teachers understand, develop, and appreciate SCK. In this chapter, findings are summarized and expanded into implications for secondary mathematics teacher preparation programs. The chapter ends with recommendations for practice and suggestions for further research.

Research Questions

This study was designed to investigate one overarching question with three sub questions. This section discusses to what degree these research questions were answered and describes how the questions were modified based on a shift in the focus of the study from SCK Assessment scores to experiences that develop SCK. The research questions are restated below.
What kinds of experiences support the development of Specialized Content Knowledge (SCK) for beginning algebra among pre-service secondary mathematics teachers?

a. In what ways do secondary mathematics pre-service teachers exhibit Specialized Content Knowledge on an assessment measuring SCK in beginning algebra?

b. What teacher preparation experiences do secondary mathematics pre-service teachers identify as aiding in the development of Specialized Content Knowledge as indicated on the SCK Assessment?

c. What experiences outside of teacher preparation do secondary mathematics pre-service teachers identify as aiding in the development of SCK?

Sub Question A

The first sub question is, “In what ways do secondary mathematics pre-service teachers exhibit Specialized Content Knowledge on an assessment measuring SCK in beginning algebra?” At the start of the study, it was anticipated that combining interviews of secondary mathematics pre-service teachers with quantitative analyses of their SCK Assessment scores would reveal trends related to the level of SCK that participants exhibited based on their various experiences. However, as data collection and analysis progressed, the focus moved away from how pre-service teachers exhibited SCK on the assessment. When assessment items were discussed during interviews, questions focused on the experiences that helped participants select an answer choice for those items and did not aim to measure the participants’ levels of SCK development.
Furthermore, pre-service teachers often changed their answers as they worked through items in the interview setting. This indicated that participants’ scores on the SCK Assessment were not as reliable as the researcher had hoped. The inability of the SCK Assessment to measure levels of SCK became evident early in the study when the assessment was analyzed using the framework for measuring SCK developed by Rich and Bair (2011). Dr. Bair (personal communication, August 19, 2015) noted that the multiple choice items make it difficult to assign a level of SCK measured by individual items.

The SCK Assessment could, however, be used as a conversation starter to build SCK. Participants enjoyed answering the items and saw their relevancy to teaching. As participants explained their reasoning for selecting an answer, the researcher did not engage in any discussion about whether they were correct or incorrect. Even so, participants often modified their reasoning as they came to more fully understand the problem, showing that pre-service teachers may develop SCK by spending time thinking about the items. Whether used in a methods course, a content course, or in professional development with practicing teachers, discussing these items is a way to potentially develop SCK.

Sub Question B

The second sub question is, “What teacher preparation experiences do secondary mathematics pre-service teachers identify as aiding in the development of Specialized Content Knowledge as indicated on the SCK Assessment?” Looking back on this question, it became apparent that “as indicated on the SCK Assessment” was unclear.
When this phrase was added to the research question, it was intended to explain that as participants answered questions about experiences that develop SCK, their conception of SCK might be based on the assessment items. Following discussions about selected SCK Assessment items and looking over the remaining items, interviewees were told, “SCK has been defined as the mathematical knowledge and skill unique to teaching and the mathematical knowledge not typically needed for purposes other than teaching. The items in this exam are assessing some examples of SCK.” This phrasing did not mean that pre-service teachers’ SCK Assessment scores would be examined for a participant’s understanding of SCK. A better phrasing of this sub question is, “What teacher preparation experiences do secondary mathematics pre-service teachers identify as aiding in the development of Specialized Content Knowledge as they conceptualize it after discussing items on the SCK Assessment?”

Sub Question C

The third sub question is, “What experiences outside of teacher preparation do secondary mathematics pre-service teachers identify as aiding in the development of SCK?” This is a parallel to the second sub question and was designed to elicit data regarding experiences with family, work, childhood, and K-12 schooling that are not attributable to teacher preparation programs. Although participants were invited to discuss experiences outside of teacher preparation that aided in their development of SCK, they discussed very few experiences that were in isolation of teacher preparation, likely because the participants were further removed from those experiences. For example, many pre-service teachers discussed tutoring as a helpful experience, but would
they be tutoring if they were not in a teacher preparation program? Despite the limited data, this sub question was retained in order to capture valuable experiences without direct ties to teacher preparation.

Findings and Implications

Findings presented in this section include two discussions related to coursework, three discussions concerning interactions with learners, and a brief treatment of additional findings not directly related to the research questions. Implications for practice are introduced along with each finding, and are also summarized as a list of specific recommendations in a later section.

Coursework

Learning Mathematics in the Context of Teaching. Eleven of the interviewed participants had completed coursework focused on mathematics in the context of teaching; none of these questioned the relevancy of those courses and in fact found them beneficial. One pre-service teacher’s appreciation of Higher Mathematics for Secondary Teachers increased as he discussed the SCK Assessment items, recognizing the course as more valuable than he had initially thought. All interviewees were asked what they would include in a teacher preparation program for secondary mathematics teachers. In many cases, pre-service teachers who had not completed coursework focusing on mathematics in the context of teaching expressed a difficulty in seeing the relevancy of their own coursework. One explained, “I feel like we spend a lot of time on calculations
and learning new things and we don’t spend enough time going back and being like, ‘Okay…I know you already know how to do algebra, but can you explain to me why this would be wrong or why this is also correct?’” Eight of the interviewees alluded to wanting coursework that examines student work, looks closely at high school mathematics, and explores student thinking like the items on the SCK Assessment. This is especially interesting since these interviewees were typically unaware that courses examining mathematics in the context of teaching existed or what they entailed.

It is reasonable to assume that when pre-service teachers perceive a strong connection between their coursework and their future careers, they are more likely to invest in those courses. Secondary mathematics teacher preparation programs should ideally offer courses that examine mathematics in the context of teaching. While participants in this study described courses that would focus specifically on high school level mathematics content, this material may not be appropriate for conventional higher mathematics coursework. However, secondary mathematics pre-service teachers could benefit from upper level mathematics courses that present content, activities, and assessments in the context of teaching.

Conventional mathematics courses at all levels of college typically enroll undergraduates from many different majors. When pre-service teachers are included in that audience, these courses should aim to implement activities in the context of teaching when possible. Bass (2005) suggested that Mathematical Knowledge for Teaching can be viewed as a kind of applied mathematics. Historically, examples of mathematics
applied to engineering, biology, and other disciplines are often included in conventional mathematics courses; mathematics applied to teaching could also be included.

Unfortunately, supplemental materials that highlight mathematics applied to teaching are not readily available. Currently, instructors of conventional mathematics courses who are willing to incorporate mathematics in the context of teaching into their lessons would need to create those problems and activities themselves, and those who do invest time and effort in this process are left to ask themselves if the materials they create are effective. A solution might be to develop units or tasks for conventional mathematics courses that include problems and activities emphasizing mathematics in the context of teaching. This could encourage instructors of such courses to take advantage of this new perspective on a unique application of mathematics.

Shifts in Perspective. Pre-service teachers in this study were asked, “How, if at all, has your viewpoint of the specialized mathematics knowledge needed for teaching high school mathematics changed over time?” The opportunity to reflect on this question revealed some important changes in participants’ perspectives about the mathematics knowledge needed for teaching. Although the question allowed participants to reflect on changes that may have occurred at any point in their lives, the majority of interviewees described changes that took place during teacher preparation. Mostly as a result of various coursework, but also through tutoring, pre-service teachers in this study came to three big realizations. First, they realized that to be an effective teacher they needed a better understanding of the high school mathematics they will teach. Second, they gained a deeper understanding of the nature of mathematics as a coherent body of knowledge.
Finally, they began to appreciate the connections between advanced mathematics and the mathematics taught in high school.

It appeared that at least for some pre-service teachers, these realizations had never been expressed before, and perhaps even emerged as a result of interview conversations. Such understandings are important components of the professional practice of teaching. Teacher preparation programs have the potential to help pre-service teachers emerge as professionals through providing them with opportunities to discuss and reflect on their changes in perception as they move through teacher preparation programs. This could be accomplished through seminars, structured peer discussions, or reflective interviews with advisors and instructors.

Interactions with Learners

Field Experience. In this study, pre-service teachers with extensive field experience found those interactions helpful in developing SCK. One said that it was during field experience that he was able to "see how it's really beneficial for [the students] to think through problems without telling them how to do it." Overall, pre-service teachers desired more field experience or more interaction with students during their field experience. Increased access to learners gives pre-service teachers more opportunity to apply what they are learning in mathematical situations, and to specifically practice some aspects of SCK (e.g., analyzing student solutions).

Tutoring. In addition to organized field experience, some pre-service teachers had the opportunity to interact with learners through tutoring. They found these
interactions helpful in developing SCK, perhaps even more so than field experience. When one pre-service teacher was asked whether field experience helped her develop SCK, she responded, "Not as well as tutoring, because a lot of times, you know, you're just observing." Pre-service teachers in the study conveyed that tutoring allowed them to learn about different approaches to problems, practice their teaching, revisit old material, and ultimately answer items on the SCK Assessment.

Pre-service teachers should be encouraged to tutor, whether their own peers, college students taking other mathematics courses, or students at the K-12 level. During field experiences, pre-service teachers could offer individual tutoring to students who are struggling in class or who need an extra challenge. Pre-service teachers at later stages in teacher preparation could be assigned to tutor local high school students, or hired to tutor high school students by their parents.

One program in the study took this concept to the extreme in a unique methods course. At Anderson, a private college with only two mathematics education majors, the methods instructor worked alongside the two pre-service teachers in co-teaching a calculus course on campus. While this experience may not be commonplace, modified versions of co-teaching are feasible. This allows pre-service teachers to immediately apply and practice many of the things they learn, including lesson design, time management, explaining mathematical concepts (a component of SCK), and analyzing student solutions (also a component of SCK).

Interactions with Elementary Majors. Undergraduate degrees leading to teaching certification in secondary mathematics require substantial levels of mathematics
coursework, which is typically taken alongside mathematics majors. As a result, secondary teaching majors are likely to approach problems in traditional ways that are similar to their peers. In contrast, when secondary pre-service teachers are in mathematics courses alongside elementary education majors, they gain new perspectives on different, and often creative, approaches to problem solving. In this study, pre-service teachers at Bailey commended *Middle School Methods* as a course where their traditional approaches to problems were challenged by fresh ideas from future elementary teachers. For example, number talks allowed them to explore many different approaches to arithmetic and basic algebra problems that they would not have thought of as secondary mathematics education majors.

This blending of secondary and elementary education majors is an effective tool for helping secondary mathematics education majors realize that even the most basic mathematics can be viewed from different perspectives. For example, it is likely that secondary education majors have only applied standard algorithms to solve arithmetic problems. Elementary majors bring other ideas to the table. Teacher preparation programs could offer mathematics courses that are accessible to both elementary and secondary education majors. One program in the study offers middle school level content courses and a middle school methods course; middle school mathematics is a platform for the overlapping content between elementary and secondary majors.
Other Findings and Implications

Nontraditional Students. One nontraditional student in this study felt she had forgotten the high school mathematics she will be expected to teach, and did not feel as though her teacher preparation program helped fill that gap. This is a case of the “double discontinuity” described in the *Mathematical Education of Teachers II* (2012): the lack of connection between high school mathematics and college level mathematics as pre-service teachers enter college, and again as college graduates enter the high school classroom. This phenomenon is likely to have a more profound effect on nontraditional students because they are even further removed from high school mathematics.

In some cases, nontraditional students may need opportunities to refresh their knowledge of high school mathematics. One nontraditional student in the study found a way to cope with her distance from high school mathematics by reviewing a precalculus self-teaching guide before entering the teacher preparation program. Advisors of nontraditional students in secondary mathematics teacher preparation programs should be sensitive to their unfamiliarity with high school mathematics. Options to close this gap could include informal internships in high school classrooms, auditing entry-level college mathematics courses, additional coursework in basic mathematics, or self-paced reviews, which are available electronically and on paper.

Theory: Interactions with Learners
Aid in SCK Development

The ultimate goal of a grounded theory study is to develop a theory that describes the process being studied – in this case, the development of Specialized Content
Knowledge among secondary mathematics pre-service teachers. A theory comprised of *interactions with learners* connects the findings in this study. Secondary mathematics pre-service teachers identify interactions with learners, both *face to face* and *hypothetical*, as aiding in the development of SCK. Face to face interactions that contribute to SCK development include formal field placements, student teaching, internships (co-teaching), tutoring, and assisting in college courses. Instructors contacted in this study added that some coursework for pre-service teachers includes *short-term classroom visits* to observe lessons and even practice teaching. Hypothetical interactions with learners include coursework that examines mathematics in the context of teaching, analyzing student work, and grading students’ assignments at the high school or college level. Course instructors also noted that pre-service teachers in methods courses have additional opportunities to practice *tasks of teaching*, including writing lesson plans, studying curriculum, and grading student work. The diagram in Figure 11 summarizes the various interactions with learners identified in this study. Placement of interactions within the diagram does not represent relationships among those interactions.

Figure 11. Theory of SCK Development

* *Short-term classroom visits* and *tasks of teaching* were identified by instructors.
Driven by a pragmatic stance typical of grounded theory, the researcher recognizes that this diagram may fall short of describing every experience that aids in the development of Specialized Content Knowledge among secondary mathematics pre-service teachers. It should be viewed as a working theory that will be modified as new research emerges. One item of note missing from the diagram in Figure 11 is the influence of advanced coursework (e.g., abstract algebra, real analysis) on the development of SCK. With only three participants in the study who had completed Real Analysis or Abstract Algebra, this theme could not be fully explored; it remains unclear whether and how this coursework may aid in the development of SCK among secondary mathematics pre-service teachers.

The findings of this study can be linked with the analysis of Specialized Content Knowledge developed by Bair and Rich (2011). Their framework dissects the demonstration of SCK into four components: (1) ability to correctly solve a task, explain their work, justify their reasoning, and make connections; (2) ability to use multiple representations; (3) ability to recognize, use, and generalize relationships among conceptually similar problems; and (4) ability to pose problems. The following discussion shows how the learner interactions presented in Figure 11 address all four components of Bair and Rich’s (2011) interpretation of SCK.

The role of course assistant affords the greatest opportunity for pre-service teachers to engage in the first component. Mathematics coursework in the context of teaching requires solving tasks, explaining work, and justifying reasoning, but does not always lead to making connections. As course assistants assigned to support student
learning, pre-service teachers have many opportunities to solve, explain, and justify. Furthermore, the curriculum becomes transparent to the course assistants, giving them the opportunity to see how daily material is related to other content in the course.

The second component of Bair and Rich’s (2011) framework is addressed during tutoring, where pre-service teachers not only need to analyze approaches to solutions quite different from their own, but also present concepts in multiple ways to ensure understanding. The third component (recognizing, using, and generalizing relationships) is most strongly practiced during methods coursework and student teaching. Both experiences require pre-service teachers to develop lesson plans, which rely on “generat[ing] problems or tasks that will produce patterns that exhibit desired properties for generalization in multiple contexts,” (Bair & Rich, 2011, p. 301). Finally, the fourth component (ability to pose problems) is addressed through all of the interactions with learners outlined in Figure 11, both face to face and hypothetical. Problem posing is a fundamental part of teaching, tutoring, and coursework. This theory described above proposes a set of interactions with learners with the potential to productively develop SCK among secondary mathematics pre-service teachers. The next section provides suggestions for how teacher preparation programs can improve and increase such interactions.

**Recommendations for Practice**

The following statements translate implications from this study into recommendations that have the potential to improve the development of SCK among pre-
service teachers in secondary mathematics. Each broad recommendation is followed by one or more specific suggestions for implementation. In this section, “programs” refer to those programs that prepare future secondary mathematics teachers.

1. Programs could offer content courses that examine mathematics in the context of teaching.

   - Such courses could be offered at times that are convenient to both in-service and pre-service teachers, increasing enrollment while providing in-service teachers with opportunities for professional development.

2. Instructors of conventional mathematics courses could seek opportunities to implement problems and activities that examine mathematics in the context of teaching.

   - Instructors could create their own lessons and assessments that incorporate such content.

   - A library of resources related to mathematics in the context of teaching could be created and shared among instructors of conventional mathematics courses.

   - A working group of experts could perhaps be funded by grants to convene with the goal of developing teaching materials that examine mathematics in the context of teaching.
3. Programs could offer frequent opportunities for pre-service teachers to interact with learners (both face to face and hypothetically).

- Programs could work with K-12 schools and campus study/tutoring centers to provide pre-service teachers with multiple opportunities to tutor students at all levels.

- Programs could arrange field experience for pre-service teachers each semester. These may sometimes be brief interactions, such as corresponding with pen pals in K-12 or grading papers. Both activities involve analyzing student solutions, an important component of SCK.

- Pre-service teachers in methods courses could benefit from creative opportunities to teach learners of mathematics, even at the college level. If K-12 classrooms are not available, programs could consider pairing methods students as interns, course assistants, or co-instructors with instructors of lower-division college courses.

4. Programs should consider the following additions and adaptations to current coursework that serves secondary mathematics education majors.

- Offer mathematics courses that serve both elementary and secondary education majors and take advantage of their differing perspectives. Middle school mathematics provides one platform for accomplishing this goal.
• Use the SCK Assessment in methods courses, content courses, and in professional development with practicing teachers. Analysis and discussion of the items may develop Specialized Content Knowledge.

5. Advisors of nontraditional students should be sensitive to pre-service teachers’ distance from high school mathematics and seek opportunities to strengthen their knowledge of basic concepts.

• Programs may offer informal internships in high school classrooms, auditing of entry level college mathematics courses, additional coursework in basic mathematics, or self-paced reviews to help non-traditional students refresh their high school mathematics knowledge.

6. Programs could provide opportunities for pre-service teachers to discuss and reflect on their changes in perception about mathematics knowledge needed for teaching as they move through the program.

• Such reflections could take place through informal interviews with advisors and instructors, structured peer discussions, or seminars.

Recommendations for Further Research

This section first describes limitations of the SCK Assessment and recommends research to improve future measures of Specialized Content Knowledge. This is followed by recommendations for research with pre-service and practicing teachers and a description of a framework for future studies of SCK. The section concludes with a
discussion of the necessity for a distinction between the different domains of
Mathematical Knowledge for Teaching. These research recommendations are discussed
fully below, and are based on the findings and implications described above.

Limitations of the SCK Assessment

The SCK Assessment used in this study served its purpose: to select interview
participants and to elicit conversations about how experiences aid in participants’
development of Specialized Content Knowledge. However, more comprehensive
measures of SCK for high school mathematics are needed. The SCK Assessment used in
this study had a number of limitations. First, it had a strong focus on beginning algebra,
with little treatment of other domains of high school mathematics. For example, most
interviewees had completed a geometry course, but virtually no data were collected about
the treatment of geometry in the six programs, likely because the SCK Assessment did
not have any items drawing on geometry. More thorough measures of SCK at the
secondary level are needed that assess all domains of high school mathematics. Another
limitation of the SCK Assessment was its reliance on multiple choice items. Because of
this constraint, a low score on the assessment does not indicate whether the participant
only lacks Specialized Content Knowledge, or whether he or she also lacks Common
Content Knowledge. Future measures of SCK should include some open response items,
so that graders are more capable of determining what type of mathematical knowledge
respondents are drawing on to answer assessment items.
Research with Pre-service Teachers

Further research on the development of SCK among pre-service teachers should primarily include those who have completed teacher preparation. This study examined pre-service teachers at all levels of teacher preparation programs, but the richest data were collected from those who were finishing up coursework and student teaching. In the midst of coursework, pre-service teachers have a more difficult time finding the value in some experiences. For example, one pre-service teacher in the study explained that he did not see the need for taking *Methods of Proof* until a geometry course later in his program allowed him to apply proofs to the topics he would be teaching to high school students.

Research with Practicing Teachers

A great deal could be learned about SCK by following the participants in this study into their teaching careers. A researcher could re-administer the SCK Assessment and re-interview the participants, to find whether classroom teaching experience aids in the development of SCK. The practicing teachers could also be asked to reflect on their responses in the original interviews to learn whether their perspectives changed after entering the teaching field. For example, they may find some college coursework more or less helpful after being faced with classroom teaching. The practicing teachers could also be observed in classrooms to study whether participants’ scores and interview responses are aligned with their ability to apply SCK when teaching.
A Framework for Future Research

One goal of this study was to provide a framework for organizing ideas about how secondary mathematics pre-service teachers develop SCK. The findings indicate that pre-service teachers develop the majority of their SCK in coursework and through interactions with learners. Participants found coursework emphasizing mathematics in the context of teaching as more beneficial than conventional mathematics courses that do not directly address teaching mathematics. Research focused on coursework that emphasizes mathematics in the context of teaching could explore what teaching techniques and activities are most effective at integrating this type of mathematics into content coursework, both conventional courses and those designed specifically for teachers. Findings could provide suggestions for incorporating mathematics in the context of teaching into conventional mathematics courses.

Participants also found interactions with learners through tutoring and field experience to be beneficial in developing SCK. Future studies should focus on the quality and quantity of interactions with learners that are most beneficial in developing SCK. For example, should secondary mathematics pre-service teachers be encouraged to only tutor high school students, or do they benefit from teaching college students or students in elementary school? Do brief classroom field experiences aid in the development of SCK if they are rich in student interactions? Is the same true for experiences without face to face interactions, such as grading student work? What else is required of field experiences for them to be effective?
Distinction Between the Domains of MKT

As new measures of MKT and SCK are developed, it is reasonable to question whether distinctions between the different domains of MKT are important. In this researcher’s opinion, distinguishing which domains of MKT are being assessed by a particular item is an unnecessary task in the context of measuring a person’s MKT. It is important for teachers to have knowledge in all domains of MKT. Some assessment items may allow test-takers to draw on more than one domain of MKT to select an answer, but efforts to construct items that differentiate between components of MKT is unnecessary. Possessing the knowledge of mathematics and teaching needed to select the correct answer is most important, regardless of the type of MKT used to answer the item.

That said, in the context of using an assessment as a teaching tool, it still may be beneficial to distinguish between the different domains of MKT. Recall that MKT is divided into Subject Matter Knowledge and Pedagogical Content Knowledge, with SCK a subcomponent of Subject Matter Knowledge. When applying mathematics in the context of teaching to an audience of undergraduates studying in a variety of fields, it is especially important to indicate whether an item is assessing a component of Subject Matter Knowledge, as opposed to Pedagogical Content Knowledge. As an example, consider the assessment used in a Methods of Proof course in this study that asked undergraduates from various majors to analyze a fictional student’s proof to find and correct the mistakes. This is an example of SCK, which is a subcomponent of Subject Matter Knowledge. It would be less appropriate for this assessment to ask
undergraduates which mistakes are most common among students who are learning proofs; this would be considered Pedagogical Content Knowledge.

Conclusion

The findings of this study indicate that coursework and interactions with learners aid in the development of SCK among secondary mathematics pre-service teachers. More specifically, pre-service teachers value learning mathematics in the context of teaching, and they benefit from tutoring and field experience. To enable the learning of mathematics in the context of teaching, conventional mathematics courses could incorporate problems and activities that examine mathematics applied to teaching. However, supplemental material development is needed to help this process. Teacher preparation programs could also encourage pre-service teachers to tutor and provide multiple field experience opportunities. Further research could focus on the quality and quantity of tutoring and field experience that are most beneficial to secondary mathematics pre-service teachers.

The SCK Assessment used in this study was limited in its ability to measure participants’ SCK; however, follow up interviews offered insights into how secondary mathematics pre-service teachers develop Specialized Content Knowledge through teacher preparation. By combining different approaches, this study revealed participants’ understanding, development, and appreciation of Specialized Content Knowledge.
REFERENCES CITED


Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers II*. Providence, RI.


APPENDICES
APPENDIX A

PROSPECTIVE TEACHER SURVEY AND SCK ASSESSMENT
1. Thank you for taking the time to visit this survey and assessment! This survey and assessment should take less than one hour to complete. You will NOT be able to save your progress; it must be completed all in one sitting! Do NOT use the back button on your web browser, or you will have to start the survey over from the beginning! – Danielle Pettry

2. Name (First and Last)

3. E-mail Address (Your e-mail address is needed so that the researcher can contact you, if needed.

4. Phone Number (If you do not want to provide your phone number, type “n/a.”

5. Best way to contact you: E-mail Phone Call Text Message
1. Mr. Wright asked his students to solve the equation $6 - 3(x - 5) = 24$. After reviewing his students’ work, he found two interesting methods for solving the equation and asked those two students to present their methods on the board.

<table>
<thead>
<tr>
<th>Brenda’s method</th>
<th>Daniel’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 - 3(x - 5) = 24$</td>
<td>$6 - 3(x - 5) = 24$</td>
</tr>
<tr>
<td>$6 - 3x + 15 = 24$</td>
<td>$-3(x - 5) = 18$</td>
</tr>
<tr>
<td>$-3x + 21 = 24$</td>
<td>$x - 5 = -6$</td>
</tr>
<tr>
<td>$-3x = 3$</td>
<td>$x = -1$</td>
</tr>
<tr>
<td>$x = -1$</td>
<td></td>
</tr>
</tbody>
</table>

After Brenda and Daniel presented their methods, Mr. Wright’s class discussed these two methods. One student, Steve, compared them and then said, “I like Daniel’s method because there are less steps. However, if it is a harder division, Brenda’s method would be easier.” For which of the following equations would Steve be most likely to use Brenda’s method?

A) $5 - 2(x - 3) = 21$

B) $10 - 7(x + 5) = 6$

C) $6 - 3(x + 1) = 9$

D) $8 + 4(x - 3) = 14$

2. A lesson in Ms. Hagerman’s textbook defines the distributive property, but the exercises merely ask for its definition. To motivate her students to learn the definition, Ms. Hagerman tells them that the distributive property can often be used to simplify the evaluation of expressions.

She wants to give her students an example that will focus their attention on how the distributive property can be useful in evaluating expressions. Of the following expressions, which would best serve her purpose?

A) $12 \times \left(\frac{3}{4} + \frac{1}{4}\right)$

B) $18 \times \left(\frac{3}{5} - \frac{1}{10}\right)$

C) $36 \times \left(\frac{5}{12} + \frac{2}{9}\right)$

D) Each of these expressions would serve her purpose equally well.
During a lesson on solving multistep equations, Ms. Kane asked her students to solve the equation \(-5x + 8 = 13x - 10\). While walking around the classroom looking at what the students were writing, she noticed several different strategies. For each of the following student solutions, indicate whether or not the work provides evidence that the student is reasoning correctly about this problem.

<table>
<thead>
<tr>
<th></th>
<th>Provides Evidence of Correct Student Reasoning</th>
<th>Does Not Provide Evidence of Correct Student Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) (-5x + 8 = 13x - 10) &lt;br&gt; (8 = 18x - 10) &lt;br&gt; (18 = 18x) &lt;br&gt; (1 = x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) (-5x + 8 = 13x - 10) &lt;br&gt; (8 = 13x - 10 - 13x + 10) &lt;br&gt; (8 = 26x) &lt;br&gt; (x = \frac{8}{26})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C) (-5x + 8 = 13x - 10) &lt;br&gt; (\frac{8 + 5x}{x} = 3x) &lt;br&gt; (\frac{8}{8} = \frac{5x}{x}) &lt;br&gt; (1 = x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D) (-5x + 8 = 13x - 10) &lt;br&gt; (-13x + 8 = 13x - 8) &lt;br&gt; (-13x = -16) &lt;br&gt; (x = 1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Mr. Brownstein’s class was solving the equation:

\[ 3(x - 2)^2 = 6(x - 2)(x + 5). \]

Kenneth suggested dividing both sides by 3 to get:

\[ (x - 2)^2 = 2(x - 2)(x + 5). \]

Then he suggested dividing both sides by \((x - 2)\), but Sandra said, “You cannot divide both sides by \((x - 2)\).” In response, Kenneth asked, “If you can divide both sides by 3, why can’t you divide both sides by \((x - 2)\)?”

Of the following statements, which best explains why you cannot divide both sides of the equation by \((x - 2)\) as Kenneth suggested?

A) You cannot cancel \((x - 2)\) because it represents a real number.

B) It is better to expand the expressions on both sides of the equation first to obtain

\[ x^2 - 4x + 4 = 2(x^2 + 3x - 10), \]

and then you won’t have to worry about \((x - 2)\).

C) Division by zero is not defined, so you would have to consider the case of \(x = 2\) separately.

D) Because \(x\) is a variable, it can vary—you may not be canceling the same amount from both sides.
5. During a unit on solving linear equations, Ms. Martino asks her students to write one question that she could use on the unit test at the end of the chapter. Joe writes the following question.

If Joe had to solve the sentence \(8y - 9 = 0\) for \(y\), what would be the value of \(y\)?

Ms. Martino thinks the equation would be a good one for her students to solve. However, she decides to revise the question because she is concerned that the wording may cause some of her English Language Learner (ELL) students to answer this question incorrectly, even if they understand the mathematics involved. Of the following revisions, which one best addresses Ms. Martino’s concern?

A) When Joe solved for \(y\) in the equation \(8y - 9 = 0\), what was the value of \(y\)?

B) Joe solved the sentence \(8y - 9 = 0\) for \(y\). What is the value of \(y\)?

C) What is the value of \(y\) in the equation \(8y - 9 = 0\)?

D) If \(8y - 9 = 0\), what would be the value of \(y\)?
6. Before teaching a lesson on multiplying two trinomials, Ms. Ryan wants a better sense of what her students know about multiplying two binomials. She asks them to find the product \((2x + 1)(x - 4)\) and explain their methods. While walking around the class, she notices several different methods. For each of the following, indicate whether or not the student response provides evidence that the student has an understanding of the multiplication of binomials that would support the development of a strategy for multiplying two trinomials.

<table>
<thead>
<tr>
<th></th>
<th>Would Support a Strategy for Trinomials</th>
<th>Would Not Support a Strategy for Trinomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>I used the FOIL method. The product of the first terms is (2x^2), the product of the outer terms is (-8x), the product of the inner terms is (x), and the product of the last terms is (-4). Then, I combined the two like terms, the (-8x) and the (x), to get (-7x), so my final answer is (2x^2 - 7x - 4).</td>
<td></td>
</tr>
<tr>
<td>B)</td>
<td>I just multiplied like you do with regular numbers. I put the (2x + 1) on the top and the (x - 4) on the bottom. Then I multiplied each term on the bottom by each one on the top. Finally, I added up the like terms to get my answer, (2x^2 - 7x - 4).</td>
<td></td>
</tr>
<tr>
<td>C)</td>
<td>I drew a box with two columns for the (2x) and the (1) and two rows for the (x) and the (-4). Then, I multiplied the terms that went with the row and column that each small box was in. Then, I wrote out what I got in each small box, combined the like terms, and got (2x^2 - 7x - 4).</td>
<td></td>
</tr>
</tbody>
</table>
7. Mr. Anderson asked his students to simplify the following algebraic expression.

\[
\frac{2(a + 1)}{3a} + 3 - \frac{2}{3a} - \frac{6a - 2}{6} =
\]

One of his students gave the incorrect solution shown below.

\[
\frac{2(a + 1)}{3a} + 3 - \frac{2}{3a} - \frac{6a - 2}{6} = \frac{2a + 2}{3a} + 3 - \frac{2}{3a} - \frac{6a}{6} + \frac{2}{6}
\]

\[
= \frac{2a}{3a} + \frac{2}{6} + \frac{2}{6} - a
\]

\[
= \frac{2}{3} + 2 \left( \frac{\frac{1}{6}}{\frac{1}{6}} + \frac{1}{6} \right) - a
\]

\[
= \frac{2}{3} + \frac{7}{6} - a
\]

\[
= \frac{4}{6} + \frac{14}{6} - a
\]

\[
= \frac{18}{6} - a
\]

\[
= 3 - a
\]

Of the following descriptions, which best characterizes what is wrong with this student’s work?

A) This student used the distributive property incorrectly.

B) This student confounded mixed fractions with factors.

C) This student forgot to cancel common factors in several places.

D) This student needs to apply a more formal procedure by finding the common denominator and then adding all terms.
8. Having nearly finished a chapter on linear equations, Mr. Hassan’s students seem quite proficient in generating standard formats for linear equations, using techniques for graphing linear equations, and solving systems of two linear equations. However, he is concerned that his students are applying these techniques in routine ways and tend to think only algebraically or only geometrically without reasoning fluidly in both ways.

Mr. Hassan wants to give his students a problem that would require an understanding of the topic that goes beyond the set of procedures students have learned and that would support his students’ ability to work and talk across algebraic and geometric interpretations. Of the following problems, which would best serve this dual purpose?

A) Describe in your own words a procedure for finding the point of intersection given the equations of two lines.

B) Find the intersection of the following two lines and graph them.

\[ y = 2x + 3 \]
\[ y = 2x - 7 \]

C) Consider two linear functions, where \( a \) and \( b \) are negative.

\[ y = x + 3 \]
\[ y = ax + b \]

What can you say about the point of intersection of their graphs?

D) Using ideas about solving systems of two linear equations, solve the following system of three equations and explain what the solution means.

\[ x + 4y + z = 0 \]
\[ x - 4y + 2z = 3 \]
\[ x = 4y + z \]
9. Having taught her students to factor quadratics with integer coefficients, integer roots, and a leading coefficient of 1, Ms. Quezada explained that she was going to give them a harder problem. She then asked them to solve the following.

\[ 3x^2 - 3x - 6 = 0 \]

After a few minutes of work, the class discussed their solutions. Letitia said that \( x \) was \(-1\) or 2 and explained, “I added 3x to both sides and divided by 3.”

\[
\begin{align*}
3x^2 - 6 &= 3x \\
3x &= x^2 - 2
\end{align*}
\]

She then continued, “The parabola’s just down a little and the line’s at 45 degrees, so it’s just below zero and about 2 to the right. \( x \) can be \(-1\) and 2, and those are the only possible ones.”

Of the following, which best characterizes Letitia’s approach to this problem?

A) Letitia’s **method** is wrong because she should have first divided by 3 and then factored the left side of the equation.

B) Letitia’s **method** is wrong because this is a parabola and you could graph it, but you would have to graph the original equation and look for the roots.

C) Letitia’s **reasoning** is correct, but her method often leads to points of intersection that might be hard to determine visually.

D) Letitia’s **reasoning** is correct, but her method requires knowledge of calculus.
10. Ms. Lang’s class had been studying the concept of slopes of lines, so she asked them to consider all of the lines passing through one point and how the slopes of those lines vary. The students had used geoboards in some earlier work, so they started talking about the slopes of lines on an “infinitely extended” geoboard. (Geoboards are flat blocks of wood, roughly one foot square, with pegs laid out on a grid where rubber bands can be hooked to make lines or polygons.) The students decided that the pegs of the infinite geoboard could be thought of as the set of points with integer coordinates in the Cartesian plane.

During the discussion, students had the following exchange.

Yonah:  On the geoboard, you can’t get all of the slopes, because the geoboard points are too spread out—there are a whole bunch of lines between the ones you can make.

Andy:  I disagree. I think we can make any slope. Starting at one point, by choosing another geoboard point far enough away, we can tilt the line as much or as little as we like.

Becky:  What I was thinking was if you run a line through one geoboard point, it will always hit another one far enough out.

Of the following concepts, which is most directly related to the mathematics underlying this discussion?

A) Interpretation of the derivative—the derivative is the slope of the tangent line.

B) Density of numbers on the real line—the rational numbers are dense, but not every real number is rational.

C) The parallel postulate—given a point and a line, there is a unique line through the given point parallel to the given line.

D) Each of these concepts is equally related to the underlying mathematics.
11. During a lesson on solving multistep equations, Mr. Steinbrecher asked his students to solve the equation \(4(5x - 11) = 16\). While walking around the class looking at what the students were writing, he noticed several different strategies. For each of the following student solutions, indicate whether or not it is a valid strategy for solving this problem.

<table>
<thead>
<tr>
<th></th>
<th>Strategy Is Valid</th>
<th>Strategy Is Not Valid</th>
</tr>
</thead>
</table>
| A) | \[
\frac{1}{4} \cdot 4(5x - 11) = 16 \cdot \frac{1}{4} \\
5x - 11 = 4 \\
5x = 15 \\
x = 3
\] | |
| B) | \[
4 \cdot 4(5x - 11) = 16 \cdot 4 \\
\frac{4}{4} (5x - 11) = \frac{16}{4} \\
5x - 11 = 4 \\
+11 +11 \\
5x = 15 \\
x = 3
\] | |
| C) | \[
4(5x - 11) = 16 \\
9x - 11 = 16 \\
+11 +11 \\
9x = 27 \\
x = 3
\] | |
| D) | \[
4(5x - 11) = 16 \\
20x - 44 = 16 \\
20x = 60 \\
+11 +11 \\
x = \frac{60}{20} \\
x = 3
\] | |
| E) | \[
4(5x - 11) = 16 \\
\frac{5x}{4} - \frac{11}{4} = 4 \\
+ \frac{11}{4} + \frac{11}{4} \\
\frac{5}{4} x = \frac{16}{4} \\
x = \frac{3}{2}
\] | |
12. Mr. Jakobsen’s students were graphing the function below, where \( y \) is inversely proportional to \( x \).

\[
y = \frac{1}{x}
\]

One of his students drew the following graph.

Mr. Jakobsen has noticed that students often draw graphs with line segments like this despite frequent reminders that the graph should be curved. To get his students to discuss this issue, he asked the class what was wrong with the drawing. Of the following student explanations, which provides the best mathematical explanation of why drawing connected line segments is inappropriate for this graph?

A) When the \( x \) changes, the graph should change at the same rate all the time and it shouldn’t have corners.

B) The graph changes all the time, but it cannot have sudden changes at some of the points.

C) For any whole number, \( \frac{1}{x} \) will always be a rational number and that makes it hard to draw the graph for irrational numbers.

D) The problem is that you don’t have enough points. You need to include more points to make it look correct.
13. In a unit on simplifying expressions, one of Mr. Serrano's students wrote the following correct solution.

\[
\frac{4(a+2)}{3a} + 2 - \frac{8}{3a} - \frac{6a-1}{6}
\]

\[
= \frac{4a + 8}{3a} + 2 - \frac{8}{3a} - \frac{6a}{6} + \frac{1}{6}
\]

\[
= \frac{4a}{3a} + \frac{1}{6} - a + \frac{1}{6}
\]

\[
= 1 \frac{1}{3} + 1 \frac{7}{6} - a
\]

\[
= 1 \frac{2}{6} + 1 \frac{7}{6} - a
\]

\[
= 2 \frac{9}{6} - a
\]

\[
= 3 \frac{3}{6} - a
\]

\[
= 3 \frac{1}{3} - a
\]

Of the following descriptions, which best characterizes the student's work?

A) The student knows how to simplify expressions very well and demonstrates strategic use of standard procedures.

B) The student shows good computational skill but does not use processes efficiently.

C) The student knows how to simplify expressions very well, but in the solution the student should write all steps involved in the calculation, such as the step \( \frac{8}{3a} - \frac{8}{3a} \).

D) The student should apply a more formal procedure by first finding the common denominator and then adding all terms.
14. A lesson in Ms. Taylor’s textbook states the associative and commutative properties of addition. To motivate the students to learn the properties, she tells her students that the properties can often be used to simplify the evaluation of expressions.

She wants to give her students an example that will focus their attention on how these properties can be useful in evaluating expressions. Of the following expressions, which would best serve her purpose?

A) \((455 + 456) + (457 + 458)\)

B) \((647 + 373) + (227 + 456)\)

C) \((551 + 775) + (49 + 225)\)

D) Each of these expressions would serve her purpose equally well.
15. Ms. Kamp asks her students to consider squares of different side lengths with only the boxes along the sides shaded as in the figure below.

She asks each student to write an expression for the number of shaded boxes in a square with a side length of \( n \) boxes and to explain why the expression gives the number of shaded boxes for any size square. For each of the following explanations, indicate whether or not it provides evidence that the student understands why the expression can be used to find the number of such shaded boxes in any square.

<table>
<thead>
<tr>
<th>Explanation</th>
<th>Provides Evidence</th>
<th>Does Not Provide Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) If you start on the bottom left and go to just below the top left, then start with the top left and go to just before the top right, and keep doing that, you will get 4 groups, and each group has 1 less than ( n ), so you get ( 4(n - 1) ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) My expression is ( n + 2(n - 1) + (n - 2) ) because the top of the square has ( n ) shaded boxes, then each of the sides has ( n - 1 ) shaded boxes left, and then the bottom has ( n - 2 ) shaded boxes left.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C) Inside the square with a side length of ( n ) boxes is a square with side length of ( n - 2 ) boxes, so if you find the area of the two squares and subtract them, you will find the number of shaded boxes. So, I get ( n^2 - (n - 2)^2 ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D) I get ( 4(n - 2) + 4 ) because there are 36 boxes shaded, and when you put 10 in for the ( n ) in ( 4(n - 2) + 4 ) and follow the order of operations, the answer is 36.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E) My expression is ( 2n + 2(n - 2) ) because if you simplify ( 2n + 2(n - 2) ) you get ( 2n + 2n - 4 ), which is equal to ( 4n - 4 ), and because this doesn’t depend on ( n ), it works for any ( n ).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
16. Ms. Quinn asked her students to solve the following quadratic equation.

\[3x^2 - 6x - 24 = 0\]

Maurice explained, “I added 24 to both sides and divided by 3.”

\[3x^2 - 6x = 24\]
\[x^2 - 2x = 8\]
\[x(x - 2) = 8\]

He then concluded, “The only numbers that are 2 apart and multiply to be 8 are 2 and 4, and -2 and -4, so \(x\) has to be 4 or -2.” Students agreed that 4 and -2 work when you substitute them into the original equation, but they were unsure about his method.

Of the following statements, which best characterizes Maurice’s approach to this problem?

A) Maurice’s method is wrong because you cannot solve an equation by factoring unless one side of the equation is equal to zero.

B) Maurice’s method is wrong because he should have first divided by 3 and then factored the left side of the equation.

C) Maurice’s reasoning is correct, but his method often leads to an equation that cannot be solved by inspection.

D) Maurice’s reasoning is correct, but his method only works for equations with real roots.
17. In the last class, Mr. Rosen’s students graphed quadratics of the form \( y = x^2 + c \) for various values of \( c \) and developed a rule about shifting the graph of \( y = x^2 \) up or down. Today Mr. Rosen asked the students to predict what would happen when they graphed \( y = (x - 3)^2 \) and then to graph it. Students were surprised that the graph shifted 3 units to the right rather than left or down as they had predicted. Mr. Rosen asked them to explore a little further in groups.

As he walked around the classroom, each group explained to him the strategy they were using to explore the problem. Of the following student descriptions of a strategy for exploring the problem, which is most directly related to the underlying mathematical reason for the graph’s behavior?

A) We are trying to prove the rule, so each of us is graphing another one, \( y = (x - 2)^2 \), \( y = (x - 5)^2 \), \( y = (x + 2)^2 \), and \( y = (x + 1)^2 \), and then we will compare our results.

B) We are making a table like yesterday and putting \( x \) and \( x^2 \) and \( (x - 3)^2 \), so we can plug in different inputs and compare what the outputs are in \( x^2 \) and \( (x - 3)^2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x^2 )</th>
<th>( (x - 3)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C) We decided to look at the roots of \( y = x^2 \) and \( y = (x - 3)^2 \). The vertex of \( y = (x - 3)^2 \) is \( (3, 0) \), and the vertex of \( y = x^2 \) is \( (0, 0) \). We are looking to see what \( x \)-values we have to put in \( (x - 3)^2 \) to make the outputs the same as in \( x^2 \).

D) We are going to FOIL it out so it will look more like the ones from yesterday, and then we can graph it and compare the way we did yesterday.
18. Ms. Seidel is introducing the distributive property. To motivate her students, she wants to give them an example that will focus their attention on how using the distributive property can simplify computations. In which of the following examples will the use of the distributive property most simplify the computations?

A) $12 \times 29 + 12 \times 38 = \_\_\_

B) $17 \times 37 + 17 \times 63 = \_\_\_

C) $13 \times 13 + 15 \times 15 = \_\_\_

D) $16 \times 24 + 16 \times 24 = \_\_\_

19. Ms. Christensen asked Carla to simplify the following expression.

$$\frac{10a + 4}{2a}$$

Carla wrote the following incorrect solution.

$$\frac{10a + 4}{2a} = 10 + 2 = 12$$

Ms. Christensen then asked the class what was wrong with this solution. Which of the following student explanations characterizes what was most likely wrong with Carla’s solution?

A) She should have written $2(5a + 2)$ in the top and then canceled the 2s in the top and the bottom.

B) She saw you can break the fraction into two fractions, but the way she simplified each fraction is wrong.

C) She divided the 4 by 2, but you cannot cancel when you have more than one thing added in the top.

D) It’s not possible to cancel like this because you can only cancel factors that are the same for the top and the bottom.
20. Ms. Lindsey’s textbook uses a geoboard to model slope as rise over run. (Geoboards are flat blocks of wood, roughly one foot square, with pegs laid out on a grid where rubber bands can be hooked to make lines or polygons.) As her students explored different slopes they could make on the geoboard, Edward asked, “Since the diagonal of one of the unit squares has length \( \sqrt{2} \), does that mean you can make a line segment with slope \( \sqrt{2} \) on the geoboard?” When Ms. Lindsey asked the class whether they thought this could be done, the following exchange occurred.

Andy: Edward’s right that the diagonal of the unit square has length \( \sqrt{2} \), but its slope is 1.

Beth: Well, that doesn’t matter. We can just turn the geoboard so that the diagonal is horizontal, and then we can see squares with side length \( \sqrt{2} \).

Caitlin: Sure, but the square roots of two would just cancel. I think they always would, so you can’t get \( \sqrt{2} \) as a slope.

Dan: That’s not right, because we can make one length of \( \sqrt{2} \) and another length of 1 and use them as the rise and the run.

Which of the student statements gives the best insight into Edward’s question?

A) Andy’s statement
B) Beth’s statement
C) Caitlin’s statement
D) Dan’s statement

21. Ms. Collingwood is teaching a unit on graphing. Some of the students in her class speak a language other than English at home and when they work in small groups. As she prepares for the unit, she makes a note of vocabulary words that she believes will be challenging for her students, especially words that have different meanings in different contexts. Of the following vocabulary words that she identified, which will require the least clarification regarding differences in meaning?

A) Plane
B) Origin
C) Intercept
D) Parabola
22. Mr. Baas’ students were solving inequalities. Cheryl wrote the following solution on the board.

\[
\frac{x - 2}{x} < 1
\]
\[
x - 2 < x
\]
\[
0 < 2
\]

She concluded that because this is always true, every \( x \) would work. After some discussion, students decided that Cheryl’s solution was not correct, but they were unsure why. Of the following explanations, which best identifies what is problematic about Cheryl’s work on the problem?

A) It’s true that 2 is always greater than 0, but because you have eliminated all \( x \), we cannot say what \( x \) is.

B) We can see that the numerator always is two less than the denominator, so the fraction will always be less than 1 for all \( x \). However, we have to require the denominator \( x \neq 0 \).

C) You don’t know what \( x \) is, so when you multiply with \( x \) like this, you must assume \( x > 0 \).

D) She should have simplified the left-hand side of the inequality to \( 1 - \frac{2}{x} \) and then subtracted 1 from both sides and added \( \frac{2}{x} \) to both sides. This would yield \( 0 < \frac{2}{x} \), which is true for \( x > 0 \).

6. Class: Freshman Sophomore Junior Senior Prefer not to answer

7. How do you think you did on the assessment? Discuss your confidence with the answer choices you selected:

8. Select your teacher preparation program:

9. Which mathematics and mathematics methods courses have you COMPLETED? (Please check only the boxes for courses you’ve completed (and passed). The NEXT question will ask about classes you are currently enrolled in. If you took mathematics courses at another university, please select those courses that most closely resemble the ones you completed.)

10. Did you take any mathematics or mathematics methods courses at a DIFFERENT college or university? If so, list the titles of those courses:
11. Semester and year you plan to student teach:

12. Did you originally declare secondary mathematics education as your major (sometimes called the mathematics teaching option)?

13. What semester and year did you declare secondary mathematics education as your major (sometimes called the mathematics teaching option)?

14. What was your major(s) prior to declaring a teaching major?

15. Describe any experiences you have had working with learners, whether those experiences were related to mathematics or in other areas. Describe the learner and the subject matter for each experience. (Examples include: Math tutoring with 5th graders in an afterschool program for 3 years; 60 hour field placement in a 7th grade mathematics classroom; helping classmates/friends/siblings with mathematics homework during homeroom in high school; piano lessons given to all ages for 1 year)

16. At about what year in school did you start wanting to be a mathematics teacher? Before Kindergarten Grades K-5 Grades 6-8 Grades 9-12 After high school

17. What events and experiences influenced your decision to become a high school mathematics teacher? (These may include specific people and incidents inside or outside of school.)

18. Enter your age and gender: (If you prefer not to answer, type “Prefer not to answer”)

19. Did you graduate from a high school [in this state]? Yes No

20. Would you be willing to participate in a 90 minute interview about your teacher preparation experiences later this semester or next semester? (Some students who complete this survey will be selected for a follow-up interview. I will be able to schedule
the interview around your classes and other commitments. Anyone willing to participate in an interview will get a SECOND chance to win a Big Sky Resort LIFT TICKET for the 2015/2016 ski season or $50 CASH (your choice if your name is drawn)

Yes    No
APPENDIX B:

INTERVIEW GUIDE
1) Establish Trust

Last semester, you completed a survey about your decision to become a teacher, and completed a multiple choice exam. I would like to start by talking about some of your survey responses.

a) Are you taking any math classes this semester? (You took ____ last semester) Tell me about those classes.

b) In your view, what kind of specialized mathematics knowledge does a high school mathematics teacher need?

i) Prompt if needed: Aside from the skills needed for classroom management, what is it that a high school mathematics teacher needs to know about mathematics to be a good teacher?

ii) Prompt if needed: Aside from the typical mathematics content that you learn in a conventional mathematics course (like college algebra, pre-calculus, calculus, etc.), what other types of mathematics would a successful mathematics teacher need to know?

iii) Prompt if needed: What kind of pedagogical mathematics knowledge might a high school math teacher need?

iv) Prompt if needed: Can you think of any mathematics content that a mathematics teacher might need to know that other professions relating to mathematics would not need to know?

c) How, if at all, has your view of the specialized mathematics knowledge needed for high school mathematics teaching changed over time?

i) Prompt if needed: How have your experiences during your teacher preparation program influenced your view of high school mathematics teaching?

2) Revisit MKT Exam Items

[Items 9, 10, 12, and 22 have the most potential to reveal experiences for the development of SCK. Ask the following three questions for each item. Ask for at least 2 items, but up to 5, time permitting. When possible, start with two items that were answered correctly, and select the third question to be one that was answered incorrectly.]

9/10/12/22 c/b/b/c

a) I'll give you a minute to look over this problem. Walk me through your thought process as you select an answer.

i) Prompt (if participant changes answer): You originally chose a/b/c/d. Do you remember why you chose that answer?
i) What makes you think it's a/b/c/d now?

iii) What made you change your thinking?

b) What experiences have you had that helped you answer this item?

i) Prompt (if needed): Has there been anything in your coursework, field experiences, or other opportunities in mathematics that relate to this problem?

3) Reflection (How did you come to be able to answer the exam items?)

[Participants will be given a copy of the entire exam and a few minutes to look it over and jog their memory of the remaining items not discussed.] Take your time and look over the entire exam to help jog your memory about the types of items on the exam. We won't be talking about any more individual items in detail, but next I'd like to ask you some questions that pertain to the entire exam.

a) Tell me how you go about answering the items on this exam. What do you do? What do you think about?

b) Ball, Thames, & Phelps (2008) define specialized content knowledge (SCK) as, "the mathematical knowledge and skill unique to teaching" and the "mathematical knowledge not typically needed for purposes other than teaching." (p. 400) The items in this exam are assessing some examples of SCK. Do you understand what specialized content knowledge is, based on these definitions? [If not, further explain SCK] We just talked about your past experiences in relation to your choices on individual exam items. How do you think your past experiences and influences helped you with developing Specialized Content Knowledge?

i) Prompt if needed: On the survey, I see that you tutored/field exp/etc, how did that help you with developing specialized content knowledge [ask for each experience reported]?

c) Do you think it is important for a teacher to have specialized content knowledge?

d) As you look back on your teacher preparation and experiences you've had since you decided to become a teacher, are there any other events that stand out in your mind as preparing you for an exam like this? Please describe them and how they affected your ability to answer the exam items.

e) As you look back on your teacher preparation and experiences you've had since you decided to become a teacher, are there any events that stand out in your mind as NOT preparing you to become an effective teacher? Please describe them.
f) If you were developing a teacher preparation program for high school mathematics teachers, what experiences would you include to help prepare students for an exam like this?

g) On the survey, you mentioned that you decided to become a math teacher because __________. Tell me more about that. [Tell me about how you decided to become a teacher.]

i) Prompt if needed: When did you first decide that you wanted to become a teacher?

ii) Prompt if needed: Who, if anyone, influenced your decision to become a teacher? Tell me how they influenced you.

iii) Prompt if needed: Could you describe the events that led up to you entering the teacher preparation program.

iv) Prompt if needed: Is there anything else that you can think of that influenced you to become a teacher?

h) Is there anything you would like to ask me?

i) Is it okay if I contact you for clarification on things you said today, or for a brief follow up interview later in the year? If so, what is the best way to contact you?

Interview questions were written with "Examples of Grounded Theory Interview Questions" (Charmaz, 2001, p. 679-680) as inspiration.
APPENDIX C:

MEMO TITLES
Experiences
Being a student
Conferences
Connections between Math Subjects
Coursework that develops SCK
Experiences with (non-math) learners
Student teaching, field experiences, and similar teaching experiences
Taking coursework alongside elementary education majors
Teaching or learning SCK directly
Tutoring

Misconceptions
Mathematical Misconceptions
Misinterpreting SCK Assessment items or answer choices

Participant Profile
Coursework
Decision to become a teacher
Nontraditional students
Success as a student of mathematics

Role Models
Bad experiences with teachers
Parental influences
Quality of professors and other mentors
What makes a good teacher

SCK
Defining mathematics
Defining specialized content knowledge
Difficulty of the SCK Assessment
Necessity for SCK

Teaching Practices
Effective tutoring (and maybe teaching) strategies
Practicality of learned teaching practices

Ways of Approaching SCK Assessment Items
Approaching the items as a teacher
Approaching the items by comparing the student’s work to your own
Approaching the items by considering student thinking
Approaching the items by considering the quality of explanation and information provided
Approaching the items like proofs
Approaching the items using guess and check
Approaching the items using mathematics
Atypical approaches to items
Using test-taking strategies to answer items

Uncategorized Memos
Change in viewpoint of what teachers need to know