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Characterization of continuous bounded ordinary differential equations across all parameter choices for regulatory networks

The characterization of the long-term dynamical behavior of the solutions to a class of ordinary differential equations (ODEs) with continuous components that transition between constant threshold values has significant applications to the understanding of regulatory networks associated with both gene regulation and neural network behavior. In particular, sigmoid models are realistic representations of regulation events in regulatory networks, but being non-linear, solutions cannot be found analytically in general. We investigated the dynamics of a class of functions that transition continuously from one constant threshold value to another, by dividing phase space into a finite number of rectangular “boxes”, and analyzing the “flow” of trajectories from one box to another, which previous work has done on discontinuous “switching” models of regulatory networks. We present an algorithm that unambiguously assigns directions of flow from box to box for the general bounded continuous system, which can be used to create a “domain graph” of phase space. We also present a mapping of the Morse decompositions of the domain graph of the switching system to the Morse graph of the corresponding bounded continuous system's domain graph. By partitioning parameter space into a finite number of semi-algebraic sets, our work allows for the complete characterization of the overall dynamics of any given regulatory network's respective model over all possible parameter choices, which is a paradigm shift in the analysis of dynamical systems.