

Optimizing Cyclist Parking in a Closed System

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Abstract. In this paper, we consider the two different aspects of the bike parking problem; namely the assignment of bike racks to locations, and the selection of the minimal number of bike rack locations satisfying some maximum walking distance d . The first sub-problem considered was the assignment of bike racks to individual buildings in the attempt to satisfy the needs of the total number of cyclists expected to reside within a building during the course of an average day. We show that the case of assigning a finite number of bike racks to all buildings on a campus is NP-Hard, and propose a greedy algorithm to obtain a solution. The case of allowing for additional bike racks to be purchased is shown to be Polynomial-Time solvable. The second sub-problem, finding the minimal number of bike rack locations, is shown to be NP-Hard, and a method to use approximation algorithms for the Maximum Independent Set to find solutions is demonstrated.

1 Introduction

Historically, the process of finding adequate parking has been notoriously difficult for university cyclist commuters. Bike racks are inevitably filled, and when an individual finally does find an open slot, it is oftentimes located a great distance from the target building. The purpose of this year's data challenge is to address this issue, and find solutions to encourage the university population to use bicycles as a viable transportation option. The proposed solutions should be generic enough to be applied to other campuses.

While there are many approaches to solve this open ended problem, our group took the stance of treating the problem space as an optimization problem. Our solution only uses information about bike rack locations and counts, as well as the number of users in each subset of the system. This simplification captures the essence of the problem, but omits details such as topography, full sidewalk paths, weather conditions at local points, and the details of how foot traffic flows throughout campus over the course of a day. The omission of these details prevent us from obtaining an complete optimal solution, but our method will provide insight on improving the parking experience for cyclists by providing near optimal solutions on the simplified model.

2 The Problem Definition

The bike parking problem is multi-objective; namely, the solution should attempt to minimize the walking distance for students and employees, as well as minimize

the cost of implementation and maintenance for the university. For the remainder of the report, we will be consistent in our usage of the following variables:

- let d be the distance that has been deemed an “acceptable” walking distance.
- let L_i be the i th location for bike racks.
- let c_i be the total occupancy for building i .
- let α_i be the percentage of building i 's occupants that commute via bicycle.
- let b_j be the number of bikes that rack j can hold. Note that the existing bike racks on the campus of Montana State University only have values of $b_j = 10$ and $b_j = 18$.
- let $\lambda_{i,j}$ be the number of bike racks selected for L_i that have the capacity b_j .
- let $\sigma_{i,j}$ be the number of bike racks of capacity j at location i .

Ideally, we would like to minimize both d and the number of locations of bike racks. The astute reader will notice that these objectives are at odds. To fully minimize the number of bike rack locations, we could place all sites near the center of a university campus. The walking distance to and from buildings would likely surpass the value of d though, making this solution unacceptable. Solutions must take both objectives into account in order to be viable.

2.1 Bicycle Rack Assignment to Locations

Considering that different types of bike racks have different fixed capacities, the first problem our group chose to address was to assign the minimum number of bike racks needed to satisfy the number of cyclists for each building. The simplest case to consider is to allow for as many bikes racks of each unique capacity to be used, without limits on how many of each type can be used in the system.

Definition 1. *Bike Parking Problem with Minimum Number of Bike Racks (BPP-MNBR):*

Inputs: number of unique bike rack capacities k , number of buildings m .

Goal: Minimize the number of bike racks required to provide full service at each building.

$$\text{minimize } \sum_{i=1}^m \sum_{j=1}^k \sigma_{i,j}$$

s.t.

$$\forall i, \sum_{j=1}^k \lambda_{i,j} b_j \geq \alpha_i c_i$$

This system satisfies the constraint that the number of available bike slots at a given building is at least as large as the number of users at that building. The overall objective is to minimize the number of racks located at a given building.

Theorem 1. *Bike Parking Problem with Minimum Number of Bike Racks(BPP-MNBR) is Polynomial-Time solvable.*

Proof. Since the number of bike racks of each capacity can be treated as infinite, we can solve the constraints for each building independently. Each of these simple linear systems can be solved in Polynomial-Time by using a linear constraint algorithm such as the *Simplex* algorithm[2]. Therefore, *BPP-MNBR* is solvable in Polynomial-Time.

In reality, we oftentimes cannot treat the pool of available bike racks as infinite, and must limit ourselves to however many bike racks can be afforded in a budget. This then becomes a linear optimization problem with the additional constraint that the total number of each type of bike rack is fixed.

Definition 2. *Bike Parking Problem with Limited Number of Bike Racks(BPP-LNBR):*

Input: Given the number of bike racks n with k different bike types, where $n = n_1 + n_2 + \dots + n_k$ and the number of buildings m with occupancy c_i ($i=1,2,\dots,m$).
Question: how to deploy n bike racks over each location L_i to maximize the service to each building.

We model this problem as a minimization problem, attempting to minimize the overall number of unserved students across campus.

$$\text{minimize } \sum_{i=1}^m (\alpha_i c_i - \sum_{j=1}^k \lambda_{i,j} b_j)$$

s.t.

$$\forall k, \sum_{i=1}^m \lambda_{i,k} \leq n \tag{1}$$

$$\forall i, \sum_{j=1}^k \lambda_{i,j} \geq \theta \tag{2}$$

Where θ is some minimum number of bike slots that is guaranteed to be available at each L_i . This is to ensure that the distribution of bike racks across campus has full coverage, even in the case where the population is locationally skewed. If there are still bike racks left unassigned at the end of the assignment process, the remaining racks should be distributed evenly across the available locations to increase the coverage of as many buildings as possible.

Theorem 2. *Bike Parking Problem with Limited Number of Bike Racks(BPP-LNBR) is NP-complete.*

We conjecture that BPP-LNBR is NP-complete. The reason why we assume BPP-LNBR is NP-complete is similar to the NP-complete Knapsack problem or

subset sum problem and its running time is exponential. Currently, this proof is still incomplete. Since *BPP-LNBR* is likely NP-Complete, and therefore intractable, we propose a greedy algorithm to provide a solution to the problem. In this model, we also consider a maximal threshold function, Γ , to prevent the case of placing all available bike racks at the largest capacity building(s), and neglecting to place additional racks at medium to lower capacity buildings. A simple definition of Γ_i could be

$$\Gamma = \frac{c_i}{\sum_{j=1}^m c_j}$$

. The full description of this greedy method is detailed in Algorithm 1.

Algorithm 1 A greedy algorithm for solving BPP-LNBR

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1: procedure BPP-LNBR-GREEDY
2:   for each building  $i$  assign  $r_i$  bike racks to satisfy minimum threshold
3:   remove the  $r_i$  bike racks from the available bike rack pool,  $S$ 
4:   close;
5:   sort buildings by capacity
6:   for each building  $i$ 
7:     assign minimum number of bike racks,  $r_i$ , to satisfy  $|r_i| \geq \Gamma c_i$ 
8:     remove the  $r_i$  bike racks from the available bike rack pool,  $S$ 
9:   until  $S = \emptyset$ .
10:  goto loop.
11:  close;

```

2.2 Minimizing the Number of Locations

In addition to minimizing the number of bike racks at a location, we may also want to minimize the number of locations for groups of bike racks. The intuition is to set a maximum acceptable walking distance, d , between a bike rack and a target building. Buildings that have overlapping regions to place bike racks can share bike rack space, with the bike racks being located anywhere within the intersection of the n buildings available regions. The total number of bike slots available at this shared location should be equivalent to the sum of the demands of all of the buildings. In this way, we can completely cover the demand, as well as minimize the number of rack locations parameterized by some distance d . It is important to note that this solution is NP-Hard, and does not scale to a larger number of buildings. It can, however, be solved within an approximation factor in Polynomial-Time by using approximation algorithms for the *Maximum-Independent-Set* problem. Further details about the approximation algorithm for *MAX-IS* is described in [4].

Theorem 3. *Finding all overlapping regions between building areas is NP-Hard.*

Proof. Let M be the set of all buildings in the problem space, and d_{max} be the user defined acceptable distance.

1. Compute the pairwise distances, $d_{i,j} = distance(m_i, m_j)$ between all members of $m_i, m_j \in M$. For every $d_{i,j} \leq d_{max}$, create an undirected edge from m_i to m_j .
2. All buildings that are within d_{max} distance of each other will be a clique in the graph. In our problem space, we can treat each clique as a single shared bike rack location. This means that the *Maximum-Independent-Set* for the graph of the system is the solution to find the minimum number of locations to satisfy a minimum walking distance between each building and it's bike racks in the system.

The *Max-Independent-Set* problem is NP-Hard[4]. Since there is a 1-1 correspondence between *Max-Independent-Set* and finding the minimum number of bike rack locations, finding all the overlaps between building areas must be NP-Hard. More proof details about *Maximum-Independent-Set* can be seen in Reference[3].

Once all overlapping regions, as well as regions for the individual buildings with no overlap, are found, exact locations for bike racks can be chosen within the region. The exact locations should be chosen considering the following: proximity to a sidewalk path, proximity to a buildings entrance, and obstacles or vegetation that may be in the way of commuters or maintenance. Since each of potential bike rack locations is so small, deciding the position of the racks can be done with relative ease by ground personnel installing the racks.

3 Evaluation

Our solutions to the bike parking problem use limited information about a university campus, and provide adequate solutions to a simplified model. Due to the simplicity and tractability of our solutions, we believe that they are a viable option to attempt to improve parking for cyclist commuters. Some of the benefits of our solutions include:

- **Generality:** The solutions provided are not dependent on any concrete instance of a campus, nor do they require the problem space to be a university campus. The solutions will in fact work in any system that has a finite number of locations with a number of discrete storage slots. In addition, our solution can also minimize the number of locations by compacting locations that are within some threshold distance.
- **Scalability:** All solutions provided use algorithms that run in Polynomial-Time, making them scalable to much larger systems. In the case of Montana State University, which is expected to grow 20% within the next 5 years, the solutions can be modified by adding additional buildings, adjusting capacities of buildings, or by increasing the α_i values for the buildings. Each of these changes does not affect the overall effectiveness or performance of the solutions.

- **Feasibility:** Our solution to the case of using only existing bike racks (*BPP-LNBR*), coupled with minimizing the number of locations, is an extremely feasible solution. All of the raw materials are present in the system; they only need to be repositioned. The solution to *BP-MNBR* can be used as an upper bound for full coverage of the system, and the university can purchase additional bike racks as funding allows in an attempt to approach this full coverage of the system.

4 Conclusions

This paper provides generic and scalable solutions to the bike parking problem, at minimal cost. While abstract, both the *BPP-MNBR* and *BPP-LNBR* detail the raw number of bike racks that should be placed at each building. In addition, we provide the means for a campus to condense the number of locations for bike racks, which reduces the cost of maintenance while not taking a performance reduction in the coverage of the campus.

Future work on this problem would likely benefit from modifying the distance parameter d to be a polygon from a given buildings outer perimeter rather than a fixed distance from the approximated center of the building. This would yield results with less error for the *MIN-LOC* problem, and could potentially find additional common bike rack locations between buildings. Again, due to time constraints, this avenue was not perused.

References

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