USING ASYNCHRONOUS DISCUSSIONS TO FACILITATE COLLABORATIVE PROBLEM SOLVING IN COLLEGE ALGEBRA

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mathematics

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July 16, 2004
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ABSTRACT

This research study was conducted to investigate the nature and quality of online mathematical communication that occurred during collaborative problem solving and its effect on mathematical achievement in college algebra. Two intact sections of college algebra were randomly assigned to either a treatment group (online group work) or control group (individual seatwork). Both sections of college algebra met face-to-face and were taught by the same instructor. Students in the treatment group (n = 26) were placed into six collaborative groups. Four week-long online tasks designed according to the Treisman Workshop Model were assigned throughout the semester. These tasks were to be addressed collaboratively, with each student required to post three messages to their group’s onlinefolder. Students in the control group (n = 30) were assigned the same four tasks, but were required to work on these tasks individually.

Two content analysis techniques were utilized to answer the primary research question. The online transcripts of the treatment group were coded using the framework developed by Stacey and Gooding (1998) which examined the patterns of interactions. Results from this analysis revealed that the majority of the messages sent were coded as thinking aloud followed by responding, explaining with evidence, and questioning.

Each message was also ranked according to Gunawardena, Lowe, and Anderson’s (1997) Interaction Analysis Model. One in five messages was ranked as a high level message exhibiting evidence of the co-construction of knowledge. As indicated by the group’s average high and highest phase level reached, it was found that in 19 of the 24 problem solving episodes (six groups by four tasks) clear evidence of the co-construction of mathematical knowledge was shown.

Analysis of covariance (ANCOVA) was used to analyze mathematical achievement differences between the treatment and control groups. ANCOVA was performed on the raw scores of the final examination and researcher-designed problem solving examination using the pretest scores as the covariate. The treatment group performed as well or better on both measures of achievement. After controlling for initial differences in mathematical ability, the treatment group performed significantly better than the control group on the final examination.
CHAPTER ONE

STATEMENT OF THE PROBLEM

Introduction

The current reform movement in mathematics education has emphasized the social nature of learning and the important role that communication has in the learning and understanding of mathematics (Cobb, Boufi, McClain, Whitenack, 1997; Hiebert, 1992; Knuth & Peressini, 2001; Sfard, 2001; Steele, 2001; Wood, 1998). Professional organizations such as the National Council of Teachers of Mathematics [NCTM] and the American Mathematical Association of Two Year Colleges [AMATYC] have also recognized the value of engaging students in mathematical communication. For example, the NCTM’s Principles and Standards for School Mathematics (2000) highlights the importance of communication as a vital component in the mathematics classroom and has called for increased opportunities for students to explain, reason, and debate through mathematical communication.

It [communication] is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and the permanence for ideas and makes them public (p. 60).

At the postsecondary level, the American Mathematical Association of Two-Year Colleges reiterated this claim. In their publication Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus (AMATYC, 1995), the authors established standards and proposed recommendations for introductory college
mathematics. One such standard is the ability of students to read, write, listen to, and communicate mathematics to others.

Students will acquire the skills necessary to communicate mathematical ideas and procedures using appropriate mathematical vocabulary and notations…. Furthermore, mathematics faculty will adopt instructional strategies that develop both oral and written communication skills…. As students learn to speak and write about mathematics, they develop mathematical power and become better prepared to use mathematics beyond the classroom (p. 4).

The basic components of mathematical communication include a subject and a purpose which is focused on the learning of mathematics (McNair, 2000). In the context of classroom discourse, Pirie and Schwarzenberger (1988) extended this definition to include the following characteristics: purposeful talk concerning a mathematical subject with genuine student contributions through interactions. Although a variety of talk occurs in the mathematics classroom, teacher-led talk or the lecture model is the most prominent (Morton, 1993). According to Pirie and Schwarzenberger (1988) this type of talk is not a genuine form of mathematical communication since it does not consist of students “formulating their own opinions”, rather it consists of students “guessing the correct answers required to satisfy the questions posed by the teacher” (p. 460). Another more desirable type of talk is student-led talk which is characteristic of students working and communicating mathematics together in small groups.

**Collaborative Learning: Small Group Work**

Collaborative learning is viewed by professional organizations as an effective strategy to improve students’ learning and understanding of mathematics (AMATYC,
What students learn in the mathematics classroom “is greatly influenced by how they learn, and many students learn best through active, collaborative, small-group work” (Springer, Stanne, & Donovan, 1999, p. 22). Within this context, mathematics educators have consistently recommended the use of small group learning to facilitate mathematical communication among students (Artzt & Armour-Thomas, 1992; Civil, 1998; Curcio & Artzt, 1998; McNair, 2000; Stacey & Gooding, 1998; Yackel, Cobb, & Wood, 1991). Since the quantity of communication in small groups does not necessarily lead to the learning of mathematics (Cohen, 1994; Webb, 1991), it is imperative that true collaboration exist within these small groups. Therefore, the researcher has taken a definition of small group learning that requires two characteristics deemed necessary for true collaboration. These two characteristics are worthwhile problem solving tasks and genuine student interactions such as explaining with evidence or providing justification for one’s reasoning (Cohen, 1994; Pirie & Schwarzenberger, 1988).

Worthwhile Tasks

“Students need to work with mathematical tasks that are worthwhile topics of discussion” (NCTM, 2000, p. 60). Worthwhile tasks should provide students with the chance to enhance mathematical learning and understanding (NCTM, 2000; Pólya, 1957; Schoenfeld, 1980; Yackel, Cobb, & Wood 1991). Procedural tasks, such as those typically found in the homework section of a textbook, generally do not constitute worthwhile tasks that have the effect of stimulating mathematical discussions (Cohen,
Procedural tasks simply produce a product in which the correctness of the solution can be ascertained.

In contrast, problem solving tasks exemplify worthwhile activities that can help students solidify and extend what they know (Schoenfeld, 1980). Problem solving has been operationally defined as a ‘process’ by which students apply previously acquired skills and knowledge to new and unfamiliar situations (Branca, 1980; Krulik & Rudnick, 1989; NCTM, 2000). It is these types of worthwhile tasks that incorporate mathematical problem solving that are most often a catalyst for mathematical communication consistent with genuine student interactions.

Genuine Student Interactions

Simply providing a small group of students with a task does not necessarily supply these students with a reason to interact. In separate reviews of the literature Webb (1985) and Cohen (1994) determined that the number of interactions did not necessarily predict mathematical achievement especially when students were working on traditional school tasks. It was concluded that the relation between the total amount of interaction within a group and subsequent mathematical achievement of the group’s participants differed according to the nature of the task. Worthwhile tasks solicit genuine student interactions that are vital to the productivity of small group learning (Cohen, 1994; Hoyles, 1985; Pirie & Schwarzenberger, 1988; Stacey & Gooding, 1998; Webb, 1991). Such tasks typically require resources such as information, knowledge, problem-solving strategies, and procedural skills that “no single individual possesses” (Cohen, 1994, p. 8).
Genuine student interactions have been defined by Pirie and Schwarzenberger (1988) as consisting of a genuine contribution made to the group which, in turn, produces student interaction. Therefore, a single contribution adds to the communication by moving the conversation forward. Interactions indicate that this movement has been picked up by other members of the group. The depth and breadth of these interactions is typically dependent upon the nature of the task selected (Cohen, 1994; Stacey & Gooding, 1998; Webb, 1991).

For more routine learning, it is necessary for students to help each other to understand what the teacher or the textbook is saying, and it is helpful for them to offer each other substantive and procedural information. For conceptual learning, effective interaction should be more of a mutual exchange process in which ideas, hypotheses, strategies, and speculations are shared (Cohen, 1994, p. 4).

Productive or genuine interactions have been also defined according to various outcomes. The most common defining characteristic of interactions that are said to be “productive” is related to subsequent mathematical achievement as measured by procedurally driven tests (Cohen, 1994; Dees, 1991; O’Brien, 1993; Springer, Stanne, & Donovan, 1999). Productive or effective group interactions are also defined in association with conceptual learning (Cobb, Boufi, McClain, & Whitenack, 1997; Cobo & Fortuny, 2000; Cohen, 1994;), increased problem solving abilities (Curcio & Artzt; 1998, Dees, 1991; Norwood, 1995; Stacey & Gooding, 1998), higher order or critical thinking (Cohen, 1994; Gokhale, 1998), and the co-construction of mathematical knowledge through engagement in high levels of mathematical communication (Elbers and Streefland, 2000; McNair, 2000; Steele, 2001). Examples of high levels of mathematical communication go beyond the sharing and comparing of answers. They

Research findings on the success of small group learning in mathematics have been extremely positive at both the elementary and secondary level (Cohen 1994; Johnson, Johnson, & Smith, 1991; Webb, 1985, 1991), as well as the collegiate level (Dees, 1991; Lucas, 1999; Springer, Stanne, & Donovan, 1999). While extensive research has shown that collaborative learning is an effective means for increasing students’ understanding in K-12 mathematics, the research on the potential impact of small group collaborative learning specific to college algebra is still relatively limited. The remainder of this chapter will examine successful models of collaborative learning in college mathematics that involve students working together in close proximity or students working at a distance through computer-mediated communication.

**Collaborative Learning: The Treisman Model**

Nationwide, over a quarter of the entering freshman students do not have the necessary prerequisite knowledge to enroll in introductory collegiate mathematics courses such as college algebra (NCES, 1996). In addition, high attrition rates (failures and withdrawals) in these mathematics courses have increased this concern (Duncan and Dick, 2000). These two compounding factors have prompted mathematics educators to include supplemental forms of instruction such as collaborative small group learning.
One such strategy that incorporates both worthwhile problem solving tasks and genuine student interactions are the Treisman Workshop Model programs.

During the mid 1970’s Uri Treisman, at the University of California – Berkeley, conducted research on the cause of low performance of African American students in their first calculus course as compared to the performance of Chinese American students. He found that there was no difference in these students’ mathematical preparation, mathematical ability, socio-economic status, or family support. Instead, Treisman found a striking difference in the way these groups of students studied mathematics (Duncan & Dick, 2000; Fullilove & Treisman, 1990; Treisman, 1985, 1992). Out of the classroom, African American students tended to work on mathematics in isolation, “rarely consulting with other students” (Duncan & Dick, 2000 p. 365), whereas Chinese students often worked collaboratively in peer study groups. From this research, Treisman developed the Mathematical Workshop Program to provide students enrolled in introductory calculus with supplementary peer collaborative problem solving. The success of this program has prompted over 100 universities and community colleges to adopt such programs under the names of the Emerging Scholars Program and Math Excel Program for both introductory mathematics and science courses (Moreno & Muller, 1999).

The goal of many Emerging Scholars mathematics programs is to “increase both the success and the participation” (Duncan & Dick, 2000, p. 366) of minority students through the use of collaborative problem solving. To accommodate small group learning, students participate in discussion sections to supplement their typical class meeting times. During these collaborative sessions students are provided with problem solving
worksheets in order to stimulate both mathematical learning and mathematical understanding. Typically, these problem solving sets are composed of challenging problems to engage students in mathematical talk, as well as uncover discrepancies in a student’s mathematical understanding. The goal of these workshop sessions is for students to work collaboratively to solve and understand the ideas on these specially designed problems (Duncan & Dick, 2000; Fullilove & Treisman, 1990).

Students who participate in programs similar to the Treisman Workshop Model (such as the Emerging Scholars programs at The University of Texas-Austin and The University of Wisconsin-Madison and the Math Excel programs at Oregon State University and The University of Kentucky), have a higher probability of receiving a grade of an A or B in their mathematics course than students who do not participate (Duncan & Dick, 2000; Fullilove & Treisman, 1990; Moreno & Muller, 1999). Although these studies provide supportive evidence of the efficacy of these programs and their associated problem solving tasks, little research exists on why they work. Specifically, no study has examined the dynamics of Treisman-style collaborative groups as they solve problem sets. Moreover, no study has examined the feasibility of transferring this collaborative model to the online setting.

Collaborative Learning: The New Medium

Over the past 15 years the computer has evolved from a tool for doing mathematics to a medium through which small and large groups of learners can communicate mathematics. The use of online technologies, such as computer-mediated
communication, for educational purposes has the potential to alter interactions that occur in the distance learning environment, as well as provide a supplementary medium for interactions to take place in the traditional face-to-face classroom. Defined by Gunawardena, Lowe, and Anderson (1997), computer-mediated communication refers to the “exchange of messages among a group of participants by means of networked computers, for the purpose of discussing a topic of mutual interest” (p. 397). Based on these defining characteristics, computer-mediated communication offers new opportunities to improve the collaborative aspect of learning by facilitating communication among learners (Curtis & Lawson, 2001; Garrison, Anderson, & Archer, 2001). Specifically, computer-mediated collaborative learning creates the opportunity for a small group of students to actively co-construct knowledge (Gunawardena, Lowe, & Anderson, 1997; Kanuka & Anderson, 1998; Warschauer, 1997) and to engage in critical thinking (Garrison, Anderson, & Archer, 2001; Jeong, 2003). Furthermore, online communication can serve as a “possible cognitive amplifier that can encourage both reflection and interactions” (Warschauer, 1997, p. 472).

As traditionally viewed, collaborative learning takes place through face-to-face communication. With the use of online technologies, such communication among collaborative learners can take place in a text-based medium where interactions and the exchange of information occur among group members who are separated spatially. While extensive research has shown face-to-face collaborative learning is an effective method for enhancing mathematical performance (Dees, 1991; Springer, Stanne, & Donovan, 1999; Stacey & Gooding, 1998; Webb, 1991), research on mathematical
collaborative learning via computer-mediated communication, where communication is time-independent and text-based, is still relatively limited.

**Statement of the Problem**

Computer-mediated communication has been widely used to support small and large group communication in distance learning (Beaudrie, 2000; Gunawardena, Lowe, & Anderson, 1997; Kanuka & Anderson, 1998; Myers, 2002) and moderately used as a supplement to face-to-face mathematical courses (Kramarski, 2002; Miller, 1999). Despite its attractiveness as an instructional tool, its adoption has surpassed the understanding of how this medium best promotes collaborative learning. Generic questions related to the quantity and quality of interactions and satisfaction of participant have been answered (Berge & Mrozowski, 2001; Levin, Kim, & Riel, 1990; Mason, 1992), as well as the ‘no significant difference’ phenomenon between face-to-face courses and their online counterparts (Russell, 1999; Weems, 2002). However, few studies have examined the nature and quality of the online mathematical communication that occur during online collaborative learning, especially in introductory college mathematics courses.

In his review of evaluation methodologies for computer-mediated communication, Mason (1992) recommended broadening the research to include educational objectives such as critical thinking and the co-construction of knowledge. While many current studies have examined Mason’s suggestions in computer conferencing debates and graduate level distance education courses, the need for studies
exploring the co-construction of mathematical knowledge in online collaborative learning environments motivates the need for this research study.

The importance of communication in the mathematical classroom has received considerable attention over the past ten years. Yet, many college mathematical classrooms still remain void of student-to-student communication and collaboration (Morton, 1993). Although the Treisman Workshop Model has been shown to be an effective strategy to increase student collaboration and subsequently student achievement, no research study has investigated the quality of the mathematical communication that emerges while students are engaged in the problem solving tasks. With the introduction of online technologies, computer-mediated communication can and has allowed students and instructors to interact in fresh ways. Yet, little research regarding online mathematical communication exists, particularly related to introductory mathematics courses such as college algebra. Therefore, the question as to how online technologies can be used to enhance mathematical learning and understanding is still open for debate.

**Purpose of this Study**

The primary focus of this research study was to examine the nature and quality of online mathematical communication that occurred as college algebra students worked collaboratively in small groups. Consistent with the research in mathematics education, the nature of the online communication was defined as the basic elements or the patterns of mathematical talk (Knuth & Perissini, 2001; Stacey & Gooding, 1998). For example, Stacey and Gooding (1998) used content analysis techniques to code the nature of
mathematical communication that occurred in face-to-face small groups. They found eight categories of interaction including asking questions, responding, directing, explaining with evidence, thinking aloud, proposing ideas, commenting, and refocusing discussion. (A copy of this framework can be found in Appendix A.)

In this study, the quality of the online communication was operationally defined in accordance to the level of talk or the co-construction of knowledge that occurs during collaborative problem solving (Gunawardena, Lowe, & Anderson, 1997; Kanuka & Anderson, 1998). Gunawardena, Lowe, and Anderson (1997) proposed a constructivist model for the content analysis of computer-mediated communication in which they theorized five hierarchical levels or phases that characterized and ranked individual online communication. Based on these five phases, the Interactional Analysis Model [IAM] was developed and used to examine the co-construction of knowledge. (A copy of this instrument can be found in Appendix A.) These five phases can be summarized as

Phase I – sharing and comparing information
Phase II – discovery and exploration of dissonance
Phase III – negotiation and/or co-construction of knowledge
Phase IV – testing and modification
Phase V – phrasing of agreements/applications of the newly constructed meaning.

High level messages, according to Gunawardena, Lowe, and Anderson (1997), are messages that are ranked in Phase III, IV, or V. Specific to the discussion regarding the quality of the online mathematical communication, this research study sought to determine whether college algebra students utilizing online asynchronous discussions
could reach high levels of communication as evidenced in the co-construction of mathematical knowledge.

The fundamental idea behind mathematics communication is to enhance learning and understanding through purposeful talk and genuine student interactions. Therefore, the secondary focus of this research study was to examine the quality of the online communication in relationship to student achievement outcomes in college algebra. Specifically, this research first investigated the relationship between the quality of online communication variables and subsequent mathematical achievement based on problem solving ability and procedural skill. Online communication variables included both the quantity of messages sent, as well as the quality of messages ranked as a high level of communication according to the Interaction Analysis Model. These communication variables were determined at the individual and group level. Secondly, by employing quasi-experimental techniques, this study examined the differences in student achievement between two sections of college algebra in which one section utilized online collaborative learning and the other individual seatwork.

### Research Questions

The primary research questions for this study were shaped by the four dimensions of online collaborative learning outlined by the purpose statement. These included: 1) the nature and quality of the online mathematical communication that occurred in small groups, 2) the relationship between an individual’s quality of online mathematical communication and his or her procedural and problem solving achievement, 3) the
relationship between a group’s quality of online mathematical communication and the
dividuals’ procedural and problem solving achievements, and 4) the effect of utilizing
online asynchronous mathematical communication on student achievement outcomes in
college algebra. Specifically this study sought to address the following questions.

1. What is the nature and quality of the online mathematical communication that
   occurred in collaborative groups in a college algebra course?

2. Is there a relationship between mathematical achievement in college algebra as
   measured by a problem solving examination and the quality of online
   mathematical communication as measured by the Interaction Analysis Model?
   a. Is individual mathematical achievement as measured by a problem solving
      examination related to the online communication phase level of the
      individual?
   b. Is individual mathematical achievement as measured by a problem solving
      examination related to the online communication phase level of the group?

3. Is there a relationship between mathematical achievement in college algebra as
   measured by a procedural final examination and the quality of online
   mathematical communication as measured by the Interaction Analysis Model?
   a. Is mathematical achievement as measured by the final examination related
      to the online communication phase level of the individual?
   b. Is mathematical achievement as measured by the final examination related
      to the online communication phase level of the group?
4. How does the completion of the online collaborative tasks designed according to the Treisman Workshop Model affect mathematical achievement in college algebra?
   a. Is there a significant difference in achievement on a problem solving examination between students completing assigned mathematical tasks individually and those utilizing online collaborative group work?
   b. Is there a significant difference in achievement on the procedural final examination between students completing assigned mathematical tasks individually and those utilizing online collaborative group work?

Definition of Terms

For the purpose of this research study, the following definitions were used:

- **Active participant**: A student who posted at least one message during a single online task.
- **Asynchronous discussions/communication**: Online discussions/communication that take place through the use of a bulletin board messaging system.
- **Bulletin board system**: A computer and associated software which typically provides an electronic message database where people can log in and leave messages. Messages are typically split into topic groups. Any user may submit or read any message in these public areas. A bulletin board system may also provide archives of files, personal email and any other services or activities of interest to the bulletin board’s system operator.
• **Collaborative problem solving**: A mutual engagement of participants in a coordinated effort to solve a problem together.

• **Computer Conferencing**: The exchange of messages among a group of participants by means of networked computers, for the purpose of discussing a topic of interest. Also referred to as Computer-Mediated Conferencing.

• **Computer-mediated communication**: Communication between persons by way of computer networks. Also known as CMC.

• **Computer-mediated collaborative problem solving**: Collaborative problem solving via computer-mediated communication.

• **Content analysis**: A systematic, replicable technique for compressing many words of text into fewer content categories based on explicit rules of coding.

• **Distance education**: Any formal approach to learning in which the majority of the instruction occurs while the educator and learner are at a distance from each other.

• **Interaction**: a) The totality of interconnected and mutually responsive messages which make up a computer-mediated conferencing. b) The entire gestalt formed by the online communications among the participants.

• **Message**: A communication sent from one member of a group to other members of that group.
  
  • **Interactive message**: Any statement or question relating to the subject under discussion which is either in response to another or receives a response.
• **Non-Interactive message**: Any statement or question relating to the subject under discussion, but is not made in response to another message.

• **Threaded discussion**: A threaded discussion is an online dialog or conversation that takes the form of a series of linked messages. The series is created over time as users read and reply to existing messages.

• **Treisman Workshop Model**: A supplemental mathematical program that provides students with the opportunity to work collaboratively on problem solving exercises through well developed worksheets.

• **Turn taking**: An utterance of any length and grammatical form that is produced by a speaker without interruption.

• **Unit of meaning**: A consistent theme or idea in a message.

• **WebCT**: An online course management system (short for Web Course Tools) available for purchase by universities and other entities.
CHAPTER TWO

REVIEW OF THE LITERATURE

Introduction

The purpose of this research study was two-fold. First, this study examined the nature and quality of online mathematical communication that occurred during collaborative group work in college algebra. The focus of this component of the study was to critically scrutinize if or how students developed mathematical meaning and/or the co-construction of mathematical knowledge. Secondly, this study investigated the effects of engaging in online mathematical communication on students’ mathematical achievement as measured by both procedural and problem solving ability.

This chapter is divided into three sections. The first section depicts the theoretical framework upon which this study was founded. Following this section, the second component outlines the relevant research base in which this study was grounded. The final section provides a brief summary of the research base, as well as a summary of the methodological approaches that were used in this research.

Theoretical Foundation

Current movements in education have moved beyond the notion that information or knowledge can be simply transferred from one individual to the next. Instead, such movements have highlighted the need for students to actively co-construct their own knowledge through individual and social processes. This is the foundation of the
sociocultural approach to learning and development (Cobb & Yackel, 1996; John-Steiner & Mahn, 1996; Sfard, 2001; Sierpinska, 1998; Vygotsky, 1962). In the sociocultural perspective, communication is essential to learning. Within this framework, the focus is on the interdependence between individual and social processes during the co-construction of knowledge. Participation in cultural practices (e.g. types of classroom activities and communication), therefore, profoundly influence individual learning. Students co-construct meaning, including mathematical meaning, as they reason about ideas with others and listen to others share their thoughts (Steele, 2001). Thus, in this perspective, learning is rooted in social interactions.

The sociocultural approach to learning and development was first established and applied by Vygotsky in the 1920’s and 1930’s. For Vygotsky (1978), human activity takes place in cultural contexts mediated by language and other symbolic systems. Accordingly, all higher mental functions have their origins in social practices. Cognitive development, in the Vygotskian sense, can be depicted as the transformation of socially shared activities into internalized processes. Knowledge is constructed by the transformations from the interpsychological plane to the intrapsychological plane through the internalization of higher mental functions.

Internalization is simultaneously an individual and a social process. In working with, through, and beyond what they have appropriated in social participation and then internalized, individuals co-construct new knowledge. In contrast to facile internalization, which leads to a limited combination of ideas, internalization that involves sustained social and individual endeavors becomes a constituent part of the interaction with what is known and leads to the creation of new knowledge (John-Steiner & Mahn, 1996, p. 197).
According to Vygotsky (1962), the main importance of having students learn collaboratively is that behaviors which occur within a group can be transformed into individual processes. Therefore, the focus of sociocultural approaches has been to investigate the ways in which the co-construction of knowledge is internalized, appropriated, and transformed in formal and informal settings such as small group learning (John-Steiner & Mahn, 1996; Sfard 2001; Steele, 2001; Warschauer, 1997).

**Review of the Related Research**

In accordance with the sociocultural perspective, this literature review is focused on the nature of social interactions occurring in small group instruction and their effect on various mathematical learning outcomes. The first section provides a general overview of the effects of small group learning as reported by recent meta-analyses specific to the postsecondary level of education. The next section examines the literature on small group learning outcomes as related to procedural and problem solving ability specific to college mathematics. This section includes the research on the Treisman Workshop Model. The third section is focused on the research that involves the nature and quality of social interactions. This section includes methodologies for examining the mathematical communication. The fourth section investigates the research related to computer-mediated communication during the small group learning process.

**Small Group Learning: Meta-Analyses Results**

Small group learning has been used as a general term used to encompass both cooperative learning and collaborative learning (Johnson, Johnson, & Smith, 1991;
Although cooperative and collaborative learning have often been used interchangeably (Oxford, 1997), some researchers distinguish between the two. In the research, cooperative learning is considered more structured and systematic in which students are working in small groups towards a common specified goal. “Cooperative learning refers to a particular set of classroom techniques that foster learner interdependence as a route to cognitive and social development” (Oxford, 1997, p. 443). In contrast, collaborative learning is a relatively unstructured in which participants “negotiate goals, define problems, develop procedures, and produce socially constructed knowledge in small groups” (Springer, Stanne, & Donovan, 1999, p. 24). In agreement with this definition, collaborative learning is founded in the sociocultural theory.

For the purpose of this research review, articles reporting effects on either cooperative learning or collaborative learning have been reported. Slavin (1985) incorporated the term ‘first generational’ research for those studies that investigated outcome variables such as achievement, attitudes toward learning, and retention rates associated with small group learning. Over the past two decades hundreds of studies have compared the effects of small group versus individual learning on these related outcome variables. Many of the results of these studies confirming the value of small group learning have been reported in meta-analyses and/or reviews of the research (Cohen, 1994; Johnson, Johnson, & Smith, 1991; Springer, Stanne, & Donovan, 1999; Webb, 1985). Since this research study is focused on collaborative learning at the
postsecondary level, the results of meta-analyses associated with college students are reported.

In a meta-analysis of over 150 studies comparing cooperative, competitive, and individualistic learning at the collegiate level, Johnson, Johnson, & Smith (1991) reported cooperative learning promoted higher levels of academic achievement than individualistic learning ($d = 0.62$) or competitive learning ($d = 0.59$). These results were consistent regardless of the group task. In addition, it was found that cooperative learning promoted greater motivation to learn, more frequent use of cognitive processes such as higher-level thinking and metacognitive strategies, and retention of skills learned.

In a more recent meta-analysis on the impact of small group learning in postsecondary science, mathematics, engineering, and technology (SMET) courses Springer, Stanne, & Donovan (1999) reiterated the above results. This study sought to determine the main effects of small group learning on three outcomes among SMET undergraduates: achievement, persistence, and attitude. Of 398 studies related to small group learning in SMET courses from 1980 or later, 39 met the inclusion criteria defined by the authors. These criterions were 1) the study examined students in SMET courses; 2) the study involved small group learning composed of two to ten students inside or outside of the classroom; 3) the research was published after 1980; and 4) the researchers reported enough information to calculate an effect size. Therefore, studies that either compared small group learning with another instructional strategy or used a single sample, pre- and post-test design were included.
The main effect of small group learning were significant and positive with respect to achievement (d = 0.51), persistence (d = 0.46), and attitudes (d = 0.055). Students who worked in small groups in their SMET coursework demonstrated higher achievement, greater persistence through the course, and had more favorable attitudes than comparison students. The results based on achievement held regardless of gender or the SMET field – allied health, mathematics (including statistics and computer science courses), and science. In contrast, the positive effect size on achievement was significantly greater for groups composed of predominately African American or Latinos (d = 0.76), than primarily white (d = 0.46) or heterogeneously mixed groups (d = 0.42). The meta-analysis on achievement also found that there was a significant difference in effect sizes when students were worked in small groups outside of class such as study groups (d = 0.65) than for in-class meetings (d = 0.44). Also, various procedures for placing students in groups – self-selection, random assignment, heterogeneously matched based on achievement – was not significantly related to achievement outcomes.

Springer, Stanne, and Donovan (1999) calculated the average weighted effect size of d = 0.53 on mathematical achievement (including statistics and computer science courses). This effect size, the standardized difference between the means, provides evidence that small group learning strategies are beneficial to the learning of mathematics. Cohen (1969) specified that an effect size of 0.5 can be described as ‘medium’ and is “large enough to be visible to the naked eye” (p. 23). In statistical terms, and effect size can be described in terms of the average percentile standing of the small group learning participants as compared to non-small group participants. In this
manner, an effect size of 0.53 would indicate that the mean of the experimental group is at the 53rd percentile of the comparison group (Cohen, 1969). In other words, the differences of the two means were estimated to be a distance of 0.53 standard deviations.

**Small Group Learning in Collegiate Mathematics**

The remaining literature in this section is specific to student outcome variables such as mathematical achievement and problem solving ability as a consequence of working in small groups. All studies were conducted in the postsecondary mathematics classroom, especially intermediate and college algebra. The discussion below presents the methodological foundation for this current research project.

**Effects on Mathematical Achievement.** The effect of small group learning on mathematical achievement is often measured by mean scores on final exams (Austin, 1995; Mears, 1995; O’Brien, 1993; Searborough; 2001), pre- and post-test measures of achievement (Mears, 1995; Valentino, 1988), or student grades based on total points (Lucas, 1999; Rupnow, 1996). O’Brien (1993) in her doctoral work, conducted a study to determine the effects of cooperative learning versus the traditional lecture-discussion on the achievement, attitudes and attrition rates of students in college algebra at a community college. She used a counter-balanced quasi-experimental design with four intact sections. Two instructors taught one comparison course (lecture-discussion) and one treatment course (weekly cooperative learning activities). To control for internal validity, the lecture component, as well as the text, quizzes, and exams were the same for all four sections. The sample consisted of 47 students enrolled in two comparison classes
and 43 students enrolled in two treatment classes during a six-week summer session. Students in the cooperative learning sections, were placed into groups of four to five heterogeneously matched based on ability. Group composition remained intact throughout the semester.

Mathematical achievement was measured by the individual’s score on a final examination. An analysis of covariance was conducted to determine whether mathematical achievement was significantly different based on group membership. Although the final exams were higher for the cooperative learning sections of college algebra, this difference was not significant at the 0.01 level. Similar results revealed no significant difference in the attitude towards mathematics between the comparison and treatment groups. A Chi-Square Test of Independence revealed no significant differences with regard to attrition rates. O’Brien concluded that either method of instruction – traditional or cooperative learning – was effective in college algebra.

No significant differences were also found in a study by Austin (1995) who investigated the effect on achievement and retention of implementing cooperative learning groups throughout a semester in finite mathematics. Mathematical achievement was measured by scores on five instructor-made unit tests and retention of content was measured by scores on a common final examination. An analysis of covariance, with SAT Total Score serving as the covariate, revealed that no significant differences in mathematical achievement measures existed between the experimental (those receiving cooperative learning instruction) and the control (those receiving traditional lecture instruction) groups.
In contrast, several doctoral studies in the college mathematical classroom have shown significant and positive results pertaining to achievement when students work in small group settings when compared to students taught traditionally (Lucas, 1999; Mears, 1995; Rupnow, 1996; Searborough, 2001; Valentino, 1988). Lucas (1999) utilized an experimental design in which students \((n = 734)\) were randomly assigned into either a college algebra treatment section (16 sections with 307 total students) that implemented a cooperative learning format or a control section (27 sections with 427 total students). Student grades and point totals were analyzed to evaluate academic achievement with the quantitative ACT score serving as a covariate. The analysis of covariance for grades showed a significant difference in favor of students in the cooperative learning sections, while the ANCOVA for total points revealed no significant differences between the two groups.

Investigating cooperative learning strategies employed in the recitation component of precalculus at a large university, Searborough (2001) found that low ability students in the cooperative recitations significantly improved their score on a final examination as compared to low ability students in traditional recitations. In the treatment recitation, students were assigned to mixed-ability groups of three to four students in which they worked on weekly problem sets together for the entire semester. In contrast the traditional recitation involved students asking questions about homework problems and taking quizzes. Furthermore, it was predicted that a female who had a C, D, or F in precalculus and who was in the cooperative-learning treatment in precalculus
earned a statistically significantly higher grade in her subsequent Calculus I course than one who was in the traditional treatment in precalculus.

According to Cohen (1994), most proponents of small group learning favor the use of mixed ability heterogeneous groups because of the ‘hypothesized benefits’ to low ability students of receiving help from higher ability student (p. 10). For Vygotsky (1978), the true benefit of collaborative learning among students is to assist a student in advancing through their own zone of proximal development (ZPD). The ZPD is defined as the gap between what a student can do alone and what a student can do with the help of a more abled student. Therefore, the main importance of having students learn collaboratively is behaviors that occur within a group can be transferred to the individual. This phenomenon is especially important in the ability to solve complicated mathematical tasks such as problem solving activities (Curcio & Artzt, 1998; Steele, 2001).

**Effects on Mathematical Problem Solving Ability.** A primary goal of mathematics teaching is to develop the ability to solve a wide variety of complex mathematics problems (NCTM, 2000). Cognitive psychologists view problem solving as a series of mental operations rather than a combination of learned behaviors (Lester, 1980). Mathematical instruction, therefore, should emphasize the process (how to solve) rather than the product (getting the final answer). In this view, mathematical problem solving has two stages: understanding the problem and searching for a procedural strategy to solve the problem (Lester, 1980). Researchers have found that students working alone have the most difficulty in the first stage (Kallam, 1996; Lester, 1980; Schoenfeld, 1980).
Examining gender differences on individual problem solving ability, Kallam (1996) found that the greatest student difficulty was focused on understanding the problem. She conducted problem solving interviews on a sample of 47 students enrolled in college algebra at a Midwestern university. The results of her study indicated that only 46% of males and 30% of females understood the problem well enough to attempt to solve it. Moreover, only 17% of the males and 0% of the females solved the problem correctly, despite the fact that 71% of the males and 65% of the females believed they had. Therefore, the benefit of engaging students in collaborative problem solving is to help students become more capable of understanding the problem (Kallam, 1996; Krulik & Rudnick, 1989) by engaging in discussions about the problem and listening to others explanations or strategies.

Dees (1991) investigated the use of cooperative learning strategies in improving students’ ability to solve word problems in an undergraduate remedial mathematics course. Over 100 under-prepared students enrolled in a one-hour lecture section were randomly placed into two laboratory groups – one using cooperative learning strategies. The study was designed to answer the following research questions: 1) Are there simple, but effective, cooperative teaching strategies adaptable to the ordinary classroom by the average teacher?; and 2) Does an increase in students’ working cooperatively yield an increase in students’ ability to solve problems?

Tests of the means on three achievement measures, Arithmetic Skills test, Elementary Algebra Skills test, and an instructor-designed open-ended diagnostic test to solve word problems revealed no significant differences in mathematical ability between
the control and treatment groups. The ability to solve word problems was measured using a subset of items from the course’s algebra final examination. The results revealed that there existed a significant difference in the mean scores on this subset of questions in favor of the cooperative learning group (t = 2.80, p < 0.01). In addition, the algebra final exam scores were also significantly greater for the cooperative learning group (t = 2.29, p < 0.05).

These results demonstrated that small group learning can significantly improve mathematical problem solving and procedural abilities of students enrolled in undergraduate mathematics courses. Similar results have been shown for students working in small groups in a Treisman-style model of collaborative learning.

**Treisman Workshop Models.** During the mid 1970’s, Uri Treisman at the University of California – Berkeley conducted an observational study that examined the characteristics of successful students in first year college calculus. In the first stage of his research, Treisman interviewed calculus teaching assistants and asked them to identify strong and weak students. It was discovered that a disproportionate number of weak students were African American, whereas a large majority of strong students were Chinese American (Fullilove & Treisman, 1990; Treisman, 1985). This result shifted Treisman’s research study from examining the characteristics of strong students to the factors that explained why Chinese American students were successful at calculus and African American students were not.

To investigate the performance difference between these students, informal interviews were conducted on 20 Chinese American and 20 African American calculus
students as they prepared and studied for class. Treisman found that Chinese American students more often studied in groups, whereas African American students tended to study in isolation. “In effect, these students [African American] had compartmentalized their daily life into academic and social components” (Duncan and Dick, 2000, pg. 365). Based on these findings, Treisman developed the Mathematics Workshop Program (MWP), a supplemental program that provides students with the opportunity to work collaboratively on problem solving exercises, as well as the integration of academic life into a social venue.

The design of the Mathematics Workshop Program (or any other Treisman-style program) is characterized by four attributes. First, students are recruited and self-selected into the program based on their readiness for calculus. Specific student populations are often targeted based on race and high school characteristics (such as low income or rural). Secondly, students are required to participate in supplementary workshops throughout the week. Therefore, MWP students are involved in up to six hours of recitation per week. Thirdly, these recitation sections (typically 20-25 students) are composed of students working in small groups along with a qualified teaching assistant. Finally, each group works on a specially designed worksheet that is intended to provide students with the skills to earn a successful grade, a foundation to achieve success in the next sequential course in mathematics, and areas of mathematics that students hold misconceptions (Fullilove & Treisman, 1990). These worksheets are composed of problems that fall into one or more of the following categories:

1. “Old chestnuts” – problems that appear frequently on examinations but rarely on homework assignments;
2. “Monkey wrenches” – problems designed to reveal deficiencies either in the students’ mathematical backgrounds or in their understanding of a basic concept;

3. Problems that introduce students to motivating examples or counterexamples that shed light on or delimit major course concepts and theorems;

4. Problems designed to deepen the students’ understanding of and facility with mathematical language; and

5. Problems designed to help students master what is know as “street mathematics” – the computation tricks and shortcuts known to many of the best students but which are neither mentioned in the textbook nor taught explicitly by the instructor.

The primary responsibility of the students during the workshop is to help each other solve and understand the problems on the worksheet. “Because MWP students are asked to exhibit the same skills they will later have to demonstrate on quizzes and examinations…the workshop is uniquely suited to provide students with the tools to perform at optimum levels of proficiency” (Fullilove & Treisman, 1990, p. 469).

Fullilove and Treisman (1990) examined the success of the MWP at the University of California – Berkeley by collecting data on 646 African American students who were enrolled in first year calculus during the years of 1973-1984. They analyzed the achievement of these students in calculus segregated by whether the student attended MWP or did not (nonMWP). To help control for the effects of self-selection of the MWP participants, the authors disseminated the data into three distinct time periods: 1) 1973-1977, prior to the establishment of MWP; 2) 1978-1981, MWP created and fully financed through the Department of Education; and 3) 1982-1984, MWP partially funded through the university serving less than 23% of the African American student population.
The Chi-Square test was used to determine if there existed a significant association between group membership (MWP and nonMWP) and academic outcome variables. These variables included receiving a B- or better in calculus (honors grade), receiving a D- or less (failing grade), and persistence at the university. The authors concluded that the failure rates among MWP students (3% in 1978-1981 and 7% in 1982-1984) were significantly lower (p < .0000) than all groups of nonMWP students (33% in 1973-1977, 40% in 1978-1981, and 41% in 1981-1982). The proportion of students earning honors grades was also significantly associated with involvement in MWP. Those MWP students were two to three times more to earn a grade of B- or better than nonMWP students regardless of the year of involvement.

Based on these results, the authors concluded that the Mathematics Workshop Program was successful in “promoting high levels of academic performance among African American students” (Fullilove & Treisman, 1990, p. 472). Now called the Emerging Scholars Program (at the University of Texas – Austin and the University of Wisconsin – Madison) or the Math Excel program (at the University of Oregon and the University of Kentucky), these Treisman-style programs have been shown to increase student achievement in introductory college mathematics courses, especially for African American and Latino American students (Bonsangue & Drew, 1995; Duncan & Dick, 2000; Moreno & Muller, 1999).

The Emerging Scholars Program (ESP), established in 1988 at the University of Texas, is one adaptation of the Mathematics Workshop Program. Like most Treisman-style programs, ESP students are recruited and self-selected to participate in either first or
second semester of calculus. These students attend six additional hours of calculus workshops per week outside of their normal class meeting working collaboratively on problem solving worksheets.

Moreno and Muller (1999) investigated the effectiveness of the Emerging Scholars Program on calculus performance from 1988 to 1995. Specifically, the authors analyzed achievement data in two semesters of college calculus to determine if performance differences were associated with race, gender, or participation in ESP. The sample (n = 1,565) included all students categorized as freshman or sophomore who completed the first semester of calculus (Calc I) during the study period. Students whose quantitative SAT score was below 460 were dropped from the study, since these students typically performed “markedly less well in calculus” (Moreno & Muller, 1999, p. 36). Of the participating sample, 28.4% of the students participated in ESP for at least one semester.

The results of an ordinary least squares regression indicated that ESP students grades in Calculus I were about one letter grade higher than nonESP students. NonESP students did not attend any of the workshops. Instead, these students attended a traditional calculus recitation session. Moreover, African American, Latinos, and women participating in ESP earned significantly higher Calculus I grades than their European American or male counterparts. When ESP participation was controlled for, the authors found no racial differences in final grades. These analyses suggested that there are some program benefits after self-selection is accounted for.
Duncan and Dick (2000) examined the efficacy of the Math Excel program, a Treisman-style Workshop Model, at the University of Oregon. Although the results as measured by final grades are similar to other Treisman programs, this study differs from the others. First, the results of the Math Excel program are not restricted to performance in calculus. Performance is evaluated in four classes including college algebra and calculus. Secondly, race was not identified as a significant variable in the study.

Regression analysis revealed a significant effect on achievement favoring the Math Excel students. On average Math Excel students attained higher grades than nonMath Excel students, by over half a grade (0.671 grades points on a 4-point scale). These differences were also found after controlling for self-selection by adjusting for SAT-M scores. The difference between actual and predicted grades for Math Excel students was “almost always positive and, taken together, showed a statistically significant positive effect of Math Excel” (Duncan & Dick, 2000, p. 370).

Studies involving the effect on mathematical achievement of students participating in Treisman-style programs have received scrutiny. Some scholars argue that students involved in these programs are more motivated, study more, and subsequently receive better grades than other students. In most cases, students involved in Treisman-style programs are required to participate in an extra six hours of class preparation. Critics and non-critics of the research about these programs have called for research that goes past basic achievement data, to ask the questions as to how and why these programs work. More specifically, “what are the aspects of the peer discourse that different students find so effective?” (Duncan & Dick, 2000, p. 372).
Content Analysis Approaches to Small Group Learning

Mathematical content analysis approaches to small group learning move beyond the general questions as to the effectiveness of small group learning versus individual learning on such measures as academic achievement, problem solving ability, persistence, and/or attitudes toward learning. Instead, this approach examines the nature and quality of the interactions occurring in a small group setting and associates them with either group productivity in the ability solve mathematical problems (Artzt & Armour-Thomas, 1992, 1996; Curcio & Artzt, 1998; Gooding and Stacey, 1993; Stacey & Gooding, 1998) or the co-construction of knowledge and mathematical understanding (Cobo & Fortuny, 2000; Elbers & Streefland, 2000; Knuth & Peressini, 2001; Zack and Graves, 2001).

Cohen (1994), in her review of the research, examined the type of interactions occurring in small groups and their relationship to productive learning outcomes. Therefore, her focus was on the interactions, the nature of the task, and the productivity of the group. Articles included in this review were selected based not on the comparison between small group and individual learning, but rather on studies that “contrasted alternate forms of cooperative learning groups” (Cohen, 1994, p. 5). In this review, it was found that the most single, positive predictor of achievement was the giving of elaborate explanations.

Cohen (1994) found that for routine learning (e.g., solving a traditional mathematics problem), student interaction was typically focused of sharing and comparing of information. Therefore, task instruction, student preparation and the nature
of the teacher role were key components for the learning of procedural information. In
closest, conceptual learning required student interaction to be a shared exchange process
where ideas, strategies, and explanations are given. Less structured problems defined as
‘ill-structured’ were better suited for conceptual learning.

…limited exchange of information and explanation are adequate for routine
learning in collaborative seatwork, more open exchange and elaborated discussion
are necessary for conceptual learning with group tasks and ill-structured
problems. (Cohen, 1994, p. 10)

Webb (1985) also acknowledged that explanations were associated with
achievement. She investigated four categories of student interaction in relationship to
achievement. These categories were nonspecific interaction, giving help, receiving help,
and sequence of behaviors. Webb (1985) used partial correlations to determine that the
elaboration of ideas during small group learning produced higher achievements in
subsequent post-test measures. That is, the student who does the explaining benefits
most from collaborative learning. More importantly, Webb (1985) found that not
receiving help or receiving a terminal response (such as an answer) was consistently
negatively related to achievement.

Several researchers have developed frameworks for analyzing mathematical
discussions that occur during collaborative problem solving (Artzt & Armour-Thomas,
1992, 1996; Chiu, 1997; Garfalo & Lester, 1985; Schoenfeld, 1985; Stacey & Gooding,
1998). By utilizing content analysis techniques, these researchers were able to examine
the patterns of communication that are most often associated with effective learning.
Artzt and Armour-Thomas (1992, 1996; see also Curcio & Artzt, 1998) developed a
framework to analyze problem solving interactions occurring in small groups. They
partitioned the problem solving episode into eight episodes: read, understand, analyze, explore, plan, implement, verify, and watch and listen. An episode was considered a ‘macroscopic chunk’ of consistent problem solving behavior in which an individual is engaged. Within each episode, Artzt and Armour-Thomas categorized the behaviors as cognitive, metacognitive, or neither, as in the case of watch and listen. Episodes of cognition were involved with doing or processing information, whereas episodes of metacognition were situated in analyzing and planning.

The authors used this cognitive-metacognitive framework to describe the behaviors of 27 seventh graders working in small groups on a problem solving task. Students were placed into mixed-ability groups of four to five students. Student’s behaviors were coded on one-minute intervals by episode and by cognitive level. Of 442 behaviors coded, 38.7% were metacognitive with the greatest frequency in exploring and understanding. The authors suggested that metacognitive processes were important for task completion because they served to “enhance and propel the problem solving process” (Artzt & Armour-Thomas, 1992, p. 161). To support this claim they noted that the unsuccessful groups, those who failed to complete the task, lacked consistent metacognitive behaviors. In addition, of the behaviors exhibited, 36% were at the cognitive level with the greatest episode percentages in exploring and reading. Based on these results, the authors concluded that the continuous interplay of metacognitive and cognitive behaviors was deemed necessary from successful problem solving and maximum student interaction.
Stacey and Gooding (1998) used a similar framework to analyze mathematical communication in relationship to the successful learning of a mathematical concept. This content analysis procedure was used to determine the patterns of interaction that occurred in small group learning, whereas Artzt and Armour-Thomas’ framework was used primarily to categorize the aspects of thinking. Successful learning was defined as improved achievement on a post-test associated with division. The task was selected to reveal fifth and sixth graders misconceptions regarding division concepts. Effective groups were categorized by having at least two students who improved from pre- to post-test.

Specifically, Stacey and Gooding (1998) sought to answer the following research question: What are the nature and content of the childrens’ talk and how do they relate to what the children learn? A coding scheme was developed to analyze the nature of the interactions. Coding categories included asking questions, responding, directing, explaining with evidence, thinking aloud, proposing ideas, commenting, and refocusing discussion. The authors found that members of the effective groups interacted more and showed more interaction in every coding category except thinking aloud. Based on post-test achievement scores, groups in which students asked questions, responded to a request for clarification, and gave more detailed explanations improved significantly over ineffective groups who simply talked aloud.

Student discussions in small group learning are not only categorized by the type or nature of communication using such frameworks as described above. Discussions may also be examined for evidence of students reaching joint understanding through the co-
construction of knowledge. Elbers and Streefland (2000) used a case study approach to examine how students, working in collaborative groups, constructed shared mathematical understanding. In this study, eighth grade students worked in small groups of three alternated by whole group discussions. By investigating the sequence of verbal interactions in one mathematics lesson, the authors found that three cycles of argumentation existed in the shared construction of mathematical knowledge. These recursive cycles were “used as a tool that allowed students to propose ideas, repeat them, explore and evaluate them, and in such a form that many pupils could contribute” (Elbers & Streefland, 2000, p. 487). Moreover, these argumentation cycles allowed students to elaborate on explanations, which were taken over by other students and expanded upon. In this manner these cycles not only fostered the improvement and acceptance of knowledge, but the dispersion of ideas in these cycles provided for the expansion and construction of shared mathematical meaning.

In examining a similar question regarding how students co-construct knowledge in small group learning, Cobo and Fortuny (2000) used a case study approach to identify the models of interaction that occurred between a mixed-ability dyad of high school students solving an area-comparison problem. After establishing four models of interaction – alternative, guided, relaunching, and cooperative, the authors identified how each model influenced the students’ cognitive development. In all examples of discourse analyzed, the interactions occurring during group work significantly influenced the individual development and heuristic abilities in the problem solving process regardless of the model of interaction.
Knuth and Peressini (2001) presented a theoretical framework of examining mathematical discussions based on the work of the sociocultural theorist Mikhail Baktin. Baktin’s ‘notion of dialogicality’ presupposes that understanding is a result of the ‘interanimation’ of voices, the listener coming in contact with the speaker (Knuth & Peressini, 2001, p. 8). Understanding, according to Baktin, requires the listener to take an active role in converting the words of the speaker into their own. These utterances provided by the speaker can be differentiated into two different functions of dialogue. The univocal dialogue conveys meaning and is focused on transmitting information from speaker to listener. The dialogic dialogue is to generate or construct meaning. Knuth and Peressini (2001) used these descriptors of talk in the mathematics classroom to determine how they function in the classroom during large group and small group learning. By utilizing this framework in the mathematics classrooms, the authors found that most of the mathematical discussions were univocal in nature. That is, students and teachers tended to provide discussions that shared or compared information. Teachers more often used the initiation-reply-evaluate sequence in which a teacher initiates a question, the student replies with only a single answer, and the teacher evaluates this response. The authors argued that without the higher-level dialogic discourse students will not acquire a deeper understanding of mathematics.

**Computer-Mediated Small Group Learning**

Over the past decade research on small group learning has expanded outside the boundaries of traditional face-to-face interactions to encompass learning within small groups through the use of online technologies. Using the computer as a medium to
communicate between students and teachers offers educators a convenient tool for overcoming time and distance constraints of the typical classroom setting (Curtis & Lawson, 2001; Henri, 1992; Kaye, 1992). Through the use of computer-mediated communication (CMC), both distance education and traditional face-to-face courses can be transformed from a one-way instructional approach to a highly interactive approach to learning (Henri, 1992; Vrasidas & McIssac, 1999; Warschauer, 1997). Not only can CMC encourage collaborative learning, it significantly “alters the nature of learning and increases its quality” (Henri, 1992, p. 119).

Computer-mediated communication can be synchronous, happening in real time or asynchronous. In asynchronous communication, people are not constrained to time and space limitations of being logged in at the same time or being in the same location. According to Hara, Bonk, and Angeli (2000), asynchronous CMC has several advantages over both verbal face-to-face communication and synchronous communication in the academic arena. One such advantage of asynchronous communication is it increases the ‘wait-time’ of the response. In the face-to-face classroom setting ample wait time has been stressed to provide students with the opportunity to reflect before they respond. The combined interactivity and asynchronous nature of CMC can allow for these opportunities of reflection. Interactivity can be defined as the two-way communication between instructor and students or among students in a technical environment (Henri, 1992). This distinguishing feature of CMC is now considered the gold standard and a minimum requirement of any distance education program (Cheney, 2002).
When viewed in the sociocultural framework, which emphasizes social interactions, the text-mediated character of CMC is a powerful tool for creating the co-construction of knowledge between or among individuals (Gunawardena, Lowe, & Anderson, 1997; Kanuka & Anderson, 1998; Warschauer, 1997). Online communication can function as a ‘cognitive amplifier’ that serves to encourage both reflection and interaction. Within CMC, “the historical divide between speech and writing has been overcome with the interactional and reflective aspects of language merged into a single medium” (Warschauer, 1997, p. 472). Thus, CMC creates the opportunity for groups of students separated by time and space to co-construct knowledge by linking the reflective properties of writing with the interactive characteristics of speech.

**Use of Online Technologies in Higher Education.** In the most general sense, distance education can be defined as education which all or most of the time is in a different place from teaching, and the principal means of communication between learners and teachers is through technology (Cheney, 2002, p. 1). By this definition, distance education at the postsecondary level includes courses delivered through mail correspondence, audio and/or visual technologies, and through computer technologies such as CD-ROMs or online tools like the Internet or email. For the purpose of this study, distance education will encompass those learning experiences delivered through asynchronous computer-mediated communication.

According to the National Center of Educational Statistics (2002), 56% of all two-year and four-year degree granting institutes of higher education offered some form of distance education courses during the 2000-2001 academic year. Of these postsecondary
institutes, 90% offered Internet courses using asynchronous computer-based instruction. Cheney (2002), in his bibliography on distance education, estimated 32% of degree-granting universities offer distance education courses in undergraduate mathematics.

The most extensive literature review regarding online technologies in distance education was conducted by Berge (1997). Reporting that the characteristics of CMC have been found to be a viable means of teaching and learning, he concluded that the main advantages of CMC included:

- The asynchronous nature of an online course permits 24-hour access to other people and resources.
- The asynchronicity allows students to reflect on their own responses, as well as the comments of other students.
- CMC allows for a mentoring model of instruction in which one student or teacher can guide others in the successful completion of a task or activity.
- CMC permits interdisciplinary problem solving.

Use of Online Technologies in Collegiate Mathematics. Russell (1999) summarized over 355 research reports on technology for distance education, all of which found ‘no significant difference’ when comparing learning outcomes in traditional face-to-face courses and distance education courses. Consistent with this no significant difference phenomena reported by Russell (1999), research evaluating mathematical courses taught onsite versus online have also reported no significant differences with respect to mathematical achievement. For example, Weems (2002) conducted a study to compare two instructional approaches in beginning algebra – one taught completely online and the other taught in the traditional onsite format – on mathematical
achievement. To control for internal validity besides self-selection into the courses, each section used the same textbook, tutorial CD, course schedule, assignments, and examinations. As a substitute for the lecture component used in the onsite section, the online students were instructed to watch an instructional video and to work through their homework assignments. If students in the online section encountered difficulties with homework problems they were instructed to either post a message on the course’s bulletin board or email the instructor.

The dependent variable, mathematical achievement, was measured using a repeated measures design on three examinations that were administered to both sections of beginning algebra at an on-campus location. A total of 16 students in the online section and 18 students in the onsite section completed all three exams. Although the main effect on achievement was not significant ($F(1, 31) = 0.168, p = 0.684$) between the two sections, there existed a significant decrease in the examination performance by the online students throughout the semester.

More importantly, the investigation into the type of online communication used by the online section revealed that email between a single student and the instructor was the most prominent communication. Throughout the semester, only three students used the bulletin board messages, even when bonus points were given to students who posted on the bulletin board. To increase the interactivity of online mathematical courses, the author suggested the implementation of weekly group work assignments.

Instead of offering online mathematics courses, some instructors have supplemented traditional lecture courses with online materials/resources (Sakshaug,
2000), as well as increasing student interactions through asynchronous mathematical discussions (Beaudrie, 2000) and email (Kramarski, 2002) for small group work. Kramarski (2002) investigated the effects of using email discussions between students and teachers on the learning of graphing. This study employed a quasi-experimental design in which two classes of ninth grade students were either assigned to an EXCEL only or EXCEL/email section. The sample consisted of 25 ninth grade students in each class. In the treatment section (EXCEL/email) students were asked to not only solve for seven graphical tasks, but were also required to describe and email their solution processes to their instructor. The EXCEL only class covered the same content and students in this group were required to do the same seven tasks, there was no requirement to explain their solution processes to their instructor through email technology. Instead, these students were to write their solutions and processes on paper and turn them into their instructor.

Two quantitative measures of mathematical achievement with regard to graphing were constructed by the researcher and administered to both sections; a graph interpretation test and a graph construction test. In addition, a qualitative analysis of the email messages was conducted. An analysis of covariance, with a pretest score used as the covariate, revealed a significant difference favoring the EXCEL/email group on the graph interpretation test at the end of the study (F(1, 47) = 51.21, p < .001). In addition, an analysis of covariance also found a significant difference in favor of the treatment group on the graph construction test (F(1,47) = 3.6, p < 0.05).
The results of the qualitative analysis of the email messages as they compared to the written messages turned in by the EXCEL only group also revealed some significant differences. Content analysis was used to code ‘semantical units’ into three levels of discourse: tutorial, metacognitive, and life. At the tutorial level, a student discusses with a tutor or teachers the insights into their learning process by the use of a common language understood by both (e.g., Temperature is the dependent variable.). The metacognitive level is characterized by a student’s investigation into their own learning processes (e.g., I chose that graph because…). Finally, at the life level students relate their learning to their own life (e.g., Email motivated me to understand graphing).

Content analysis revealed that the email messages had a significantly larger percentage of messages at the tutorial (32% vs. 20%, p < 0.05) and metacognitive (48% vs. 16%, p < 0.01) levels than the written responses. Kramarski (2002) concluded that writing played a critical role in understanding graphing. Furthermore, the use of online technologies enhanced the nature of mathematical discussion between a student and the teacher.

**Transcript Analysis Approaches to Computer-Mediated Communication**

During the past decade, the most popular methodologies used in research on computer-mediated communication have been survey research (e.g., Hiltz, 1988), case studies (e.g., Mason, 1992), and more recently, content analysis of the online transcripts (e.g., Gunawardena, Lowe, & Anderson, 1997; Henri, 1992). Survey methodologies have been often used with examining student perceptions of the quality of learning that took place in the online environment. Whereas evaluative case studies have provided researchers with the means to generate theories to be tested (Mason, 1992). In contrast,
content or interaction analysis allows researchers to investigate the nature and quality of the online communication during small group learning. In particular, such analyses of the online transcripts have allowed researchers to answer the following questions:

1. What is the quality of the [online] discourse? (de Laat, 2002)

2. Was knowledge constructed within the groups by means of [online] exchanges among participants? (Gunawardena, Lowe, & Anderson, 1997)

3. Was individual achievement in problem solving related to the online communication phase level of the individual? (Beaudrie, 2000)

Henri (1992) led the way in the content analysis of computer-mediated communication. She proposed a framework for analyzing message content to provide educators with a better understanding of what is said, how it is said, and the process and strategies used during online discussions. Her analytical framework identified five dimensions to code ‘units of meaning’ within a single message. A unit of meaning was defined as a consistent theme or idea in a message. The five dimensions included 1) participation rate – the number of messages/statements sent; 2) social comments not related to content matter; 3) the type of interactive processes such as a direct or indirect response to another individual; 4) categorization of the cognitive skills used such as asking questions, making inferences, or formulating hypotheses; and 5) statements regarding metacognitive processes. Henri concluded that “an attentive educator, reading between the lines in texts transmitted by CMC, will find information unavailable in any other learning situation” (Henri, 1992, p. 118).

Modifying Henri’s content analysis framework, Myers (2002) examined the process of computer-mediated collaborative mathematical problem solving. The aim of
her research study was to distinguish the behaviors of individuals and groups that were effective in solving a non-routing mathematical task from ineffective individuals and groups. Myers coded ‘units of meaning’ within a message sent along Henri’s first three dimensions; participative, social, and interactive. The interactive dimension included statements, questions, or comments that received a response or that were made in response to another. Problem solving interactive messages were coded according to the mathematical content analysis scheme used by Stacey and Gooding (1998; see also Gooding & Stacey, 1993) for face-to-face collaborative mathematical problem solving (see discussion above). Instead of coding messages along the cognitive and metacognitive dimensions, Myers chose to code messages according to the problem solving heuristic employed. Heuristic episodes were categorized as orientation (understanding the problem), organization (planning strategies), execution, and verification.

Using this content analysis framework, Myers (2000) examined the mathematical discussions occurring between small groups of three middle school mathematics teachers enrolled in a mathematical content course. Effective problem solving groups were identified by their high scores on a mathematical task measured by an analytical rubric. It was found that these effective groups participated more with less messages categorized as social and more messages coded interactive than ineffective groups. Moreover, groups that were effective communicated more throughout the problem solving process in a recursive manner by revisiting each heuristic episode more than once. Although the sample size used in this study was relatively small, the results have common connections
with successful groups in face-to-face problem solving. (Artzt & Armour-Thomas, 1996; Stacey & Gooding, 1998)

Gunawardena, Lowe, & Anderson (1997) investigated the social construction of knowledge during an online debate utilizing CMC. More specifically, they wanted to determine whether knowledge was co-constructed within a group by means of online exchanges or if individual participants changed their understanding as a result of interactions between the group members.

The sample for this study consisted of 554 invited participants in a week-long online debate. This debate took place during the World Conference of the International Council on Distance Education. Participants were chosen because of their expertise in distance education. In addition, each participant was required to select either the affirmative or negative side of a statement regarding the role of interaction in an effective distance education course.

Several methods of content analysis, including the framework designed by Henri, were considered by the authors to examine the communication that took place. Subsequently, each of the considered frameworks were discarded in order to “overcome the shortcomings” (p. 402) in the application of these models. One such shortcoming with previous models was the definition of interactivity. Instead of defining interactions in a teacher-centered paradigm as a mechanical response to another individual, Gunawardena, Lowe and Anderson (1997) viewed interaction as “the totality of interconnected and mutually-responsive messages, which make up the conference” (p. 407). To overcome these shortcomings, the authors developed their own instrument, the
Interaction Analysis Model for Examining the Social Construction of Knowledge in Computer Conferencing (IAM). This model examined the content of each message sent and ranked it according to five hierarchical levels, called phases: 1) Sharing and Comparing of Information; 2) Dissonance or Inconsistency among Ideas, Concepts, or Statements; 3) Negotiation of Meaning/Co-Construction of Knowledge; 4) Testing and Modification of Tentative Constructions; and 5) Statement or Application of New Knowledge. The first two phases represent lower mental functioning, whereas the last three represent higher mental functioning. Moreover, Phase V incorporates metacognitive processes.

Applying this model to the online debate, Gunawardena et al. (1997), found that even though all five levels were represented, the majority of the postings occurred in Phase I. The existence of Phases II and III were indicative of “high quality” communication “as several participants were involved in exploration of dissonance or inconsistency and the negotiation of meaning on co-construction of knowledge” (p. 417). The authors concluded that the IAM had many inherent advantages over other models. These benefits included their definition of interaction as a vehicle to the co-construction of knowledge and its focus on the patterns of knowledge construction within a group. Since the development of the IAM, several researchers have used this instrument to examine the quality of interactions that occurred in computer-mediated small group learning (Beaudrie, 2000; Kanuka & Anderson, 1998;).

In his doctoral work, Beaudrie (2000) examined whether there existed differences in phase levels, as measured by the IAM, between mathematical problem solving groups
located on- or off-campus. In addition, he wanted to determine how differences in an individual’s or group’s phase levels related to academic achievement. The study was conducted during the Spring semester of 1999 in a combination geometry course for both pre- and in-service mathematics teachers. The sample consisted of 18 students (13 on campus and 5 off campus) who were placed in mixed-ability groups based on prior mathematics ability. Each group consisted of four to five students.

Throughout the semester, groups were assigned problem solving tasks that were to be addressed and solved collaboratively via online synchronous or asynchronous discussions. Messages sent were coded according to Gunawardena, Lowe, and Anderson’s (1997) Interaction Analysis Model. Beaudrie (2000) found that the majority of these messages were ranked as Phase I (41%) and Phase III (30%).

Using the chi-square test of independence, it was found that the communication phase level on an individual was independent of whether the student was on- or off-campus (p < 0.05). Thirty-six null hypotheses related to the research questions were also analyzed. Analyses were also conducted to determine if there existed a relationship between an individual’s achievement (measured by the final exam scores and average test score) and an individual’s phase level of communication. The phase level of an individual was measured in three ways: 1) the average communication phase level calculated by dividing the total amount of coded messages of the individual by the total number of messages sent; 2) the number and percentage of high level messages coded in Phase III or above sent by the individual; and 3) the total number of messages sent. Using Pearson’s correlation, no significant relationships were found between any
combination of dependent and independent variables. Analysis of group achievement on tasks and online phase levels found a significant positive relation between the rank of each group task and number of scored messages for each group on that task ($r = 0.509$, $p < 0.001$). There also existed a significant positive relations between the rank of each group task and the highest level message attained by each group for that task ($r = 0.360$, $p < 0.05$) As in Myers’ (2002) study, the more successful groups were those that interacted more and achieved higher levels of communication.

**Summary of the Methodological Principles**

To examine the nature of the online mathematical communication that occurred during collaborative learning, the study used a content analysis framework developed by Stacey and Gooding (1998). This coding technique used in this framework was slightly modified by Myers (2002) to account for computer-mediated mathematical communication. To examine the quality of the online communication, the study used the transcript analysis framework developed by Gunawardena, Lowe, and Anderson (1997). Since the development of this Interaction Analysis Model, it has been used in numerous studies to determine the quality of computer-mediated communication. These studies include Beaudrie (2000) and Kanuka and Anderson (1998). Specifically, this instrument has been utilized to determine whether or not participants engaged in computer-mediated communication reach high level messages evident in the co-construction of knowledge.

To assess the relationship between the quality of online communication and subsequent mathematical achievement, two studies have been monumental in designing
the methodological framework for this study. Webb (1985, 1991) investigated the correlation between face-to-face small group communication and achievement. Beaudrie (2000) examined the relationship between the amount and level of communication (as ranked by the Interaction Analysis Model) and achievement of students enrolled both on- and off-campus in an upper-level geometry course.

Finally, studies such as those conducted by O’Brien (1993), Austin (1995), and Lucas (1999) have aided the researcher in developing the quasi-experimental design to help investigate differences in mathematical achievement between students utilizing online collaborative learning and those who did not. These studies have highlighted the importance of using a covariate to control for initial differences in mathematical ability between groups, as well as the data collection instruments such as final exams and problem solving test.
CHAPTER THREE

RESEARCH METHODOLOGY

Introduction

This study was designed to examine the nature and quality of online mathematical communication that occurred in collaborative groups and its effects on mathematical achievement in college algebra. This chapter outlines the methodologies used in this research study. The discussion is focused on five dimensions: the selection of the sample, the initial results of a pilot study, a description of the treatment, the data collection procedures, and nature of the data analysis.

Population and Sample

The population for the research study consisted of undergraduate students enrolled in Math 105 – Algebra for College Students during the spring semester of 2004 at Montana State University. Each spring semester ten sections of college algebra are offered on campus serving approximately 300 students or about 25% of all mathematics enrollments below calculus. These students are typically freshman enrolled in their first college-level mathematics course.

The sample for this study included students enrolled in two intact sections, one treatment section and one control section, of college algebra. Of the eight sections that met three days per week, two sections were randomly selected to participate in the study. Intact sections of college algebra were used because it was impossible to assign students
randomly. The sections selected were the 9 am and 1 pm sections, which met on Monday, Wednesday, and Friday. The 1 pm section was randomly assigned as the treatment section which utilized online communication technologies for collaborative group work. This group will be referred to as the task/online group. The 9 am section was assigned as the control section in which students worked on the same assignments as the treatment group, but were required to do so individually. This group will be referred to as the task only group. A total of 56 participants, 30 in the 9 am section (control) and 26 in the 1 pm section (treatment), were initially included in this study. Of these participants, 36 were male and 20 were female. Approximately 60% of the students were previously enrolled in a remedial mathematics course prior to college algebra and 19% were repeating college algebra for at least the second time. These statistics were consistent with the other eight sections of college algebra. Table 1 shows the initial make-up of each of the two sections of college algebra at the beginning of week three.

Table 1. Initial Make-Up of the Treatment and Control Groups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Meeting Time</th>
<th>First College Math Course</th>
<th>Remedial Math Course</th>
<th>Repeating Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment Task/Online</td>
<td>1 pm MWF</td>
<td>21%</td>
<td>61%</td>
<td>18%</td>
</tr>
<tr>
<td>(n = 26)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Tasks Only</td>
<td>9 am MWF</td>
<td>21%</td>
<td>59%</td>
<td>21%</td>
</tr>
<tr>
<td>(n = 30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Over the course of the academic semester a total of seven students in the treatment group and five students in the control group did not take the final examination. These students withdrew from the class prior to this last exam. Of these seven students
who withdrew from the treatment group, two did not participate in any of the online
tasks. The remaining five students participated in one or more online tasks and their
messages were analyzed according the stated methodology. Although these student’s
online discussions were included in the section describing the nature and quality of online
communication, their data was omitted from the analysis of the other three research
questions.

Description of the Course

College Algebra is a three semester hour college credit course. The description of
the course from the Montana State University Bulletin (2000) is as follows:

*Math 105 - Algebra for College Students* – Topics include the concept of
functions and explorations of various types of functions such as linear, rational
and quadratic, selected conic sections, exponential and logarithmic functions, and
other algebra topics.

All students enrolled in college algebra during a single semester receive the same
syllabus, have the same suggested homework assignments, and take the same common
hour exams. Students entering the university with an ACT mathematics sub-score less
than 23 or a mathematics SAT score lower than 530 are required to take a mathematics
placement exam to determine appropriate placement. All students enrolled in the college
algebra course must meet one or more of these prerequisites by the end of the second
week of the semester.

Two experienced course supervisors organize the content of college algebra and
facilitate in the training of the new teaching assistants. In addition, they develop the
examinations for all sections to ensure that the same content is covered. For this study,
the course supervisors were instrumental in determining the content validity of the assigned tasks, as well as the two mathematical achievement tests.

Results of the First Pilot Study

An initial pilot study was conducted during the spring semester of 2003 at Montana State University to inform this research study. Because little data exists regarding the use online collaborative group work in introductory college mathematics courses, the goal of this pilot study was to determine whether or not undergraduates would utilize asynchronous discussions for mathematical problem solving. The main question of the pilot study was to determine the type of tasks that would elicit online mathematical talk. The secondary question included determining the appropriate group size.

One intact section of college algebra taught by the researcher was chosen to participate in the pilot study. This course met face-to-face on Monday, Wednesday, and Friday. A total of 46 students, 20 females and 26 males, participated in this study. Students enrolled in this class were assigned seven online assignments which included procedural and problem solving tasks. During the course of the semester the researcher altered the online group size, as well as the type of task that students worked on in online groups. The tasks varied from procedural problems to problem solving tasks in which students were to apply their procedural skills. The results of this pilot study were as follows:
1. Problem solving tasks and those procedural tasks that would uncover common misconceptions produced the largest response from students. This is consistent with the suggestions and research findings of studies examining face-to-face collaborative mathematics learning (Cohen, 1994; Fullilove & Treisman, 1990; Stacey & Gooding, 1998; Webb, 1985, 1991).

2. Since all groups tended to have at least one non-responsive participant, a group size of four was found to be the ideal size for online mathematical communication.

3. Gender was not related to the amount of mathematical communication in the online groups, reflecting Beaudrie’s (2000) findings that the amount and quality of online mathematical communication was not significantly different between males and females in an online geometry course for pre-service secondary mathematics teachers.

4. There were some students who did not participate in the online group work. This dilemma was remediated mid-way through the pilot study by requiring students to post at least three messages, providing them with a rubric outlining quality mathematical communication, and assessing the group activity based on individual participation.

At the end of the semester the researcher asked students to comment on their likes and dislikes about using online asynchronous discussions to work on mathematical problems. The majority of students (79%) felt positively about the use of asynchronous discussions. These students commented that the format of online activities allowed them
to interact with students (65%) and strengthen their mathematical understanding by having to explain how they solved the problem (22%). Some of the students’ comments included:

WebCT was very useful. I found that working in groups and sharing ideas and answers made me feel a lot more comfortable especially in math. To be able to work at your own pace and check your answers and processes with people benefited me a lot.

I think WebCT is a great idea because instead of having to do tedious assignments every night we had only a few problems that we could spend time on and help each other within our group. My grade was determined more from what I knew than what I could hand in on time.

Negative comments regarding the use of asynchronous discussions included the lack of feedback from some of the group members (46%) and the students’ lack of familiarity with the interface of WebCT or computers (22%). Students who participated in this pilot study also provided suggestions for improvement.

You should provide students with some initial activities to get familiar with WebCT. It was very confusing at first.

It was too easy to forget to post messages to my group. Having assignments due on the same day throughout the semester would have been easier to remember. Also, a reminder in class or a simple email to tell us to get our act together would have been helpful.

All recommendations from the participants were taken into consideration for designing the activities for this research study. Based on these results, it was determined that the structure of the online tasks and feedback from the instructor were crucial elements overlooked by the researcher in the pilot study, but critically examined in the present research design.
Description of the Treatment

Due to the inability to randomly assign participants, two intact college algebra sections were randomly assigned to be either the task/online group (treatment) or the task only group (control). These two sections of college algebra, the 9 am and 1 pm sections, were randomly selected. For this study, the 1 pm section was randomly chosen as the task/online class (n = 26). The researcher selected one experienced instructor to teach both sections of college algebra to ensure that each class received the same daily lecture and was administered the same test at the same time and location. This instructor had over eight years experience teaching college mathematics courses, but this study was his first time teaching with an online component.

Procedures for College Algebra

The treatment in this study was the use of computer-mediated communication to facilitate collaborative group work in college algebra. This communication took place asynchronously through threaded discussions using the WebCT software program. Asynchronous threaded discussions were chosen to be the sole method of online communication due to its time-independent nature.

Students in treatment group (n = 26) met face-to-face for 50-minutes three days per week. In addition to scheduled class time, students, working in small groups of four to five, were required to participate in four collaborative problem solving tasks via asynchronous threaded discussions. A total of six problem solving groups were formed. The mathematical tasks were designed by the researcher according to the Treisman
Workshop Model. This model was selected based on the positive research results on student achievement and increases in procedural skills in college level mathematics courses (Fullilove & Treisman, 1990; Duncan & Dick, 2000). A complete description of the Treisman Workshop Model can be found in Chapter Two.

The task only group (n = 30) also met face-to-face for 50 minutes three days per week. Students enrolled in this section of college algebra did not participate in online collaborative groups. Instead, they were assigned individual worksheets that were identical to the tasks in the treatment group. Specific instructions were given to students in this section to ensure that they completed the worksheets individually.

Both college algebra sections for the spring semester 2004 were 15-week courses. During the third week of the course, students in the task/online section were given individual activities to familiarize themselves with the WebCT software, including tools such as email and threaded discussions that they would be using to communicate with their groups members throughout the course. After the first examination, the collaborative groups were required to complete four week long online tasks (3-4 problems each) throughout the last eleven weeks of the course. The task only group was given the same four problem sets, but students were not allowed to work collaboratively either online or face-to-face. A copy of the four tasks can be found in Appendix D, whereas a complete syllabus for both classes can be located in Appendix E.

In the task/online section, the group tasks were to be addressed collaboratively online. Each group member was to contribute ideas, provide explanations for their strategies and procedures, and ask questions. Group members were also responsible for
seeking a consensus of the solutions for each of the tasks. Each of the six problem solving groups had one full week to complete the task as the solutions had to be emailed to the instructor by the end of the week. As part of their grade for each online task, students were required to post at least three messages per task. These postings were to contribute to the ideas presented in the group. For example, messages that simply state agreement such as “I agree with John.”, were not counted towards overall participation. Students were required to provide some explanation for their agreement or disagreement. To help students communicate in this unfamiliar environment, they were given an Online Participation Rubric designed by the researcher. This rubric can be found in Appendix F.

The online tasks were posted on WebCT the morning they were assigned, and students had until the following week by 2pm to complete it. For example, the solutions to an online task posted Friday morning were due the following Friday by class time. An extra requirement that their first contribution to solving the weekly tasks be posted within two days of the assignment encouraged students to start the problem sets as soon as possible. Assessment of the online tasks in the collaborative learning groups was based both on individual participation and the solutions to the activities. Each online activity was worth ten points in which six points was given to participation (two for each of the required messages) and four points was awarded the final solution submitted at the end of the week. The points awarded for completing the tasks and communicating online were used as incentives to participate.

In the control group, the identical tasks were handed out in-class on the same day as the online task and were due the following week during class time. These students
were given specific instructions to work on these tasks individually. To ensure that students in the task only section also started the tasks early, initial solutions to at least one of the problems were required to be submitted to the instructor during the next class period. Assessment of the four problem sets in the task only group was based solely on the correctness of the process solution, with each activity worth a total of ten points.

Selection of the Collaborative Groups

Students in the task/online section of college algebra were equally distributed into six mixed ability groups of four to five students. The members of these groups were chosen by matching on mathematical ability, with gender as a secondary matching variable. The groups remained intact throughout the course of the study. Mathematical ability was determined by a student’s combined raw score on a pretest that contained both a procedural and problem solving component. This instrument was developed by the researcher and administered during the second week of the course. In addition, this instrument would also serve as a pretest covariant. The discussion of the reliability and validity of this instrument can be found below.

Selection of the Mathematical Tasks

Since the Treisman Workshop Model of collaborative learning has been shown to be successful in college level mathematics courses, the use of similar tasks was chosen for this study. The problem sets for both college algebra sections were designed by the researcher and were composed of Treisman-style problems to engage students in mathematical communication. Moreover, each task (3-4 problems) was designed to have
two components: a procedural component to help identify misconceptions and a problem solving component that applied their procedural skills. All selected problems fell into one or more of the following groups (Fullilove & Treisman, 1990, p. 468):

- “Old chestnuts” – problems that appear frequently on examinations but rarely on homework assignments;

**Example:** A city recreation department plans to build a rectangular playground 810 square meters in area. The playground is to be surrounded by a fence, which is 3 meters longer than twice the width. (Adapted from a Math Excel Worksheet)
  a. Express the length of the fence in terms of its width.
  b. Express the width of the fence in terms of its length.
  c. Find the dimensions of the fence.

- “Monkey wrenches” – problems designed to reveal deficiencies either in the students’ mathematical backgrounds or in their understanding of a basic concept;

**Example:** Solve the following linear equation: (Adapted from a Math Excel Worksheet)
  a. \((m + 3)^2 = (m - 9)^2\)

- Problems that introduce students to motivating examples or counterexamples that shed light on or delimit major course concepts and theorems;

**Example:** If \(P(x) = ax^2 + bx + c\) and \(Q(x) = cx^2 + bx + a\), then what is the relationship between the zeros of \(P(x)\) and \(Q(x)\)? (Schoenfeld, 1980)

- Problems designed to deepen the students’ understanding of and facility with mathematical language;

**Example:** The spread of oil leaking from a tanker is in the shape of a circle. The radius \(r\) (in feet) of the oil spread after \(t\) hours is \(r(t) = 200\sqrt{t}\). The area of a circle is given by the function \(A(r) = \pi r^2\). (Adapted from a Math Excel Worksheet)
  a. Find \((A \circ r)(t)\)
  b. Interpret what \((A \circ r)(t)\) models.
  c. What is the domain of \((A \circ r)(t)\)?
Problems designed to help students master what is know as “street mathematics” – the computation tricks and shortcuts known to many of the best students but which are neither mentioned in the textbook nor taught explicitly by the instructor.

Example: The price of a dress is reduced by 40%. When the dress still does not sell, it is reduced by 40% of the reduced price. If the price of the dress after both reductions is $72, what was the original price? (Blizter, 2003, p. 107)

To ensure that the tasks incorporated the characteristics of a Treisman-style worksheet, the researcher attended a three-day training at The University of Texas at Austin for their Emerging Scholars Program. The sessions at the conference aided the researcher in designing the tasks to be well matched with the Treisman model. In addition, many items included on the worksheets were adapted from other universities that incorporated the Treisman Workshop Model into their college algebra curriculum such as the Math Excel programs at Oregon State University and The University of Kentucky. A copy of each of the four worksheets can be found in Appendix D.

Content validity of the mathematical tasks was established by the two course supervisors of college algebra. These instructors were asked to examine the four problem sets to judge whether the items measured both procedural and problem solving ability as warranted by the course description. The agreement rate was 100%.

Research Design and Data Collection

The purpose of this research study was two-fold: a) to investigate the nature and quality of the online mathematical communication and its relationship to student achievement outcomes and b) to examine the effects on mathematical achievement of incorporating online collaborative learning groups into a traditional face-to-face college
algebra course. According to Slavin (1985) studies that examine the effects of small group learning on student achievement and/or other student attributes are called ‘first generational’ studies which typically employ quantitative methodologies. On the other hand, studies that examine the processes of small group learning are termed ‘second generational’ studies which normally utilize qualitative methods. Therefore, this research study utilized a mixed-methodology design in which both quantitative and qualitative methods were employed to address each research question. This methodology was chosen to add “scope and breadth” (Creswell, 1994, p. 175) to the study, as well as aid in the triangulation of the data collected. Furthermore, the mixed methodology design was “advantageous to a researcher…to better understand a concept being tested or explored” (Creswell, 1994, p. 177).

Mathematical Content Analysis

The transcripts of the online communication were analyzed to determine the nature of the mathematical communication. Data on the basic components of collaboration was collected by performing a content analysis on the transcripts of all the online messages posted within the six problem solving groups. A coding scheme, described below, was used to analyze the online communication based on the nature of the mathematical talk that occurred within a group. Several schemes for coding the online communication described in the literature review were tried. These included the frameworks of Curcio and Arztz (1998), Knuth and Peressini (2001), and Stacey and Goeding (1998) based on face-to-face dialogue, as well as Henri (1992) and Myers (2002) based on online communication.
The most satisfactory coding scheme was developed by Stacey and Gooding (1998). This framework was used to analyze and code face-to-face interactions in small groups. Figure 1 gives examples of eight coding categories employed in this framework.

Figure 1. Stacey and Gooding’s Categorical Coding Scheme.

<table>
<thead>
<tr>
<th>Asking Questions</th>
<th>Responding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Of a previous speaker</td>
<td>To a request for clarification</td>
</tr>
<tr>
<td>From own thinking or working</td>
<td>Agreeing</td>
</tr>
<tr>
<td>Reading a word problem</td>
<td>Disagreeing</td>
</tr>
<tr>
<td></td>
<td>Repeating</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Directing</td>
</tr>
<tr>
<td></td>
<td>Explaining with Evidence</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Thinking Aloud</td>
</tr>
<tr>
<td></td>
<td>Proposing Ideas</td>
</tr>
<tr>
<td></td>
<td>Commenting (Affective)</td>
</tr>
<tr>
<td></td>
<td>Refocusing Discussion</td>
</tr>
</tbody>
</table>

Due to the synchronous nature of face-to-face communication, Stacey and Gooding chose ‘turn taking’ as the unit of analysis. *Turn taking* was defined as “an
utterance of any length and grammatical form that is produced by a speaker without interruption” (Stacey & Gooding, 1998, p. 196). In contrast, an asynchronous turn (message) may contain multiple ideas or topics. Adapting Stacey and Gooding’s framework to an online setting, Myers (2002) used the ‘unit of meaning’ as the unit of analysis. A unit of meaning was defined as consistent theme or idea in a message (Henri, 1992). Accordingly, a single message could be coded into multiple categories depending upon the number of ‘units of meaning’ it contained. Consistent with Myers (2002) coding scheme, the researcher chose the unit of meaning to the unit of analysis.

**Interaction Analysis Model**

To assess the quality of the online mathematical communication, the transcripts were also coded using the Interaction Analysis Model (IAM) that was developed by Gunawardena, Lowe, and Anderson (1997). This framework was designed to examine the co-construction of knowledge in computer conferencing. This model analyzes the entire online transcript to determine: a) the type of cognitive activity such as asking questions, clarifying statements, negotiating agreements and disagreements, and synthesizing information; b) resources used by the participants for exploring and negotiating the problem; and c) evidence of changes in understanding or the co-construction of knowledge as a result of collaborative interactions (Gunawardena, Lowe, & Anderson, 1997). To examine the online transcripts for evidence of the co-construction of knowledge, the designers of this instrument defined five distinct and hierarchical phases. Figure 2 summarizes the five phase levels as well as provides examples of each of the levels. (A copy of the instrument can be found in Appendix A.)
Figure 2. Gunawardena, Lowe, and Anderson’s Interaction Analysis Model.

<table>
<thead>
<tr>
<th>Phase I: Sharing and Comparing</th>
<th>Observation, opinions, agreement, corroborating evidence, clarification, and/or identification of a problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase II: Discovery and Exploration of Dissonance</td>
<td>Identification of differences in concepts, understanding, or schemas, and questions to clarify disagreement.</td>
</tr>
<tr>
<td>Phase III: Negotiation of Meaning/ Co-construction of Knowledge</td>
<td>Negotiation/clarification of meaning of terms, identification of areas of agreement, or proposal of co-construction.</td>
</tr>
<tr>
<td>Phase IV: Testing and Modification</td>
<td>Testing against cognitive schema, personal experience, formal data experimentation, or contradictory information.</td>
</tr>
<tr>
<td>Phase V: Agreement Statement(s)/Application of Newly Constructed Knowledge</td>
<td>Summarization of agreement, or metacognitive statements.</td>
</tr>
</tbody>
</table>

Messages ranked in Phase I and Phase II are considered to “represent the lower mental functions”, while messages rated in Phase III, Phase IV, and Phase V “represent the higher mental functions” (Beaudrie, 2000, pg. 74).

The author used the IAM to rate an entire message, rather than rate different sections of the message. Consistent with the research design of this study, segmenting an entire message into ‘units of meaning’ has been used in other online transcript analysis to determine the nature of communication. According to Gunawardena, Lowe, and Anderson using this kind of analysis “merely describes the pattern of connection among messages, and not the entire gestalt to which the messages contribute” (1997, p. 407). In accordance with the designers of this instrument, the author rated a complete message.
The authors of this instrument also noted that through use in different formats, the instrument has been proven valid.

In addition, to determine the number of messages scored at each of the five phase levels, the research used the Interaction Analysis Model to determine an individual’s three online communication variables. These variables consisted of their total number of ranked messages posted, their percentage of high level messages, and their average high phase level score. The percentage of high level messages was defined as the ratio of messages coded as Phase III, IV, or V by the total number of ranked messages sent for that individual. This ratio was shown valid in a study by Beaudrie (2000) who examined the quality of the communication phase level of four groups in an online geometry course for pre-service and in-service teachers. A message was not ranked if it did not include mathematical content. Unranked messages included social messages and those involving issues of emailing the final solutions or the use of technology. The average high phase level score was calculated by first determining the highest phase level reached by the individual for each of the four online tasks. These four scores were then added together and divided by the number of tasks in which the individual actively participated.

Accordingly, a group’s communication variables were also determined using the Interaction Analysis Model. These group variables included the total number of ranked messages posted by the group, the percent of high level messages sent by the group, and the group’s average high phase level score. The group’s average high phase level was calculated for each online task by determining the highest level of message sent by each
active member of the group. These numbers were added together and then divided by the
total number of active participants.

Reliability Issues. According to Weber (1990), there are two types of reliability
pertinent to content analysis. These include stability and reproducibility. Stability can be
defined as the extent to which the results of the coding are stable over time. This type of
intra-rater reliability was established for both coding schemes, Stacey and Gooding’s
framework and the IAM, by recoding 35% of the original messages. A random cross-
section of messages from Task 1 to Task 4 was selected. In addition, the researcher
chose to recode over one-third of the original messages to encompass the same quantity
of messages that was typically sent for a single task. An agreement rate over 90% was
chosen by the researcher to indicate that she was consistently coding the messages
according to the two content analysis frameworks. This agreement rate was chosen based
on the reliability procedures found in Chapter Two (e.g., Beaudrie, 2000; Myers, 2002).

The second type of reliability is referred to as reproducibility which is the extent
to which the coding classification produces the same results when the messages are coded
by another coder. As with intra-rater reliability, 35% of the messages were randomly
selected to be coded by a second rater. Cohen’s Kappa statistic was used to assess the
inter-rater reliability for both coding schemes. A Kappa greater than .70 was considered
satisfactory (Fleiss, 1981).

In order to determine whether the Interaction Analysis Model would accurately
measure the quality of mathematical discussions such as the co Construction of
knowledge in a college algebra course, a second pilot study was conducted the semester
prior to this research study. This second pilot study occurred during the fall semester of
2003 and the structure was identical to the design of this research study with the
exception that the researcher taught both sections of college algebra. In addition to
testing the feasibility of using the IAM instrument, the pilot study served as a method to
judge the reliability and validity of the other researcher-designed instruments and the
Treisman–style mathematical tasks.

After eliminating messages that did not pertain to mathematical content, a total of
397 messages were scored. Table 2 shows the total number of ranked messages sent at
each phase level, as well as the percentages.

<table>
<thead>
<tr>
<th><strong>Phase Level</strong></th>
<th><strong>Total Ranked Messages</strong></th>
<th><strong>Percentage</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase I – Sharing/Comparing</td>
<td>172 messages</td>
<td>43%</td>
</tr>
<tr>
<td>Phase II – Discovery/Explanation of Dissonance</td>
<td>111 messages</td>
<td>28%</td>
</tr>
<tr>
<td>Phase III – Negotiation of Meaning/Co-construction of Knowledge</td>
<td>68 messages</td>
<td>17%</td>
</tr>
<tr>
<td>Phase IV – Testing and Modification of Proposed Synthesis or Co-construction</td>
<td>46 messages</td>
<td>12%</td>
</tr>
<tr>
<td>Phase V – Agreement Statement(s)/Application of Newly-Constructed Meaning</td>
<td>0 messages</td>
<td>0%</td>
</tr>
</tbody>
</table>

The rating of all messages for this second pilot study during the fall semester of
2003 was done by the researcher. To establish whether the researcher was effectively
utilizing the IAM instrument, another individual was trained and rated a substantial
amount of messages. The researcher and the second rater met twice to compare their
ranking of messages and discuss any discrepancies in scoring. After this initial training,
the researcher and the second scorer individually rated 35% of the messages (139
messages) to establish inter-rater reliability. Cohen’s Kappa was calculated at 0.787, which exceeded the satisfactory limit.

**Pretest – Initial Mathematical Ability**

All students in both sections of college algebra were administered a mathematical pretest that consisted of a 16-item procedural test and a three-item problem solving skills test during the second week of the semester. Due to the typical high rates of adding and dropping the course by students during the first week of class, the second week of class was chosen to minimize the risk of a large number of students adding after the test was given.

The procedural component of the pretest was designed by the researcher based on the released questions from the Mathematics Association of America (MAA) Algebra Examination. The questions were designed to assess the students’ knowledge of concepts and basic mathematical skills relating to numbers and operations, as well as algebra (refer to Appendix B). The original MAA Algebra Exam could not be administered due to the high cost associated with using this exam. This exam was used to match students into mixed ability groups. Since this test also served as a pretest, as the raw scores were used to control for initial differences between the control and treatment groups, it was designed to incorporate the MAA Algebra Examination’s high predictive validity score.

The second pilot study informed the researcher about the validity of the 16-item procedural pretest. To determine the predictive validity of this pretest, the researcher administered the test to 92 students enrolled in two sections of college algebra during the fall semester of 2003. A student’s raw score on their pretest was correlated with their
scores on the final examination for the course. The Pearson’s Product Moment Coefficient (PPMC) was calculated to determine the strength of the relationship between the pretest and the final examination. The PPMC returned a value of 0.679, with a p-value of 0.000. Therefore, the pretest was deemed to be a strong predictor of subsequent mathematical achievement.

The problem solving component of this pretest was composed of three open-ended tasks that were repeated on the problem solving post-test. These open response items were scored using a rubric. Validity and reliability of these items are discussed below.

**Procedural Ability – Final Examination**

At the end of the semester, one common comprehensive final examination was administered to students in both sections of college algebra. This test was used to determine a student’s procedural ability at the end of the course. Since the same instructor taught both the treatment and control groups, both sections were administered the exam at the same time and in the same room. The comprehensive final was worth 150 points and contained 15 procedural-based problems. The procedural component of the pretest consisted of prerequisite material covered on the first examination. Therefore, some of the items on the final exam were similar to the problems selected on the procedural component of the pretest.

The course supervisor of college algebra designed the questions for the final examination. In addition, each instructor for the individual sections gave input into the validity of the questions by verifying that the test covered the course objectives.
Therefore, content validity was established by the course supervisor and the instructors of college algebra. All instructors graded the same problem on the exam for all students enrolled in college algebra.

**Problem Solving Ability – Problem Solving Examination**

Prior to the final examination students in both sections of college algebra were administered a problem solving examination (See Appendix B). This test was constructed by the researcher of this study and contained tasks similar to the problem solving tasks that were given to each section during the course of the semester. Many items on the problem solving examination were used or adapted from college algebra textbooks, previous Treisman style worksheets, and the NCTM’s *Problem Solving in School Mathematics 1980 Yearbook*.

In addition, the researcher sought the advice of the two course supervisors of college algebra to help evaluate the content of the problem solving items. To establish the content validity of the problem solving ability instrument, each course supervisor was asked to examine the instrument to judge whether the items measured mathematical problem solving skills. The content areas included perimeter and area, maximum and minimum problems, the use of the Pythagorean Theorem, and percentage problems. The researcher and the two course supervisors agreed that all five items were consistent with the goals of the college algebra curriculum.

The second pilot study revealed the necessity of using a rubric to score each constructed-response item. The scoring rubric was borrowed and adapted from the dissertation work of Runde (1997), who used it to award partial credit on both a pretest
Data Analysis

Multiple phases of data collection occurred throughout the course of the research study. Qualitative and quantitative methods were employed to critically examine the four major research questions. This section outlines how these data collection procedures were analyzed in accordance with these research questions.

Nature and Quality of the Online Communication

Two distinct coding schemes were used to answer the following research question:

1. What was the nature and quality of the online mathematical communication that occurred in collaborative groups in a college algebra course?
Stacey and Gooding’s framework (1998) was used to investigate the nature of the online mathematical communication. Each ‘unit of meaning’ within a message was coded as one of the following eight categories: asking questions, responding, directing, explaining with evidence, thinking aloud, proposing ideas, commenting, and refocusing discussion. The Interaction Analysis Model (Gunawardena, Lowe, & Anderson, 1997) was used to examine the quality of the online mathematical communication. Each posted message from an individual was ranked according to five phase levels. Specifically, this study sought to determine whether there existed evidence of the co-construction of mathematical knowledge.

**Online Communication and Mathematical Achievement**

A second focus of this research study was to examine the relationship between the quality of an individual’s or group’s communication variables and mathematical achievement. To determine if there were significant relationships between an individual’s procedural skills as measured by the final examination or problem solving ability as measured by the problem solving examination and an individual’s communication variables, a Pearson’s Product Moment Correlation Coefficient was used. The dependent variable for these hypotheses was an individual achievement variable (either the final examination score or problem solving test score). The independent variable was the individual’s communication scores (total number of messages sent, the percentage high level messages, and average high phase level score).

The same procedure was used to determine if there were significant relationships between an individual’s procedural skills as measured by their score on the final
examination or problem solving ability as measured by their score on the problem solving examination and their group’s communication scores (total number of messages sent by the group, percentage of high level messages sent by the group, and the group’s average high phase level score). Specifically, the Pearson’s Product Moment Correlation Coefficient (tested at the 0.05 alpha level) was used to answer the following twelve research hypotheses:

2a.1) There was no significant relationship between an individual’s problem solving examination score and his or her total number of scored messages.

2a.2) There was no significant relationship between an individual’s problem solving examination score and his or her percentage of high level messages.

2a.3) There was no significant relationship between an individual’s problem solving examination score and his or her average high phase level score.

2b.1) There was no significant relationship between an individual’s problem solving examination score and the total number of scored messages sent by their group.

2b.2) There was no significant relationship between an individual’s problem solving examination score and the percentage of high level messages sent by their group.

2b.3) There was no significant relationship between an individual’s problem solving examination score and their group’s average high phase level score.

3a.1) There was no significant relationship between an individual’s procedural final examination score and his or her total number of scored messages.

3a.2) There was no significant relationship between an individual’s final examination score and his or her percentage of high level messages.
3a.3) There was no significant relationship between an individual’s procedural final examination score and his or her average high phase level score.

3b.1) There was no significant relationship between an individual’s procedural final examination score and the total number of scored messages sent by their group.

3b.2) There was no significant relationship between an individual’s procedural final examination score and the percentage of high level messages sent by their group.

3b.3) There was no significant relationship between an individual’s procedural final examination score and their group’s average high phase level score.

Since three correlations were computed for each subset of hypotheses, if a significant difference was found a corrected significance level was calculated. To minimize the chances of making a Type I, the researcher chose to use the Bonferonni approach which requires dividing the 0.05 alpha level by the number of computed correlation. Therefore, a p value less than 0.017 (0.05/3) would be declared significant.

**Differences between Two College Algebra Sections**

Due to the inability to randomly assign participants, the researcher chose two intact sections of college algebra. Following Campbell & Stanley’s (1963) methodology, the researcher utilized a quasi-experimental nonequivalent control group design. As “one of the most widespread experimental designs in educational research”, this quantitative research design is effective when the “control group and experimental group do not have pre-experimental sampling equivalence” (Campbell & Stanley, 1963, p. 47). The use of this design helps minimize threats to the internal validity of this study such as the effects of history, testing, instrumentation and maturation (Campbell & Stanley, 1963). For the
purpose of this study, one intact section of college algebra (n = 26) was randomly assigned to utilize online collaborative learning groups (treatment group). As a control group, one intact section (n = 30) was randomly assigned as a task only group. Specifically, this phase of data analysis sought to answer the following research hypotheses:

4a) After controlling for initial differences in mathematical ability, there was no significant difference in achievement on the problem solving examination between students completing assigned mathematical tasks individually and those utilizing online collaborative group work.

4b) After controlling for initial differences in mathematical ability, there was no significant difference in the achievement on a procedural final examination between students completing assigned mathematical tasks individually and those utilizing online collaborative group work.

Means and standard deviations were reported for the pretest (combined procedural and problem solving components) and a t-test was conducted to test for significant differences in the scores between the treatment and control groups. Based upon these results, the pretest was used as a covariant for the mathematical achievement measures. Means were calculated based on the scores of the final examination to determine a procedural score for each section. Means were also calculated based on the scores of the problem solving examination. An analysis of covariance (ANCOVA) was used to test for significant differences in procedural skills and problem solving ability between the two sections of college algebra. All quantitative hypotheses were examined at the 0.05 alpha level.
Summary of Research Methodologies

This study employed a mixed-methodology approach, combining quantitative and qualitative techniques for the data analysis. To answer the first research question, two different content analysis frameworks were employed. The coding framework developed by Stacey and Gooding (1998) was used to describe the nature of the online communication. The interaction analysis framework developed by Gunawardena, Lowe, and Anderson (1997) was used examine the quality of the online communication. Specifically, this model was used to determine whether there existed evidence of the co-construction of knowledge during the online group work.

To address the second and third research questions, twelve correlational research hypotheses were examined. Specifically, the Pearson’s Product Moment Correlation Coefficient was used to determine the relationship between the quality of the online mathematical communication and mathematical achievement. Finally, a quasi-experimental nonequivalent control group research design was utilized to answer the fourth research question. An analysis of covariance was applied to determine whether significant differences in mathematical achievement existed between the control and treatment sections of college algebra.
CHAPTER FOUR

RESULTS OF THIS STUDY

Introduction

The main research questions were divided into four categories to aid in the discussion of the results for this study. These categories included: 1) the nature and quality of online mathematical communication in small groups at both the individual and group level, 2) the relationship between the quality of online mathematical communication and mathematical achievement based on a problem solving examination, 3) the relationship between the quality of online mathematical communication and mathematical achievements based on a procedural final examination, and 4) the investigation of the differences in mathematical achievement between two sections of college algebra.

Nature and Quality of the Online Communication

This section contains both a qualitative description regarding the nature of the online mathematical communication and a quantitative evaluation of the quality of this communication. The nature of the online mathematical communication includes both a description of the total participation and the results of a mathematical content analysis. The nature of the mathematical communication was examined utilizing a framework developed by Stacey and Gooding (1998), which explores the basic characteristics of the individual and group interactions. The quality of the online mathematical communication
was evaluated using Gunawardena, Lowe, and Anderson’s (1997) Interaction Analysis Model describing the phase levels of the co-construction of mathematical knowledge.

**Total Participation**

Over the course of the four online tasks, a total of 278 messages were posted. Participation varied significantly among each individual. On average, each student posted 2.8 messages per online task, with a range of 0 – 7 messages. The median number of messages sent per individual during a single online task was three. Recall, each participant was required to send a minimum of three messages per task to receive full credit for the assignment.

Of the original 26 students enrolled in the treatment group, only two did not participate in any of the group tasks. These students subsequently dropped from the course. Of the remaining 24 students, three of them participated in only one of the four tasks. Incidentally, the task in which these students participated varied from Task 2 to Task 4. Two of these three students posted only one message apiece to their group’s online folder, whereas the third student posted four messages.

As a result of differences in individual participation, the number of messages sent in the six problem solving groups (referred to as the Blue Group, the Purple Group, the Orange Group, the Yellow Group, the Green Group and the Red Group) also varied significantly. Table 3 on the following page shows the number of messages sent per group for each of the online tasks. The number in parenthesis indicates the number of
active participants in the group for each of the online tasks. An *active participant* was defined as a student who posted at least one message during the online task.

Table 3. Total Messages Sent for each Group Per Task

<table>
<thead>
<tr>
<th>Group</th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
<th>Task 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>9 (3)</td>
<td>9 (3)</td>
<td>8 (3)</td>
<td>6 (2)</td>
</tr>
<tr>
<td>Purple</td>
<td>21 (4)</td>
<td>17 (4)</td>
<td>11 (4)</td>
<td>8 (3)</td>
</tr>
<tr>
<td>Orange</td>
<td>11 (3)</td>
<td>7 (2)</td>
<td>11 (3)</td>
<td>7 (3)</td>
</tr>
<tr>
<td>Yellow</td>
<td>18 (4)</td>
<td>15 (4)</td>
<td>14 (3)</td>
<td>7 (2)</td>
</tr>
<tr>
<td>Green</td>
<td>13 (3)</td>
<td>13 (3)</td>
<td>10 (3)</td>
<td>9 (4)</td>
</tr>
<tr>
<td>Red</td>
<td>18 (4)</td>
<td>15 (4)</td>
<td>9 (4)</td>
<td>12 (4)</td>
</tr>
</tbody>
</table>

In all six of the problem solving groups, the total number of messages sent declined from Task 1 to Task 4. This phenomenon occurred even when the number of active participants remained constant from one task to the next. For example, from Task 1 to Task 3, the Purple Group’s number of messages significantly declined from 21 messages to 11 messages even with all of the group’s members actively participating.

Two reasons may account for the decline in the number of sent messages for the Purple Group, as well as the remaining groups. First, the messages sent during the final three online assignments contained information on more than one problem within each task. Therefore, instead of simply discussing problem one, an individual might supply ideas for multiple problems. This fact made each message considerably larger, while decreasing the number of messages posted. Secondly, the number of messages sent regarding group processes, such as deciding who would send the final solutions to the instructor, declined significantly in all problem solving groups.
Table 3 also illustrates that in the majority of the groups the number of active participants declined over the course of the semester. It should be noted that four of the seven students who withdrew from the class did so prior to the third or fourth task. The remaining three students who did not take the final exam participated in all four online assignments.

**Intra-Rater and Inter-Rater Reliability**

The two coding procedures used to determine the nature and quality of the online mathematical communication introduced elements of subjectivity. To ensure that the coding instruments were utilized correctly by the researcher both types of reliability, stability and reproducibility, were considered. Two months after the initial coding of the online transcripts using both the Stacey and Gooding framework and the Interaction Analysis model, 35% of the messages sent were re-coded to determine the extent to which the coding was stable over time. An agreement rate was calculated to determine the intra-rater reliability for each of the two coding schemes. The agreement rates for Stacey and Gooding’s framework was 92.9% and the agreement rate for the IAM was 89.6%.

The Cohen Kappa statistic was used to determine the extent to which the coding schemes produced the same results when the messages were coded by a second rater. This inter-rater reliability statistic was calculated for both the Stacey and Gooding’s framework and the IAM, the Kappa statistic returned values of 0.869 and 0.809 respectively. It was concluded that the inter-rater reliability was satisfactory.
Mathematical Content Analysis

To examine the nature of the computer-mediated communication, the research utilized a framework developed by Stacey and Gooding (1998; see also Gooding & Stacey, 1993) for coding individual and group behaviors that occurred in collaborative groups. The goal of this section is to report the results of the mathematical content analysis that explored the basic components and patterns of the online communication. The coding categories include: asking questions, responding, directing, explaining with evidence, thinking aloud, proposing ideas, commenting, and refocusing the discussion.

As discussed in Chapter Three, the unit of analysis was modified from Stacey and Gooding’s selection of turn taking. For this study, the unit of meaning, a consistent theme or idea in a message, was selected as the unit of analysis for coding the online transcripts. Accordingly, a single message could be coded into multiple categories depending upon the number of units of meaning it contained. A complete description of this framework can be found in Figure 1 in Chapter Three.

Of the 278 messages sent during the four online assignments, 355 units of meaning were coded into one of the eight categories. Note, 77 messages were double coded because these messages contained two units of meaning. Since these categories were intended to describe the nature of the online mathematical communication, an example of each of the eight categories from the data analyzed for this study is provided in Figure 3 on the following page.
Figure 3. Examples of the Coding Categories from the Online Transcripts.

**Asking Questions**
- Of a previous speaker

  “...But I don’t understand your answers for 1 and 2? Could you tell us again how you got them?”
- From own thinking or working

  “I think that #3 would have something along the lines of one side is x and the other leg is x+7. What would the equation look like? I suck at starting these.”

  - Reading a word problem

    “The problem says we have to find the area when we know the perimeter. Is the perimeter 2w*2L?”

**Responding**
- To a request for clarification

  [S1: “Does any one know how to set up problem 4?? do you plug it into the quadratic equation?”]

  S2: (Response): “for problem four you foil both sides of the equation then add like terms then put into the quadratic”

- Agreeing

  “Hey i agree with number 3.”

- Disagreeing

  “If you plug 5.75 into you equation it doesn’t work. I got a different answer . here is how i did it.” (this student proceeds to explain her answer by providing evidence for the disagreement)

- Repeating

  - No messages were coded in this category.

**Directing**

“John can you check to see if my answers are correct.”
Explaining with Evidence

“not sure why you plugged 391 into your equation.

Start with these equations: Perimeter = 80 = 2l + 2w and Area = 391 = lw

Get the first equation in terms of l only: 80 = 2l + 2w factor out 2
80 = 2(l+w) now divide both sides by 2
40 = l+w so you can subtract w
1 = 40 – w plug this into lw=391
(40-w)w = 391 …”(keeps going with explanation)

Thinking Aloud

“1a. -3 & + -square root of ½”

OR

“Here is what I got for the second problem
Here is what I got for the second problem:
(x-.4x) - .4x +.16x = 72
x-.8x+.16x = 72
.36x = 72
x = 200
Original Price = $200.00”

Proposing Ideas

“for # b im still trying to figure out any ideas i think i have to use cube root formula to solve?”

Commenting

“Hi mark, christine. hank im just checkin this thing out im going to look over the problem a little later and right back. I take it we only have to #4?. talk ta ya all soon”

Refocusing Discussion

“Instead of trying to figure out the last problem, could all of you just post your answer keys for the first three problems.”
During this phase of the mathematical content analysis it became apparent that a stronger distinction between the categories explaining with evidence and thinking aloud was necessary. Although both represent cognitive strategies (Stacey & Gooding, 1998), these categories differ according to their degree of interaction and level of elaboration. Stacey and Gooding (1998) referred to thinking aloud statements as non-interactive, whereas explaining with evidence statements were typically interactive statements. An interactive statement can be defined as those statements that are made in response to another or that receive a response. The researcher chose to further classify thinking aloud contributions according to Myers (2002) definition of a “statement that exhibit a person’s thoughts, actions, or processes while working.”

Two types of thinking aloud contributions emerged from the data. The first type was the statement of an answer only such as “1a. -3 & +-square root of ½”. The second type was the posting of a mathematical answer key characterized by step-by-step mathematical equalities. An example of a mathematical answer key includes:

“Here is what I got for the second problem:
(x-.4x) -.4x +.16x = 72
x-.8x+.16x = 72
.36x = 72
x = 200
Original Price = $200.00”

This latter type of thinking aloud contributions also highlight the differences in the level of elaboration between a thinking aloud statements and a explaining with evidence contributions. Elaboration during the process of solving the problem in the thinking aloud category was limited to mathematical symbols. Whereas, a combination of
mathematical symbols and everyday language was provided during the process of explaining with evidence.

The percentage for each contribution was calculated by dividing the number of units of meaning coded in a category by the total number of units of meaning of all the messages sent during the four online assignments. Table 4 shows the percentage of contributions at each of the eight coding categories.

Table 4. Percentage of Problem Solving Contributions.

<table>
<thead>
<tr>
<th>Coding Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questioning</td>
<td>11.9%</td>
</tr>
<tr>
<td>Responding</td>
<td>22.5%</td>
</tr>
<tr>
<td>Directing</td>
<td>0.3%</td>
</tr>
<tr>
<td>Explaining with Evidence</td>
<td>14.3%</td>
</tr>
<tr>
<td>Thinking Aloud</td>
<td>36.3%</td>
</tr>
<tr>
<td>Proposing Ideas</td>
<td>6.2%</td>
</tr>
<tr>
<td>Refocusing Discussion</td>
<td>0.3%</td>
</tr>
<tr>
<td>Commenting</td>
<td>8.2%</td>
</tr>
</tbody>
</table>

The content analysis revealed three major characteristics of the online communication. The majority (36.3%) of contributions during the online assignments were classified as non-interactive thinking aloud statements. Secondly, interactive statements such as questioning, responding, and explaining with evidence accounted for half (49.7%) of the units of meaning. These were behaviors found by Stacey and Gooding (1998) to be frequent contributions of effective problem solving groups. Effective groups where defined as those who had at least two improvers from pretest to post-test. Third, there was little evidence of the coding categories directing and
refocusing the discussion. It may be that these categories occur more frequently in face-to-face communication or when the online discussions take place synchronously.

Overall, the total percentage found in each of the eight coding categories did not differ significantly from those found in face-to-face mathematical communication as reported in Stacey and Gooding (1998). Although Stacey and Gooding selected fifth and sixth graders as their sample, the nature of their mathematical task was not different than the tasks selected for this research study. In both cases, the tasks were designed to elicit inconsistencies in a student’s mathematical knowledge or uncover common misconceptions. Recall, the tasks in this present study were developed according to the Treisman Workshop Model and designed to give students the opportunity to identify areas of weakness in their mathematical knowledge (Fullilove & Treisman, 1990).

In contrast, the total percentage of units of meaning coded in one of the eight coding categories did differ from those found in online mathematical communication as reported by Myers (2002). In her study, Myers (2002) had larger percentages of messages coded as asking questions and proposing ideas and a much smaller percent of messages in the thinking aloud category. This may be explained by the nature of the task, as well as the type of communication selected by the participants. Myers (2002) selected a single open-ended mathematical task that was to be solved collaboratively either utilizing asynchronous or synchronous format.
Interaction Analysis Model

To assess the quality of the computer-mediated mathematical communication, Gunawardena, Lowe, and Anderson’s (1997) Interaction Analysis Model was used. This framework ranks a single message as one of five hierarchical phases describing the levels of the co-construction of mathematical knowledge. Of the 278 messages sent, 23 were coded as Not Ranked (NR). These messages were not ranked because they did not include mathematical content. Of these 23 messages, two were of the social nature and 21 pertained to the sending the final solutions by email. Although these messages were not vital to the problem solving process, they were important in establishing group processes. The remaining 255 messages were coded into one of five phases as discussed in Chapter Three (See Figure 3).

A major area of interest of this research study was whether or not undergraduate students enrolled in an introductory mathematics course would reach the high levels of communication that are associated with the co-construction of knowledge (Phase III, IV, or V). Table 5 shows the total number of ranked messages at each of the five phase levels.

Table 5. Total Number of Messages Scored at each Phase Level.

<table>
<thead>
<tr>
<th>Messages</th>
<th>Low Level: Sharing/Comparing or Discovery of Dissonance</th>
<th>High Level: Evidence of the Co-construction of Mathematical Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase I</td>
<td>Phase II</td>
</tr>
<tr>
<td>154</td>
<td>53</td>
<td>40</td>
</tr>
</tbody>
</table>
Low level messages (Phase I or II) accounted for 81% of the total scored messages sent. Over half (60.4%) of the total number of scored messages were classified as the sharing and comparing of information (Phase I). The mathematical content of these Phase I messages tended to be students stating their answer or solution process or students agreeing without evidence. The majority of the Phase I messages could be classified as ‘thinking aloud’. An example of a Phase I message from the online transcripts includes:

“I have worked the second problem and came up with $11274.91 being own at 6.5% interest and $7225.09 being own at 12% interest for a total of $18500. Let x = amount own at 6.5% amount own at 12% (18500-x)

>.065x+.12(18500-x)=1599.88
.065x+2220-.12x=1599.88
-.055x= -620.12
-620.12/- .055=11274.91=x
18500-11274.91=7225.09”

This message epitomizes a Phase I message as it is a statement put forth by an individual so that comments can be made regarding the correctness of the solution. Typically, Phase I messages consisted of participants providing ideas or solutions to the group. Phase I messages may also include observation, opinion, agreement statements, or corroborating examples.

The second largest percentage of scored messages (20.8%) was classified as the discovery and exploration of dissonance or inconsistencies with one’s own or another’s solution strategy (Phase II). Typically, the content of Phase II messages was ‘asking questions’ of another or about the problem, or ‘responding’ with a disagreement
statement as described in Stacey and Gooding’s (1998) framework. An example of a Phase II message from the online transcripts includes:

“hey all, I have tried to work out number 4 but I am having a bit of trouble. Here is what I have so far. Mabey someone could help me out a little.
I took the point (3,-2) at the center (5,k) with radius of 6, I used the equation  \((x-h)^2+(y-k)^2=r^2\) After I plug in the numbers I get  \((3-5)^2+(-2-k)^2=6^2\)
I have had a hard time figuring out what to do from here. I know that I need to square everything but I start getting wierd numbers and square roots that don't seem to work out. Mabey someone could help me and give me a little of direction. Mabey I am using the wrong formula?”

This message was ranked a step above the Phase I message because there exists evidence of dissonance in a learner’s existing framework of knowledge or skills. This dissonance has been acknowledged by the sender leading him or her to pose a question to the entire group in an attempt to subdue this dissonance.

High level messages (Phase III, IV, or V) showing evidence of the co-construction of knowledge accounted for 19% of the total messages sent. The third largest percentage of scored messages (15.7%) was coded as the negotiation of meaning and/or the co-construction of knowledge (Phase III). These Phase III messages were typified by students ‘explaining with evidence’ their problem solving process. The example from the online transcript below shows the transition from a Phase II message (the one above) to the Phase III message.

Andrew, I was setting up #4 so that 6 is the distance, so using the distance formula:  \(6= \text{All in the square root:}(x2-x1)^2 +(y2-y1)^2\) so set it up as;
6= sq. rt of (5-3)^2 + (k+2)^2....so 2^2 and k^2+4k+4 ....
then combine, so 6= sq. rt of k^2 +4k+8....
so if you square both sides, you can drop the radical...
so 6^2=k^2+4+8....
So then here is where I got lost. I tried to set it up by factoring and it wouldn't work out. K^2+4k-28=0, and then I tried to complete the square and it come out:
K^2+4k+2^2=28+2^2 \ I ended up with K=+/- 2 x square root of 2. So then when I tried to do the distance formula again with -40 or 30 it didn't work out. This checked out with the quad formula when I realized I could use it.

This message was coded at Phase III not because the individual stated the correct answer, but because of the explanation he or she provided in response to their group members’ request for help. This type of message aids in the co-construction of mathematical knowledge among all of the readers.

The fourth largest percentage of scored messages (3.1%) was classified as the testing and modification of a proposed synthesis or solution process (Phase IV). An example of a Phase IV message from the online transcripts includes:

“I like your way of working through the problems. i was trying to do it another way. i was just trying to plug everything into the perimeter equation for the triangle. I didn't work. Now I see why I have to use the pythagreums therum. When i try it your way it works.”

In alignment with the mathematical content of a traditional college algebra course, a Phase IV message can be clearly seen when an individual takes a proposed solution method and tests it by using the method to re-calculate a solution.

No messages in the online transcripts were coded as a Phase V message. Phase V messages require the synthesis or application of new knowledge. This fact may be partially explained by the requirements set-up by the instructor to successfully complete the online mathematical tasks. It was not required for students to extend what they learned in the online communication to additional small group tasks.

Overall, the data in Table 4 provides evidence to suggest that undergraduate students enrolled in college algebra were capable of the co-construction of mathematical
knowledge during online collaborative learning. In one out of every five messages posted there was evidence of the co-construction of mathematical knowledge as measured by the Interaction Analysis Model.

In addition to the total percentage of messages scored at each phase level, an indication of the co-construction of knowledge can be found by examining the percentage of high level messages, messages ranked as Phase III, IV, or V. Figure 4 shows a line graph of the percentage of high level messages (Phase III, IV, or V) sent for each of the four online tasks.

Figure 4. Percentage of High Level Messages per Online Task.

Figure 4 revealed two characteristics of the overall communication. First, the line graph supports the claim that student in college algebra reached high levels of communication during each of the online tasks. Secondly, the percentage of high level
messages fluctuated significantly throughout the four tasks. Task 3 had the smallest percentage of high level messages. This fact was partially due to the type of problems associated with this task. The problems were mostly procedural in nature and differed from the problem solving tasks found in the remaining assignments.

Although the number of scored messages at each phase level and the percentage of messages scored at Phase III and above shed significant light on how individuals within groups communicated, it was the highest level reached by each individual during the collaborative group work that provided the greatest insight into a group’s co-construction of mathematical knowledge. As described in Chapter Three, a group’s average high phase level score was calculated to provide a more accurate representation of the co-construction of mathematical knowledge that took place during the online tasks. This communication score was used to identify any occurrences of the co-construction of mathematical knowledge for each task within each of the six problem solving groups. A group’s average high phase level was calculated for each task by first identifying the highest level message reached by each individual. Then, these scores were added together and divided by the total number of active participants. Table 6 on the following page shows each group’s average high communication score, as well as the highest phase reached in the group for each of the online tasks. The highest phase reached in a group was found by determining the highest phase level for each group during each of the four online tasks.
Table 6. Group Average High Phase Level Scores for each Task.

<table>
<thead>
<tr>
<th>Group</th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
<th>Task 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg High</td>
<td>High Phase</td>
<td>Avg High</td>
<td>High Phase</td>
</tr>
<tr>
<td>Blue</td>
<td>3.33</td>
<td>4*</td>
<td>2.33</td>
<td>3*</td>
</tr>
<tr>
<td>Purple</td>
<td>2.5</td>
<td>3*</td>
<td>3</td>
<td>4*</td>
</tr>
<tr>
<td>Orange</td>
<td>2.67</td>
<td>3*</td>
<td>2.5</td>
<td>3*</td>
</tr>
<tr>
<td>Yellow</td>
<td>2</td>
<td>3*</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>Green</td>
<td>3.33</td>
<td>4*</td>
<td>2.67</td>
<td>4*</td>
</tr>
<tr>
<td>Red</td>
<td>2.25</td>
<td>4*</td>
<td>2.5</td>
<td>4*</td>
</tr>
</tbody>
</table>

* Indicates high phase message was sent.

In the majority of the 24 problem solving episodes (six groups times four tasks), there was evidence of the co-construction of mathematical knowledge. Designated by the asterisks (*), in 19 episodes of online collaborative group work, at least one individual reached high levels of communication thereby showing evidence of the co-construction of knowledge. For example, three of the problem solving groups, the Blue Group, the Purple Group, and the Green Group had their group average high communication scores at or near Phase III. This fact would be indicative of most, if not all, of the group’s members reaching the negotiation of mathematical meaning and/or the co-construction of mathematical knowledge during each of their online assignments. On the other hand, during five of the problem solving episodes no individual in the group reached the co-construction of mathematical knowledge (Phase III or above). As an example, the Orange Group, the Yellow Group, and the Red Group had a group average high communication score at or near Phase II or Phase I. The drop in the group’s average high communication score implies that two or less of the group’s members reached the co-construction of mathematical knowledge.
In addition, Table 6 illustrates that the highest phase level reached during the 24 problem solving episodes varied from Phase I to Phase IV. For some groups, the quality of their online communication never extended past the sharing and comparing of information (Phase I). This can be seen in two of the problem solving episodes in which the group’s average high phase level score was equal to one. On the other hand, there was evidence that some groups transitioned through the first four phase levels from sharing and comparing of information to the testing of a proposed solution. This can be seen in six of the problem solving episodes in which the group’s highest phase level reached was at Phase IV.

A Mapping of the Nature onto the Quality

It became apparent during the two separate content analyses that there existed some degree of overlap among the different coding categories. For example, thinking aloud contributions as distinguished by Stacey and Gooding’s mathematical content analysis were consistently ranked as Phase I messages according to Gunawardena, Lowe, and Anderson’s framework. Therefore, the researcher chose to map the units of meaning coded according to the mathematical content analysis into the phase levels of the co-construction of knowledge. Recall, a message could have more than one unit of meaning, but only one associated phase level. Specifically, the researcher wished to determine if any significant pattern emerged in this mapping. Table 7 on the following page shows the number of units of meaning coded into each of the five phase levels.
Table 7. Mathematical Content Analysis Scores in Relation to IAM Phases.

<table>
<thead>
<tr>
<th>Coding Category</th>
<th>Asking Questions</th>
<th>Responding</th>
<th>Explain w/ Evidence</th>
<th>Thinking Aloud</th>
<th>Proposing Ideas</th>
<th>Commenting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase I</td>
<td>4</td>
<td>36</td>
<td>5</td>
<td>121</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>Phase II</td>
<td>29</td>
<td>17</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Phase III</td>
<td>9</td>
<td>19</td>
<td>35</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Phase IV</td>
<td>0</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No Rank</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>21</td>
</tr>
</tbody>
</table>

Analysis of the data found in Table 7 revealed many interesting patterns with regard to this mapping. Typically, contributions coded as responding, thinking aloud and proposing ideas tended to be ranked as a Phase I message. Disagreement statements (responding by disagreement) and asking questions from one’s own thinking were consistently mapped into Phase II. Phase III messages, were largely made by students explaining with evidence as they responded to those questions from another group member. On the opposite end of the continuum, Phase IV messages were consistently coded as agreeing with evidence. That is, Phase IV messages were a combination of responding by agreement and explaining with evidence.

This mapping was also consistent with the findings in Table 6 which shows the average high phase level scores for each of the six problem solving groups. Together Tables 6 and 7 serve as an indication of the differences in the nature and quality of communication among the six groups. For example, groups whose highest level reached during a task was Phase I consistently exchanged mathematical solutions. The online communication that occurred within these groups was typically marked by a string of non-interactive thinking aloud statements. Groups whose highest level reached during a
task was Phase III or Phase IV exhibited a larger percentage of ‘intellectual interactions’ (Stacey and Gooding, 1998, p. 199) such as responding with evidence.

**Individual Achievement and Online Communication**

This section reports the results regarding the relationship between the quality of the online mathematical communication as measured by the IAM and mathematical achievement as measured by the end-of-the-semester final and problem solving examinations. The quality of the online communication was determined by calculating six online communication variables. These six variables including the total number of ranked messages sent, the percentage of high level messages sent (Phase III, IV, or V), and the average high phase level score were calculated at both the individual and group level.

Table 8 on the following page shows the descriptive statistics for each individual student. These statistics include each student’s initial ability level, as well as their raw scores on the problem solving and final examinations. In addition, Table 8 gives both an individual’s average high phase level score and his or her group’s average high phase level score. The average high phase level score was calculated by first determining the highest phase level reached by the individual for each of the four online tasks. These four scores were then added together and divided by the number of tasks in which the individual actively participated.
As depicted in Table 8, an individual’s average high phase level score was not dependent of his or her initial mathematical ability. High, medium, and low ability students tended to communicate at equal levels of mathematical talk. Other important results were drawn from Table 8. First, low performance on the problem solving
examination was evident. This performance did not depend on initial mathematical ability either, especially for high and medium ability students. Secondly, there appeared to be a positive relationship between an individual’s average high phase level score and his or her raw score on the final examination. With the exception of two cases, an individual with an average high phase level score at or above 2.5 typically scored above average on the final examination. The statistical results of this hypothesis, as well as eleven others will be presented in the following sections.

Problem Solving Examination

Data gathered from each individual’s problem solving score and their individual and group online communication variables were used to answer the second research question: Is there a relationship between mathematical achievement in college algebra as measured by a problem solving examination and the quality of online mathematical discussions as measured by the Interaction Analysis Model?

The research hypotheses, 2a.1 – 2a.3 were tested for all students based on their individual online communication variables, disregarding their group membership.

Hypothesis 2a.1: There was no significant relationship between an individual’s problem solving examination score and his or her total number of ranked messages.

Test: The Pearson’s correlation returned a value of 0.390, p = 0.109. Therefore, hypothesis 2a.1 was not rejected at the $\alpha = .05$.

Hypothesis 2a.2: There was no significant relationship between an individual’s problem solving examination score and his or her percentage of high level messages.
Test: The Pearson’s correlation returned a value of 0.164, p = 0.530. Therefore, hypothesis 2a.2 was not rejected at the $\alpha = .05$.

Hypothesis 2a.3: There was no significant relationship between an individual’s problem solving examination score and his or her average high phase level score.

Test: The Pearson’s correlation returned a value of 0.230, p = 0.385. Therefore, hypothesis 2a.3 was not rejected at the $\alpha = .05$.

Although there existed positive relationships between an individual’s problem solving examination scores and their three individual online communication variables, the results of the three hypotheses tests revealed that these relationships were not significant at the 0.05 level.

The research hypotheses, 2b.1 – 2b.3 were tested for all students based on their group online communication variables, disregarding the number of tasks in which the individual participated.

Hypothesis 2b.1: There was no significant relationship between an individual’s problem solving examination score and the total number of ranked messages sent in their group.

Test: The Pearson’s correlation returned a value of -0.048, p = 0.859. Therefore, hypothesis 2b.1 was not rejected at the $\alpha = .05$.

Hypothesis 2b.2: There was no significant relationship between an individual’s problem solving examination score and the percentage of high level messages sent by their group.
Test: The Pearson’s correlation returned a value of 0.446, p = 0.084. Therefore, hypothesis 2b.2 was not rejected at the $\alpha = .05$.

Hypothesis 2b.3: There was no significant relationship between an individual’s problem solving examination score and their group’s average high phase level score.

Test: The Pearson’s correlation returned a value of 0.068, p = 0.803. Therefore, hypothesis 2b.3 was not rejected at the $\alpha = .05$.

Although there existed positive relationships between an individual’s problem solving examination scores and two of their three group online communication variables, number of messages sent by the group and group average high communication score, the results of the three hypotheses tests revealed that these relationships were not significant at the 0.05 level.

**Final Examination**

Data gathered from each individual’s final examination score and online communication variables were used to answer the third research question: Was there a relationship between mathematical achievement in college algebra as measured by predominately procedural final examination and the quality of online mathematical communication as measured by the Interaction Analysis Model?

The research hypotheses, 3a.1 – 3a.3 were tested for all students based on their individual online communication variables, disregarding their group membership.

Hypothesis 3a.1: There was no significant relationship between an individual’s final examinations score and his or her total number of ranked messages.
Test: The Pearson’s correlation returned a value of 0.355, \( p = 0.136 \). Therefore, hypothesis 3a.1 was not rejected at the \( \alpha = .05 \).

Hypothesis 3a.2: There was no significant relationship between an individual’s final examination score and his or her percentage of high level messages.

Test: The Pearson’s correlation returned a value of 0.506, \( p = 0.032 \). Therefore, hypothesis 3a.2 was rejected at the \( \alpha = .05 \).

Hypothesis 3a.3: There was no significant relationship between an individual’s final examination score and his or her average high phase level score.

Test: The Pearson’s correlation returned a value of 0.374, \( p = 0.204 \). Therefore, hypothesis 3a.3 was not rejected at the \( \alpha = .05 \).

There existed a significant positive relationship between an individual’s percentage of high level messages sent and their scores on the final examination. As stated in Chapter Three, to control for Type I error each significant correlational result was corrected using the Bonferonni method. The resulting correlational analysis was not significant, \( p > 0.05/3 \). Although there existed positive relationships between an individual’s final examination scores and their two other individual online communication variables, the results of these hypotheses tests revealed that these relationships were not significant at the 0.05 level.

The research hypotheses, 3b.1 – 3b.3 were tested for all students based on their group online communication variables, disregarding the number of tasks in which the individual participated.
Hypothesis 3b.1: There was no significant relationship between an individual’s final examination score and the total number of ranked messages sent by their group.

Test: The Pearson’s correlation returned a value of 0.026, \( p = 0.920 \). Therefore, hypothesis 3b.1 was not rejected at the \( \alpha = .05 \).

Hypothesis 3b.2: There was no significant relationship between an individual’s final examination score and the percentage of high level messages sent by their group.

Test: The Pearson’s correlation returned a value of 0.170, \( p = 0.514 \). Therefore, hypothesis 3b.2 was not rejected at the \( \alpha = .05 \).

Hypothesis 3b.3: There was no significant relationship between an individual’s final examination score and their group’s average high phase level score.

Test: The Pearson’s correlation returned a value of 0.204, \( p = 0.432 \). Therefore, hypothesis 3b.3 was not rejected at the \( \alpha = .05 \).

The results of the three hypotheses tests revealed that there were no significant relationships between an individual’s final examination score and their group’s three online communication variables.

Achievement Differences between Sections

This section reports the means of the pretest covariate for both sections of college algebra including the t-test results. In addition, this section reports the analysis of covariance results for both the problem solving and final examinations.
Pretest Results

During the second week of class, students enrolled in both sections of college algebra took a mathematical pretest to determine prior mathematical ability. This pretest consisted of a 16-multiple choice test based on the MAA Algebra Exam and three problem-solving task. The pretest results can be found in Table 9. The treatment group consisted of the students enrolled in the college algebra section that utilized online collaborative problem solving during the semester. Only students who completed the course in both sections were included in the final sample.

Table 9. Independent Samples T-Test for Pretest Results.

<p>| SUMMARY |</p>
<table>
<thead>
<tr>
<th>Groups</th>
<th>Count</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment Group</td>
<td>17</td>
<td>12.40</td>
<td>4.12</td>
</tr>
<tr>
<td>Control Group</td>
<td>25</td>
<td>15.18</td>
<td>2.38</td>
</tr>
</tbody>
</table>

| T-Test |
|--------|-------|------|------|------|
| Levene’s Test | df | t | P-value |
| F | Sig |
| Equal Variances Assumed | 4.172 | .048 | 40 | 2.765 | .009 |
| Equal Variances Not Assumed | 23.29 | 2.507 | .020 |

An independent-samples t-test was conducted to evaluate whether or not the pretest scores differed significantly between the treatment and control groups. The Levene’s test for equality-of-variance assumption was violated, $F(1,40) = 4.172$, $p = .048$. Therefore, equal variances were not assumed. The t-test was significant, $t(23.29) = 2.507$, $p = .02$. Students in the treatment group ($M = 12.40$, $SD = 4.12$) on average
performed at a lower mathematical ability than students in the control group (M = 15.18, SD = 2.38).

Problem Solving Examination

Data gathered from each individual’s problem solving score for each section of college algebra were used to answer the following research hypothesis:

Hypothesis 4a: There was no significant difference in the achievement on the problem solving examination between students completing assigned mathematical tasks individually and those utilizing online collaborative group work.

Based on the results of the t-test above, an Analysis of Covariance (ANCOVA), with the pretest variable as the covariate, was employed to analyze this hypothesis. The scores on the problem solving exam served as the dependent variable. A summary of the ANCOVA is shown in Table 10.

Table 10. ANCOVA for Problem Solving Examination by Group.

<table>
<thead>
<tr>
<th>Source</th>
<th>Problem Solving Examination</th>
<th>Adjusted Problem Solving Exam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>M</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>16</td>
<td>8.94</td>
</tr>
<tr>
<td>Control Group</td>
<td>22</td>
<td>8.32</td>
</tr>
</tbody>
</table>

a. Evaluated at covariates appeared in the model: Pretest = 13.88

**ANCOVA**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>2</td>
<td>34.45</td>
<td>17.22</td>
<td>1.60</td>
<td>.084</td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
<td>44.08</td>
<td>44.08</td>
<td>4.09</td>
<td>.105</td>
</tr>
<tr>
<td>PRETEST</td>
<td>1</td>
<td>30.90</td>
<td>30.90</td>
<td>2.87</td>
<td>.099</td>
</tr>
<tr>
<td>SECTION</td>
<td>1</td>
<td>14.38</td>
<td>14.38</td>
<td>1.34</td>
<td>.256</td>
</tr>
</tbody>
</table>
A one-way analysis of covariance (ANCOVA) was conducted. The independent variable was the section in which each student was enrolled. As noted above, the dependent variable was the scores on the problem solving examination. The ANCOVA was not significant, $F(2, 35) = 2.87$, $p = .099$. The adjusted mean of the treatment group ($M = 9.35$) was slightly higher than the adjusted mean of the Control Group ($M = 8.02$), but this difference was not significant at the $\alpha = 0.05$ level. Based on these results, hypothesis 4a was not rejected.

Final Examination

Data gathered from each individual’s final examination score for each section of college algebra was used to answer the following research hypothesis:

Hypothesis 4a: There was no significant difference in the achievement on the final examination between students completing assigned mathematical tasks individually and those utilizing online collaborative group work.

Based on the results of the t-test for the pretest results, a one-way analysis of covariance (ANCOVA), with the pretest variable serving as a covariate, was used to analyze this research hypothesis. The individual scores on the final examination served as the dependent variable. A summary of the ANCOVA is shown in Table 11.

Table 11. ANCOVA for Final Examination by Group.

<table>
<thead>
<tr>
<th>Source</th>
<th>Final Examination</th>
<th>Adjusted Final Examination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$</td>
<td>$M$</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>17</td>
<td>65.00</td>
</tr>
<tr>
<td>Control Group</td>
<td>25</td>
<td>64.52</td>
</tr>
</tbody>
</table>

$^b$. Evaluated at covariates appeared in the model: Pretest = 14.05
### ANCOVA

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>2</td>
<td>1061.88</td>
<td>530.94</td>
<td>2.52</td>
<td>.094</td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
<td>3688.66</td>
<td>3688.66</td>
<td>17.50</td>
<td>.000</td>
</tr>
<tr>
<td>PRETEST</td>
<td>1</td>
<td>1059.54</td>
<td>1059.54</td>
<td>5.03</td>
<td>.031*</td>
</tr>
<tr>
<td>SECTION</td>
<td>1</td>
<td>208.44</td>
<td>208.44</td>
<td>.89</td>
<td>.326</td>
</tr>
</tbody>
</table>

* Indicates significance at the α = 0.05 level.

A one-way analysis of covariance (ANCOVA) was conducted. The independent variable was the section in which each student was enrolled in. As noted above, the dependent variable was the scores on the final examination. The ANCOVA was significant, $F(2, 41) = 5.03, p = .031$. The adjusted mean of the treatment group ($M = 67.66$) was significantly higher than the adjusted mean of the control group ($M = 62.71$). Based on these results, hypothesis 4a was rejected. After controlling for initial differences in mathematical ability, there existed a significant difference between the college algebra sections on the final examination.

### Summary of Important Results

Chapter Four presented the data and results of this research study. After the testing of all research hypotheses, the following results were deemed significant.

1. The nature of the online communication in this study resembled that found in studies examining face-to-face and online mathematical communication.

2. Almost 20% of the scored messages sent during the online tasks were ranked at or above a Phase III message. Moreover, in 19 of the 24 problem solving episodes at least one group member reached a high level of communication. These
provided evidence that the co-construction of knowledge took place in these collaborative online problem solving groups.

3. There existed a significant positive relationship between an individual’s percentage of high level messages sent and their scores on the final examination. After controlling for Type I error, this result was not significant.

4. There existed a significant difference between the treatment and control groups on the final examination. The treatment group who utilized online collaborative problem solving groups significantly outperformed the control group, after controlling for initial differences.
CHAPTER FIVE

CONCLUSIONS

Introduction

Chapter Four discussed the results of this research study. In this chapter, a brief overview of the purpose of the study, the sample, and the research methodologies will be presented. Secondly, the conclusions and interpretations of the results will be addressed by the researcher. The implications for further action regarding content analysis in computer-mediated mathematical communication will also be discussed. Finally, recommendations for future research will be offered.

Overview of the Study

The purpose of this research study was to investigate the nature and quality of online mathematical communication that occurred during collaborative problem solving and its effects on mathematical achievement in college algebra. The significance of this study was two-fold. First, many national organizations in mathematics education emphasize the importance of productive communication in the learning of mathematics at both the secondary and post-secondary levels (AMATYC, 1998, NCTM, 2000). Secondly, there has been an increased concern regarding the high failure rates among undergraduate students in introductory mathematics courses such as college algebra and calculus (Duncan & Dick, 2000; Morton 1993).
The sample consisted of students enrolled in two intact sections of college algebra. Both sections were taught by the same instructor and the content of the courses was identical. The treatment section originally consisted of 26 students who were placed into six collaborative work groups with four to five members. The members of these groups were balanced by matching on prior mathematical ability as measured by a researcher-designed pretest. Throughout the course these six groups, as well as the control section, were assigned four mathematical tasks. Students in the treatment section were to address these tasks collaboratively while working in online groups. To receive full credit for each of the four assignments, each student in the treatment group was required to post three online contributions during the week to their group folder. A complete transcript of the online communication for each group was recorded in electronic form using the threaded messaging tool of WebCT. The control group (n = 30) worked on identical assignments during the week, but they were instructed to work on these tasks individually.

Each message sent within the online collaborative groups was analyzed for the nature of the communication and coded according to the framework developed by Stacey and Gooding (1998; see also Gooding & Stacey, 1993). This framework allowed the researcher to examine the basic elements of the online communication and the patterns of interaction that emerged. Units of meaning within each message were coded into one of the following eight categories: asking questions, responding, directing, explaining with evidence, thinking aloud, proposing ideas, commenting, and refocusing the discussion. This coding scheme was used to examine the nature of the online communication that
occurred in the six collaborative groups. A copy of this framework can be found in Appendix A.

Secondly, each message sent was analyzed and ranked according to the Interaction Analysis Model developed by Gunawardena, Lowe, and Anderson (1997). This transcript analysis framework was used to examine the quality of the online communication that occurred in the six collaborative groups. Specifically, this model allowed the researcher to determine whether or not the co-construction of mathematical knowledge occurred. Messages, therefore, were ranked as one of the following:

- Phase I: Sharing and Comparing of Information
- Phase II: The Discovery and Exploration of Dissonance of Inconsistency among Ideas, Concepts, or Statements
- Phase III: Negotiation of Meaning/Co-construction of Knowledge
- Phase IV: Testing and Modification of Proposed Synthesis or Co-construction
- Phase V: Agreement Statement(s)/Application of Newly-Constructed Meaning

In addition, data on the six individual and group online communication variables were examined to determine if there existed a relationship between these variables and mathematical achievement. These communication variables included the number of scored messages sent, the percent of high level messages (messages scored at Phase III, IV, or V), and the average high phase level score for both the individual and the group.

Mathematical achievement data was gathered for students enrolled in the two sections. This data consisted of an individual’s pretest, final exam and problem solving examination scores. The final sample of students included 17 students in the treatment group and 25 students in the control group. Academic achievement data for the treatment group was correlated with the twelve online communication variables to address Research Questions 2 and 3. In addition, the achievement data for all students in both the
control and treatment groups was used to evaluate Research Question 4 pertaining to the differences in mathematical achievement between the treatment and control section of college algebra.

Results from the first content analysis procedure revealed that the majority of the messages sent were coded as thinking aloud followed by responding, explaining with evidence, and questioning. Moreover, the results of the second transcript analysis revealed that one in five messages was ranked as a high level message exhibiting evidence of the co-construction of knowledge. As indicated by the group’s average high and highest phase level reached, it was found that in 19 of the 24 problem solving episodes (six groups by four tasks) clear evidence of the co-construction of mathematical knowledge was shown.

An analysis of covariance (ANCOVA) was used to analyze mathematical achievement differences between the treatment and control groups. ANCOVA was performed on the raw scores of the final examination and researcher-designed problem solving examination using the pretest scores as the covariate. The treatment group performed as well or better on both measures of achievement. After controlling for initial differences in mathematical ability, the treatment group performed significantly better than the control group on the final examination.

Conclusions and Inferences

Based on the results in Chapter Four, conclusions were reached for each of the four research questions. This section is separated by reporting these conclusions along
with the researcher’s inferences regarding the results for each of the four research questions.

Research Question 1 – Nature and Quality

Question 1 addressed the nature of the online mathematical communication that occurred during the collaborative group work. Data for this conclusion was drawn from examining the total participation of students working in online groups, as well as a mathematical content analysis of the messages sent. A content analysis framework developed by Stacey and Gooding (1998) was utilized. (see Appendix A). Three significant findings regarding the nature of the online communication were found.

Over the course of the semester the total number of messages posted for each task significantly declined. Evidence for this conclusion can be found in Table 3 which shows the number of messages sent per group during each of the four online tasks. Two causes are associated with this decline. First, the number of messages declined even when the number of active participants remained the same. This phenomenon can be explained by examining both the type of messages sent, as well as the length of these messages in the later tasks. There was a significant decline in the number of social messages and messages regarding group processes throughout the four tasks. In addition, the length of the messages in the later tasks tended to be larger. Instead of containing ideas regarding a single problem, messages in these later tasks tended to contain ideas on multiple problems. However, as depicted in Figure 4, these longer messages did not necessarily mean higher quality.
Another explanation for the decrease in the number of total messages sent may be attributed to the Hawthorne effect. As students became used to working in their online groups, the newness of this environment might have worn off. Another likely possibility was that the availability of university computers may have decreased. Task 1 and Task 2 occurred during the beginning of the semester, whereas Task 3 and Task 4 occurred close to the university’s midterm week and final’s week, respectively. Therefore, Task 3 and Task 4 occurred during the peak times in which the university computer labs are typically full. This can be seen in the following comment made during the third task.

“Sorry guys for the lateness of my answers. I have tried to use the computers in the lab for almost a week, but everytime I go there is too many people there.” (This student proceeds to give their answers to three of the problems.)

The second cause of the decline in the total number of messages sent was that three students withdrew from the course prior to the fourth task. These students had previously been active participants in the prior tasks. The reason(s) why these students withdrew from the course were beyond the scope of this research study.

The nature of communication exhibited in the online transcripts of this study was consistent with those found in studies of face-to-face mathematical communication (Pirie & Schwarzenberger, 1998; Stacey & Gooding, 1998), as well as computer-mediated mathematical communication (Myers 2002). Evidence to support this conclusion can be drawn from Table 4. The data in this table shows the percentage of contributions in eight categories of mathematical talk: asking questions, responding, directing, explaining with evidence, thinking aloud, commenting and refocusing discussion.
The patterns of face-to-face interaction that have been associated with higher achievement were also found in this online medium (Webb, 1991; Cohen, 1994; Stacey & Gooding, 1998.) Responding to another, asking questions, explaining with evidence accounted for half (49.7%) of the mathematical contributions. These were interactive statements in which there was a clear transition from sharing answers to discussing and resolving inconsistencies and dissonance. Stacey and Gooding (1998) termed such patterns as intellectual interactions.

The largest percentage (36.3%) of problem solving contributions was categorized as non-interactive thinking aloud. These statements were defined as contributions that exhibited a person’s thoughts, actions, or processes while working. The majority of the thinking aloud contributions consisted of a student posting a mathematical answer key. This key provided not only the answer to the problem, but also a step-by-step solution process consisting of mathematical symbols. These responses were typical of the work required for in-class homework assignments, quizzes, and examinations. Therefore, it is not surprising that the mathematical answer key was the largest percentage of contributions to the online groups.

A quarter of the non-interactive thinking aloud contributions were statements that contained only an answer. One explanation for these types of postings may lie in the nature of instructor-led mathematical discourse found in many college classrooms; the initiate-respond-evaluate sequence (Knuth & Peressini, 2001)). In this pattern of mathematical communication the teacher initiates the discourse by asking the whole class a question. A student then responds by stating an answer from their own thinking.
Finally, the teacher evaluates the feasibility of the answer provided by his or her student. In the online communication the question is posed at the beginning of the assignment. Students were then simply posting their response to this question and awaiting their peers’ evaluation of its correctness.

An alternative explanation for the large percentage of thinking aloud contribution may lie in the nature of the tasks. The goal for each of the four online tasks was to reach a consensus regarding the solutions to the problems. Therefore, students may have spent a large proportion of their time working individually to solve each problem before posting a message to the group. Under this assumption, the nature of the tasks could be categorized as those typified by routine learning. For routine learning, it is necessary for students to help each other solve the problem. This may be best accomplished through offering “each other substantive and procedural information” (Cohen, 1994, p.4) consistent with thinking aloud statements.

A smaller percentage of contributions to the problem solving groups were proposing ideas and commenting. Throughout the four online tasks only one message was coded as directing and one message as refocusing the discussion. It may be that these contributions are seen more frequently in face-to-face interactions. For example, in the asynchronous environment, the time spent online is typically time on task. That is, students log onto the server with a purpose of posting a message that will contribute to their group’s efforts in solving the task. Therefore, there is little evidence of being off task in the asynchronous environment and little need to refocus the discussion.
A more detailed analysis of the online transcripts revealed differences in the dynamics of the groups. The communication among members of the six problem solving groups varied from a simple exchange of mathematical answer keys to discussions in which genuine student interactions occurred. As described in Chapter One, genuine interactions were those in which students share more than solutions, they share their knowledge, problem solving strategies, and procedural skills within their group. The difference between the groups can be further explained by the percentage of non-interactive thinking aloud statements versus the percentage of intellectual interactions. To highlight these differences consider the following excerpts from two of the six problem solving groups during the first task.

**Subject: Hank’s #4**

**Message no. 12**

**Author:** Hank

**Date:** Sunday, February 8, 2004 9:16pm  

\[(m+3)(m+3)=(2m-1)(2m-1)\]  
\[m^2+6m+9=4m^2-4m+1\]  
\[6m+9=3m^2-4m+1\]  
\[9=3m^2-10m+1\]  
\[0=3m^2-10m-8\]  
\[(3m+2)(m-4)\]  
\[m=-\frac{2}{3} \text{ or } 4\]

**Subject: marks #4**

**Message no. 73**

**Author:** Mark

**Date:** Friday, February 13, 2004 10:35am  

\[(m+3)(m+3)=(2m-1)(2m-1)\]  
\[m^2+6m+9=4m^2-4m+1\]  
\[m^2+9=2m^2-6m+1\]  
\[0=3m^2-10m-8\]  
\[(3m+2)(m-4)\]

and now iam stuck i dont even know if i got that right you can take the square root of \(m^2\) and root the other side as well but i am not sure how to set that up
In this problem solving group it was evident that there was a lack of true collaboration as exchanges between participants were limited to the sharing of information. As the second message above was posted after the first message, it could not be assumed that individuals read the messages posted by their group. Although participation is an important component of collaboration, in asynchronous discussions, reading a message is also an important component of effective collaboration.

The type of interactions that occurred within the groups was determined to be most critical component of effective collaboration. Groups that exhibited true collaboration interacted more through asking questions, responding to questions, and explaining with evidence as seen in the next excerpt.

**Subject: Problem 1**

**Message no. 4**

**Author:** Susan

**Date:** Sunday, February 8, 2004 11:06am

I was unable to solve problem 1 because I don't know how to finish the equation: Let x be width, and length is 2x+3, the equation is A=L(W), so (x)(2x+3)=810meters squared. Once we solved for x I thought we could plug it in to the equation for perimeter; P=2L+2W. So anyway (x)(2x+3)=810 is (I think)2x^2+3x=810. I don't know what to do with it after that....I was able to solve problems 2 and 3, but not 4, but I'll take any help with 1 first.

**Subject: Problem 1**

**Message no. 7**

**Author:** Andrew

**Date:** Sunday, February 8, 2004 5:33pm

Hey all, I am also having trouble with problem one. I also set the question up as: Let x be width, and let (3+2x) be length. As Susan stated the equation is A=L(W), therefor(x)(3+2x)=810^2.

After I distribute the x I get 3x+2x^2=810^2, I go further and get 3x+2x^2=656100

After this I don't know what to do. Anyone have any ideas????
Subject: stuck on 1
Message no. 10
Author: Ted
Date: Sunday, February 8, 2004 9:12pm
Hi everyone, I am also having trouble with number 1. I think that Susan's process is correct, but when you get $2x^2 + 3x = 810$, you subtract the 810 and you get $2x^2 + 3x - 810$ and that is a quadratic equation. But I am still not sure how to solve it. Andrew, I am not sure why you squared the 810, but it might correct- I'll keep trying and post another message if I get it.

Subject: Problem 1 and 4
Message no. 54
Author: Susan
Date: Thursday, February 12, 2004 6:27pm
I am so stuck on number 1. All my answers have huge decimal places, and I have pages of work on it alone. Let me know if you all get this or something else. And Thanks Ted for saying you'll E-mail the answers, I'll try to come up with something for #1 before 2 tomorrow.

Subject: problem 1 solved
Message no. 57
Author: Ted
Date: Thursday, February 12, 2004 7:14pm
Hi everyone,
I got an answer to problem one. I went to the math tutoring center and got some help. We were right! The equation $(2w^2+3w-810)$ we agreed on in class is correct. Unfortunately, the dimensions of the fence are irrational numbers and that is just how these numbers work out after you plug them in to the quadratic formula. Here’s what went down, $a=2$, $b=3$, $c=-810$ (I plugged these into the quadratic formula) $-b + or -$ the square root of $b^2 - 4ac$, all over $2a$. In this case you don’t need to worry about $-b +$ the square root of …… because we aren’t going to have negative dimensions. (Continues on by explaining the solution)
The dimensions of the fence are $41.7716474$(length) by $19.38858237$(width).

Subject: Problem 1
Message no. 61
Author: Andrew
Date: Thursday, February 12, 2004 8:34pm
Hey all, Sorry for the lateness of my response but here it goes. So for number 1 I was able to work it all out the same way ted did. However, I did notice that he didn't use x. Which might be what is confusing you Susan. So I will explain one more time how I, and ted both got the answers we did. First, Let x be the width then $(2x+3)$ is the length If A=LW then, $(x)(2x+3)=810$ you then get $2x^2+3x=810$ subtract 810 from both sides and then get $2x^2+3x-810=0$ Then we use the quadratic equation to solve for x $a=2$, $b=3$, $c=-810$ The equation is as follows $x= -3$ plus or minus the square root of $3^2-4(2)(-810)$, all divided by $2(2)$. You then get $x= -3$ plus or minus the square root of $9-(-6480)$, all divided by $4$ then $x=-3$ plus or
minus the square root of 6489, divide by 4 Like John said since we can't have negative lengths then we can do away with the minus between the -3 and the square root of 6489 From here when I first plugged it into my calculator it didn't work right. Shellie if this is where you are running into problems then you might have been doing what I was doing. If you punch in the equation as is in your calculator -3 + the square root of (6489) divided by 4, you get numbers a little smaller than what John got. This is what I was doing. To fix it I didn't divide by 4 until after I let the calculator figure out -3 + the square root of 6489, then divide by 4. I then got x=19.38858237 ft for width. L=2x+3 so 2(19.38858237)+3=41.77716475 ft for length. (19.38858237)(41.77716475)=809.99 repeating. Like John said. Close enough for me. So, a) if x is the width then (2x+3)= Length b) if Length is 2x+3 then (2x+3)-3 all divided by 2 is the width. c) Width is 19.38858237 ft Length is 41.77716475 ft Andrew :)

This pattern of computer-mediated communication was consistent with the definition of mathematical communication outlined in Chapter One. It was purposeful talk on a mathematical subject containing genuine student contribution through interactions (Pirie & Schwarzenberger, 1988). The data from this group also revealed that the members interacted more, with a larger percentage of intellectual interactions. In addition, this group collaborated throughout the problem solving process providing clear evidence of high levels of mathematical thinking (Pirie & Schwarzenberger, 1988).

Question 1 also assessed the quality of the online mathematical communication, specifically to address whether or not the co-construction of mathematical knowledge occurred during the online collaborative group work. Data used to address this research question came from the coding of all messages sent using the Interaction Analysis Model developed by Gunawardena, Lowe, and Anderson (1997).

The level of online communication employed by college algebra students during collaborative problem solving tasks did reach high levels of communication such as the negotiation of meaning and the co-construction of mathematical knowledge. Partial evidence that supports this conclusion is presented in Table 5 which shows the number of scored messages at each of the five phase levels. Approximately one in five messages
was indicative of the co-construction of mathematical knowledge. As depicted in Figure 4, these high level messages (Phase III and Phase IV) were evident in all four of the online tasks. In addition, there was evidence of the co-construction of mathematical knowledge in 19 of the 24 problem solving episodes. These results support the research findings that online communication can function as a ‘cognitive amplifier’ (Warschauer, 1997, p. 472) that creates opportunities for small groups of students to co-construction knowledge (Gunawardena, Lowe, & Anderson, 1997; Kanuka & Anderson, 1998; Warschauer, 1997).

No messages were ranked as Phase V. This highest level consists of students applying recently constructed mathematical knowledge to new problems. One explanation for the lack of Phase V messages was the nature of the assigned tasks. These tasks required the students to find a solution. It was not required, either by the instructor or the assignment, to apply the concepts learned during the collaborative problem solving to different tasks. An alternative explanation for the lack of Phase V messages could have been that students were unfamiliar as to how to communicate in the online environment. Although the researcher provided each student with an online participation rubric at the beginning of the semester, it was unclear if this rubric helped students communicate effectively. It is suggested that instructor coaching during the online task may help students reach higher phases of mathematical communication.

It is also important to note that the majority of messages occurred in the lowest phase of communication, Phase I. The researcher believes that messages involving the sharing and comparing of mathematical information may be the easiest form of online
communication. These messages closely resemble the individual seatwork involved in doing homework or taking quizzes and exams that make up a large portion of the traditional college algebra course.

Another explanation for the large percentage of Phase I messages may lie in the assumption that the co-construction of knowledge may not always be an observable phenomenon in the online medium. Kanuka and Anderson (1998), theorized that knowledge construction may take place over time, after the final message has been posted. Therefore, instead of student’s jointly constructing knowledge during exchanges of messages, the online environment might serve as a mechanism for the parallel construction of mathematical knowledge. It also may be possible that students who posted their mathematical answer keys may have internally processed through the stages of the construction of knowledge. Under this assumption, the construction of knowledge occurred prior to posting their message.

Research Questions 2 and 3 – Online Communication and Mathematical Achievement

Research Questions 2 evaluated the relationships between the three individual and three group online communication variables and their mathematical achievement as determined by their problem solving examination. Whereas, Research Question 3 examined these relationships in regards to the procedural final examination. The six online communication variables included the number of scored messages sent, the percentage of high level messages, and the average high phase level score for both the individual and the group. Based on the results of Chapter Four, conclusions were made regarding twelve research hypotheses.
There was no significant relationship between an individual’s three online communication variables (total number of messages sent, percentage of high level messages, and average high phase level score) and their score on the problem solving examination. Moreover, there was no significant relationship between a group’s three online communication variables (total number of messages sent by the group, percent of high level messages sent by the group, and the group’s average high phase level score) and a group member’s score on the problem solving examination. This may have been caused by the low performance on this examination. Out of a total of 20 points, the average score on the problem solving examination was 8.89 (SD = 2.61).

Webb (1991), in her review of the literature regarding face-to-face mathematical cooperative learning, found that task-related talk was significantly related to mathematical achievement. This phenomenon may help explain the results of this study. Although the communication in the online collaborative groups was focused on the mathematical task, the nature of the communication was largely a discussion of the procedural components of the problem solving task. Little communication focused on setting up the problem, but rather it focused on the solving of a specific equation. This failure by the groups to discuss the heuristics of problem solving may partially explain these results.

There was one significant finding regarding the correlation between an individual’s three online communication variables and their score on the procedural final examination. There was a significant positive correlation between the percentage of high level messages (ratio of high level messages to total number of messages) sent by an
individual and their score of the final examination. When testing the other two research hypotheses related to an individual online communication variables (total number of messages sent and average high phase level score) and their score on the procedural final exam, no significant relationships were found. Moreover, no significant relationships were found between a group’s online communication scores (total number of messages sent, percent of high messages, and group average high score) and a group member’s score of the procedural final.

High level messages sent by an individual tended to provide the whole group with elaborate explanations regarding the procedural components of the problem. It is these elaborate explanations that have been found in other studies regarding face-to-face collaborative learning to be the most consistent, positive predictor of achievement (Webb, 1991; Cohen, 1994). One explanation for the significant relationship between an individual’s percentage of high level messages and their subsequent mathematical achievement on the procedural final exam is that the student who explains how to solve a mathematical equation benefits the most. Students gain insight into their mathematical thinking when they explain and justify their reasoning (NCTM, 2000). The result of this correlational analysis must be taken with caution. Although the Pearson’s Product Moment Correlation returned a significant result \((r = 0.506, p = 0.032)\), the corrected Bonferonni method did not reveal a significant relationship \((p > .05/3)\). The Bonferonni method was used to control for Type I error.
Research Question 4 – Differences in Mathematical Achievement

Research Question 4 investigated the effects of working on the online collaborative tasks designed according to the Treisman Workshop Model on mathematical achievement. Mathematical achievement data, which included the raw scores on the pretest, problem solving examination, and procedural final examination, was collected for both the treatment and control sections of college algebra. The treatment section was required to work on the mathematical tasks collaboratively through online threaded discussions, whereas the control group worked on the same tasks individually.

The treatment group did as well or better than the control group on the two measures of mathematical achievement, the problem solving examination and the procedural final examination. After controlling for initial differences in mathematical ability, the treatment group performed better on the procedural final examination than the control group. The treatment group had a significantly higher adjusted mean score (M = 67.66) than the control group (M = 62.71). No significant difference was found in regards to the problem solving examination.

This result for the final examination was consistent with findings from other studies at the postsecondary level that found academic achievement was significantly enhanced by the use of collaborative learning (Fullilove & Treisman, 1990; Dees, 1991; Duncan & Dick, 2000; Springer, Stanne, & Donovan, 1999). The researcher believes that the increased performance of the treatment group over the control group on the procedural final examination was achieved through the discussion of the Treisman-style
tasks. Several direct implications relating to the treatment may help explain the results of this quasi-experimental analysis. First, the tasks were designed to identify common misconceptions and areas of mathematical knowledge that a student must strengthen. Through collaboration, these “misconceptions can be identified and addressed” (NCTM, 2000, p. 61). The online discussions served as a mechanism for assessment in which students were provided feedback regarding the correctness of their solution strategies. Due to this feedback students in the treatment group had opportunities to test, reflect upon, and make permanent procedural skills before the assignment was graded. In contrast, students in the control group may not have had these opportunities.

Secondly, by reading the responses posted by the students during the week-long task, the instructor of this research project was able to get immediate feedback whether or not the students in the treatment group understood the mathematical concepts addressed in-class. Therefore, the online discussions served as a tool of assessment in which the instructor could revise and re-teach concepts for the treatment group that he discovered were not well understood. This was the case for certain procedural skills such as finding the inverse of a function and determining whether a function was even or odd.

Thirdly, the nature of the online communication was largely focused on procedural skills such as solving an equation. Little talk was associated with the heuristics of problem solving. This fact may also explain why no significant difference was found with regard to the mean scores on the problem solving examination.

Finally, it became apparent in the online transcripts at the beginning of the research that many students lacked the confidence in their solution strategies. That is,
students often ended their online messages by adding that the solution they obtained was “probably not right” or “way off”. By comparing and sharing answers and procedural strategies in online discussions, students in the treatment group may have overcome these issues of confidence. The control group who worked on the tasks individual did not receive feedback on the correctness of their solutions by their peers.

Due to the extraneous variables beyond the control of this study, the positive benefit found in the quasi-experimental analysis can not be generalized. One variable included the selection of the afternoon section as the treatment group instead of the morning section. It is believed that the second class may benefit from the instructor teaching the same mathematical concept earlier in the day. Under this assumption, the instructor has time to reevaluate his or her teaching strategies to help students overcome misconceptions that were discovered previously. Another variable was the amount of time on task between the control and treatment groups. It was impossible to ascertain whether or not the time of task for both groups was equal. Finally, the pretest scores used as the covariate in the analysis of covariance may not have been a sufficient control variable for initial mathematics ability. This explanation seems to have the weakest standing as the first examination scores mirrored the results of the pretest indicating that the treatment group performed lower than the control group.

**Remarks and Suggestions for Further Action**

In this study, the transcript analysis procedures consisted of two components. First, each message was divided into ‘units of meaning’ and then coded using Stacey and
Gooding’s eight coding categories. This enabled the researcher to determine the nature of the online mathematical communication. This content analysis procedure did not allow the researcher to determine the level of mathematical talk. That is, which responses showed evidence of knowledge construction or mathematical learning. In addition, the framework, developed for face-to-face mathematical talk, did not substantially allow for non-interactive responses such as providing a mathematical answer key. For this study non-interactive messages that either simply stated an answer or provided an answer key were combined as ‘thinking aloud’ statements. The researcher believes that these two types of messages serve different functions in the group processes. For example, a mathematical answer key can help another group member figure out how to solve the problem. Whereas, a simple answer does not allow for this response. Moreover, Webb (1985, 1991) concluded that simply stating an answer during small group learning was consistently negatively related to mathematical achievement and giving help, such as an answer key was positively related to mathematical achievement.

Secondly, each message was coded using Gunawardena, Lowe, and Anderson’s Interaction Analysis Model (IAM). This transcript analysis procedure was specifically designed for computer-mediated conferencing and allowed the researcher to determine whether there existed co-construction of mathematical knowledge. Therefore, it was possible to determine the level of each message in relationship to mathematical learning. This coding procedure did not allow the researcher to examine the type or nature of the mathematical talk.
Moreover, the IAM was developed to document the social construction of knowledge in an online debate regarding the use of computer-mediated communication. The nature of the task in this online debate allowed participants to take either an affirmative or negative stance on the issue. This was remarkably different than the tasks for this current research study. The tasks for this research study were designed to uncover common mathematical misconceptions and areas of weaknesses in an individual’s mathematical understanding. Messages that provided a mathematical answer key were consistently coded as Phase I messages without regard to whether the individual constructed mathematical knowledge prior to posting their message or if the parallel construction of knowledge occurred after the final message was sent. In some cases, a similar response was coded as a Phase III message because the author concluded that they had difficulty in solving the problem and another group member’s ideas had helped them solve the problem. This can be seen in the example below.

Subject: stuck on 1
Message no. 10
Author: Ted
Date: Sunday, February 8, 2004 9:12pm

Hi eveyone,

I am also having trouble with number 1. I think that Shellie's process is correct, but when you get 2x^2 + 3x=810, you subtract the 810 and you get 2x^2+3x-810 and that is a quadratic equation. But I am still not sure how to solve it. Andrew, I am not sure why you squared the 810, but it might correct- I'll keep trying and post another message if I get it.

Therefore, the researcher of this study found a need to combine both frameworks to account for some of these discrepancies. Table 7 showed the number of units of meaning coded according to Stacey and Gooding’s framework mapped into each of the
five phase levels. The patterns in this table allowed the researcher to determine the nature of the messages coded at each of the five phase levels. For example, messages coded as Phase IV were generally those that responded with evidence to support an agreement with another’s response. As discussed above, thinking aloud statements were typically ranked as Phase I messages, whereas statements asking a question were generally coded as Phase II messages. Phase III messages were consistently coded as explaining with evidence.

While both transcript analysis procedures provided an objective method to help deal with the abundance of text in the online mathematical communication, it is suggested that a new framework be developed to aid in the interpretation of the basic elements of the mathematical talk in terms of the co-construction of knowledge. The researcher suggests this new content analysis procedure be a combination of the useful components of both Stacey and Gooding’s framework and the Interaction Analysis Model. This new framework will be relevant to those developing and teaching online mathematical courses as a mechanism to provided feedback and monitor the value of online collaborative problem solving.

Limitations and Delimitations

The limitations of the study were as follows:

1. The population of this study was drawn from students who enrolled in two of ten sections of college algebra at Montana State University in the spring of 2004.
2. The researcher was unable to randomly assign students to each of the college algebra sections: the section utilizing online mathematical discussions and the section that did not.

3. The period of the study was for only one semester.

4. Throughout the semester, only four week long tasks were assigned to both the treatment and control groups.

5. Although methods were employed to ensure that students enrolled in the control section of college algebra worked individually on their mathematical problem sets, complete control was never achieved.

6. Individual messages were given one score only, based on the highest phase level attained in the message.

7. Messages were scored only for the person who sent it, disregarding how reading the message might contribute to the co-construction of knowledge.

The delimitations for this study were as follows:

1. Any student whose personal history resulted in him or her not participating in the online group activities for an extended period of time were excluded from the sample. In total two, students did not participate in any of the four online tasks.

2. Students in either section who dropped from the course or failed to take the required final examination were excluded from the quantitative portion of this study.

3. Students were placed in heterogeneous groups by matching the students based on their pretest score, with gender as a secondary matching characteristic. Each
group consisted of one high math ability student, two medium ability students, and one low ability student. Due to the make-up of the course, only half of the problem solving groups were composed of two males and two females.

4. The final examination was designed by the course supervisors of college algebra. The researcher did not have any control as to the types of items selected for this examination.

**Recommendations for Future Research**

There were several pertinent issues that arose from the discussion of the results of this research study. Many of these issues have been recommended for future studies. These recommendations are discussed below.

Few studies exist that examine online small group learning in undergraduate mathematics courses. Therefore, there is a need to replicate this study with a larger sample size and multiple sections of college algebra. This would help assess the generalizability of the results found in this current study. Secondly, a series of similar studies in other undergraduate mathematics courses such as calculus and statistics can help expand the research base on the dynamics of online mathematical communication. Since much of the Treisman research is focused on calculus, a study examining online collaborative problem solving in calculus would further this research base.

With the transfer of collaborative learning to the online environment, additional studies should focus on how this new medium influences interactions. Is there a difference in the dynamics of face-to-face mathematical communication versus online
mathematical communication when groups of students are working on the same tasks?

The notion of the co-construction of knowledge in an online environment is partially based on the assumption that other group members actively participate in the discussion, by reading the responses of others. A study should be developed to ascertain effect of reading a response on an individual’s construction of mathematical knowledge. For example, how is the reading of a message internalized into the reader’s existing knowledge base?

Webb (1991) determined that receiving no response to a question or receiving a terminal response to a question was consistently negatively related to achievement. In this study, many of the messages asking for help did not receive a response. A study should be conducted to determine the effect of receiving no help in the online environment. For example, do students who receive no answer to their request overcome this shortcoming or do they ‘learn’ less than a peer who received a response?

Moreover, in an asynchronous online environment, responses, if given, are often time-delayed. These responses can be given over a period of minutes, hours, or even days. A study should also be conducted to examine how time-delayed responses affect the learning of mathematical concepts. This may be accomplished through a series of interviews with the active participants of the study.

**Concluding Comments**

The results of this study are pertinent to those interested in distance education. Over the past five years the number of distance courses in mathematics has grown
significantly (Sakshaug, 2000). With this increased growth comes an increased concern as to how to effectively teach mathematics in this new environment. Data from this study show that, if given the opportunity, students can co-construct mathematical knowledge in the text-based medium of online technologies. Moreover, the opportunities to communicate mathematically online may allow students to clarify their understanding through explanation and justification.


Russell (1999). *No significant difference phenomenon*. Montgomery, AL: International Distance Education Certification Center.


Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. Educational Studies in Mathematics, 46(1-3), 13-57.


APPENDICES
APPENDIX A

CONTENT ANALYSIS FRAMEWORKS
### Asking Questions
- Of a previous speaker
  - How many \( \frac{1}{2} \)’s in 4?
- From own thinking or working
  - Who’s got 2? That goes there.
- Reading a word problem
  - \( \frac{1}{2} \) km split into four sections. How long is each section? We’ve got to work that out

### Responding
- To a request for clarification
  - S1: How many \( \frac{1}{2} \)’s in 4?
    - S2: There are 4 wholes, right?
  - Agreeing
    - Yeah
  - Disagreeing
    - You’re wrong
  - Repeating
    - S3: No, 8 goes there.
    - S4: No, 8 goes there.

### Directing
- Put it down.

### Explaining with Evidence
- No, 4, How many \( \frac{1}{2} \)’s in 8? (\( 4 \div \frac{1}{2}; \) Pointing left to right and replacing 1/8 card with 8)

### Thinking Aloud
- 12 divided by 6. All right. 12 divided by 6 is there.

### Proposing Ideas
- It’s 2 so they’re not the same answer so we can’t put them in.

### Commenting (Affective)
- We’ve finished. Done it.

### Refocusing Discussion
- Yeah. That’s what you’re meant to do. You’re meant to set it out under this (pointing to headings).
INTERACTION ANALYSIS MODEL FOR EXAMINING SOCIAL CONSTRUCTION
OF KNOWLEDGE IN COMPUTER CONFERENCING
(Gunawardena, Lowe, and Anderson, 1997)

Phase I: Sharing/Comparing of Information. Stage one operations include:
A. A statement of observation or opinion
B. A statement of agreement from one or more other participants
C. Corroborating examples provided by one or more participant
D. Asking and answering questions to clarify details of statements
E. Definitions, description, or identification of a problem

Phase II: The Discovery and Exploration of Dissonance or Inconsistency among Ideas, Concepts, or Statements. Operations which occur at this stage include:
A. Identifying and stating areas of disagreement
B. Asking and answering questions to clarify the source and extent of disagreement
C. Restating the participant’s position, and possible advancing arguments or considerations in its support by references to the participant’s experience, literature, formal data collected, or proposal of relevant metaphor or analogy to illustrate point of view

Phase III: Negotiation of Meaning/Co-construction of Knowledge
A. Negotiation or clarification of the meaning of terms
B. Negotiation of the relative weight to be assigned to types of arguments
C. Identification of areas of agreement or overlap among conflicting concepts
D. Proposal and negotiation of new statements embodying compromise, co-construction
E. Proposal of integrating or accommodating metaphors or analogies

Phase IV: Testing and Modification of Proposed Synthesis or Co-construction
A. Testing the proposed synthesis against “received fact” as shared by the participants and/or their culture
B. Testing against existing cognitive schema
C. Testing against personal experience
D. Testing against formal data collected
E. Testing against contradictory testimony in the literature

Phase V: Agreement Statement(s)/Application of Newly-Constructed Meaning
A. Summarization of agreement(s)
B. Applications of new knowledge
C. Metacognitive statements by the participants illustrating their understanding that their knowledge or ways of thinking (cognitive schema) have changed as a result of the conference interaction
APPENDIX B

MATHEMATICAL ACHIEVEMENT INSTRUMENTS
For problems 1-16, please circle the correct answer.

1. The first operation performed in simplifying the expression \( \frac{[-6 + 3(4)]^2}{2} \) is:
   (a) division
   (b) addition
   (c) multiplication
   (d) squaring

2. Simplify: \( 3a - (5b - 4a) \)
   (a) \( 7a - 5b \)
   (b) \( -a - 5b \)
   (c) \( -15ab - 12a^2 \)
   (d) \( -15ab + 12a^2 \)
   (e) \( -6a^2b \)

3. If \( x/2 = y \) and \( y = z - 4 \) what is the value of \( x \) when \( z = 28? \)
   (a) \( x = 48 \)
   (b) \( x = 40 \)
   (c) \( x = 24 \)
   (d) \( x = 12 \)
   (e) none of these

4. If \( (4/3)x - (1/2) = 0 \), then \( x = \)
   (a) \( 3/8 \)
   (b) \( 2/3 \)
   (c) \( 3/2 \)
   (d) \( 11/6 \)
   (e) \( 8/3 \)
5. \(\frac{6m^2 + 3m}{3m} = \)

(a) 3m  
(b) 6m^2  
(c) 2m + 1  
(d) 6m^2 + 1  
(e) 5m

6. \(\frac{x - 3}{8} - \frac{7}{4} = \frac{5}{8}\) has a solution of

(a) -12  
(b) -6  
(c) 15  
(d) 16  
(e) 22

7. If \(x^2 + 2x = 3\), then \(x\) could equal

(a) -3  
(b) -2  
(c) -1  
(d) 0  
(e) 3

8. \(4 \frac{1}{2} - 3 \frac{2}{3} = \)

(a) 0  
(b) 5/6  
(c) \(\frac{1}{6}\)  
(d) \(\frac{5}{6}\)  
(e) 2

9. \((4x^2y)(-3x^5y^4) = \)

(a) \(-12x^7y^5\)  
(b) \(-12x^{16}y^4\)  
(c) \(x^3y^3\)  
(d) \(-12x^7y^4\)
10. Solve: $4(k - 2) = k/3 + 5$. The solution is
(a) less than -3
(b) between -3 and 0
(c) between 0 and 3
(d) greater than 3

11. Solve $5pq - 10p = 7q$ for p.
(a) $p = 2q / -5$
(b) $p = 7q / (5q - 10)$
(c) $p = (7q + 10) / 5q$
(d) $p = 7q / (q - 2)$

12. Which statement is true regarding the two lines whose equations are:
   \[ x - 2y = 4 \quad \text{and} \quad 2x + y = 2 \]
(a) the lines are parallel
(b) the lines are perpendicular
(c) the lines coincide
(d) none of these

13. $\sqrt[4]{50x^8y^{12}} =$
(a) $5x^2y^3\sqrt{2}$
(b) $25x^8y^{12}$
(c) $5x^6y^{10}\sqrt{2}$
(d) $5x^4y^6$

14. The graph of $x - 4y + 8 = 0$ crosses the y-axis at $y =$
(a) -8
(b) -2
(c) 0
(d) 2

15. Multiply: $(2 - r)(5 - 3r)$
(a) $-3r^2 + 11r + 10$
(b) $-3r^2 - r + 10$
(c) $3r^2 + 11r + 10$
(d) $3r^2 - 11r + 10$
16. When completely factored, one of the factors of $9A^2 - 4B^2$ is:

(a) $3A + 4B$
(b) $9A - 2B$
(c) $3A - 2B$
(d) $9A + 4B$
(e) The binomial cannot be factored

Part II – Constructed Response

In the space provided, answer the following problems. Please show all your work to receive full credit.

1. Including an 8% room tax, the Percent Inn charges $162 per night. Find the inn’s nightly cost before the tax is added. (Adapted from Blitzer, 2003, p. 107)

2. Julia’s soybean field is 3 meters longer than its width. To increase her production, she plans to increase both the length and width by 2 meters. If the new field is 46 square meters larger than the old field, then what were the dimensions of the old field? (Dugopolski, 1996)
3. The width of a rectangular gate is 2 feet more than its height. If a diagonal board of length 8 feet is used for bracing, what are the dimensions of the gate? (Dugopolski, 1996)
In the space provided, answer the following problems. Please show all your work to receive full credit.

1. Find the length and width of a rectangle with a perimeter of 80 that has the maximum possible area.

2. Including an 8% room tax, the Percent Inn charges $162 per night. Find the inn’s nightly cost before the tax is added.
3. Julia’s soybean field is 3 meters longer than its width. To increase her production, she plans to increase both the length and width by 2 meters. If the new field is 46 square meters larger than the old field, then what are the dimensions of the old field?

4. You inherit $18,750 with the stipulation that for the first year the money must be placed in two investments paying 10% and 12% annual interest, respectively. How much should be invested at each rate if the total interest earned for the year is to be $2117.

5. The width of a rectangular gate is 2 feet more than its height. If a diagonal board of length 8 feet is used for bracing, what are the dimensions of the gate?
APPENDIX C

PROBLEM SOLVING SCORING RUBRIC
The following rubric was used to score students’ constructed-response items on the problem solving examination. Dennis Runde (1997) used this rubric to score students’ work on a pre-test and post-test word problem examination. It was loosely adapted from the work of Malone, Douglas, Kissane, and Mortlock (as cited in Runde, 1997).

**Score of 0: Non-commencement**

Non-commencement refers to the inability of the student to begin to make meaningful progress towards the solution of the problem. Some examples of non-commencement follow.

a. The student leaves the question blank or has work that is meaningless.

b. The student recopies data from the problem, but does not organize it in a meaningful fashion.

c. The student makes a statement unrelated to the mathematical solution in the problem such as “I have no idea.”

d. The student copies a formula which does not apply to the problem.

e. The student assigns a variable to a quantity that is already known.

f. The student writes an equation, but none of the terms of the equation correctly match those of the correct equation.

**Score of 1: Approach**

The student shows some attempt at approaching the problem, but reaches an early impasse. For example:

a. The student correctly assigns a variable to an unknown quantity and then stops.

b. The student correctly identifies a formula necessary to solve the problem, but fails to implement it correctly.
c. The student is unable to use a variable or an equation, but shows by trial and error she or he has some understanding of what is involved to solve the problem without successfully solving the problem.

d. The student has a picture drawn that conveys some aspect of the problem, but errors exist in its accuracy.

e. The student has a table set up with some accurate data in place, but not all table entries are accurate.

f. The student writes an equation, but only one of the terms of the equation correctly match those of the correct equation.

Score of 2: Substance

Sufficient detail is shown that demonstrates procedure toward a rational solution, but major conceptual errors exist. For example,

a. The student has a completed picture which accurately represents the constraints of the problem but fails to successfully continue further.

b. The student has accurately completed a data table but fails to successfully continue further.

c. The student writes an equation, but only two of the terms of the equation correctly match those of the correct equation.

d. The student has successfully solved the problem using numerical trial and error methods, but has shown no use of a variable or equation.

e. The student has correctly written an equation to solve the problem, but major errors exist in their calculations such as $(x + 2)^2 = x^2 + 4$.

Score 3: Result

The problem is very nearly solved, but minor errors produce an invalid solution.

The types of errors here can be considered computational, not conceptual.
a. The student has written the correct equation, but has either stopped or made a computational error.

b. The student has written an equation that would have been correct, except she or he used data other than those given in the problem. The student then goes on to correctly solve this flawed equation.

c. The student solves the proper equation correctly, but fails to complete the solution problem (e.g., Finds one of the unknowns, but fails to do the necessary computation to find the second unknown.)

Score of 4: Completion

The student uses an appropriate equation and produces the valid problem solving process and solution. If the student fails to label the answer with correct units a score of 4 is still given.
APPENDIX D

TREISMAN MATHEMATICAL TASKS
Online Worksheet #1 – Due 2/13/04 at 5pm

Directions: You and your group are responsible for completing this assignment by the due date. You must post your first response to your discussion session by Monday at 5pm. To receive full credit you must post at least two additional responses to your group such as answering questions and/or explaining how you solved a problem. A final solution key must be turned in by the due date by email.

1. A city recreation department plans to build a rectangular playground 810 square meters in area. The playground is to be surrounded by a fence, which is 3 meters longer than twice the width.

   d. Express the length of the fence in terms of its width.
   e. Express the width of the fence in terms of its length
   f. Find the dimensions of the fence.

2. You owe $18,500 in two separate student loan accounts. Since you are still in school your loans will be deferred so you don’t have to pay any money for the entire year, but interest will still accrue. One of your loans has an annual interest rate of 6.5% and the other has an annual interest rate of 12%. After one year $1,599.88 of interest has accrued on your loans. How much money did you owe in each of the two student loan accounts?

3. The price of a dress is reduced by 40%. When the dress still does not sell, it is reduced by 40% of the reduced price. If the price of the dress after both reductions is $72, what was the original price?

4. Solve the following quadratic equation:  
   \((m + 3)^2 = (2m - 1)^2\)
Online Worksheet #2 – Due 2/20/04 by 5 pm

**Directions:** You and your group are responsible for completing this assignment by the due date. You must post your first response to your discussion session by Monday at 5 pm. To receive full credit you must post at least two additional responses to your group such as answering questions and/or explaining how you solved a problem. A final solution key must be turned in by the due date by email.

1. Solve the following equations:
   a. \(2x^3 + 6x^2 = x + 3\)  
   b. \((3m - 1)^{3/5} = 1/8\)

2. The perimeter of a rectangle is 80 inches and its area is 391 inches squared. Find the length and width of this rectangle.

3. A sign in the shape of a right triangle has one leg that is 7 inches longer than the other leg. What is the length of the shorter leg if the perimeter is 30 inches?
Online Worksheet #3 – Due 3/8/04 at 5 pm

Directions: You and your group are responsible for completing this assignment by the due date. You must post your first response to your discussion session by Wednesday at 5pm. To receive full credit you must post at least two additional responses to your group such as answering questions and/or explaining how you solved a problem. A final solution key must be turned in by the due date by email.

1. The snow depth in Michigan’s Isle Royal National Park varies throughout the winter. In a typical winter, the snow depth in inches can be approximated by the following function:

   \[
   f(x) = \begin{cases} 
   6.5x & \text{if } 0 \leq x \leq 4 \\
   -5.5x + 48 & \text{if } 4 < x \leq 6 \\
   -30x + 195 & \text{if } 6 < x \leq 6.5
   \end{cases}
   \]

   Let x represent the time in months with x = 0 representing the beginning of October.

   a. Determine f(1) and explain what this means in terms of the graph.
   b. On which interval is the snow depth increasing? On which interval is the snow depth decreasing?
   c. In what month is the snow the deepest? What is the deepest snow depth?
   d. In what months does the snow begin and end?

2. Determine if the following functions are even, odd, or neither. Explain your reasoning.

   a. \( f(x) = x^2 + 4|x| \)
   b. \( g(x) = x^3 + 1 \)

3. Find the value of \( k \) so that the distance between (3, -2) and (5, \( k \)) is 6.

4. If \( g(x) = 2x - x^2 \), find \( g(-1) \) and \( g(x + 2) \).
5. The graph of \( f(x) \) is given at the right. Please answer a, b, and c with interval notation.

a. What is the domain of \( f(x) \)?
b. What is the range of \( f(x) \)?
c. Over what interval(s) of \( x \) is the functions increasing?
d. Determine the ordered pair(s) that represent the relative maximum.
e. If \( f(x) = 4 \), then \( x = \) ________
f. Does \( f(x) \) have an inverse? Justify your answer.
Online Worksheet #4 – Due 4/5/04 at 5 pm

Directions: You and your group are responsible for completing this assignment by the due date. You must post your first response to your discussion session by Wednesday at 5pm. To receive full credit you must post at least two additional responses to your group such as answering questions and/or explaining how you solved a problem. A final solution key must be turned in by the due date by email.

1. Given  \( f(x) = \frac{3x - 2}{x + 1} \) find \( f^{-1}(x) \).

2. Determine the quadratic function which has a vertex at (-2, 3) and passes through the point (-5, -6).

3. A projectile is fired from a cliff 200 feet above the water at an inclination of 45 degrees to the horizontal, with a muzzle velocity of 50 feet per second. The height of the projectile above the water is given by:

   \[
   h(x) = \frac{-32x^2}{50^2} + x + 200
   \]

   where \( x \) is the horizontal distance of the projectile from the base of the cliff.

   a. Find the maximum height above the water reached by the projectile.

   b. How far from the base of the cliff will the projectile strike the water?

4. Find the length and width of a rectangle with a perimeter of 80 that has the maximum possible area.
APPENDIX E

THE COURSE SYLLABUS
<table>
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<tr>
<th>MONDAY</th>
<th>TUES.</th>
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## CONTROL GROUP

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APPENDIX F

ONLINE PARTICIPATION RUBRIC


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<th>Criteria</th>
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<td><strong>Initial Solutions &amp; Explanations</strong></td>
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<tr>
<td>Students will submit solutions to each problem assigned. Students will also be asked to provide an explanation on how they solved each problem. This explanation must be thorough enough for the group members to understand.</td>
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<tr>
<td>I submitted initial solutions and explanations to all problems. My explanations were complete and on time so that everyone could understand my thinking process.</td>
<td>I submitted all the initial solutions, but did not give all the explanations. Or I submitted all the solutions, but gave explanations such as I followed the example in class. I did not complete the assignment. I provided my group with some of my initial solutions, but gave no explanations or the explanations were incomplete. No Response submitted</td>
</tr>
<tr>
<td>My responses thoroughly answered all aspects of the discussion and went beyond what was required by bringing original information to the discussion or by further asking questions. All responses were on time and I submitted three responses to my group.</td>
<td></td>
</tr>
<tr>
<td>My responses were good and addressed all aspects of the discussion. All responses were on time and I submitted three responses to my group.</td>
<td>My responses were brief. My typical response was &quot;I agree&quot; or &quot;I don't follow what you are saying.&quot; Or I responded less that three times.</td>
</tr>
<tr>
<td>Each group MUST submit the solutions to all the problems.</td>
<td>All solutions were turned into the instructor before the due date. Only some of the solutions were submitted or solutions were submitted after due date.</td>
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