The Learning Difficulties Faced by Community College Algebra II Students in Understanding Algebraic and Symbolic Notation

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An assignment from Dr. Arthur Bangert's EDU 607 Quantitative Educational Research: To design, implement and complete a quantitative research project which shows evidence of the advanced professional's skills in accessing and interpreting literature and applying information from research to solve or discover more about a practical problem encountered in his or her professional practice.

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Abstract

This research project is aimed to find out students' common errors and misconceptions regarding to the understanding of the algebraic and symbolic notation and what factors affecting them in their understanding. 18 students from a semester-long algebra II class from a community college were invited to participate in this study by taking a pre- and a post- diagnostics tests. Students' answers on the tests were analyzed. 15 students were interviewed afterwards to explain their errors. Students' answers on the tests and responses from the interviews have shown that students did have some misconceptions concerning learning of algebra. They generally had difficulty in recognizing the roles of the variables that were used in algebraic expressions or equations. They also ignored the order of operations. Most of them had difficulty in knowing the difference between algebraic expressions and equations. The other purpose of this research was to study the effectiveness of adopting collaboration as an instructional strategy on the students' conceptual understanding of the algebraic and symbolic notation. A paired-samples *t*-test was conducted to compare the results between the pre- and the post-diagnostic tests in adopting collaboration as an instructional strategy. The results suggested that the collaboration did have an impact on the students' learning in algebraic and symbolic notation. Specifically, the results suggested that adopting collaboration could help students' conceptual understanding on the algebraic and symbolic notation. The teaching strategies that might help students combat misconceptions and overcome learning difficulties when learning basic algebra are also discussed in this research report.

CHAPTER 1: INTRODUCTION

Background

Algebra is a special language with its own conventions (Stacey & MacGregor, 1999). It is a generalized form of arithmetic and for the purpose of generalization of arithmetic; algebraic and symbolic notation, variables (letters), symbols and signs, is used. Algebra is indeed a difficult area of Mathematics because of its nature of generalization and abstraction. The variables, symbols and signs used in Algebra have different meanings and interpretations in different situations. Therefore, it is not surprising that many students find Algebra very hard to learn. Research indicates that students particularly experience difficulty with the concept of variable, a difficulty that might partially be explained by the fact that within mathematics, variables can be used in many different ways (Rosnick, 1981; Schoenfeld and Arcavi, 1988; Wagner, 1993).

The concept of variable is one of the most fundamental ideas in mathematics from elementary school through college (Davis, 1964; Hirsch and Lappan, 1989). This concept is so important that its invention constituted a turning point in the history of mathematics (Rajaratnam, 1957). Algebraic symbols play a critical role in the mathematics curriculum. It is essential to make sure that students understand the learning of algebra for them to be able to succeed in Mathematics which is known as one of the gate keepers for success in all fields of life (Samo, 2008).

Students can only learn algebra well when they can function abstraction. Abstraction makes life easy, makes it possible. Words and language are conveniently vague so that the word car, for example, could cover all cars, not just one. So, anyone who has learned to speak, demonstrates that he can use concepts. There are no words without concepts (Mason, 2005). Therefore, how can we deny that children are already the masters of abstraction, as soon as they use language, and that they of course bring this mastery and the algebraic concepts with them when they come to school (Gattegno, 1970, pp.23). Unfortunately, this is not really the case in reality. The algebraic and symbolic notation is highly abstract in nature. Collis (1975) argued that the difficulties children have in Algebra actually relate to the abstract nature of the elements in Algebra.

Despite the vast research by researchers, namely Collis (1975), Kuchemann (1981) and Kehkashan (1998), on the students' difficulties in understanding variables in Algebra, the overall image that emerges from the literature is that students have **misconceptions** about the use of algebraic and symbolic notation. Students interpreted the variables in different ways which caused them to have misconceptions about the meanings of the variables. If we would like to help students develop a correct concept of variables and succeed in advanced math classes, misconceptions about variables students hold will have to be investigated.

Errors and misconceptions in algebra are not random - There are reasons behind students' understanding of variables when they demonstrate misconceptions in reading variables and show difficulty in interpreting them. In most of the cases the letter is regarded by the learners as shorthand or abbreviation for any object or as an object in its own right (Collis, 1975). It is a common misconception among students. Early experiences with Algebra often lead students to develop this misconception where letters stands for abbreviations of objects (Samo, 2008). Kuchemann (1981) investigated in one of his research where a group of students' response to the following problem:

> *Shirts cost **s** dollars each and pants cost **p** dollars a pair. If I buy 3 shirts and 2 pairs of pants, what do 3**s** + 2**p** represent?*

Response of the most of the students suggested *3 shirts and 2 pairs of pants.* This shows that students perceive ***s*** as shirts and ***p*** as pants rather than ***s*** for the number of shirts and ***p*** for the number of pair of pants. This response suggests that students have a strong tendency to conceive letters as labels denoting specific sets, which seems to be a result of the students' attempt to accommodate their previous arithmetic experience with letters to the new meanings assigned to letters within an algebraic context (Samo, 2008). MacGregor and Stacey (1997) found that many eleven-year-olds who had never been taught Algebra thought that the letters were abbreviations for words such as *h* for height or for specific numbers. Further, they found that students have a misconception that these numbers were the "alphabetical value" of the letter such as $h = 8$ because it was the eighth letter of the alphabet. Another misconception stems from Roman numerals. For example, 10*h* would be interpreted as "ten less than *h*" because IV means "one less than five." Another misconception can be found from the other problem given by Kuchemann (1981): *What can you say about p if* $p + q = 12$ *and p is a natural number greater than q?* Most of the students replied $p = 7$ which indicates that students think the letter *p* can only have a specific value.

Other researchers have also found the following common pitfalls or difficulties in manipulating and interpreting algebraic and symbolic notation by students:

- A tendency to mix numbers and letters. For example: 'add 4 onto 3*N*' as 7*N* (Hart, 1981).
- A tendency to ignore operations in generalizations. Kieran (1992) calls 'one of the most common errors made in algebra' the inappropriate generalization of $7a \div 7 = a$ to $7a - 7 = a$.

- Exhibit a strong proclivity toward performing a left-to-right word-order match. For example, in the well-known "Students and Professors" problem below, 37% of college engineering students answered incorrectly by writing an equation as $6S = P$:

  Write an equation using the variables S and P to represent the following statement: There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors.

  Students frequently write the reverse of what they intend (Clement, Lochhead & Monk, 1981).

For the learning of Algebra, learners should have a conceptual understanding about the use of the algebraic and symbolic notation and the context in which it is used. In other words, they should know the situation in which the algebraic expressions or equations are made. Foster (2007) says that, "when we memorize rules for moving symbols around on paper we may be learning something but we are not learning Mathematics" (p.164). Moreover, the use of symbols without an understanding cannot develop students' relational understanding of Algebra. Foster (2007) highlighted that if students are taught abstract ideas without meaning, this might not develop their understanding. He suggested that if teachers want students to know Algebra then they must be given a deeper understanding of the use of symbols. Since same letter can be used in different contexts with different meanings. The different meanings of the same letter or symbol in different contexts create problems in conceptual understating of the concepts of Algebra and in solving the algebraic problems (Zahid, 1998). It is believed that students always do not receive enough support when they transfer their learning from arithmetic to algebra. For example, Matz (1980) points out, when young people move from arithmetic to algebra in their schooling, they are quietly expected to take a giant leap in their mathematical problem-solving strategies: while they have learned to expect in arithmetic to simply apply algorithms like long division, in algebra they must compose and carry out plans for solutions. Teachers are encouraged to pay more attention to the conceptual development instead of emphasizing manipulation skills (Foster, 2007). Lochhead & Mestre (1999) recommended teachers to provide opportunities for students to actively participate in the process of overcoming their misconceptions.

Problem Statement

Algebra is an important area of Mathematics. Because of the heavy use of the algebraic and symbolic notation and its nature of generalization and abstraction, Algebra is considered to be a difficult area of Mathematics for many students, including community college students. About 50 percent of students do not pass college algebra with a grade of C or above, as noted in a recent report (May 2016), *Common Vision*, from the Mathematical Association of America (MAA). The report called Americans' struggle with math "the most significant barrier" to finishing a degree in both STEM and non-STEM fields. In the worst-case scenarios, students can get stuck in remedial classes (elementary algebra classes) and fall so far behind that they drop out of college all together. Many college students have experienced mathematics as a roadblock to other fields — such as science, technology and engineering.

Algebra, a language that is composed by algebraic and symbolic notation, is the first in a series of higher-level mathematics classes students need to succeed in college and life. Students can only succeed in the advanced mathematics classes like College Algebra and Calculus which involve a tremendous use of algebraic and symbolic notation when they have strong algebraic skills.

Kieran (1992) stated that students' misconceptions and common errors in algebra are rooted generally from the meaning of symbols. It is believed that students who hold misconceptions on the meaning of the algebraic and symbolic notation are not given enough opportunities in class to participate in the discussions on the meaning of the symbols (Lochhead & Mestre,1999). Researchers (Lannin, Townsend, Armer, Green & Schneider, 2008) suggested that teachers could adopt collaboration as a learning strategy when teaching algebra: Giving students opportunities to construct their own algebraic representations, discuss the meaning of these representations with classmates, and debate the advantages and limitations of each are important elements in the process of developing and refining algebraic representations.

Purpose Statement

The aim of this research study is to investigate if the researcher's students hold the same or similar common misconceptions that are mentioned in the literature review and what factors causing those misconceptions. In addition, this research will also attempt to identify other misconceptions that are not mentioned in the literature. Collaboration as a learning strategy will be experimented for any improvement on the students' understanding of algebraic and symbolic notation.

Research Questions

(1) What are the difficulties most community college Algebra II students have in understanding algebraic and symbolic notation?

- What types of conceptual and procedural errors regarding algebraic and symbolic notation are common among community college Algebra II students?
- In what ways do community college Algebra II students misinterpret algebraic and symbolic notation?

- What factors contribute to community college Algebra II students' misconceptions (or misunderstandings) about algebraic and symbolic notation?

(2) Does the use of collaboration as an instructional strategy improve students' understanding of algebraic symbols?

Limitations

The verbal protocol analyses of interviews depends on how articulate the students are in being able to tell the researcher what they are thinking. Some of them might not be able to describe what they exactly are thinking in their mind. The data collected from the field notes of interviews may somehow be subjective as they are simply based on the researcher's own interpretations of students' responses. The results in this study might not be able to apply to a larger population as the sample is conveniently taken from the researcher's single-semester-long algebra II class from a community college.

Delimitations

Students participated in this study are community college students enrolled in a developmental mathematics class, Algebra II. The prerequisite for the Algebra II class is Algebra I. It is assumed that those students are all have similar foundation in algebra. Results from this study are only generalizable to other community college students who are enrolled in a semester long Algebra II course.

Significance of the Study

Teachers who teach algebra would find the information collected in this study very helpful for addressing the "Expressions and Equations" topic which is in an important domain in the mathematics curriculum. Students who like to succeed in advanced math classes like College Algebra and Calculus will be required a strong foundation of algebra. Based on the identified common errors and the students' thinking that leads to those errors, teachers will have better ideas on the possible pitfalls when students are learning algebra and can plan lessons accordingly to make sure that students will not fall into those pitfalls. Some effective teaching strategies might be able to be identified in order to help the college students get through the Algebra barrier and finish their degree in both STEM and non-STEM fields.

Summary

This research project investigates students' misconceptions and common errors regarding to the understanding of algebraic and symbolic notation. Lochhead and Mestre (1999) pointed out that conceptual understanding is the key to mend the misconceptions. Students should be given opportunities in class to collaborate with classmates on developing meaning for symbolic representations in order to understand the concepts of the use of the algebraic and symbolic notation (Lannin et al, 2008). However, there are no studies that have determined the effectiveness for adopting collaboration as a learning strategy when teaching algebra. The present research study seeks to understand the relationship between the collaboration as instructional strategy and the students' conceptual understanding on the algebraic and symbolic notation.

CHAPTER 2: LITERATURE REVIEW

The following literature review will discuss what common errors most students make in the learning of algebra, what factors or reasons causing those errors and what teaching strategies that might help combat students' difficulty in learning algebra.

Common Mistakes Students Usually Make In Their Understanding Of Algebraic And Symbolic Notation

Errors and misconceptions in algebra are not random. There must be reasons behind students' understanding of variables when they demonstrate misconceptions in reading variables and show difficulty in interpreting them. Getting a picture on what students' misconceptions about variables are will definitely help teachers have a better idea of how their students understand algebra. Therefore, teachers should first diagnose what the common mistakes are when students interpret variables and "treat those misconceptions early enough for students not to inhibit the learning of higher mathematics" (Lochhead & Mestre, 1999, P.326). According to Kieran (1992), misconceptions and common errors are rooted generally from the meaning of symbols. Different researchers have shown the following common pitfalls or difficulties in manipulating and interpreting algebraic and symbolic notation by students:

- A tendency to mix numbers and letters. For example: 'add 4 onto $3N$' as $7N$ (Hart, 1981).
- A weakness in dealing with fractions in algebraic equations. While 70% of Swedish 16-year-olds were able to solve $3(3x - 2) = 2x$, less than 30% were able to solve $\frac{3x-2}{2} = \frac{x}{3}$ (Ekenstam & Nilsson, 1979).
- A tendency to ignore operations in generalizations. Kieran (1982) calls 'one of the most common errors made in algebra' the inappropriate generalization of $\underline{7a \div 7 = a}$ to $\underline{7a - 7 = a}$.
- A tendency to ignore the hierarchy of operations in solving an equation. '$2 + 3 \times 5$' is read as a string from left ('2 + 3 is 5 times 5 equals 25'), rather than an expression tied to a hierarchy ('$2 + 3 \times 5$ equals $2 + (3 \times 5)$ and that equals 2 + 15 which is 17') (Kieran, 1982).
- Exhibit a strong proclivity toward performing a left-to-right word-order match. For example, in the well-known "Students and Professors" problem below, 37% of college engineering students answered incorrectly by writing an equation as $6S = P$:

Write an equation using the variables S and P to represent the following statement: There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors.

Students frequently write the reverse of what they intend (Clement, Lochhead & Monk, 1981).

Ways That Students Misinterpret Algebraic And Symbolic Notation

Why do students make the above mistakes? It is necessary for teachers to study what students are actually thinking when they see variables. In mathematics, a letter in a mathematical expression can mean a specific number ($2x + 5 = 9$), a range of numbers ($3 + x < 7$), specifically related numbers ($2x - 5 = y$), any number ($ab = ba$), or no number at all (23 m long). Kuchemann (1978) listed seven different interpretations that algebra students might attach to the use of letters in equations:

(1) Labels	f, y in $3f = 1y$ (3 feet in 1 yard)
(2) Constants	π, e, c
(3) Unknowns	x in $5x - 9 = 91$
(4) Generalized Numbers	a, b in $a + b = b + a$
(5) Varying Quantities	x, y in $y = 9x - 2$
(6) Parameters	m, b in $y = mx + b$
(7) Abstract Symbols	e, x in $e * x = x$

Much of the difficulty students encounter with variables may be related to their inability to recognize the correct role of the literal symbol (letters) (Philipp, 1992), which cause them to interpret algebraic and symbolic notation improperly. It was found that only a small percentage of students were able to consider algebraic letters as generalized numbers or as variables, with the majority interpreting letters as specific unknowns for all cases (Kieran, 2007).

Students are told that in algebra, letters stand for numbers. However, they see letters used with other meanings. Letters are used in many contexts, both within and outside mathematics, as abbreviated words or as labels (Satcey & MacGregor, 1999). Students often confuse the distinction between variables and labels (Lochhead & Mestre, 1999). Teachers must use precise language to address the right meaning of the variables in the right context so students can interpret the use of the variables correctly.

Factors That Contribute To Students' Misconceptions About Algebraic And Symbolic Notation

Teachers may think that students come fresh to algebra, not considering that they already have ideas about the uses of letters and other signs in familiar contexts (Stacey & MacGregor, 1999). In order to have a clearer picture of students' misconceptions about algebra, teachers should investigate what factors affect how students learn and understand algebra and what causes those misconceptions. Teachers who recognize the many sources of misunderstanding and point them out in their teaching can improve students' performance (Stacey & MacGregor, 1999).

Hart's report (1981) showed that students very often do not use teacher taught algorithms. In many cases, students replace the algorithms they are taught by their own methods. There are two reasons that might explain why students ignore the formal methods taught by teachers but cling to earlier learned strategies or invent new strategies: **(i)** Teachers introduce algebraic algorithms too soon and assume once taught the students would remember them; **(ii)** Mathematics teaching is often seen as initiation into rules and procedures which, though very powerful (and therefore attractive to teachers) are often seen by students as meaningless (Kuchemann, 1981).

Stacey and MacGregor (1999) also suggested the following causes of students' common misunderstandings:

- Students' interpretations of algebraic symbolism are based on other experiences that are not helpful.
- The use of letters in algebra is not the same as their use in other contexts.

- The grammatical rules of algebra are not the same as ordinary language rules.
- Algebra cannot say a lot of things that students want to say.

Teaching Strategies That Can Help Students Communicate With The Notation More Effectively

It is believed that students always do not receive enough support when they transfer their learning from arithmetic to algebra. Matz (1980) points out, when young people move from arithmetic to algebra in their schooling, they are quietly expected to take a giant leap in their mathematical problem-solving strategies: while they have learned to expect in arithmetic to simply apply algorithms like long division, in algebra they must compose and carry out plans for solutions. As Clement (1982) claims, moving from arithmetic to algebraic generalizations is a process that has been found to take time. Kieran's research (1992) has convinced her that the impression among most adolescents that equations are what they appear to be in arithmetic – expressions of a process that begins with a computation and ends with an answer – is an impression that resists change. The overwhelming emphasis on the manipulation skills by teachers is one of the major factors to strengthen this impression onto the students. Teachers are encouraged to pay more attention to the conceptual development instead of emphasizing manipulation skills.

The research literature consistently indicates that misconceptions are deeply seated and not easily dislodged. It is clear that simply telling students that their conceptual understanding of a particular mathematical topic is incorrect and then giving them an explanation is often not sufficient to extirpate the misconception. Teachers need to provide opportunities for students to actively participate in the process of overcoming their misconceptions (Lochhead & Mestre, 1999).

Collaboration as an Instructional Strategy

Researchers (Lannin, Townsend, Armer, Green & Schneider, 2008) recommended teachers to adopt collaboration as an instruction strategy to enhance students' conceptual understanding when learning algebra. They argued that giving students opportunities to construct their own algebraic representations, discuss the meaning of these representations through discussions with classmates, and debate the advantages and limitations of each representation are important elements in the process of developing and refining algebraic representations. They believed that collaborating with classmates can help the student who creats the representation to better understand the general nature and meaning of the representation. This strategy allows other students to consider representations that they might not have independently developed. In addition, through discussion, students can link verbal and symbolic representations and this can help enhance their understanding in the roles of the variables. Teaching instruction plays a vital role in improving students understanding about the generalization and in helping students in developing their algebraic thinking in the result they can develop algebraic expressions and equations.

Summary

The literature reviews that most students do have misconceptions and common errors in the understanding of the algebraic and symbolic notation. The problem is rooted from the lack of the conceptual understanding of the notation. Since a variable could have a different meaning in a different context, this makes Algebra a hard subject for numerous students. As the literature suggests, giving students opportunities to collaborate and discuss with each other about the meaning of the algebraic representations used in an algebraic expression or equation could help students develop conceptual understanding on the meaning of the algebraic and symbolic notation.

CHAPTER 3: METHODS

Introduction

This research paper is to study for the learning difficulties faced by community college Algebra II students in understanding algebraic and symbolic notation. The goals are to investigate students' misconceptions and common errors regarding to the learning of the notation and to understand the relationship between the collaboration as an instructional strategy and the students' conceptual understanding on the notation. Two research questions guide this study:

(1) What are the difficulties most community college Algebra II students have in understanding algebraic and symbolic notation?

- What types of conceptual and procedural errors regarding algebraic and symbolic notation are common among community college Algebra II students?
- In what ways do community college Algebra II students misinterpret algebraic and symbolic notation?
- What factors contribute to community college Algebra II students' misconceptions (or misunderstandings) about algebraic and symbolic notation?

(2) Does the use of collaboration as an instructional strategy improve students' understanding of algebraic symbols?

Research Design

This study utilizes a mix-methods design in which the quantitative portion will collect data prior to the intervention and after the intervention. In addition a survey will be distributed to the participants to measure their perception of the intervention. Lastly, follow–up interviews will be conducted with students to gather more information about students' misconceptions and errors that they made on the assessments.

Participants

Study participants were identified using nonrandom, convenience sampling in a semester-long Algebra II course offered at a community college with a total degree seeking population of 1600 students attending a community college in the Northwestern United States. The sample population represented 80% of the overall population of students who are taking the Algebra II course during the semester the research study is taken. The study participants consist of 67% female and 33% male. The prerequisite of the Algebra II course is Algebra I. Therefore, it is assumed that the study participants have a similar algebra background.

Data Collection

Pre-and Post Tests. The diagnostic test (Appendix A) is adopted from the Misconception Diagnostic Test that was used in the "A Formative Assessment of Students' algebraic Variable Misconceptions" study (Lucariello, Tine & Ganley, 2014). The test was made up of a total nine multiple-choice items. Three items (1 – 3) were designed with a misconception response that reflected the erroneous understanding that a variable can be ignored. Three items (4 – 6) were designed with a misconception response that reflected an erroneous understanding that a variable is a label for an object. Three items (7 – 9) were designed with a misconception response that reflected the erroneous interpretation that a variable was a specific unknown, as opposed to one that can hold varying values. Responses within each group of items were significantly correlated with one another, with $p < 0.05$ in all cases. The nine items will be presented to each participant in a random order. A Cronbach's Alpha was estimated as 0.77

indicating that the Misconception Diagnostic Test has high internal consistency reliability (Lucariello, Tine & Ganley, 2014).

Validity of the Misconceptions Diagnostic Test was established in two ways. First, to examine predictive validity, a hierarchical multiple regression analysis was run which determined the extent to which misconception responding can predict an external criterion (specially, Algebra Ability Test performance). Second, the construct validity of the diagnostic test items was determined by examining students' transfer of their correct, incorrect, and misconception responses on the diagnostic test to the near and far transfer problems. Specifically, it was expected that transfer levels would be above chance for correct and misconception responding, reflecting that correct and misconception understandings are entrenched forms of thinking. The results of the hierarchical multiple regression analysis were statistically significant ($p < 0.0001$). The correlations between the scores on the diagnostic test items and scores on the far transfer items were 0.73 for proportion of correct responses, 0.56 for incorrect responses, and 0.42 for misconceptions, which provided evidence of construct validity (Lucariello, Tine & Ganley, 2014).

Survey. The survey (Appendix B) is developed by the researcher as a pilot instrument. There are three parts in the survey. Part I is for gathering information about the participants' algebra background. Part II consists of six Likert scale questions focused on the participants' perceptions of the collaboration as an instruction strategy on the conceptual understanding of the algebraic and symbolic notation. Part III includes three open-ended questions for the participants to elaborate or clarify their responses in Part II. The researcher will use a reliability test in ANOVA (Analysis of Variance) to estimate the reliability of this instrument.

Interviews. Based on the responses collected from the diagnostic test (pre- and post-test), semi-structured interviews will be conducted with individual students so the researcher can have a better understanding on the participants' thinking process leading to the misconceptions or errors. These interviews will be transcribed and coded to find common themes. Interview questions can be referred to Appendix C.

Procedures

The consent form (Appendix D) will be sent out to the target students in the 7th week of the semester. It is stated within the consent form that participation is completely voluntary and will not have a negative or positive impact on the student. The study participants will then be asked to take the diagnostic test as a pre-test in the beginning of the 8th week. The researcher will identify the misconceptions and errors based on the responses of the pre-test. During the 8th week to 10th week, the researcher will intervene the class by adopting collaboration as the instruction strategy when she teaches the "Algebraic Expressions and Equations" topic: Giving students opportunities to construct their own algebraic representations, discuss the meaning of these representations with classmates, and debate the advantages and limitations of each representation. In the beginning of the 11th week, the study participants will be asked to take the same diagnostic test again as the post-test. The researcher will then compare the results between the pre- and post- test to see if there is any improvement in the scores after the intervention. During the 11th week, the participants will also be requested to take the survey which is to measure student perceptions on the use of the collaboration as instruction strategy. Semi-structured interviews will also be arranged with individual students so they can explain to the researcher how they came up with those misconceptions and errors on the pre- or post- test.

Data Analysis

The scores collected in the pre- and post- test will be entered into SPSS for analysis. A *t*-test will be used to compare the mean scores of the two tests. The mean scores are to compare students' conceptual understanding of the algebraic and symbolic notation before and after the adoption of the collaboration as an instructional strategy. A survey will also be used to measure student perception of the collaboration as an instructional strategy. Results from the survey will be reported using descriptive statistics. Based on the students' responses on the tests, interviews will be arranged to have students elaborate their thinking process that leads to the errors that they made on the tests. Field notes collected from the interviews will be transcribed and coded to find common themes.

Summary

A diagnostic test will be used as a pre- and post- test to identify participants' misconceptions and errors on the algebraic and symbolic notation and the effectiveness of the collaboration as an instruction strategy on the participants' conceptual understanding on the notation. A survey is utilized to collect participants' own perceptions on the use of the collaboration as an instruction strategy in class. Interviews are to be arranged to obtain the data about what kind of participants' thinking process that lead to the misconceptions and errors.

CHAPTER 4: RESULTS

Introduction

The purpose of this research study was to identify students' common misconceptions and errors in interpreting algebraic and symbolic notation by observing students' diagnostic test results (steps and answers students wrote on the tests) and interviewing students. The study also sought to determine the effectiveness of the collaboration as an instructional strategy on the students' conceptual understanding on the notation by comparing the results between pre- and post- diagnostic test. In addition, students' perception of collaboration was measured by using a survey. 18 students from a semester-long Algebra II class from a community college have participated in this study.

Research Question 1

What are the difficulties most community college Algebra II students have in understanding algebraic and symbolic notation?

- What types of conceptual and procedural errors regarding algebraic and symbolic notation are common among community college Algebra II students?
- In what ways do community college Algebra II students misinterpret algebraic and symbolic notation?
- What factors contribute to community college Algebra II students' misconceptions (or misunderstandings) about algebraic and symbolic notation?

Based on the steps and answers students wrote in the diagnostic tests and the dialog with students in the interviews, the following types of conceptual and procedural errors regarding algebraic and symbolic notation are common among the community college Algebra II students:

(1) 82% of the students appeared to assume algebraic products must be greater than algebraic sums. e.g. $2d$ must be great than $d + 2$.

(2) 30% of the students made sense of a problem by choosing a numerical example. e.g. Test item #3: They replaced m by a random number.

(3) More than 60% of the students interpreted algebraic letters as abbreviated names of objects. e.g. Test item #5: They thought S stands for students instead of number of students.

(4) More than half of the students showed inadequate reading skills when solving algebraic problems, such as recognition of words, parsing ability and awareness of the effect of syntax on meaning. Like **Student D** gave the answer $b = 2d$ in the test item #6 instead of the correct answer $d = 2b$. When asked, she explained, "because d is twice the value of b".

(5) About 30% of the students made errors while handling the operations of expressions associating numbers and letters.

e.g. $3m + \underline{5 - 2m} + 1 = 3m + \underline{3m} + 1$ (test item #3)

(6) 100% of the students failed to distinguish algebraic expressions from equations. None of the interviewees could explain correctly the difference between algebraic expressions and equations. Most of them thought that they were the same.

(7) About 20% of the students could not handle a variable could be a representation of numbers. For example, in the interview with **Student C** about the test item #4, he expressed his dislike of believing the t in the expression '$t + 4$' could only represent numbers. He thought t could mean other objects, like *time*.

(8) About 20% of the students assumed the value of a variable must be a positive non-zero integer. For example, some students believed that the k in the expression '$k + 8$' could only represent positive non-zero integers. They felt uncomfortable that the value of a variable would be a decimal, an irrational number, zero or being negative.

Ways Of Making Conjectures With Algebraic And Symbolic Notation By Students

The following ways of making conjectures with algebraic and symbolic notation can explain why students made the common mistakes mentioned above:

(1) Students believed that **a numerical answer must be given to any algebraic problems** and therefore tried to 'produce an answer" in their own ways; they found hard to accept the lack of closure of the answers. '*The answer looks better without a letter*', '*If there is a variable in the answer, it looks rather incomplete*', students explained in the interviews.

(2) Students used a **Syntactic Word Order Matching Process** and used a variable as a letter in translating word problems into algebraic equations. They simply took the variables as objects but did not think of the meaning of the equations they constructed (i.e. they did not think if the value of the expression on the L.H.S. was equal to that on the R.H.S.).

(3) Treating Letters As Specific Values. For example, to answer the question "Which one is larger, 2*d* or *d* + 2?", the students need to treat variables as variables instead of specific values. However, students have a strong resistance to accepting that a letter could represent a generalized number.

(4) Alter The Rules To Fit The Unfamiliar Problem. Students generally tried to change the rules to rewrite an unfamiliar problem so that they can feel like they could "solve" the problem. For example, **Student F**, she knew the value of "*r* – *r*" was 0 but she changed it to 2*r* so that *r* would not be missing in the equation. She felt uncomfortable to see the *r* gone in the answer.

(6) Over-generalization

For example, apply the rule in $7a \pm 7 = 7\ (a \pm 1)$ to $7a \div 7 = 7\ (a \div 1)$ or $7a \times 7 = 7\ (a \times 1)$.

(7) Tend To Handle Numbers And Variables Separately

> **T**: Can you explain how you got 3m – 2m = 1?
>
> **Student A**: I eliminated the letter *m* first, then calculate 3 – 2.

Factors Affecting Students In Their Understanding Of Algebraic And Symbolic Notation

From the findings and analyses in the research, the following factors might explain what contributes to students' failure in adapting algebraic techniques:

(1) Students Adopt Their Own Methods Rather Than Formal Methods Taught By Teachers

Students tended to invent their ways to solve problems and manipulate algebraic expressions rather than using the formal ways taught by teachers. For example, from the explanation given by **Student G** in the interview, it is clear that he created his own way so that the problem could be solved:

T: Why did you give $t - t = 2t$?

Student G: I don't think that $t - t$ would be equal to 0 here, otherwise, t will be missing in the equation. Therefore, I tried to put $2t$ there (because there are two "t"s in the equation) so that I could "solve the equation".

(2) Letter Phobia

Psychologically, some students might have 'letter phobia' and therefore always try to ignore them. They do not feel comfortable that a variable does not have a particular value. They found variables 'mysterious' and did not know how to handle them. Some students even thought that no matter what kind of calculation was involved, the variable has to be existed. Like the following interview with **Student K**:

T: Why did you think that $2Y + Y - 3y = Y$?

Student K: Because $Y = y$.

T: Then, that means that $2Y + Y - 3y$ could be equal to $2y + y - 3y$.

Student K: yes.

T: What do you think that the value of $2y + y - 3y$?

Student K: $0y$

T: What is the value of $0y$?

Student K: *y*

T: What is the result if 0 is multiplied by any numbers?

Student K: 0

T: Then, do you still think that $0y = y$?

Student K: hm… yes, because the letter *y* should not be suddenly disappeared.

From the conversation with **Student K**, he would feel insecure if the letter was missing. On the other hand, he also could not handle the fact that the letter *y* is a representation of numbers.

Research Question 2

Does the use of collaboration as an instructional strategy improve students' understanding of algebraic symbols?

A paired-samples *t*-test was conducted to compare the results between the pre- and the post-diagnostic tests in adopting collaboration as an instructional strategy. There was a significant difference in the scores for pre-test (M = 55.56%, SD = 0.16) and post-test (M = 78.40%, SD = 0.13), t (17) = 10.87, p = 0.000 (see Table 1). These results suggested that the collaboration as an instructional strategy does have an impact on the students' learning in algebraic and symbolic notation. Specifically, the results suggested that adopting collaboration could help students' conceptual understanding on the algebraic and symbolic notation. Table 2 shows the percentages of the correct response students had given in each item of the diagnostic pre- and post-tests.

Table 1

Descriptive Statistics of Results of Pre- and Post- Diagnostic Tests

Intervention of using Collaboration as an Instructional Strategy

Pre-Test N = 18		Post-Test N = 18	
M	SD	M	SD
55.56%	0.16	78.40%	0.13

Table 2

Percentage of students giving correct response on the nine diagnostic items.

Item Number	Correct Response (Pre-Test)	Correct Response (Post-Test)
1	57%	72%
2	72%	88%
3	70%	82%
4	79%	92%
5	39%	73%
6	45%	66%
7	76%	91%
8	56%	75%
9	64%	78%

A survey was used to measure students' perception on the adoption of collaboration as an instructional strategy. The survey items were found to be highly reliable (6 items; $\alpha = .86$).

Table 3

Descriptive Statistics of Results of Survey

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
Algebra is hard to learn.	0%	0%	12%	44%	44%
Algebraic and symbolic notation used in an expression or equation is confusing to me.	0%	0%	0%	39%	61%
I find discussing the algebraic and symbolic notation in class with classmates help me better understand the meaning of the notation.	0%	0%	11%	72%	17%
I find having the time discussing the algebraic and symbolic notation in class with classmates help me better develop the concepts of the notation.	0%	0%	6%	88%	6%
I find the time on the collaboration with classmates on learning the algebraic and symbolic notation well spent.	0%	0%	6%	83%	11%
I hope to have more opportunities in class to collaborate with classmates on the learning of algebra.	0%	0%	0%	72%	28%

Table 3 shows the descriptive statistics of the results of the survey. Based on the responses collected from the survey, 100% of the participants had taken Algebra I in the past semester or school year. This indicated that all the participants had a similar algebra background. More than 85% of the students thought that algebra was a hard subject. The reported reasons were ‘Algebra is abstract’, ‘Algebraic symbols are confusing’, ‘The algebraic language is similar to our daily language but the rules are different’ and ‘Symbols are simple yet can have complex meanings’. Over 90% of the students agreed that the collaboration time in class was well spent and collaborating with classmates could assist their understanding and learning on the algebraic and symbolic notation. Students reported in the survey that when collaborating with other classmates, they got the opportunity to explain their thinking to peers. Also, they could listen to the other classmates' thoughts which helped them develop the understanding on the algebraic and symbolic notation. 100% of the students hoped to have more opportunities in class to collaborate with classmates on the learning of algebra.

Summary

The current study aimed at finding out students' common misconceptions and errors in interpreting algebraic and symbolic notation. Results from the diagnostic tests show that there does exist common misconceptions and errors among students when reading the notation. In addition, this research study looked at the effectiveness of adopting collaboration as an instructional strategy on the students' conceptual understanding on the algebraic and symbolic notation. Analysis results suggested that there was a significant difference in the mean scores of the pre- and post-diagnostic test before and after the intervention of using collaboration as an instructional strategy.

CHAPTER 5: DISCUSSION

Introduction

Algebra is so significant as a part of Mathematics that its foundation must begin to be built in the very early grades. It must be a part of an entire curriculum which involves creating, representing, and using algebraic and symbolic notation for relationships (Samo, 2008). The aim of this study was to explore students' perceptions about the use of the algebraic and symbolic notation and the effect of their perceptions on their learning of Algebra. The study revealed that the students had many misconceptions in the use of the algebraic and symbolic notation, which affect their learning of Algebra. For better understanding the concepts of Algebra and use of

Algebra as a tool to use it in real world situations, it is important that the teachers should develop students' algebraic thinking and symbol sense. One of the suggestions for teachers is to adopt collaboration as an instructional strategy in order to enhance students' algebraic thinking and the use of the algebraic and symbolic notation (Lannin, Townsend, Armer, Green & Schneider, 2008). Findings from this study showed that using collaboration as an instructional strategy did have a significant impact on students' conceptual understanding on the algebraic and symbolic notation.

Research Question 1

What are the difficulties most community college Algebra II students have in understanding algebraic and symbolic notation?

- What types of conceptual and procedural errors regarding algebraic and symbolic notation are common among community college Algebra II students?
- In what ways do community college Algebra II students misinterpret algebraic and symbolic notation?
- What factors contribute to community college Algebra II students' misconceptions (or misunderstandings) about algebraic and symbolic notation?

The findings and analyses in this study have revealed that students do have common misconceptions regarding to the use of the algebraic and symbolic notation, as we have discussed in Chapter 4. The following factors might further explain what contributes to students' misconceptions (or misunderstandings) about algebraic and symbolic notation:

(1) Children Adopt Their Own Methods Rather Than Formal Methods Taught By Teachers

What makes the students prefer using their own methods to the ones formally taught by the teachers? The reason could be that some teachers place more emphasis on pre-formalization work and thus encouraged the students to use their own informal methods. Unfortunately, methods of solution in pre-formalization work did not always directly match or bear resemblance to the process of the formal method. If the teachers do not provide an appropriate bridge to help students translate from pre-formalization work to the formal method, students will still stick to the own informal method. The other reason why students like to adopt their own ways to solve Mathematics problems is that they might see Mathematics teaching involve too many rules and procedures that they find meaningless.

(2) Levels Of Meaning And Levels Of Readiness

Why students think learning algebra especially difficult and teachers find teaching algebra hard? One of the reasons is the matching of levels of meaning with the levels of learner readiness. Each of the primary algebraic concepts – variable and equation – can have several different meanings, depending upon context, and a learner's ability to understand and make use of a particular meaning depends in good part on that learner's cognitive development (Driscoll, 1983). When children are in their early adolescence (aged 12 to 14), most of them are concrete operational (in Piagetian terminology) and their thinking is largely tied to their perceptions. Clement (1982) reminds us that understanding an equation in two variables appears to require an understanding of the concept of variable at a deeper level than that required for a one-variable equation. Unfortunately, before children enter the formal operational stage of cognitive development, they are unable to keep two or more variables in mind at one time, nor think about their own thinking. That explains why both students and teachers find the learning and teaching of algebra rather difficult.

(3) Some Algebraic Errors Have Already Deeply Rooted

There have been studies documenting students' errors in parsing algebraic expressions (for example, Matz, 1979 & Sleeman, 1984). The kind of error like simplifying $7a - 3$ to $4a$ has frequently been seen and most of the errors are not restricted to the beginning algebra students. The so-called 'deletion' error (e.g. $7a - 3 = 4a$ or 4), was the most prevalent one that students made when simplifying expressions. Some students are over-generalizing certain mathematically valid operations, arriving at a single generic deletion operation that often produces incorrect results. It seems that this kind of errors is deep-rooted and could be carried from junior high school to college.

(4) Lack Of Support On The Teaching Of Algebra

Textbooks which are commonly used by schools always emphasize only the simplification and manipulation of algebraic expressions, substitutions of values in formulas, and getting a solution of an equation which could help the students in completing the homework or preparing for tests. The teaching of algebra then became skills driven. But what students actually need, besides the procedural knowledge, is the conceptual understanding on the use of the algebraic and symbolic notation.

Research Question 2

Does the use of collaboration as an instructional strategy improve students' understanding of algebraic symbols?

Developing meaning for symbolic representations is an important part of the mathematics experience for all students. Researchers (Lannin, Townsend, Armer, Green & Schneider, 2008) recommended teachers to adopt collaboration as an instruction strategy to enhance students' conceptual understanding when learning algebra. They suggested that before introducing formal symbolic representations, students should develop their own nonstandard means for representing algebraic situations. Through student-to-student and student-to-teacher discourse, meaningful formal representations will emerge. As meaning continued to develop, students then will focus on the generality of their representations. Through the development of meaning of formal representations, we can reduce the pitfalls associated with instruction that focuses on manipulating symbols without connection to the meaning behind the symbols. The results of the pre- and post-diagnostic tests collected in this study did suggest that adopting collaboration as an instructional strategy did have a significant impact on the students' conceptual understanding on the algebraic and symbolic notation. The survey responses also indicated that students found collaborating with peers helpful on developing the meaning of the algebraic symbols.

Limitations of the study

The sample population was non-random and conveniently taken from the researcher's single-semester-long algebra II class from a community college, making the results specific to the selected population and no necessarily transferrable to a larger population. In addition, the verbal protocol analyses of interviews depended on how articulate the students were in being able to tell the researcher what they were thinking. Some of them might not be able to describe what they exactly were thinking in their mind. The data collected from the field notes of interviews may somehow be subjective as they were simply based on the researcher's own interpretations of students' responses.

Conclusions and Implications

The Teaching Approaches of Helping Students Understand Algebraic and Symbolic Notation more Effectively

It cannot be denied that most of the students see Mathematics as a foreign language which involves multiple levels of meaning and various visual and symbolic representations. Like any foreign language, the key to success is to grasp the translation skills well. Algebra students have to learn to translate between visual and symbolic representations and among the several levels of meaning for variables and equations (Driscoll, 1983). Without doubt, teachers are the ones who should carry the biggest responsibility in helping students learn these translation skills. Effective mathematics teachers can identify and communicate the continuity of mathematics to their students so as to help both learning and teaching of algebra. Here are some suggested approaches for educators on helping students learn algebra more effectively:

(1) The Correction Of Students' 'Invented' Methods

It is interesting to see that students tend to use their own methods to solve problems instead of using the formal methods taught by teachers. Their methods might be able to let them find the right answers for a certain types of questions, but fail them when they apply the same methods to other questions. Unfortunately, detecting this kind of 'methods' is not easy. Teachers need to be aware of the possible wrong methods and try to correct them with the students. Researchers also drew the teachers' attention to the formalization of method in the teaching of algebra that 'a fuller understanding of algebra … requires that attention be paid to the kind of methods that children use, and to ways of assisting children to become aware of the uses and limitations of different kinds of procedure… Only when children become aware of the limitations of their own methods, it is suggested, will they be prepared to contemplate the value of the more formal methods which the teacher is attempting to teach.' (Booth, 1984, p. 93).

(2) To Engage Students In An Early, Pre-Algebra Process Of Constructing Meaning For Equations And Variables (Driscoll, 1983)

The teaching of algebra should be based on concrete activities. Given the evidence that cognitive development is an essential influence on a young child's initial learning of algebra, teachers have to recognize to parallel the bridge from concrete representations to abstract representations. Researchers (Herscovics & Kieran, 1980) suggested teachers should avoid an early plunge into a totally symbolic approach to equations and unknowns. Instead, work with students on strictly arithmetic equations, focusing on the notion of equivalence and investigating the effects of various operations on equivalence. The advantage of this approach is to give students room to construct meaning for the concept of an equation. To bridge students from concrete representations to abstract representations is an essential step for students to build up correct concepts of algebra. Herscovis & Kieran (1980) suggested the following example in their research:

The first step was to cover one of the numbers in an arithmetic identity with a finger and to define ‘equation’ as ‘an arithmetic identity with a hidden number.’

“What’s the hidden number in this equation?”

$$+5+4=12-4+4$$

At the next level, the hidden number was represented pictorially, namely, with a box:

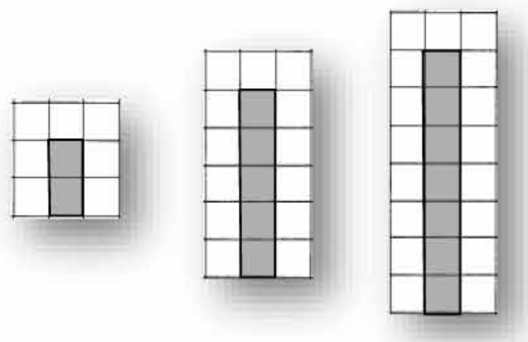
$$6+\square-1=5\times 2$$

Finally, after working with equations represented in these concrete and pictorial ways, the students were ready to deal with the abstract representation:

$$2\times a+5=7+6$$

(3) Providing Opportunities For Students To Express Generality

Students should be given chances to generalize for themselves rather than the teachers presenting the generalizations to them. Hart and her colleagues realized from the CSMS study that so many British teenagers had little access to three of the seven interpretations of letters (interpretations referred to p.13 of this paper) used in equations. The reason probably is that students have not yet developed beyond the concrete operational stage. They saw the need to provide more room for students to engage in more activities that are pictorial than symbolic and let them express generalities themselves so that they can construct meaning for the concept of variables on their own. Moreover, knowing how expressions are built up helps to clarify the process of 'undoing' which is the concept students need to grasp in solving equations. Here is a suggested exercise (Hart, 1981, p.118):



Ask students to find the number of white tiles needed for perhaps 10, 20, 40, and eventually 100 black tiles. Challenge them to come up with a rule that expresses the relationship between the numbers of black and white tiles.

(4) Asking Students To <u>'Find As Many Ways As You Can'</u> and Providing Room For Discussion

Encourage students to write as many algebraic expressions as possible in different ways so that students can realize that the same general relationship can be expressed in more than one way and one expression is equivalent to another. Besides, students should be given time to collaborate with peers to discuss the patterns constructed and how they achieve the answers. This exercise can help create a learning atmosphere in which students can be familiar with transforming expressions in various ways and can discuss the variable relationships.

(5) Teachers Should Not Be Too Directive In Teaching

Teachers should avoid throwing out predetermined set of rules to students. Instead, let students learn to construct and manipulate algebraic expressions and equations on the basis of their own understanding of mathematical relationships. The benefit of this approach is to help students learn how to choose the methods and the sequence of operations needed to solve an equation. The teacher's task is to facilitate not instruct, which means that the following statements should be avoided: "Watch me, this is how to do it."; "That's right, good, now you're getting it!"; "You made a mistake, fix it now."; "Remember to use your favorite strategy.", etc. Teachers are recommended to bring the students to the point of recognizing the importance of strategies rather than giving direct instructions.

(6) Teachers Should Monitor Their Own Use Of Language In Algebra And Emphasis The Concept Of Variables

Teachers can also guard against the growth of misconceptions by using appropriate language in algebra classes. Keep away from saying something like 'Let P = professors'. Teachers should state clearly that variables represent numbers and an equation expresses the equivalence resulting from the interaction of variables and numbers. Correct usage of language by teachers could change the misconceived tendency among many veteran algebra learners to perceive variables as labels for objects and equations as statements of correspondence between the labeled objects.

Recommendations for Further Research

The purpose of this research was to identify students' common misconceptions regarding to the learning of algebra and the effectiveness of adopting collaboration as an instructional strategy on the students' conceptual understanding of the algebraic and symbolic notation. The results collected from this study were only limited to a small sample taken from a semester-long algebra II class from a community college. To make the research results transferable and usable to other schools, it would be beneficial to seek a random sample from a larger population for future studies.

Summary

From this research study, it could be seen that there are misconceptions among students in their understanding of algebra. These misconceptions include the meaning of variables, misunderstanding of what an equation or an algebraic expression signifies, careless and informal and imprecise use of mathematical concepts, and poorly developed concepts of the operations of arithmetic. All of these misconceptions affect students' understanding of algebra and contribute to their inability to adapt algebraic techniques. The longer an algebraic misconception persists, the harder it is to remove from the students. Some students were even found to use their own methods which enable them to find correct answers of some low level problems, but thus hinder their learning of the formal taught methods for solving problems that need higher level skills. Teachers should definitely take some precautions to prevent those misconceptions about algebra and students' wrong methods on learning algebra from becoming so deeply rooted. Textbooks fall very short of guiding students to understand the meaning of the variables used in an equation or an expression. Mathematics teachers must bear the major responsibility for helping students to develop these understanding skills. Therefore, through this research, I hope that the results and the analyses can help Mathematics teachers on how to guide students to articulate their experience of Mathematics and to model their behavior in understanding the algebraic and symbolic notation on what they experience in classroom interactions.

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APPENDIX A

1) n is a whole number greater than 0 and less than 5. How many values of $3n$ can there be?

A 0
B 3 (m)
C 4*
D 5

2) m is a positive whole number. How many possible values can $10m$ have?

A 5
B 10 (m)
C 20
D Infinitely many*

3) Simplify $3m + 5 - 2m + 1$.

A 7 (m)
B 10
C $m + 6$*
D $7m + 8$

4) In the expression $t + 4$, what does t represent?

A 10
B 20
C time(m)
D Any number*

5) At a university, there are six times as many students as professors. This fact is represented by the equation $S = 6P$. In this equation, what does the letter S stand for?

A number of students*
B professors
C students (m)
D none of the above

6) Latoya and Keith dropped a ball from various heights and measured the height of each of the bounces. They recorded their data in the chart below.

Height from which ball was dropped (d)	40in.	50in.
Height of each bounce (b)	20in.	25in.

Which equation best shows the relationship between the height from which the ball was dropped and the height of the ball's bounce?

A $hb = hd + 20$(m)
B $b = 2d$
C $b = d + 30$
D $b = 1/2d$*

7) How many different values can the expression $k + 8$ have if k can be replaced by any number?

A One (m)
B Infinitely many*
C Eighty
D Zero

8) Trees are cut and new ones are planted. The data are shown below.

Number of trees planted (p)	Number of trees cut (c)
3	6
5	10

Which equation that will allow you to predict the number of trees planted (p) given the number of trees cut (c)?

A $c = 2p$*
B $c = p + 3$ (m)
C $c = 4p$
D $c = 2p + 100$

9) Rita put some hummingbird feeders in her backyard. The table shows the number of hummingbirds that Rita saw compared to the number of feeders. Bird-Watching

Number of Feeders (f)	Number of Hummingbirds (h)
1	3
2	5

Which equation best describes the relationship between h, the number of hummingbirds, and f, the number of feeders?

A $h = 11f$
B $h = 2f + 1$*
C $h = f + 2$ (m)
D $h = f + 6$

APPENDIX B

Questionnaire

Confidentiality

All information that is collected in this study will be treated confidentially. While results will be analyzed and reported, you are guaranteed that none of your personal information will be identified in any report of the results of the study. (Participation in this questionnaire is voluntary and any individual may withdraw at any time.)

About the Questionnaire

- This questionnaire asks for information about the relationship between the collaboration as instructional strategy and the students' conceptual understanding on the algebraic and symbolic notation.

- When question refers 'Algebraic and Symbolic Notation' it means by 'variables/letters', 'symbols' or 'signs' that are used in an algebraic expression or equation.

- This questionnaire should take approximately 30 minutes to complete.

- When you have completed this questionnaire, please return to Amy Kong, the PI of this research study.

- When in doubt about any aspect of the questionnaire, or if you would like more information about it or the study, you can reach Amy Kong by phone at 406-447-6364 or by email at amy.kong@umhelena.edu.

Thank you very much for your cooperation!

Part (I)

Please indicate if you have taken the following class in the past school year 2015-2016 or in the past semester (Fall 2016). Check all that apply.

PreAlgebra ☐

Algebra I ☐

Part (II)

Please indicate the extent to which you agree or disagree with the following statements. Mark (X) EACH item.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
Algebra is hard to learn.					
Algebraic and symbolic notation used in an expression or equation is confusing to me.					
I find discussing the algebraic and symbolic notation in class with classmates help me better understand the meaning of the notation.					
I find having the time discussing the algebraic and symbolic notation in class with classmates help me better develop the concepts of the notation.					
I find the time on the collaboration with classmates on learning the algebraic and symbolic notation well spent.					
I hope to have more opportunities in class to collaborate with classmates on the learning of algebra.					

Part (III)

What do you think is the most difficult component when learning algebra?

____
____
____
____
____

Why do you think that the collaboration with classmates in class can or cannot help you better understand the meaning of the notation?

____
____
____
____
____

Why do you think that the collaboration with classmates in class can or cannot help you better develop the concepts of the notation?

____
____
____
____
____

- END -

APPENDIX C

Interview questions

Can you tell me what the letters / symbols / signs in this algebraic expression / equation mean to you?

What does this algebraic expression / equation mean to you?

How did you set up this algebraic expression / equation?

Why did you think that $5 \times p \div q$ and $p \div q \times 5$ were unequal?

Why did you write $12m \div m = 12m$?

Why did you think that way?
How did you get $16m - 6m \times 2 = 10m \times 2$?

Why did you think $(8 - 8)\, x = x$?

Can you explain how you got $4a - a \div 2 = 21$ into $3a \div 2 = 21$?

When you were asked to use an algebraic expression to represent ______________, why did you give ______________?

Can you explain how you got the answer for this problem?

Can you explain why you think that?

Why did you say $c = 2$ in this question? Besides 2, do you think that c can be other values?

Why did you think that ________________ is true / not true?

APPENDIX D

SUBJECT CONSENT FORM FOR PARTICIPATION IN HUMAN RESEARCH AT MONTANA STATE UNIVERISTY

Principal Investigator (PI): Amy Kong
Phone: (406) 447-6364
Project Title: The Learning Difficulties Faced by Community College Algebra II students in Understanding Algebraic and Symbolic Notation

You are invited to participate with no obligation in a research study which is to investigate what learning difficulties Algebra II students have when interpreting algebraic and symbolic notation (variables/letters, symbols and signs used in equations and expressions). After the investigation, teaching strategies will be devised by the PI in order to help students combat those difficulties when learning algebra.

If you choose to participate in this research study, you would be asked to do a list of algebraic problems twice as a pre-test and a post-test. Based on the responses collected from the problems, you might be invited for an individual interview with the PI so more information can be gathered for the research study. You also would be invited to do a survey to comment a particular teaching strategy the PI will use in class when teaching the "Algebraic Expressions and Equations" topic. The information obtained from this study (including the results from the tests, survey and interviews) will be kept confidential and will only be reported in analyses with no specific connections made to individuals. At no point will your identity be revealed. All data will be stored in a locked file cabinet, accessible only by the PI.

Your decision whether or not to participate will not interfere with your course grade, or current or future relationships with your instructor. You may choose to withdraw from the study at any time without penalty, and the PI may choose to cancel your participation at any time.

The risks and inconveniences for participants will be minimal; the only additional requirement might be to complete an approximately half-hour-long interview with the Investigator at the participant's convenience, plus any further interviews needed for clarifications purposes only.

Do you have any questions? (Circle one) **NO** **YES**

If you circled YES, please contact the PI, Amy Kong, at the above phone number or by email at Amy.Kong@umhelena.edu before signing this form. If you have questions or concerns regarding your rights as a research participant, you may also contact the Chair of the Institutional

Review Board at MSU at (406) 994-4707, or at mquinn@montana.edu. Do not sign this form until these questions have been answered by your satisfaction.

I **AGREE** **DO NOT AGREE** (circle one) to participate in this research study. Participation is voluntary.

Participant's Name (please print):______________________________
Date: ____________________

Participant's Signature:____