

JOURNAL OF THE ENVIRONMENTAL ENGINEERING DIVISION

TECHNICAL NOTES

Proc. Paper 13790

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MICROBIAL REACTION RATE EXPRESSIONS

By William G. Characklis¹

The subject of this technical note is the equation for saturation kinetics that has been used to describe adsorption (6), enzyme conversions (13), microbial growth (10,11), and other rate phenomena. All of these processes can be characterized as possessing active reaction sites. Mar (7), in a previous communication, deals with microbial growth, yet uses the terms "Michaelis-Menten model" and "Monod theory" interchangeably. The relationship between the two models is tenuous at best. Monod (11) indicates that the models are of the same form but suggested that the saturation constant, K_s , ". . . should . . . be expected to bear some more or less distant relation to the apparent dissociation constant of the enzyme involved in the first step of the breakdown of a given compound." Kessick (5) presents a convincing argument in this regard.

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A "Monod theory" does not exist. Monod (11) states that ". . . relatively simple empirical laws are found to express conveniently the relation between exponential growth rate and concentration of an essential nutrient." He also points out that ". . . several mathematically different formulations could be made to fit the data. But it is both convenient and logical to adopt a hyperbolic equation . . . similar to an adsorption isotherm or to the Michaelis-Menten equation." In summary, Monod was the first to use a hyperbolic equation to describe microbial growth rate as a function of concentration of the limiting nutrient. The curve-fit was good and the method has received wide acceptance although indiscriminate application of the method may yield divergent results. Several examples may be instructive.

Mass transfer rate is frequently mistaken for reaction rate in processes where diffusion limits the overall conversion. Consider a microbial cell or floc suspended in a growth media where the bulk limiting substrate and its concentration, C_s ,

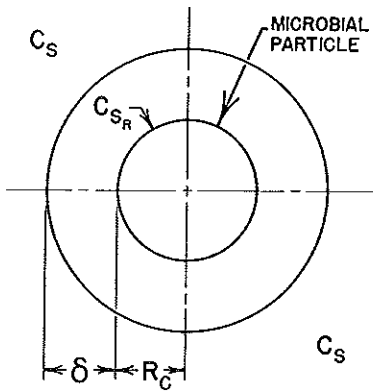


FIG. 1.—Schematic Diagram of Microbial Cell in Growth Medium of concentration C_s Surrounded by Capsular or Hydrodynamic Boundary Layer δ

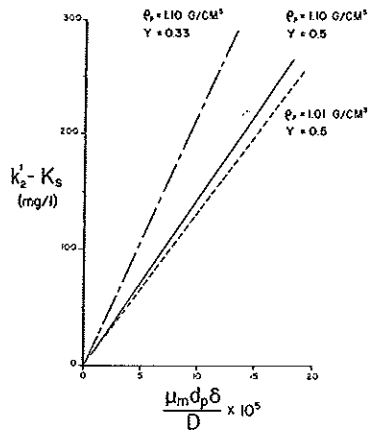


FIG. 2.—Effect of Mass Transfer Limitations on Saturation Constant Obtained from Microbial Growth Experiments ($k'_2 = K_s + [\mu_m C_b / (YA_s)]$)

are known (Fig. 1). The cell is surrounded by a stagnant liquid layer, or a polymeric matrix (e.g., capsular material), of thickness δ . The concentration of limiting substrate at the cell surface is C_{sR} . Substrate must diffuse through the boundary layer before it can "react" at the cell surface. The rate of diffusion may have an appreciable effect on the observed substrate removal rate. Note that substrate removal rate refers to the rate of decrease of C_s and could be different from substrate reaction rate which depends on C_{sR} , the concentration "seen" by the cell.

Assuming one-dimensional transfer, the substrate flux due to diffusion is

$$N_s = \frac{D}{\delta} (C_s - C_{sR}) \dots \dots \dots (1)$$

in which D is the molecular diffusivity of substrate in the growth media.

If the surface reaction rate obeys saturation kinetics, then

$$RA_s = - \frac{dC_{sR}}{dt} = \frac{1}{Y} \frac{dC_b}{dt} = \frac{\mu}{Y} C_b \dots \dots \dots (2)$$

and $RA_s = \frac{C_b}{Y} \left(\frac{\mu_m C_{sR}}{K_s + C_{sR}} \right) \dots \dots \dots (3)$

or $R = \frac{k_1 C_{sR}}{K_s + C_{sR}}$ in which $k_1 = \frac{\mu_m C_b}{YA_s} \dots \dots \dots (4)$

in which R = reaction rate per unit surface area of the cell or floc; A_s = cell or floc surface area per unit reactor volume; Y = microbial cell yield; μ = specific microbial growth rate; μ_m = maximum specific microbial growth rate; C_b = biomass per unit reactor volume; and K_s = saturation "constant."

At steady state $R = N_s$ and from Eqs. 1 and 4

$$\frac{k_1 C_{sR}}{K_s + C_{sR}} = \frac{D}{\delta} (C_s - C_{sR}) \dots \dots \dots (5)$$

Eliminating C_{sR} from Eq. 5 results in the following quadratic equation:

$$R^2 - R \left[k_1 + \frac{D}{\delta} (C_s + K_s) \right] + \frac{D}{\delta} C_s k_1 = 0$$

The quadratic equation of the form $R^2 - aR + b = 0$ has the approximate solutions $a - b/a$ or b/a . The former solution cannot be used since the value of R becomes infinite when δ approaches zero. Consequently

$$R = \frac{k_1 C_s}{\left(K_s + \frac{k_1 \delta}{D} \right) + C_s} \dots \dots \dots (6)$$

Eq. 6 demonstrates the possible effect of mass transfer on the apparent saturation constant, i.e., the saturation constant determined from measurement of bulk substrate concentrations.

If $k_1 \delta / D \gg K_s$ and $k_1 = \mu_m C_b / (YA_s)$ is substituted into Eq. 6, then Contois' equation (3) is obtained

$$RA_s = \frac{k'_1 C_s}{k_2 C_b + C_s} \dots \dots \dots (7)$$

in which $k'_1 = \mu_m C_b / Y$; and $k_2 = \mu_m / (YA_s)$.

Fig. 2 indicates the effect of mass transfer limitation on the saturation constant in terms of the dimensionless variable $\mu_m d_f \delta / D$. The development suggests that Contois' equation is a specific case of saturation kinetics applied to a mass transfer-limited microbial growth system. Other researchers have considered the effect of mass transfer limitations with respect to specific microbial systems in more detail (2,4,8,12). Atkinson (1) presents a more general development which accounts for discrepancies in many microbial growth systems.

The development presented indicates the importance of distinguishing between

reaction rates and mass transfer rates in microbial growth rate and substrate consumption rate measurements. Reaction rate data, if expressed as an intensive property (i.e., independent of system mass), can be used for prediction or design, whereas mass transfer rate data depend on system mass, geometry and flow regime.

ACKNOWLEDGMENTS

Conversations with M. A. Kessick, Darrell King, J. M. King, and B. M. Thomson were helpful in preparing this note.

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