

# Chapter 7

## Systems Analysis of Microbial Adaptations to Simultaneous Stresses

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**Abstract** Microbes live in multi-factorial environments and have evolved under a variety of concurrent stresses including resource scarcity. Their metabolic organization is a reflection of their evolutionary histories and, in spite of decades of research, there is still a need for improved theoretical tools to explain fundamental aspects of microbial physiology. Using ecological and economic concepts, this chapter explores a resource-ratio based theory to elucidate microbial strategies for extracting and channeling mass and energy. The theory assumes cellular fitness is maximized by allocating scarce resources in appropriate proportions to multiple stress responses. Presented case studies deconstruct metabolic networks into a complete set of minimal biochemical pathways known as elementary flux modes. An economic analysis of the elementary flux modes tabulates enzyme atomic synthesis requirements from amino acid sequences and pathway operating costs from catabolic efficiencies, permitting characterization of inherent tradeoffs between resource investment and phenotype. A set of elementary flux modes with competitive tradeoffs properties can be mathematically projected onto experimental fluxomics datasets to decompose measured phenotypes into metabolic adaptations, interpreted as cellular responses proportional to the experienced culturing stresses. The resource-ratio based method describes the experimental phenotypes with greater accuracy than other contemporary approaches and further analysis suggests the results are both statistically and biologically significant. The insight into metabolic network design principles including tradeoffs associated with concurrent stress adaptation provides a foundation for interpreting physiology as well as for rational control and engineering of medically, environmentally, and industrially relevant microbes.

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## Abbreviations

$^{13}\text{C}$	carbon 13
$C_{\text{fumarate}}$	concentration of fumarate
$C_{\text{inv}}$	carbon investment
$\text{Cmole}$	carbon mole
$\text{Cmol glc}$	$\text{Cmol glc}$ , carbon moles of glucose
$\text{Cmol X}$	$\text{Cmol X}$ , carbon moles of biomass
$C_{\text{op,X}}$	carbon operating cost for growth
$\text{C}\mu\text{M}$	micromoles of carbon per liter of cytosol
$E$	matrix containing ecologically important elementary modes
$[\text{E}]$	enzyme concentration
EFM	elementary flux mode
EFMA	elementary flux mode analysis
FBA	flux balance analysis
$k_{\text{cat}}$	catalytic turnover number
$K_{\text{m}}$	half-saturation constant
MFA	metabolic flux analysis
$N_{\text{inv}}$	nitrogen investment
$N_{\text{inv,X,1:1}}$	nitrogen investment for growth, minimalist flux-to-enzyme approach
$O_{2,\text{op,X}}$	oxygen operating cost for growth
$\mathbf{v}$	vector containing fluxes
$v_{\text{max}}$	maximum enzyme-catalyzed reaction rate
$\mathbf{w}$	vector containing weighting factors

## 7.1 Introduction: Resources and Life

Life is driven by mass and free energy extracted from the environment. The immense diversity of life highlights the variety of these mass and energy forms as well as the array of successful strategies for exploiting them. Metabolic networks channel these resources under an array of environmental constraints and stresses using competitive sets of gene products (Elser et al. 2007). The discipline of ecology has explicitly and implicitly studied mass and free energy flows on a continuum of size scales for more than 100 years. Central themes of many ecological analyses include resource investment strategies, energetic efficiencies of different physiologies and the tradeoffs between the two. These theories, developed and tested on macroscale systems, are useful for decoding the rapidly accumulating omics databases representing microbial-scale systems.

Scarcity of resources over evolutionary time is thought to have influenced the elemental make-up of microbes (Dekel and Alon 2005; Elser et al. 2011; Makino et al. 2003; Sterner and Elser 2002; Zinn et al. 2004). For instance, elemental availability is believed to have influenced the evolution of microbial DNA and amino acid sequences (Bragg and Hyder 2004; Bragg and Wagner 2009). The chromosome sequence of *Pelagibacter* is biased toward low nitrogen containing codons, presumably as a response to the extreme nitrogen scarcity in oceanic ecosystems (Giovannoni et al. 2005). Proteins involved with carbon, nitrogen, and sulfur acquisition contain less of the respective element than average proteins in both prokaryotic and eukaryotic microorganisms (Baudouin-Cornu et al. 2001; Elser et al. 2011). Highly expressed proteins are also thought to be influenced by ‘elemental sparing’ where the relative material synthesis costs of limiting resources are reduced (Bragg et al. 2012). Strategic use of limiting resources also extends to enzyme cofactors. Respiratory chains require large investments of iron; microbes native to low iron environments often shift from iron containing enzymes to non-iron requiring enzymes such as from ferredoxin to flavodoxin under conditions of iron scarcity (Erdner and Anderson 1999).

It is hypothesized that not only are macromolecule sequences and cofactor requirements influenced by resource scarcity, but also the structure and regulation of entire biochemical networks. Selective pressures have eliminated microbes that fail to allocate limiting resources to cellular functions providing the most competitive return on investment (Carlson 2007, 2009; Dhurjati et al. 1985; Molenaar et al. 2009; Wessely et al. 2011). The return on investment can reflect a multitude of fitness properties including toxin production or resistance, attachment to surfaces or chemotaxis, osmolyte synthesis or salvage, and high-yielding or high-rate extraction of free energy. The role of high-yielding, metabolic efficiency on cellular physiology has been established both theoretically and experimentally (Carlson and Sreenc 2004b; Varma and Palsson 1993; Westerhoff et al. 1983).

This chapter details a theory for decoding the multiplicity of microbial strategies to extract and channel mass and energy flows under simultaneous environmental stresses, with a focus on resource scarcity. Elucidating tradeoffs between resource availability and microbial physiology provides a theoretical basis for systemic analyses of omics data, and a rational basis for controlling microbes in medical, environmental and industrial applications.

## 7.2 *In Silico* Analysis of Metabolic Systems

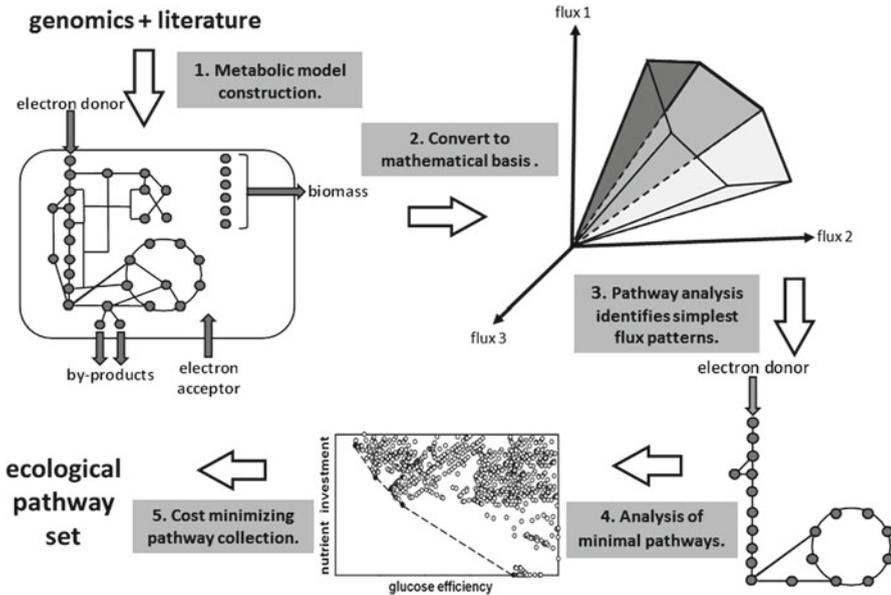
Notable advances have been made in understanding the basis for metabolic functioning, but there is still need for guiding network design principles (Huang 2000; Papp et al. 2009). These principles need to be investigated through computer-based analyses integrating the numerous metabolic components and their interactions into testable models. For instance, any broad search of the interconnected fluxes of atoms, electrons, and energy in large-scale reaction networks is simply beyond manual feasibility.

Applications of biochemical kinetic models, typically systems of coupled ordinary differential equations describing the evolution of concentrations in time, have illuminated microbial processes for decades (e.g. Kargi and Weissman 1987; Nielsen and Villadsen 1992; Smallbone et al. 2010; Steuer et al. 2006; Straight and Ramkrishna 1994; Varner 2000). Unfortunately, these approaches when applied to metabolic networks are typically either simplistic in terms of the number of processes considered, or in terms of the mechanistic model, or require extensive sets of condition-specific kinetic parameters. Published kinetic parameters, as compiled by databases like BRENDA (Chang et al. 2008; Schomburg et al. 2004), can vary over several orders of magnitude for the same enzyme, requiring expert knowledge for appropriate incorporation into biological models and interpretation of results (Teusink et al. 2000).

An alternative class of successful *in silico* methods, circumventing requirements for large condition-sensitive parameter sets, is known as stoichiometric modeling. Stoichiometric models extract systemic information from conservation relationships and molecular-level network structure, frequently deduced from omics datasets (Reed and Palsson 2003; Trinh et al. 2009). There are three major stoichiometric modeling approaches: metabolic flux analysis (MFA), linear programming methods (also known as flux balance analysis or FBA), and metabolic pathway analysis (including elementary flux mode analysis or EFMA). All three methods define a high-dimension solution space of physiologically permissible metabolic flux distributions based on a stoichiometric matrix specifying system conservation relationships. This solution space is often represented as a pointed convex cone, although the shape can vary depending on the reversibility properties of the reactions (Llaneras and Picó 2010; Wagner and Urbanczik 2005). The three approaches differ in how metabolic flux distributions are selected from the solution space. Detailed descriptions and comparisons of these methods can be found elsewhere (Blank and Kuepfer 2010; Reed and Palsson 2003; Schilling et al. 1999; Trinh et al. 2009).

The continuous solution space defined by stoichiometric methods such as FBA or EFMA contains an effectively infinite number of mathematical possibilities. Identifying biologically and ecologically relevant flux distributions in this continuum requires selection criteria. Maximization of biomass yield on substrate is a widely assumed basis for identifying a cellular physiology. The measure has an appealing simplicity, reflecting an ecologically reasonable structure-function relationship for metabolic networks. This criterion has been utilized successfully to predict and interpret microbial behaviors, including *Escherichia coli* grown in glucose-limited chemostats at modest dilution rates (Carlson and Srienc 2004b; Fong et al. 2003; Fong and Palsson 2004; Varma et al. 1993; Varma and Palsson 1994). However, biomass yield maximization does not always adequately describe metabolic behaviors such as batch growth, or chemostat growth under conditions of nitrogen limitation (Carlson 2007, 2009; Papp et al. 2009; Schuetz et al. 2007; Schuster et al. 2008, 2011).

Because of its role in the current chapter, EFMA will be discussed briefly here. A more detailed description can be found in Chap. 2 of this book which is dedicated to EFMA. EFMA begins with the construction of a stoichiometric matrix representing



**Fig. 7.1** Schematic representation of elementary flux mode analysis (EFMA) model building and cost-benefit analysis. EFMA model building and cost-benefit analyses involve the following steps: (1) metabolic model construction using genomic information and literature resources; (2) model conversion into a mathematical basis, whereby the reaction stoichiometries and reversibilities constrain steady-state cellular phenotypes to flux distributions within a space known as a flux cone; (3) EFMA decomposition of a reaction network into its simplest steady-state pathways, called elementary flux modes (EFMs); (4) cost-benefit analysis translates the EFMs into resource costs based on biochemical and genomic information; (5) minimizing combinations of these resource costs identifies competitive EFMs, collected as ecologically relevant pathway sets

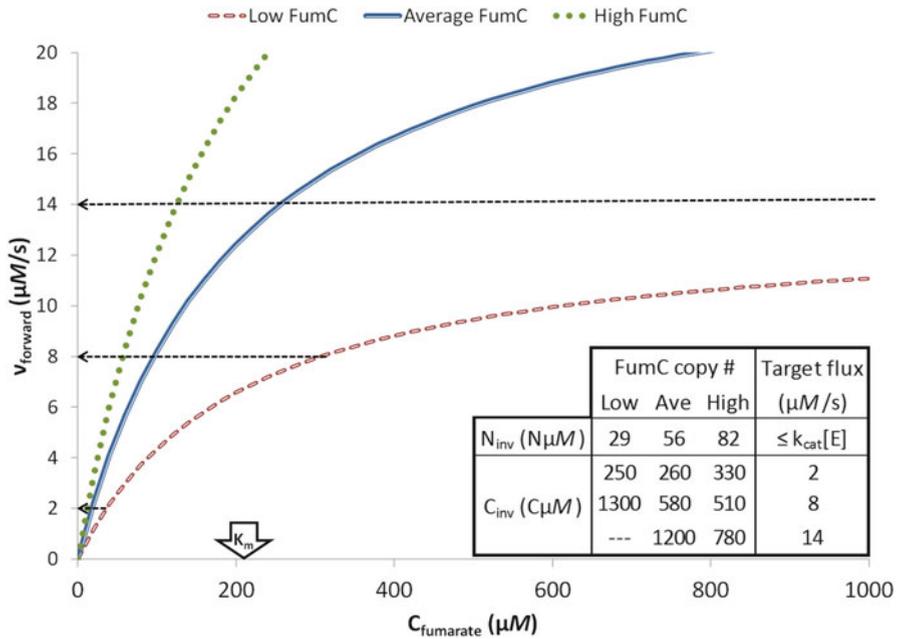
the set of biochemical reactions to be considered. These are frequently compiled based on enzymes annotated from a genomic or metagenomic dataset (Fig. 7.1). The mathematical representation of the model encompasses all thermodynamically relevant system flux distributions. The complete set of enzymatically unique, minimal steady-state pathways spanning this permissible space is known as the elementary flux modes (EFMs) (Gagneur and Klamt 2004; Klamt et al. 2005; Schuster and Hilgetag 1994; Schuster et al. 2000). EFMs allow straightforward investigation of a network's metabolic potential from the bottom up (Klamt and Stelling 2003; Llaneras and Picó 2010; Papin et al. 2004; Trinh et al. 2009). These simplest pathways represent building blocks for steady-state metabolism, and their linear combinations allow system-level analyses of individual microbes, pure cultures, and consortia (e.g. Carlson et al. 2002, 2005; Carlson 2007, 2009; Klamt et al. 2008; Poolman et al. 2004; Taffs et al. 2009; Trinh et al. 2006, 2008; Wlaschin et al. 2006; Zhao and Kurata 2009).

### 7.3 Analyzing Metabolic Networks with Molecular Level Resource-Ratio Theory

The shortcomings of the often used ‘biomass maximization on substrate’ criterion highlight the need for additional theoretical frameworks for explaining microbial behaviors. The ecological fields are rich in theoretical approaches for analyzing fitness in various environments. In addition, decades of economic research have highlighted the importance of strategic resource allocation. These theoretical frameworks are well-suited for describing and interpreting many levels of biological organization, from molecular-level metabolic systems to entire ecosystems (Carlson and Taffs 2010). Resource-ratio theory, from the discipline of ecology, is a unifying approach for understanding shared resource competition and investment (Tilman 1980). A recent meta-analysis found that experimental tests supported predictions based on resource-ratio theory 75% of the time, making it one of the most successful theories in ecology (de Mazancourt and Schwartz 2010; Miller et al. 2005).

An approach similar to resource-ratio theory has been applied to *in silico* metabolic networks to quantify the fitness of the genome encoded metabolic potential subjected to pairs of scarce resources (Carlson 2007, 2009). It is hypothesized that evolutionary selection has favored regulation schemes directing resource investment toward effective, stress-specific metabolic pathways. The methodology examines tradeoffs between possible genome-encoded physiologies, represented as EFMs. Cells with regulation schemes permitting effective use of genomic potential would gain a fitness advantage under the relevant stress. To date, the approach has been applied to combinations of two limiting resources which can be either catabolic or anabolic in nature (Carlson and Srienc 2004b; Carlson 2007, 2009). This metabolism-focused application is a departure from traditional resource-ratio theory, which analyzes competition between different organisms for shared resources, rather than comparing potential phenotypes available to a single organism. The approach also shares conceptual similarities with the efficient frontier curve traditionally utilized in economic risk-return analysis and recently applied to biological systems (Kitano 2010).

Case studies in molecular level resource-ratio analysis decomposed *in silico* stoichiometric models of *E. coli* into EFMs (Carlson and Srienc 2004b; Carlson 2007, 2009). As mentioned previously, EFMs are minimal biochemical pathways comprised of metabolite transport and chemical reactions; the enzyme-based steps require an investment of anabolic resources like nitrogen or iron. EFMs, in this context, represent theoretical proteomes for which investment requirements can be tabulated. Calculation of investment requirements necessitates an assumed relationship between fluxes and enzyme concentrations. This relationship varies depending on specific enzyme properties, as well as the chemical environment. Two scenarios are proposed as bounds on the pathway-level flux-to-enzyme concentration relationships. The first approach is a minimalist relationship between pathway enzymes: the concentration ratio of every enzyme pair is set to one. This scenario would utilize varying metabolite pool size and activity modulation including allosteric regulation to achieve a specified flux distribution (this flux-to-enzyme model



**Fig. 7.2** Forward flux through the *E. coli* fumarase FumC (ordinate) as a function of fumarate concentration (abscissa), for three enzyme concentrations (*curved traces*). Each of the dashed *horizontal lines* represents a target flux, i.e. a fitness objective. The intersections of these lines with the various traces represent alternative investment strategies equally capable of achieving the fitness objective. The carbon and nitrogen investments ( $N_{\text{inv}}$  and  $C_{\text{inv}}$ ) are quantified in the inset table; they represent a sum of both enzyme and metabolite pool investments. Michaelis-Menten kinetics were assumed, and the reverse reaction was neglected. Kinetic parameters for *E. coli* FumC were taken from the literature (Estévez et al. 2002), and the substrate concentration associated with half-saturation of FumC is indicated on the x-axis with an *arrow*. The plotted range of fumarate concentrations is well within intracellular measurements of *E. coli* from the literature (Bennett et al. 2009). The ‘low’, ‘average’, and ‘high’ enzyme concentrations were estimated based on intracellular copy numbers measured in *E. coli* (Taniguchi et al. 2010), assuming a cylindrical cell 2  $\mu\text{m}$  long with a 1  $\mu\text{m}$  diameter

was previously termed the first order method). The second flux-to-enzyme approach assumes a flux proportional relationship. Enzyme concentration ratios for any pathway enzyme pair are proportional to flux ratios through the corresponding reactions. The proportional scenario would use differences in enzyme concentration as the major controller of flux (this model was previously termed the zeroth order method). A comparison of the minimal and proportional flux-to-enzyme approaches to experimental fluxomic data suggests the minimalist approach is a better approximation for *E. coli*, although many additional, potentially relevant, relationships are imaginable (Carlson 2009).

The two flux-to-enzyme approaches are compared in Fig. 7.2 using experimental data for the *E. coli* fumarase enzyme FumC. A flux, representing a fitness objective, can be realized by a number of different proteomes, depending on the associated

metabolome. Two central concepts are illustrated in Fig. 7.2 by the intersections of the fitness objective fluxes (dashed lines) and the enzyme kinetics traces. First, each target flux can be driven by multiple combinations of enzyme and substrate concentrations. Second, low enzyme concentration may not support a high target flux, no matter how high the substrate concentration. These ideas illuminate tradeoffs between alternative investment strategies for achieving the same phenotype.

A straightforward tradeoff exists between investment of substrate into enzymes and into metabolites. Steady state metabolite pools are an investment because stationary concentrations represent substrate that is unavailable for other processes. The optimal strategy in terms of this tradeoff isn't universal: it depends in complex fashion on kinetic parameters, enzyme and substrate stoichiometry, and the target flux. Comparing the second and third lines of the inset table from Fig. 7.2 shows that it takes less carbon investment to drive very small fluxes through FumC with a large fumarate pool, but larger fluxes can be driven more economically with a large enzyme pool. This effect, which may seem counterintuitive initially, is a result of extreme differences in investment per molecule of enzyme versus per molecule of substrate.

Tradeoffs are environment specific and investment of an abundant resource into metabolite pools may not represent a fitness burden. The first line from the Fig. 7.2 inset table illustrates this point: if nitrogen limitation is the only relevant stress, it may be competitive to express the smallest FumC concentration capable of supporting a given target flux, even if that requires a heavy investment of carbon into metabolite pool. Additional environmentally-dependent tradeoffs are also noteworthy. In an environment that changes rapidly, the target flux may increase faster than a microbe can reorganize its proteome. In those environments, it may be competitive to overbuild enzymes for the current condition. As an example, the second line of the inset table from Fig. 7.2 implies that, in a carbon-limited environment with a low flux target, it is effective to drive that flux via the metabolite pool ( $C_{inv} = 250$  vs.  $330$   $C_{\mu M}$ ). This conclusion, however, rests on the assumption that the flux target won't suddenly and drastically increase. If it did, the final line of the inset table from Fig. 7.2 demonstrates that the new flux target might exceed the capacity of the small enzyme pool, providing a competitive advantage to the 'overbuilt' phenotype.

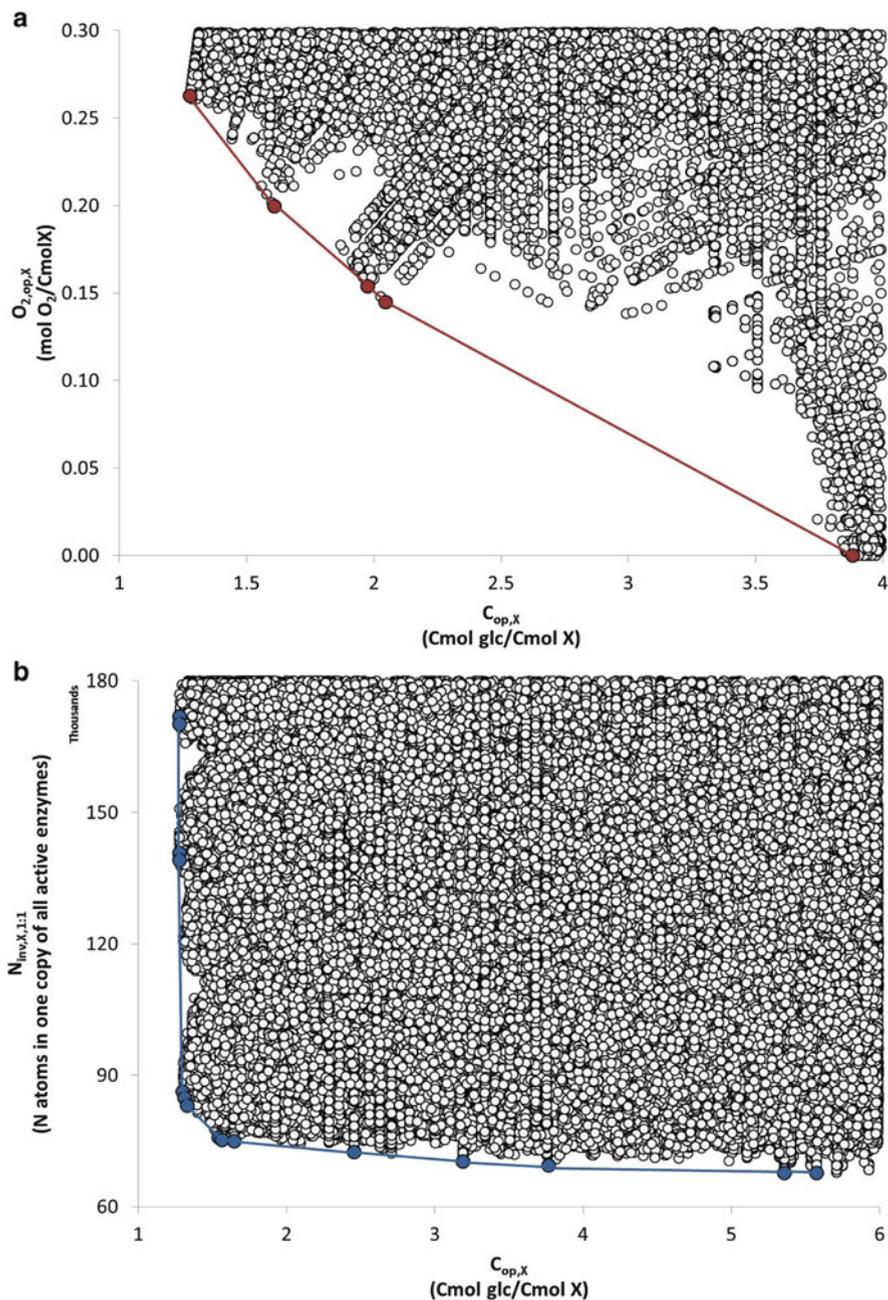
## 7.4 Constructing Resource Pair Tradeoff Curves

EFM anabolic resource investment requirements can be analyzed in concert with a catabolic physiological fitness metric termed an operating cost. EFM operating costs are the amount of substrate, typically an electron donor or acceptor, required to synthesize a specified amount of cellular product, such as one Cmole of biomass or one mole of ATP: equivalent to inverted yields. The operating cost reflects the net conversion efficiency of the pathway. The relationship of investment requirements against operating costs ranks EFMs efficiently based on combined resource use minimization.

Minimizing combined EFM resource costs for the same cellular product identifies a tradeoff curve, also known as a ‘cost minimization envelope’ (Carlson and Sreenc 2004a, b). Figure 7.3 illustrates two tradeoff curves for the *E. coli* central metabolism. Figure 7.3a is an electron donor (glucose) vs. electron acceptor (oxygen) tradeoff curve while Fig. 7.3b is an electron donor (glucose) vs. anabolic resource (nitrogen) investment cost tradeoff curve. The plots are graphical illustrations of resource-ratio planes defined by the genome encoded metabolic potential. Each circle represents one distinct EFM. The EFM coordinates are the costs in terms of the two plotted resources to synthesize one Cmole of biomass. Physiologies that minimize requirements for resources are hypothesized to represent ecologically and economically competitive strategies. The leftmost EFM along the electron donor axis (abscissa in Fig. 7.3a, b) minimizes the cost of biomass synthesis on glucose; however, this reduction is offset by a higher requirement for the resource plotted on the ordinate. High tradeoff curve resource-ratios (paired resource/electron donor) permit increasingly efficient extraction of substrate free energy, up to a system constrained maximum. Moving right along a tradeoff curve maps competitive physiologies as the resource-ratio of either oxygen or nitrogen decreases relative to glucose. Lowering the resource-ratio increases glucose operating costs (Fig. 7.3).

Tradeoff curves represent combinations of enzymatic steps that provide the highest possible free energy yield based on the interaction between network stoichiometry and resource availability ratios. When the ratio of oxygen or nitrogen to carbon is small, tradeoff analysis indicates it is competitive for *E. coli* to down regulate the citric acid cycle, extracting easily accessible free energy from glycolysis while secreting partially oxidized metabolic intermediates (e.g. acetic or lactic acid). Retaining the ‘lost’ material and energy through further oxidation would represent a poor investment of the scarce resource. When the same resource ratios are high, the EFMs populating the left extreme of the tradeoff curve indicate it becomes ecologically competitive to completely oxidize glucose. The slopes of a tradeoff curve formalize the diminishing return of free energy extraction from substrate relative to efficient use of the second resource. These specific results are based on the physiological potential of *E. coli*, and general results depend on an organism’s genome-encoded potential.

Physiological responses consistent with resource-ratio theory can have tradeoff properties that seem initially contradictory. For instance, the phenomena of microbial overflow metabolisms have been reported under a variety of conditions, including both nutrient excess and scarcity. The widespread occurrence of this metabolic strategy suggests there are fundamental adaptive principles guiding microbial responses (El-Mansi and Holms 1989; El-Mansi 2004; Majewski and Domach 1990; Neijssel et al. 1996; Straight and Ramkrishna 1994; Teixeira de Mattos and Neijssel 1997). The presented resource-ratio approach provides a single theory to explain overflow metabolism for both conditions based on small resource availability ratios. Under conditions of unrestricted environmental resources, the cellular resource ratio is constrained by the ratio of the corresponding transporter capacities ( $V_{\max}$ ). An imbalance in transporter capacity can create nutrient limitation stress responses even when



**Fig. 7.3** Tradeoff curves for biomass synthesis in *E. coli* MG1655. Each circle represents an elementary flux mode (EFM) producing one carbon mole of biomass; dark filled circles and line segments represent flux distributions minimizing the combined costs plotted on the respective axes. Both abscissae represent glucose operation costs ( $C_{op,X}$ ): the amount of glucose required to

the limiting resource is present at high levels, relative to enzyme half-saturation values. Under conditions of nutrient scarcity, however, availability depends on environmental supply and transporter affinity ( $K_m$ ). The resource-ratio-based analysis of metabolic potential predicts a physiology consistent with an overflow metabolism in both cases.

Additional *in silico* studies have investigated alternative enzyme investment proxies, including the minimization of total flux or number of biochemical steps (de Figueiredo et al. 2009; Hoffmann et al. 2006; Holzhütter 2004; Poolman et al. 2004; Stelling et al. 2002), a summation of enzyme set molecular weights or volume (Beg et al. 2007; Vazquez et al. 2008a, b; Wessely et al. 2011), and a number of alternatives (Schuetz et al. 2007). While the approaches don't explicitly consider resource-ratio theory, they do consider allocation of resources, and sometimes arrive at similar predicted physiologies. An explicit consideration of resource investment has advantages when the resource is not uniformly distributed across proteins like the cofactor iron. Recent simulations and experiments in our laboratory demonstrated that the iron-limited physiological response is distinct from nitrogen limitation in *E. coli*, and that these differences map well to a resource-ratio interpretation.

## 7.5 Decomposing Fluxomics Data into Simultaneous Stress Adaptations

Microbes live in complex environments and are exposed frequently to simultaneous demands (Elser et al. 2007). Adaptations to maximize fitness require allocating limiting resources in optimal proportions between multiple cellular responses. These adaptations are proposed to be predictable using economic, evolutionary, and ecological theory (Bloom et al. 1985; Carlson and Taffs 2010; Kitano 2010; Papp et al. 2009; Perrin and Sibly 1993). In fact, dynamic modeling methods have been applied for decades to study simultaneous environmental pressures (Bader 1978; Kooijman 2000; Molenaar et al. 2009; Straight and Ramkrishna 1994), but these models are not yet able to fully capitalize on the omics revolution. Continuing advances in stoichiometry-based network models, however, are enabling systems-wide modeling efforts to understand microbial resource allocation.

A recent study used the stoichiometric modeling approach EFMA to decode potential *E. coli* adaptations to multiple stresses (Carlson 2009). EFMs from the *E. coli* model were translated into resource costs, and a set of ecologically relevant EFMs were assembled from cost minimizing tradeoff curves (Fig. 7.3). Each individual

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**Fig. 7.3** (continued) build one Cmole of biomass using a pre-existing proteome. The ordinate in subfigure (a) represents oxygen operation cost ( $O_{2,op,x}$ ); a similar metric for oxygen. The ordinate in subfigure (b) represents nitrogen investment cost ( $N_{inv,x}$ ): the amount of nitrogen contained in the proteome supporting each EFM flux distribution. The plotted investment costs were calculated using the minimalist flux-to-enzyme method

EFM consists of a unique enzyme pattern, providing a basis for flux fingerprinting. The flux distributions were projected onto experimental  $^{13}\text{C}$  fluxomics datasets (Schuetz et al. 2007) to decompose measured phenotypes into metabolic responses, interpreted as proportional to the experienced culturing stresses.

The mathematics behind this analysis used experimentally measured  $^{13}\text{C}$ -based flux distributions ( $\bar{v}$ ) and the set of ecologically competitive elementary modes ( $E$ ) identified through paired cost minimizations. The fluxomic data described the *E. coli* central carbon metabolism using ten degrees of freedom; the EFMs were projected into the same ten dimensional space to permit comparison. The proportional contribution of each individual EFM to the overall phenotype is given by a weighting vector  $\bar{w}$  such that:

$$\bar{v} = E\bar{w} \quad (7.1)$$

where  $E$  is a matrix containing the elementary modes as column vectors. The vector  $\bar{w}$  was determined using nonnegative least squares analysis to solve the following problem:

$$\text{Min}_{\bar{w}} \|E\bar{w} - \bar{v}\|; \forall i : w_i > 0 \quad (7.2)$$

Only positive elements were considered for  $\bar{w}$  to limit considerations to biologically meaningful solutions; biomass- and ATP-synthesizing EFMs are not reversible. The magnitude of each weighting element represents the proportional contribution of the corresponding EFM to the mathematical description of measured cellular physiology.

In the described study (Carlson 2009), the *E. coli* metabolic network contained 197,018 distinct EFMs; the set  $E$  consisted of 38 distinct pathways identified from tradeoff curve analysis. Only three or four of these pathways were needed to best describe each of the four experimentally measured metabolic phenotypes ( $\bar{v}$ ). These phenotypes comprised unrestricted batch growth, two carbon-limited chemostats (dilution rates of 0.2 and 0.4  $\text{h}^{-1}$ ), and a nitrogen-limited chemostat (dilution rate of 0.2  $\text{h}^{-1}$ ). The best fit of  $\bar{v}$  for each of the four experimental distributions included EFMs from multiple tradeoff curves, suggesting that every culture investigated was responding to simultaneous stresses. For instance, the batch growth flux distribution was best described as a combination of metabolic strategies likely to be competitive for oxygen and nitrogen limitation, as well as an optimal biomass synthesis pathway. The description is attractive considering the often-reported overflow metabolism associated with *E. coli* unrestricted batch growth (e.g. Meadows et al. 2010; Xu et al. 1999).

In addition to its promising qualitative properties, the accuracy of the *in silico* description was compared quantitatively to an extensive collection of FBA-based flux distributions using the Euclidean distance metric. The set of ecologically selected EFMs described the experimental data with better accuracy than any of the tested FBA-based distributions. The best FBA descriptions required different objective functions for different culturing conditions. The most accurate reported FBA-based

description for the experimental batch growth data utilized the maximization of ATP yield per flux unit objective function (Schuetz et al. 2007).

The nonnegative least squares-based predictions were tested for statistical and biological significance by comparing the ability of 10,000 randomly selected EFM sets (of the same size) to describe the experimentally measured phenotypes. Not one random set described the four tested experimental datasets as accurately as the EFM set selected by minimization of combined costs. A perturbation analysis was performed to characterize the sensitivity of the approach to experimental uncertainty in the measured flux distributions. The predictions were remarkably stable, with the magnitude of the weighting factors falling into a tight distribution and no more than 6% of 10,000 simulations selecting EFMs different than the initial batch growth analysis (Carlson 2009).

## 7.6 Future Directions

An important consideration in stoichiometric modeling of metabolism is the robustness of phenotype. For every metabolism our group has investigated, the resource-ratio space very near the tradeoff surface includes many additional EFMs (Fig. 7.3). Laboratory chemostat experiments have shown that yield differences as small as 0.5% are selectable (Dykhuizen and Hartl 1980), but selection pressures vary in both magnitude and kind over evolutionary time, suggesting that even greater margins in 'optimal' metabolic behavior are likely fixed in nature. While perhaps not 'competitive' in strictly-controlled, non-varying environments, versatility and resilience matter in the long game, and tradeoffs between robustness and optimal performance are well known in nature as well as engineering (Csete and Doyle 2002; Kitano 2010). Our laboratory is experimenting with different margins of optimality and clustering approaches to find an appropriate balance between the concepts of metabolic robustness, resource requirements and cellular performance for selecting important metabolic phenotypes.

Not all investments produce a benefit that is easy to compare objectively between EFMs. For example, extensive theoretical and experimental work supports the notion that rate-based strategies can outcompete yield-based strategies in certain environments (MacLean 2008; Schuster et al. 2008). Unfortunately, quantifying the relative benefit of rate differences between pathways would obviate the main benefit of stoichiometric modeling: wide applicability without extensive parameterization. Similarly, some investments can be difficult to tabulate; cofactor requirements for enzymes are frequently unknown and potentially flexible with likely fitness consequences, causing difficulties in analysis of trace element scarcity. Differential investments in non-metabolic functions (e.g. chaperones, sensory proteins, or even ribosomal synthesis) can require special effort to be integrated into the tradeoff curve framework, although simple dynamic models are being developed for that purpose (Molenaar et al. 2009). With eukaryotic systems, organelles and their associated biochemical pathways have additional investment requirements that

should be accounted for to adapt the presented methodology. Questions remain: for instance, what additional membrane investment cost should be considered for organelle enzymes, or is this investment negligible when compared to the outer cellular membrane synthesis costs? Detailed ecological models exist for cellular resource budgets, accounting for uptake and growth machinery, storage compounds, and baseline physiologic functions. These are important considerations: while the metabolic enzyme collection is flexible in microbial generalists, the resources invested into other processes are critical throughout microbiology and presumably non-uniform. In addition microbes often express enzymes that are not required for their current circumstances. In a form of ‘hedging metabolic bets’, microbes may divert a fraction of their resources to a backup plan if the environment suddenly changes. Economic models associated with hedge funds surely will be useful for interpreting what fraction of a resource pool should be invested into risk-management strategies. For these reasons, development of suitable proxies for various types of investments and benefits will continue as an open area in the field of metabolic analysis. The best proxies will be calculable without extensive laboratory work, but effective in selecting a set of modes that is responsive to a specific stress.

Finally, microbial communities can be highly regulated and stable structures based on extensive trading of resources (Miller et al. 2010; Pfeiffer and Bonhoeffer 2004; van der Meer et al. 2005; Wintermute and Silver 2010). Stoichiometric analysis of community metabolism is a growing field (Dias et al. 2008; Stolyar et al. 2007; Taffs et al. 2009; Zhuang et al. 2011; Zomorodi and Maranas 2012), and aspects of the presented analysis have been extended to coevolved microbial consortia to examine whether members forgo individual optimality for community-related benefits (Taffs et al. 2009). Resource-ratio theory has also recently been expanded to interpret resource exchanges between species (de Mazancourt and Schwartz 2010). Refining this theory to allow stoichiometry-based modeling of exchanges between and relative abundance of interacting microbes will play an important role in deciphering geochemical cycles. Highly efficient natural systems will also provide design blueprints for robust artificial consortia in bioprocess applications (Bernstein et al. 2012).

## 7.7 Conclusion

Stoichiometric network models are important theoretical tools, facilitating systems-wide modeling efforts for interpreting how microbes allocate resources in response to different environmental demands. This chapter explored microbial phenotypes that maximize fitness in multi-factorial environments through an economic analysis of metabolism. The economic analysis identified costs associated with every genome-encoded phenotype, which were used with ecological resource-ratio theory to characterize phenotypic tradeoff surfaces. These metabolic strategies support the notion of fundamental adaptive principles to the regulation of metabolism. For instance under the seemingly opposite scenarios of unrestricted and scarce resources, the resource-ratio based analysis predicts an overflow metabolism is ecologically

competitive. The presented schemata, developed primarily to understand metabolic network structure and function, provide a rational basis for investigations in environmental ecology, as well as for control of microbial processes in medical and industrial applications.

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