

“MATHEMATICAL MODELING AND PERFORMANCE EVALUATION OF  
SOLITON-BASED AND NON-SOLITON ALL-OPTICAL  
WDM SYSTEMS”

by

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## ABSTRACT

This thesis presents a performance evaluation of soliton and non-soliton based all-optical Wavelength Division Multiplexed (WDM) networks assuming the existing infrastructure (e.g., fiber and other physical layer components). The performance evaluation is carried out by a conveniently defined Quality (Q) factor, which is a measure of the signal to noise ratio, and indirectly, the bit error rate (BER) of the system. A solution to the Nonlinear Schrödinger Equation (NLSE), describing the propagation of light inside a fiber with linear and nonlinear impairments is found mathematically from which the Q factor is calculated for both solitons and non-solitons. We also compare the accuracy of the Regular Perturbation (RP) based method for analytically calculating the Q factor, with that of the standard Split Step Fourier (SSF) method. Results show that the RP based method gives more accurate results than the widely used SSF method for reasonably low power levels. Results also show that the soliton based systems perform much better than the non-soliton systems for typical system parameters in the existing infrastructure for bit rates of up to 10Gbps per channel. In this thesis, we present a sample WDM based optical network with mesh topology and show that the end-to-end Q factor of a soliton system is higher than that of non-soliton systems.

## CHAPTER 1

## INTRODUCTION

With the growing popularity of the all-optical networks for realizing high speed, low error communications, several techniques and standards have been proposed, aimed at improving the capacity of the all-optical networks. The wavelength division multiplexing (WDM) technique provides a solution to the ever-increasing demand for bandwidth and is fast replacing existing systems in providing high bit rates[2] of over 40Gbps at very low Bit Error Rates (BER's) of about  $10^{-9}$ .

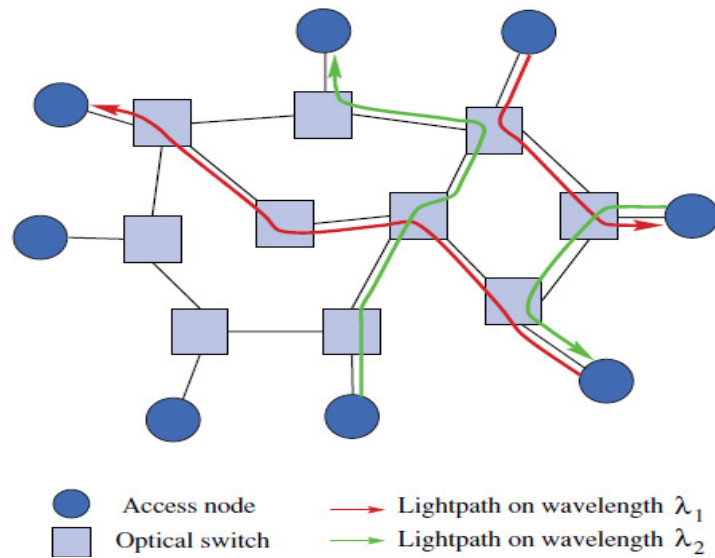


Figure 1.1 An all-optical WDM network with light path connections[2]

There are several routing and wavelength assignment (RWA) issues involved in such networks. In order to analyze and evaluate various RWA algorithms, it is desired to have a compact mathematical model for evaluating the Quality factor (Q factor) which reflects the performance of the WDM network. The performance of the all-optical WDM

systems is limited by several physical layer impairments arising out of the quantum nature of light. These physical layer impairments include both linear and nonlinear effects and play a vital role in limiting the total capacity of the system.

In conventional fiber systems, the non-return to zero (NRZ) transmission format is widely used[5] and is preferred over the return-to-zero (RZ) format because of its more efficient use of the electronic bandwidth and timing jitter tolerance. The performance of both NRZ and RZ formats is hugely limited by the nonlinear impairments that become prominent in multi-channel systems. However, nonlinear effects are not always detrimental to the performance of an optical system. For certain special pulse profiles, some nonlinear effects actually counteract the effects of the linear impairments and balance the signal distortion caused by them to maintain the shape of the pulse propagating in the fiber. Thus, the issue of pulse shape is central to the designing of any all-optical communication system. These special pulses, known as solitons, have the ability to propagate without distortion to infinite distances or retain their shape periodically in ideal loss-less fibers. The solitons which maintain their shape without distortion are called the fundamental solitons while those which retain their shape periodically are termed as higher order solitons and are dealt with in detail in chapter 3. Studies show that the performance of a system improves significantly when solitons are used for fiber links with bit rates up to 20Gbps [1].

However most of the papers on solitons reported the performance of systems using solitons for long-distance point-to-point links and we believe that the use of solitons for dynamically configured WDM networks has not been sufficiently addressed.

Hence, we examine the feasibility of improving the system performance of a mesh topology WDM all-optical network using solitons while focusing on the performance evaluation of point-to-point links as well.

The goal of our present research is to develop a mathematical model for evaluating the performance of the WDM all-optical networks by calculating the end to end Q factors in the network for both soliton based and non-soliton pulse shapes. We have developed theoretical models for calculating Q factors for systems using both soliton and non-soliton pulses and verified them with simulations run using PHOTOSS software. We have further calculated the end-to-end quality factors for some sample networks and compared the performance of soliton and non-soliton based systems in a WDM all-optical network for different bit rates and system parameters.

### Background of the Research Topic

The performance of any optical system is limited by the impairments induced on the light propagating in the physical layer. These impairments are broadly classified into two types. They are:

- Linear impairment effects and
- Nonlinear impairment effects

To understand the origin of these effects, we need to look at the Maxwell electromagnetic equations that govern the propagation of light in a fiber. The basic Maxwell equation set for a fiber free of charges is

$$\nabla \times E = - \frac{\partial B}{\partial t} \quad (1.1.1)$$

$$\nabla \times H = \frac{\partial D}{\partial t} \quad (1.1.2)$$

$$\nabla \times D = 0 \quad (1.1.3)$$

$$\nabla \times B = 0 \quad (1.1.4)$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic field vectors respectively,  $\mathbf{D}$  and  $\mathbf{B}$  the electric and magnetic flux densities and  $t$ , the time parameter.

For any dielectric the flux densities are given by

$$D = \epsilon_0 E + P \quad (1.1.5)$$

$$B = \mu_0 H + M \quad (1.1.6)$$

where  $\epsilon_0$  and  $\mu_0$  are the vacuum permittivity and permeability respectively,  $\mathbf{P}$  and  $\mathbf{M}$  are the induced electric and magnetic polarizations respectively. Since an optical fiber is a non-magnetic material,  $\mathbf{M} = 0$  whereas the polarization vector  $\mathbf{P}$  is related to the electric field vector  $\mathbf{E}$  in the fiber.

### Linear Impairment Effects

At relatively low intensities of light, the polarization of a dielectric (fiber in this case) is linearly related to the magnitude of the electric field vector  $\mathbf{E}$  and is given by the following equation

$$P(r,t) = \epsilon_0 \int_{-\infty}^{\infty} \chi(r,t-t') E(r,t') dt' \quad (1.1.7)$$

where  $\chi(r,t-t')$  is the linear susceptibility, which is a second rank tensor

The impairments that result when the above substitution is made in equation (1.1.5) of the Maxwell equations are termed as linear impairments. The major linear

impairment effect that is of interest in high speed optical communications is the group velocity dispersion (GVD).

Group Velocity Dispersion in optical fibers:

Since any optical signal has a finite spectral width in practice, different frequency components of the optical pulse travel at different velocities, resulting in pulse broadening at the end of the fiber span. This is called group velocity dispersion or chromatic dispersion. Mathematically, the group velocity of a pulse  $V_g$  is given by

$$V_g = \left( \frac{d\beta}{d\omega} \right)^{-1} \quad (1.1.8)$$

where ' $\omega$ ' is the angular frequency and ' $\beta$ ' is the propagation constant given by

$$\beta = \frac{n\omega}{c} \quad (1.1.9)$$

where  $c$  is the velocity of light in vacuum

Thus

$$V_g = \left[ \frac{d \left( \frac{n\omega}{c} \right)}{d\omega} \right]^{-1} = \left( \frac{1}{c} \right) \left[ \frac{d(n\omega)}{d\omega} \right]^{-1} \quad (1.1.10)$$

The GVD has two origins:

- a. Material dispersion, which is the result of the frequency dependence of the fiber index  $n$  and
- b. Waveguide dispersion which arises due to the irregular distribution of light within the fiber for different frequencies.

Hence the total GVD,  $D$  can be written as

$$D = D_M + D_W \quad (1.1.11)$$

where  $D_M$  and  $D_W$  are given by

$$D_M = \left( \frac{-2\pi}{\lambda^2} \right) \frac{dn_{2g}}{d\omega} = \left( \frac{1}{c} \right) \frac{dn_{2g}}{d\lambda} \quad (1.1.12)$$

$$D_M = \left( (-2\pi) \frac{\Delta}{\lambda^2} \right) \left[ \left( \frac{n_{2g}^2}{n_2 \omega} \right) V \frac{d^2 V_b}{dV^2} + \frac{dn_{2g}}{d\omega} \frac{dV_b}{dV} \right] \quad (1.1.13)$$

where  $n_{2g}$  is the group index of the cladding material given by

$$n_{2g} = n_2 + \omega \left( \frac{dn_2}{d\omega} \right) \quad (1.1.14)$$

The material dispersion  $D_M$  is the major component of chromatic dispersion. Taking into account the chromatic dispersion, the Maxwell equations are solved to get the following propagation equation for a pulse traveling in a fiber[1]:

$$\frac{\partial A(z,t)}{\partial z} + \beta_1 \frac{\partial A(z,t)}{\partial t} + \frac{j\beta_2}{2} \frac{\partial^2 A(z,t)}{\partial t^2} - \left( \frac{\beta_3}{6} \right) \frac{\partial^3 A(z,t)}{\partial t^3} = 0 \quad (1.1.15)$$

Here, 'z' is the distance traveled by the pulse along the fiber, 't' is the time,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  are the first, second and the third derivatives of ' $\beta$ ' at  $\omega = \omega_0$  (center frequency), and  $A(z,t)$  is the slowly varying amplitude of the pulse envelope given by

$$A(z,t) = \frac{B(z,t)}{e^{j(\beta z - \omega t)}} \quad (1.1.16)$$

where  $B(z,t)$  is the pulse envelope. Now in a reference frame that travels with the group velocity of the pulse, the equation (1.1.15) changes to

$$\frac{\partial A(z,t')}{\partial z} + \frac{j\beta_2}{2} \frac{\partial^2 A(z,t')}{\partial t'^2} - \left( \frac{\beta_3}{6} \right) \frac{\partial^3 A(z,t')}{\partial t'^3} = 0 \quad (1.1.17)$$

where  $t' = t - \beta_1 z$  is the transformed time.



### Nonlinear Impairment Effects

For fairly intense electromagnetic fields, the response of the silica dielectric of an optical fiber becomes nonlinear. The polarization vector in this case will have a nonlinear component in addition to the linear component

$$\mathbf{P} = \mathbf{P}_L + \mathbf{P}_{NL} \quad (1.1.18)$$

where  $\mathbf{P}_L$  is the linear component and  $\mathbf{P}_{NL}$  is the nonlinear component of the polarization vector. The nonlinear component  $\mathbf{P}_{NL}$  is given by

$$\mathbf{P}_{NL} = \epsilon_0 (\chi^2 : \mathbf{E}\mathbf{E} + \chi^3 : \mathbf{E}\mathbf{E}\mathbf{E} + \dots) \quad (1.1.19)$$

where the  $j^{\text{th}}$  order susceptibility  $\chi^{(j)}$  is, in general, a tensor of order  $(j+1)$ .

The second order susceptibility  $\chi^{(2)}$  has non-zero components only for media which do not have inversion symmetry. Since an optical fiber shows symmetry, the second order nonlinearity vanishes in fibers. Thus the lowest order nonlinear term in the polarization vector is due to the third order susceptibility tensor  $\chi^{(3)}$ . Since the higher order susceptibility terms have negligible effect on the polarization term, they can be ignored for our present study. The nonlinear polarization vector then becomes

$$\mathbf{P}_{NL} = \epsilon_0 \chi^3 : \mathbf{E}\mathbf{E}\mathbf{E} \quad (1.1.20)$$

When the above equation is substituted in the Maxwell's equations, it gives rise to several nonlinear effects, the significant ones being cross phase modulation (XPM), self phase modulation (SPM), four wave mixing (FWM) and other less-significant effects like the Brillouin scattering and Raman scattering effects.

Self phase modulation (SPM):

The cubic dependence of the nonlinear polarization vector on the electric field in nonlinear fibers results in the linear power dependence of the refractive index of the fiber[1]

$$n' = n + n_2 \left( \frac{P}{A_{eff}} \right) \quad (1.1.21)$$

where  $n'$  is the modified refractive index taking into account the nonlinear effects,  $n$  is the linear refractive index of the fiber and  $n_2$  is the nonlinear index coefficient,  $P$  is the optical power and  $A_{eff}$  is the effective area of the fiber core.

Since the propagation constant of a fiber depends on this refractive index, it becomes power dependent as well. The propagation constant  $\beta$  is given by[1]

$$\beta' = \beta + k_o n_2 \frac{P}{A_{eff}} \quad (1.1.22)$$

where  $k_o$  is the wave number, given by  $k_o = 2\pi/\lambda$

The factor “ $k_o n_2 / A_{eff}$ ” is called the nonlinear coefficient and is denoted by ‘ $\gamma$ ’. The nonlinear parameter  $\gamma$  has values typically ranging from  $1W^{-1}/Km$  to  $51W^{-1}/Km$ . Due to this power dependence of the propagation constant, the optical phase changes along the fiber as well. This optical phase change  $\Phi_{NL}$  can be found from

$$\Phi_{NL} = \int_0^L (\beta' - \beta) dz \quad (1.1.23)$$

This phase shift which is induced by the optical pulse on itself is termed self phase modulation.

Cross phase modulation (XPM):

In a WDM system, when more than one channel propagates through the fiber, the intensity dependence of the refractive index in one channel modulates the phase of the signal in the neighboring channels as well. This nonlinear phenomenon that occurs due to the interaction of different co-propagating channels in a WDM system is called the cross phase modulation (XPM) effect. The XPM effect is a dominant effect in WDM systems with higher intensities of optical input and closely spaced channels. When we substitute equation (1.1.20) in the Maxwell equations and solve them, we get an equation describing the propagation of the light pulse in a loss-less fiber with nonlinearities[3] given by

$$\frac{\partial A_m}{\partial z} + \left( j \frac{\beta_2}{2} \right) \frac{\partial^2 A_m}{\partial t^2} = j\gamma |A_m|^2 A_m + 2j\gamma \sum_{n=1}^m |A_n|^2 A_m \quad (1.1.24)$$

The summation of the last term extends over the total number of channels.

The first term in the right hand side of equation (1.1.24) accounts for the SPM term whereas the second term includes the XPM term. The factor of 2 for the XPM term indicates that the XPM term effects are twice as large as the SPM effects. Here we have neglected the third order dispersion term and assumed that the fiber is loss-less. Equation (1.1.24) is known as the nonlinear Schrödinger equation (NLSE) and it describes the propagation of light in a nonlinear fiber.

Four Wave Mixing (FWM):

Apart from the phase modulation effects, the third order susceptibility tensor  $\chi^{(3)}$  also gives rise to another nonlinearity called four wave mixing (FWM)[4]. In a WDM system, when three channels with frequencies  $f_1$ ,  $f_2$  and  $f_3$  co-propagate, the third order polarization term gives rise to a fourth channel with frequency  $f_4$ , given by

$$f_4 = \pm f_1 \pm f_2 \pm f_3 \quad (1.1.25)$$

Among the several frequencies that are possible corresponding to the different combinations of plus and minus signs in the above equation, only a few frequencies appear in practice in a nonlinear fiber. This is due to the phase-matching condition that is required for the newly formed frequency to be sustained. The phase mismatch  $\Delta$  is related to the dispersion coefficient  $\beta$  as

$$\Delta = \beta_2 f_{ch}^2 K \quad (1.1.26)$$

where K is a proportionality constant.

Hence the phase completely matches when  $\beta_2$  is close to zero. The FWM effect can thus be seen to be dominant in fibers with a very low dispersion constant or systems with closely spaced channels. To avoid this FWM effect to a large extent in the nonlinear fibers, modern systems use a technique called dispersion management where each fiber section has high local dispersion of about 5ps/Km-nm while maintaining a low overall dispersion coefficient[1].

### Fiber Losses

Fiber losses play an important role in limiting the performance of a communication system. Long haul fiber systems became practical only after the advent of the Erbium doped fiber amplifiers (EDFAs) which compensate for the fiber losses by amplifying the optical signal at regular intervals along the path. Several factors contribute to losses in a fiber, the prominent ones being material absorption and Rayleigh scattering.

Material absorption:

Material absorption is again divided into intrinsic absorption which is caused by the absorption of the material of the fiber (usually SiO<sub>2</sub>) and extrinsic absorption corresponding to the losses due to the absorption by the impurities in the fiber. Material absorption is considerably insignificant (typically less than 0.05dB/Km) in the wavelength range of 1.3μm to 1.6 μm typically used for fiber optic communication

Rayleigh scattering:

Rayleigh scattering occurs due to the fluctuations in the refractive index value of the fiber. These fluctuations arise out of the density fluctuations of the fiber that occur due to imperfect manufacturing.

Rayleigh scattering decreases with increasing wavelength. At sufficiently high wavelengths, it can drop down to as low as 0.05dB/Km (which occurs at wavelengths > 1.8μm), but in the commonly used range of 1.3 μm-1.6 μm, it is as high as 0.16 dB/Km. Thus in a typical fiber system the losses are usually dominated by Rayleigh scattering.

Apart from the above two factors, waveguide imperfections also contribute to the fiber losses.

All these different loss parameters can be represented by a single factor called the attenuation coefficient, denoted by  $\alpha$  and given by

$$a = \left( -\frac{1}{P} \right) \frac{dP}{dz} \quad (1.1.27)$$

where P is the power of the optical signal.

The output power  $P_{out}$  is thus given by

$$P_{out} = P_{in} e^{-aL} \quad (1.1.28)$$

where  $L$  is the length of the fiber and  $P_{in}$  is the input optical power launched into the fiber.

$\alpha$  is usually expressed in terms of dB/Km and is calculated using the equation

$$a \left( \frac{dB}{km} \right) = \left( \frac{10}{L} \right) \log_{10} \left( \frac{P_{out}}{P_{in}} \right) \quad (1.1.29)$$

The variation of  $\alpha$  with the wavelength of the signal is shown below in the loss-spectrum of a typical single-mode fiber (SMF) for a wavelength range that extends well across the commonly used range of  $1.3\mu\text{m}$  to  $1.6\mu\text{m}$

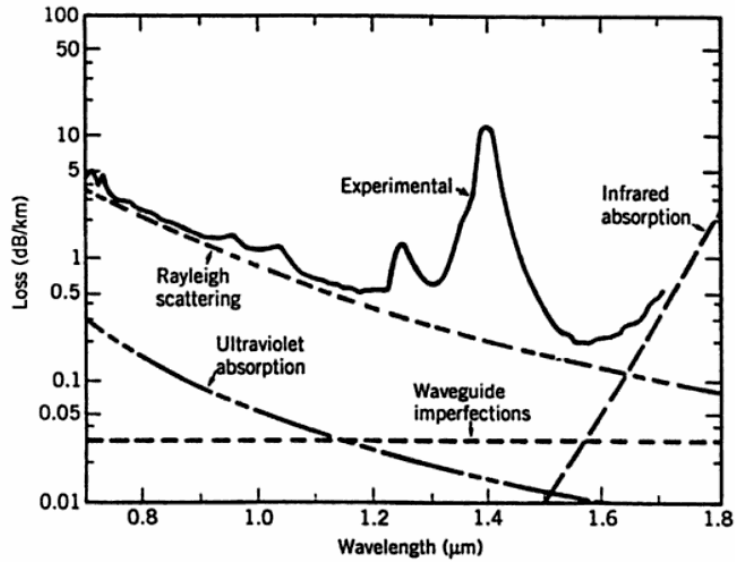


Figure 1.2 Loss spectrum of a single-mode fiber produced in 1979[1a]

When we take into account the fiber attenuation, the NLSE will have an extra term that corresponds to the loss  $\alpha$ . Thus the NLSE for a lossy nonlinear fiber is given by

$$\frac{\partial A_m}{\partial z} + \left( j \frac{\beta_2}{2} \right) \frac{\partial^2 A_m}{\partial t^2} = - \left( \frac{a}{2} \right) A + A j \gamma |A_m|^2 A_m + 2 j \gamma \sum_{n=1}^m |A_n|^2 A_m \quad (1.1.30)$$

Quality Factor (Q factor)

The performance of a fiber optic system is evaluated in terms of a conveniently defined Q factor that reflects the BER of the system. This can be calculated by solving the NLSE given by equation (1.1.30) above.

The quality factor is defined in general terms as[1f]

$$Q = \frac{I_1 - I_o}{d_1 - d_o} \quad (1.2.1)$$

where  $I_1$  and  $I_2$  are the mean currents for bit 1 and bit 0 respectively, and  $\delta_1$  and  $\delta_o$  are the standard deviations for the bits 1 and 0 respectively. Since we denote a '0' bit with no signal in the bit slot, we can safely assume that

$$I_o = 0 \text{ and } d_o = 0 \quad (1.2.2)$$

And if we assume that the responsivity of the detector  $R = 1AW^{-1}$ , then

$$I_1 = RP = P \quad (1.2.3)$$

Hence the quality factor reduces to

$$Q = P / \delta_1 \quad (1.2.4)$$

It is customary to express the quality factor 'Q' in terms of dB.

$$Q_{dB} = 10 \log_{10} \left( \frac{P}{d_1} \right) \quad (1.2.5)$$

The Q factor definition may vary slightly depending on the type of system under consideration. For instance, while calculating the Q factor of a soliton based system, we calculate the standard deviation of the position fluctuations of the pulse in the bit slot whereas for a non-soliton system, we calculate the standard deviation of the amplitude fluctuations of the pulse envelope. The need to define a Q factor to evaluate the

performance of an optical system arises out of the fact that the BERs involved here are less than  $10^{-9}$  and an accurate measurement of the BER through simulations requires more than  $10^{10}$  bits, which is impractical. Hence the system performance is evaluated by the calculation of the Q factor.

The relationship between the Q factor and the BER of the system varies with the modulation format and the detection scheme used at the receiver. For instance, for a Gaussian decision variable at the receiver, the Q factor is related to the BER as[6]

$$Q = \sqrt{2} \operatorname{erfc}^{-1}(2BER) \quad (1.2.6)$$

Different impairments introduced above have different impacts on the pulse propagating in the fiber. For instance, the group velocity dispersion (GVD) changes the width of the pulse while the phase modulation effects (SPM and XPM) introduce nonlinear noise to the pulse and hence induce amplitude fluctuations to it, and four wave mixing (FWM) creates a new frequency component that satisfies the phase-matching condition. Hence the challenge here is to quantify all these effects and to account for all these effects while calculating the Q factor. Before analyzing the performance of the soliton-based system, we should first study the performance of non-soliton systems, which forms the topic of our next chapter.



## CHAPTER 2

## SOLVING THE NLSE

As discussed in the last section of the previous chapter, a solution to the NLSE (1.1.30) gives us a way to calculate the quality factor of an optical system. In this chapter, we will examine the commonly used methods to solve the NLSE[7] and then introduce the technique that we use in our study.

The NLSE falls under the category of a more general class of nonlinear differential equations given by

$$j \frac{\partial A}{\partial z} + \left( \sigma \frac{d}{2} \right) \frac{\partial^2 A}{\partial t^2} + A \times A^2 = G \quad (2.1)$$

A solution to this generalized equation has been extensively studied and reported in various papers[8][9]. The term  $G$  on the right side is called the perturbation term. In the case of fiber optics, this term depends on the physical properties of the fiber, such as the attenuation  $\alpha$  and the nonlinear term  $\gamma$ . Equation (2.1) becomes integrable only for the special case when  $G = 0$  which pertains to loss-less fibers. In this case, it is possible to get a complete solution of the NLSE by analytical techniques. Since the attenuation in a fiber has a considerable effect on the propagation of light through the fiber, it is useful to solve equation (2.1) for a more general case when  $G \neq 0$ . In such a case, the NLSE cannot be solved analytically and requires some numerical approximations to get to the solution. Many such numerical techniques have been employed to achieve a close approximation to the solution. The most widely used approximation is the split-step Fourier (SSF) method[7].

SSF Method

The basic idea of the SSF method is to solve the NLSE by considering only one nonlinear effect at a time. This can be done by dividing the fiber span into a large number of segments with small size and solving the NLSE with only the linear term for each of the small segments. The nonlinear term is then added at the end of each segment, thus giving a close numerical approximation to the true shape of the pulse.

Suppose the fiber span is divided into several small steps of size  $x$ , the NLSE can be re-written for each step as

$$\frac{\partial A(z,t)}{\partial z} = (L + N) A(z,t) \quad (2.1.1)$$

where  $L$  is the linear part and  $N$  is the nonlinear part

With this split step approximation, after the pulse  $A(z,t)$  traverses the length  $x$  through the fiber, it is given by

$$A(z+x,t) = e^{xL} e^{xN} A(z,t) \quad (2.1.2)$$

The step-size  $x$  is selected depending upon the accuracy required and the computational complexity in the numerical calculations.

The implementation of the SSF method may get quite complicated when all the nonlinear effects are considered together. Since several frequencies co-propagate in a single fiber in a WDM system, for a closely spaced set of frequencies, a large number of fourier terms is required to accurately represent the signal numerically in each channel. Moreover, if the phase matching condition for the FWM effect is satisfied, new frequencies are introduced to the existing set which increases the bandwidth and thereby

the computational complexity of the system. In addition to this, a technique called dispersion management is sometimes used to suppress the nonlinear effects to some extent, by varying the dispersion constant along the fiber continuously according to the attenuation in the signal[11]. Thus, a very small step size is required to account for the dynamically varying dispersion effect. All these factors contribute to the computational complexity in the realization of the SSF method and sometimes make it ineffective for practical implementation. Other methods for obtaining an approximate solution to the NLSE include linearization techniques such as perturbation methods with parametric gain, small signal analysis and the variational method[10]. All these methods, however, give acceptable results only for small signal powers well below 5dBm, which is the typical laser power per channel in a WDM system, or for very low fiber losses.

We present here a variation to the Regular Perturbation (RP) method based on the power series expansion of the pulse-envelope in  $\gamma$ . This method has been suggested and implemented by Enrico Forestieri and Marco Secondini[11], where only the SPM was considered among the nonlinear effects. Here, we extend the method to include XPM as well, which is the dominating effect for high power levels in WDM systems. We demonstrate that this method, when used for complex WDM systems with a large number of channels, not only improves the accuracy of the solution, but also reduces the computational complexity as compared to the widely used SSF method.

A Variation of the Regular Perturbation (RP) Method

We will first develop an integral representation of the NLSE with SPM and XPM terms and finally get a recurrence relation for the pulse envelope  $A(z,t)$  using the power series expansion in  $\gamma$ . We start by making the following substitution to the equation (1.1.30)

$$A(z,t) = e^{-\frac{az}{2}} u(z,t) \quad (2.2.1)$$

Equation (1.1.30) then reduces to

$$\frac{\partial u_1}{\partial z} + \left( j\beta \frac{2}{2} \right) \frac{\partial^2 u_1}{\partial t^2} = \left( j\gamma |u_1|^2 u_1 + 2j |u_2|^2 u_1 \right) e^{-az} \quad (2.2.2)$$

where  $u_1(z,t)$  corresponds to the channel under consideration and  $u_2(z,t)$  is related to the adjacent channel which has the highest XPM effect on  $u_1(z,t)$ . Note here that we considered the XPM effect of only one other channel with pulse envelope corresponding to  $u_2$ . The results can then be extended to the multi-channel case just by adding more XPM terms. Now let us make the following substitutions

$$f_1(z,t) = e^{-az} |u_1(z,t)|^2 u_1(z,t) \quad (2.2.3)$$

$$f_2(z,t) = e^{-az} |u_2(z,t)|^2 u_1(z,t) \quad (2.2.4)$$

After taking the Fourier transform of equation (2.2.2) and substituting equations (2.2.3) and (2.2.4) in it, we have

$$\frac{\partial U_1(z,\omega)}{\partial z} - \left( j\beta_2 \frac{\omega^2}{2} \right) U_1(z,\omega) = j\gamma F_1(z,\omega) + 2j F_2(z,\omega) \quad (2.2.5)$$

where  $U_1(z,\omega)$  is the Fourier transform of  $u_1(z,t)$  and  $F_1(z,\omega)$ ,  $F_2(z,\omega)$  are the Fourier transforms of  $f_1(z,t)$  and  $f_2(z,t)$  respectively. Again, substituting

$$U_1(z, \omega) = -\exp\left(-j\beta_2 \omega^2 \frac{z}{2}\right) V_1(z, \omega) \quad (2.2.6)$$

into equation (2.2.5), we get

$$\frac{\partial V_1(z, \omega)}{\partial z} = \left(jF_1(z, \omega) + 2j\gamma F_2(z, \omega)\right) \exp\left(j\beta_2 \omega^2 \frac{z}{2}\right) \quad (2.2.7)$$

which, after integrating from 0 to z, becomes

$$V_1(z, \omega) = V_1(0, \omega) + j\gamma \int_0^z \exp\left(j\beta_2 \omega^2 \frac{\zeta}{2}\right) \left(F_1(\zeta, \omega) + 2F_2(\zeta, \omega)\right) d\zeta \quad (2.2.8)$$

This, after substituting back the value of  $U_1(z, \omega)$  from equation (2.2.6), reduces to

$$U_1(z, \omega) = U_{1(0)}(z, \omega, \omega) + j\gamma \int_0^z \exp\left(-j\beta_2 \omega^2 \frac{z-\zeta}{2}\right) \left(F_1(\zeta, \omega) + 2F_2(\zeta, \omega)\right) d\zeta \quad (2.2.9)$$

where  $U_{1(0)}(z, \omega, \omega) = U_1(0, \omega) \exp\left(-j\beta_2 \omega^2 \frac{z}{2}\right)$  is the Fourier transform of the solution of equation (2.2.2) for  $\gamma = 0$ .

Letting  $H(z, \omega) = \exp\left(-j\beta_2 \omega^2 \frac{z}{2}\right)$  and taking the inverse Fourier transform of equation (2.2.9), we get

$$u_1(z, t) = u_{1(0)}(z, t) + j\gamma \int_0^z \left[ \left( |u_1(\zeta, t)|^2 + 2|u_2(\zeta, t)|^2 \right) u_1(\zeta, t) \right] \otimes h(z - \zeta, t) e^{-\alpha\zeta} d\zeta \quad (2.2.10)$$

where  $h(z, t)$  is the inverse Fourier transform of  $H(z, \omega)$ ,  $\otimes$  denotes the temporal convolution and  $u_{1(0)}(z, t) = u_1(0, t) \otimes h(z, t)$  is the signal at  $z$  in a linear loss-less fiber.

Now we expand the optical field complex envelope  $u_1(z, t)$  in the power series at  $\gamma$

$$u_i(z, t) = \sum \gamma^k u_{i(k)}(z, t) \quad (2.2.11)$$

where  $i = 1, 2$

Substituting this expansion into equation (2.2.10) and comparing the coefficients of equal exponents of  $\gamma$  on both sides, we finally arrive at the recursive relation given by

$$u_{1(n)} = j \int_0^{\tilde{z}} \left( \sum \sum u_{1(i)} \{ u_{1(k-i)} u_{1(n-k)}^* + 2 u_{2(k-i)} u_{2(n-k)}^* \} \right) \otimes h(z - \zeta, t) e^{-a\zeta} d\zeta \quad (2.2.12)$$

for  $n \geq 1$

For example, the first two  $u_{1(n)}$ 's are given below:

$$u_{1(1)} = j \int_0^{\tilde{z}} \left( |u_{1(0)}|^2 u_{1(0)} + |2 u_{2(0)}|^2 u_{1(0)} \right) \otimes h(z - \zeta, t) e^{-a\zeta} d\zeta \quad (2.2.13)$$

and

$$u_{1(2)} = j \int_0^{\tilde{z}} \left( 2|u_{1(0)}|^2 u_{1(1)} + |u_{1(0)}|^2 u_{1(1)}^* + 4|u_{2(0)}|^2 u_{1(1)} + 2 u_{1(0)} u_{2(0)} u_{2(1)}^* \right) \otimes h(z - \zeta, t) e^{-a\zeta} d\zeta \quad (2.2.14)$$

Hence, the final  $u_1$  is given by

$$u_1 = u_{1(0)} + u_{1(1)} \gamma + u_{1(2)} \gamma^2 + u_{1(3)} \gamma^3 + \dots \quad (2.2.15)$$

### Comparison With the SSF Method

With the RP method, we can achieve the desired level of accuracy just by adding more terms in the power series expansion whereas in the SSF method, for achieving higher accuracy, the step size needs to be decreased which increases the number of iterations in the numerical evaluation.

The RP method is computationally more efficient than the SSF method only if the required number of terms in the power series expansion is below 3. As the number of

terms increases, the computational difficulty associated with the RP method increases rapidly and exceeds that of the SSF method.

However, the main advantage of our method is that in most of the cases, the desired accuracy can be obtained by just considering the first two terms of the power series expansion. As long as the power level is not much higher than 10dBm, this method gives better results than the SSF method with less computational complexity as shown by the results below.

### Results

The quality factor is calculated by the RP method described above and by SSF method for different values of the GVD parameter. The values considered for the second order dispersion coefficient  $\beta_2$  range from  $-0.5\text{ps}^2/\text{km}$  to  $0.5\text{ps}^2/\text{km}$ . The results are plotted together in the same graph shown in figure (2.1) below along with the results of simulation run using PHOTOSS software[24]

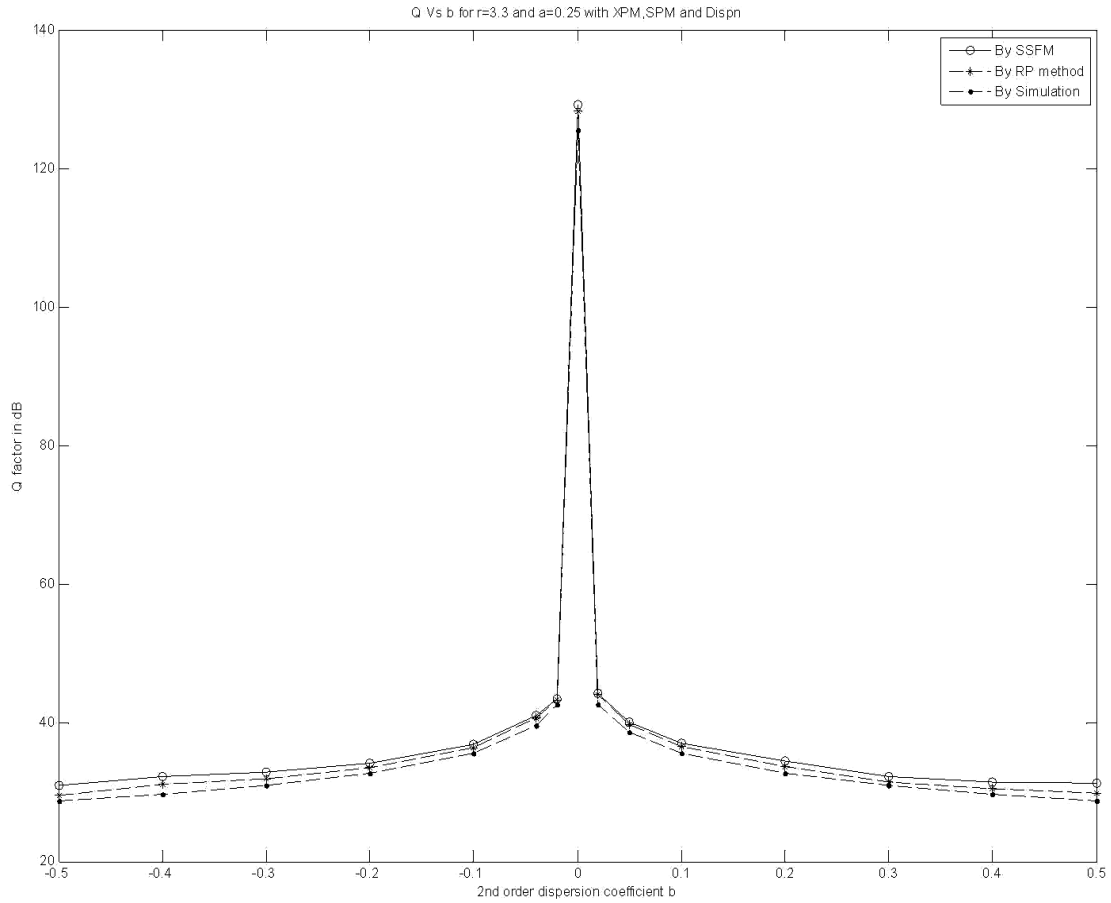


Figure 2.1 Result showing a comparison of the Q factor calculated by the RP method and by the SSF method, plotted against the second order GVD parameter

Other fiber parameters assumed here are  $\gamma = 3.3\text{W}^{-1}\text{km}^{-1}$ ,  $\alpha = 0.25\text{dB/km}$  and an input signal power of about 2mW. The figure shows an improvement of about 2dB by the RP method with that of the general SSF method at  $\beta_2 = 0.5\text{ps}^2/\text{km}$  which is typical for the fibers used in communications. Furthermore, the results show a peak at  $\beta_2 = 0$  which is apparent because the dispersion-induced pulse broadening effect is absent at  $\beta_2 = 0$ . The huge difference between the peak value of the Q factor and the rest of the values is due to the fact that GVD is the dominant effect at power levels as low as 2mW and hence the Q



factor is largely limited by the dispersion. The dependence of the Q factor on other fiber parameters is discussed along with some results in later chapters.

Thus, a complete analytical solution of the generalized NLSE is not always possible and several numerical and analytical approximations are used to solve the nonlinear equation. However, a complete and practical solution of the NLSE is possible under certain conditions that depend upon the launched pulse shape and other fiber parameters[1]. These solutions always give the same pulse shape however far the pulse may travel and this shape is the same as that of the input pulse shape. This happens under highly selective situations and has been studied extensively in recent years, since it makes the behavior of the fiber system more predictable and reliable, thus giving high performance. Systems using these kinds of pulse shapes are called *soliton* systems and is the topic of our next chapter.

## CHAPTER 3

## SOLITONS

Introduction

There is a long history associated with solitons and their origin. As far as optical systems are concerned, a soliton is a specific pulse profile, which, when launched into a nonlinear optical fiber, maintains its shape as it travels down the fiber under ideal conditions. The existence of solitons in fiber systems and their advantages were first demonstrated in 1980[12] and by the late 1990's, they have become a potential candidate for modern long-haul optical systems.

In this chapter, we examine various kinds of solitons, propagation of these solitons through different nonlinear fibers, the effects of different fiber parameters, the performance evaluation of a soliton-based fiber link and some other design issues. We will also look at the effect of the Amplified Spontaneous Emission (ASE) noise due to the EDFAs on the performance of a soliton system.

Origin of Solitons

In the August of 1834, near the union canal at the outskirts of Edinburg, a Victorian engineer observed that a wave of water, created due to the motion of a boat, traveled down the channel without losing its shape for a very long distance[12]. He called it a solitary wave of translation since it carried along water with it while maintaining its shape. Following some laboratory experiments confirming the existence of the solitary

waves in many circumstances, theoretical studies were conducted to understand the formation and behavior of these waves.

The main breakthrough in understanding the theory behind the solitary waves came in the form of a 1895 paper from Amsterdam by researchers Korteweg and de Vries [13], where they were successful in developing a nonlinear equation representing the hydraulics of the moving water, the solutions of which represent the solitary waves. This equation can be simply written as[13],

$$\frac{\partial \eta}{\partial t} + \eta \frac{\partial \eta}{\partial x} + \frac{\partial^3 \eta}{\partial x^3} = 0 \quad (3.2.1)$$

where  $\eta$  represents the change in height,  $t$  the time and  $x$ , the distance traversed.

This equation is popularly called the KdV equation and similar in structure to the NLSE that we are studying. Studies on the KdV equation have shown that the solutions of the KdV equation and thus the NLSE behave more like particles than waves. For instance, the interaction between two solitary waves is similar to that of an elastic collision between two elementary particles. This prompted Kruskal and Zabusky to first use the term “soliton” for a solitary wave [12].

A soliton pulse is described by the NLSE when the dispersive effects and the phase modulation effects balance out over a period of time, in a loss-less fiber, which occurs for a sech pulse profile. Now we will mathematically solve the NLSE for different soliton solutions.

### Solving for Fundamental Solitons

As mentioned earlier, a soliton pulse is created when the dispersive effects balance out with the SPM effects in a loss-less fiber. Hence we will start with the NLSE without the XPM and attenuation terms for deducing the fundamental solitons and then study the effect of XPM and attenuation on the soliton propagation through the fiber. The NLSE without the XPM term is given by

$$\frac{\partial A_m}{\partial z} + \left( j \frac{\beta_2}{2} \right) \frac{\partial^2 A_m}{\partial t^2} - \left( \frac{\beta_3}{6} \right) \frac{\partial^3 A_m}{\partial t^3} = - \left( \frac{\alpha}{2} \right) A + j\gamma |A_m|^2 A_m \quad (3.3.1)$$

Neglecting the third order dispersion term,  $\beta_3$  and setting  $\alpha = 0$ , we have

$$\frac{\partial A_m}{\partial z} + \left( j \frac{\beta_2}{2} \right) \frac{\partial^2 A_m}{\partial t^2} = j\gamma |A_m|^2 A_m \quad (3.3.2)$$

Now we make the following substitutions

$$\tau = \frac{t}{T_o} \quad (3.3.3)$$

$$\xi = \frac{z}{L_D} \quad (3.3.4)$$

$$\text{and } U = \frac{A}{\sqrt{P_o}} \quad (3.3.5)$$

where  $T_o$  is the pulse-width parameter and is related to the FWHM width  $T_s$  of the pulse

as  $T_o = \frac{T_s}{2 \log(1 + \sqrt{2})}$  and  $L_D = \frac{T_o^2}{|\beta_2|}$  is the dispersion length of the fiber

Equation (3.3.1) then becomes

$$j \frac{\partial U}{\partial \xi} - \left( \frac{s}{2} \right) \frac{\partial^2 U}{\partial \tau^2} + N^2 |U|^2 U = 0 \quad (3.3.6)$$

where the dimensionless parameter  $N$  is given by

$$N^2 = \gamma P_o L_D = \gamma P_o \frac{T_o^2}{|\beta_2|} \quad (3.3.7)$$

and  $s = \pm 1$ , corresponding to  $\beta_2$  being positive or negative

This leads us to the concept of bright solitons and dark solitons. Depending upon the sign of  $\beta_2$ , two kinds of soliton solutions are possible: bright and dark solitons.

The pulse-like solitons that are widely used come under the category of bright solitons. They occur when the second order dispersion  $\beta_2 < 0$ . The solutions of equation (3.3.6) exhibit a dip in the intensity at their peak values for  $\beta_2 > 0$ . These solitons are referred to as dark solitons and are seldom used in fiber-communications[1]. Hence we consider bright solitons for our study. From now on, whenever we say solitons, unless otherwise mentioned, we refer to the bright solitons.

Therefore, assuming  $\beta_2 < 0$  and substituting  $s = -1$ , equation (3.3.6) can be re-written as

$$j \frac{\partial u}{\partial \xi} + \left(\frac{1}{2}\right) \frac{\partial^2 u}{\partial \tau^2} + N^2 |u|^2 u = 0 \quad (3.3.8)$$

where  $u = NU$  is the renormalized amplitude.

The dimensionless parameter  $N$  which depends on the fiber and pulse parameters gives the order of the soliton. A fundamental soliton, corresponding to  $N = 1$  exhibits the ideal behavior of maintaining its shape under suitable conditions in loss-less fibers whereas the higher order solitons initially lose their shape while propagating, but recover it after traversing a distance  $z_o$  along the fiber, given by

$$z_o = \left(\frac{\pi}{2}\right) L_D \quad (3.3.9)$$

where  $z_0$  is called the soliton period and plays an important role in the design of a soliton-based optical system.

We will solve equation (3.3.8) only for the fundamental solitons and extend the results to higher order solitons. So let us substitute  $N=1$  in equation (3.3.8).

Now the solution to the above equation is assumed to be of the form

$$u = V(\tau) e^{j\phi(\xi)} \quad (3.3.10)$$

Substituting this in equation (3.3.8) and solving, we finally get

$$U = \text{sech}(\tau) e^{j\frac{\xi}{2}} \quad (3.3.11)$$

which gives

$$A(\xi, \tau) = \sqrt{P_o} \text{sech}(\tau) e^{j\frac{\xi}{2}} \quad (3.3.12)$$

where we have used the following initial conditions

$$V(\infty) = V(-\infty) = 0 \quad (3.3.13)$$

$$\frac{\partial V(\infty)}{\partial \tau} = \frac{\partial V(-\infty)}{\partial \tau} = 0 \quad (3.3.14)$$

$$V(0) = 1 \quad (3.3.15)$$

$$\text{and} \quad \left[ \frac{dV(\tau)}{d\tau} \right]_{\tau=0} = 0 \quad (3.3.16)$$

to arrive at the above solution

Figures (3.1a) and (3.1b) below shows the propagation of a fundamental and a second order soliton along a fiber without loss. We can easily see how the pulse profile of the second order soliton varies periodically while the fundamental soliton remains the same as it propagates through the loss-less fiber.

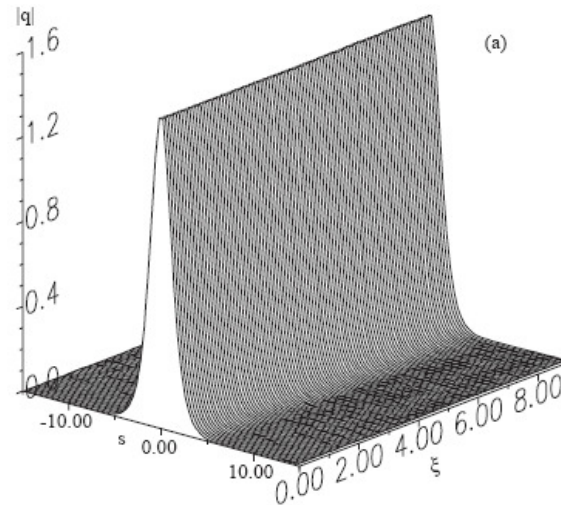


Fig 3.1a Evolution of a first order soliton in a nonlinear loss-less fiber[14]

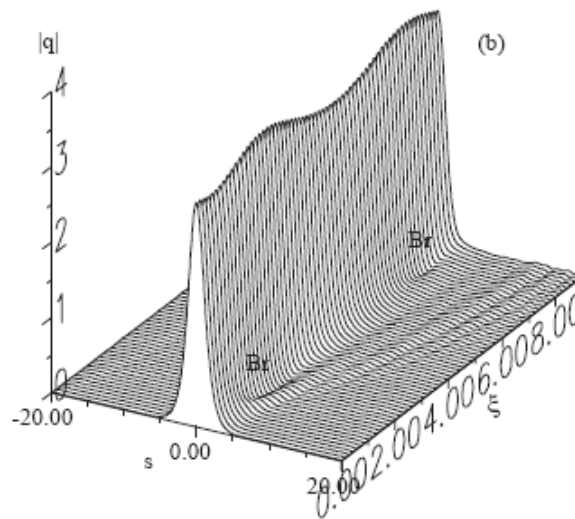


Fig 3.1b Evolution of a second order soliton in a nonlinear loss-less fiber[14]

### Design Issues

Even under ideal conditions, sustaining the shape of a soliton propagating in a fiber is not easy and involves many design issues. Assuming a nonlinear loss-less fiber

with negative second order dispersion, we have already seen that a fundamental soliton is possible when the parameter  $N$  equals 1. This is given by

$$N^2 = \gamma P_o \frac{T_o^2}{|\beta_2|} = 1 \quad (3.4.1)$$

As can be seen from the above equation, the fiber parameters like the time period of the launched soliton  $T_o$  and its input power  $P_o$  should be selected in such a way as to satisfy the above condition. However, there is one distinct advantage of solitons that helps in the designing of the system. One of the characteristics of solitons is that they are capable of adapting to a non-conductive environment to a certain extent. This means, even if the conditions required for a smooth propagation of a soliton deviate slightly, the soliton shape evolves over a period of time and eventually regains its original shape after traversing a certain distance, depending upon the extent of deviation.

Hence, even if the value of  $N^2$  oscillates between 0.5 and 1.5, a fundamental soliton can still adapt to this, and after traveling through a sufficient length of the fiber and retains its sech profile. Clearly, the distance after which a fundamental soliton regains its original shape depends on how close the value of  $N$  is to unity. This distance normally corresponds to  $\xi \gg 1$ . This stability of a soliton under small perturbations makes it practically feasible even in non-ideal conditions

A similar behavior is shown when the shape of the input pulse deviates slightly from the sech profile required to launch the soliton into the fiber. However, this is possible only if the value of  $N^2$  does not deviate much and is as close to unity as possible.

Another challenge while designing a soliton-based system is to make sure that the input pulse contains no frequency chirp. Frequency chirp is defined as the deviation of



the frequency of the light signal from its center frequency through the width of the pulse. This deviation can either be increasing, decreasing or both. A chirped sech pulse with a constant chirp is given by

$$A(0, \tau) = \text{sech}(\tau) e^{-jC \frac{\tau^2}{2}} \quad (3.4.2)$$

where  $C$  is the chirp factor.  $C$  can take both positive and negative values corresponding to up-chirp and down-chirp. For a chirp-free pulse,  $C=0$ . Commercial lasers producing ultra-short width pulses usually generate chirped pulses and can be detrimental to a soliton system. But again, if we can maintain the value of  $N^2$  to be 1, the fundamental soliton evolves after a certain distance to regain its original shape. However, there is a critical range for the chirp factor, outside which solitons cannot regain their original shape even after considerably long distances. This range is given by the condition  $|C| < C_{\text{crit}}$  where the critical value of the chirp factor  $C_{\text{crit}}$  depends on the order of the soliton. For a fundamental soliton,  $C_{\text{crit}} = 1.64$ [15]. Figure (3.2) below shows the evolution of a fundamental soliton with initial chirp

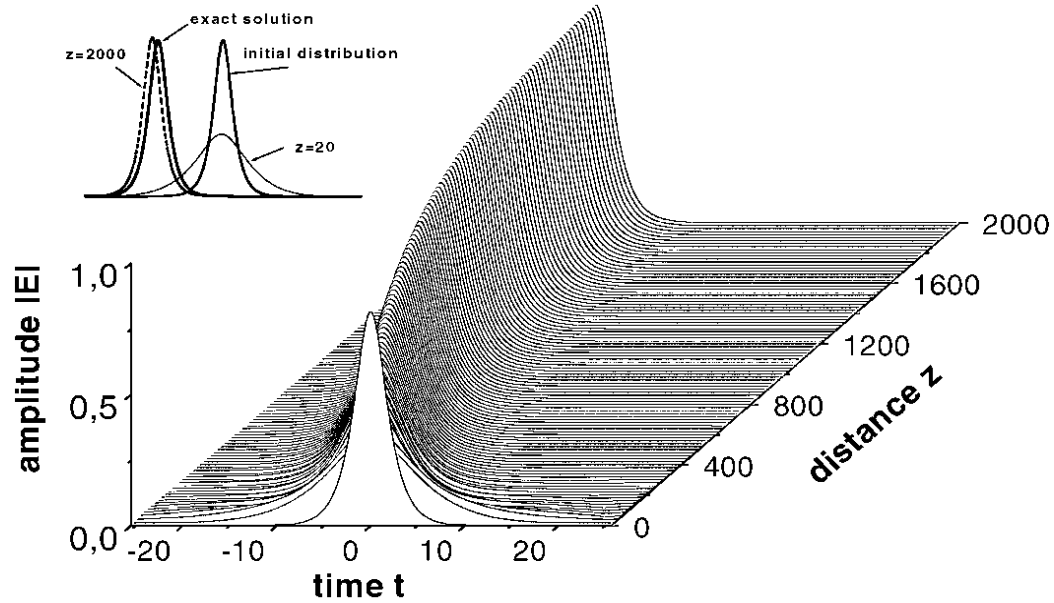


Figure 3.2 Evolution of an initially chirped soliton into a chirp-less soliton in a nonlinear fiber[20].

Apart from these effects, for the high performance of a soliton based communication system, two adjacent solitons in a single channel must be conveniently spaced in the time domain. This is because soliton pulses fluctuate in their respective bit slots while propagating down the fiber due to the timing jitter induced by various sources, which will be discussed in the later sections, and this may result in the pulse moving into the adjacent bit-slot. This can either result in an intra-channel collision if there is a pulse in the adjacent bit slot or may result in an error in reading the bit at the receiver, both of which increase the BER of the system. Hence the soliton pulses should be well-separated in time to avoid intra channel collisions. The pulse width parameter  $T_o$  is related to the bit period  $T_B$  as

$$T_B = (2q_o) T_o \quad (3.4.3)$$

where  $q_0$  is called the soliton separation parameter and is typically chosen to be about 4-5[1]. The bit rate of the system  $R_b$  thus depends on  $q_0$  as

$$R_b = \frac{1}{2q_0 T_o} \quad (3.4.4)$$

Hence, the parameter  $q_0$  puts a limit on the performance of a soliton-based communication system and plays an important role in the design of the system.

Even in a carefully designed system, solitons generally tend to interact with each other during propagation due to various reasons and these interactions can be quite detrimental to the performance of the system, and hence need to be studied in detail.

### Soliton Interaction

As mentioned earlier, solitons behave as particles especially when interacting with each other. Soliton interaction can be studied under two categories:

- Intra channel soliton interaction, and
- Inter channel soliton interaction

#### Intra Channel Interaction

Intra channel soliton interactions play an important role in limiting the bit rate of a soliton system. For the theory developed for a single soliton pulse to be applicable to a train of pulses, the soliton pulses need to be sufficiently separated because the combined field effect of two adjacent solitons is different from that of a single soliton and hence the soliton shape is affected. Soliton interaction in the same channel can be studied by

solving the NLSE by replacing the input pulse with a combination of two adjacent solitons, whose initial separation is given by the normalized parameter  $2q_0$ .

$$u(0, \tau) = \text{sech}(\tau - q_0) + r \text{sech}[r(\tau + q_0)] e^{j\theta} \quad (3.5.1.1)$$

where  $\theta$  is the relative phase of the two solitons and  $r$  the relative amplitude. With this initial condition,  $u(\xi, \tau)$  can be evaluated by solving equation (3.4.2) using the inverse scattering method (ISM)[16]. Results show that for certain values of  $r$  and  $\theta$ , the soliton separation  $q_s$  oscillates with a certain time period. Specifically for  $r = 1$  and  $\theta = 0$ , the separation between the adjacent solitons oscillates with period

$$\xi_p = \left(\frac{\pi}{2}\right) e^{q_0} \quad (3.5.1.2)$$

This variation in the separation between solitons results in a timing jitter at the receiver. As explained earlier, this causes the receiver to incorrectly decode the bit-position of the soliton. Since the presence of a pulse in the bit period means a ‘1’ and its absence means a ‘0’ in an intensity modulated fiber system, the receiver assumes that the soliton pulse belongs to the adjacent bit slot and decodes ‘1’ instead of ‘0’ in the adjacent bit-slot and a ‘0’ instead of ‘1’ in the original bit-slot, thus increasing the BER of the system. To avoid this, the period of oscillation of the soliton separation must satisfy the following condition

$$\xi_p L_D \gg L_T \quad (3.5.1.3)$$

which can be written as a simple design criterion as

$$R_b^2 L_T \ll \frac{\pi (e^{q_0})}{8q_0^2 |\beta_2|} \quad (3.5.1.4)$$

where  $L_T$  is the transmission distance.

### Inter Channel Interaction

Until now we have considered only the dispersion and self phase modulation effects on soliton pulses. But in case of WDM systems, the XPM effect becomes prominent and influences the shape of the propagating soliton. The effect of XPM on the soliton pulses can be perceived as the interaction of soliton pulses belonging to adjacent channels in a WDM system.

Suppose we have two soliton pulses  $u_1$  and  $u_2$  belonging to adjacent channels having frequencies  $f_1$  and  $f_2$ , with a spacing of  $f_{ch}$  between them. Since the group velocity of a pulse is frequency-dependent, pulses  $u_1$  and  $u_2$  travel with different velocities and hence collide with each other at some point in the fiber. This collision is similar to an elastic collision between elementary particles. The length of the collision is given by

$$L_{coll} = 2T_s \frac{T_o}{|\beta_2| \Omega_{ch}} \quad (3.5.2.1)$$

where  $\Omega_{ch} = 2\pi f_{ch} T_o$  is the normalized frequency spacing of the solitons. During the collision period, the frequency of each of the soliton pulses changes dynamically, thus speeding up or slowing down each of the solitons. After the collision, however, the solitons recover their original shape, frequency and the speed just as in an elastic collision between particles. Thus, an inter-channel soliton collision results in the temporal shift of the soliton pulses in their bit positions, which is given by

$$t_{shift} = \frac{4T_o}{\Omega_{ch}^2} \quad (3.5.2.2)$$

Thus, XPM has the effect of inducing a timing jitter to the soliton pulses in their bit slots and this imposes a limit on the bit rate of the system.

### Effect of Attenuation on Soliton Transmission

The basic reason that a soliton maintains its shape during propagation is the balancing of the SPM term with the dispersion term which remains constant through the length of a typical single mode fiber. In case of a lossy fiber though, the attenuation in the signal amplitude decreases the effect of the SPM on the pulse and this disrupts the balance between the SPM and the dispersion terms and the soliton shape thus gets disturbed. This has the effect of increasing the width of the soliton pulse propagating through the fiber.

In order to maintain the soliton shape, we thus need to somehow maintain the balance between the two terms. This can be done in two ways:

- 1 By using dispersion managed fibers and
- 2 By implementing loss-managed systems

#### Dispersion Managed Fibers

The basic idea of a dispersion managed fiber is to vary the second order dispersion parameter  $\beta_2$  along the length of the fiber in order to compensate for the attenuation to maintain the balance between the group velocity dispersion (GVD) and the SPM terms. Dispersion management is achieved by using either dispersion decreasing fibers (DDFs) or by using periodic dispersion maps (PDMs).

In the first method, fibers with decreasing GVD profiles are used to compensate for the decreasing SPM effect at each point in the lossy fiber. The NLSE in this case is given by

$$j \frac{\partial u}{\partial \xi} + \left(\frac{1}{2}\right) \left[ \frac{\beta_2(\xi)}{\beta_2(0)} \right] \frac{\partial^2 u}{\partial t^2} + N^2 |u|^2 u = 0 \quad (3.6.1.1)$$

where  $\beta_2(\xi)$  is the GVD which decreases with  $\xi$  along the fiber length.

The main disadvantage of using DDFs is that the overall dispersion value in a fiber span is high which degrades the system performance. Thus PDMs provide a better solution to the problem of attenuation in soliton systems.

In the PDM method, several short links of constant dispersion are used. The average dispersion of a fiber span made up of several such small links balances the varying SPM effects caused by the attenuation. This reduces the average dispersion of a fiber span, even though the dispersion is quite large in the individual map periods that make up the whole span. In addition, the pulse propagates linearly through each map period since the nonlinear effects are negligible for such short lengths of the fiber[3f].

### Loss Managed Systems

As the name clearly suggests, the soliton shape in this case is maintained by compensating for the losses throughout the fiber by amplifying the signal using EDFAs at regular intervals. While doing so we must ensure that the non-compensating segments of the fiber are not long enough to distort the soliton shape. There are two schemes of amplification usually employed:

- Lumped amplification, in which the signal is amplified by a series of EDFAs, placed at regular intervals across the fiber. This is similar to the way amplification is done even for non-soliton systems, and

- Distributed amplification, where the loss is locally compensated at each point in the fiber by amplifying the attenuated signal continuously through out the fiber link. This can be achieved by deploying erbium-doped fiber segments throughout the system and pumping them at regular intervals.

The distributed amplification scheme is preferable for high speed systems with long transmission distances since the losses are constantly compensated at every point in the fiber which means there is no residual distortion on the soliton pulses at the end of each fiber span[18]. However, the distributed amplification scheme is not always practically feasible since it requires modifications in the existing infrastructure. Lumped amplification, on the other hand is the conventional amplification scheme used for most of the communication networks with minor design adjustments[1]. Hence, we focus on the lumped amplification scheme in our present study.

### Lumped Amplification

As mentioned previously, the lumped amplification scheme is widely used in many networks. The only difference while using it for a soliton system is the spacing between the amplifiers, which should be as low as possible. This ensures that the attenuation effects in the individual spans do not accumulate enough to distort the soliton shape before the signal is amplified at the end of each span. The amplifier spacing for a typical loss-managed soliton system should thus be much smaller than the usual 80 to 100 Km spacing used for the conventional non-soliton systems[19]. Consider a fiber link with total transmission distance  $L_A$  and let there be  $N_A$  number of uniformly spaced amplifiers



throughout the link. If the gain of each amplifier is given by  $G$ , then the NLSE for a lumped amplification scheme is given by

$$j \frac{\partial u}{\partial \xi} + \left(\frac{1}{2}\right) \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = \left(-\frac{j}{2}\right) \Gamma u + \left(\frac{j}{2}\right) g(\xi) L_D u \quad (3.7.1)$$

where  $\Gamma = \alpha L_D$  is the loss across one dispersion length and  $g(\xi)$  is the gain factor given by

$$g(\xi) = \sum G \delta(\xi - m\xi_A) \quad (3.7.2)$$

where the summation extends over  $m$  terms.

To solve equation (3.7.1), we substitute

$$u(\xi, \tau) = \sqrt{p(\xi)} v(\xi, \tau) \quad (3.7.3)$$

Substituting this in equation (3.7.1) and proceeding, we get two equations containing  $p(\xi)$  and  $v(\xi, \tau)$

$$j \frac{\partial v}{\partial \xi} + \left(\frac{1}{2}\right) \frac{\partial^2 v}{\partial \tau^2} + p(\xi) |v|^2 v = 0 \quad (3.7.4)$$

and

$$\frac{dp}{d\xi} = \left[ g(\xi) L_D - \Gamma \right] p \quad (3.7.5)$$

Solving them, we get an equation which reduces to the NLSE for loss-less fibers if the following two conditions are satisfied:

- 1  $P_s$  (input peak power of the launched soliton) =  $\frac{G \log(G)}{G-1} P_o$

where  $P_o$  is the input peak power in loss-less fibers and

- 2  $L_A \ll L_D$  which in other terms can be written as

$$T_0 \gg \sqrt{|\beta_2| L_A}$$

The factor  $\frac{P_s}{P_o}$  is called the energy enhancement factor for loss-managed solitons and is denoted by  $f_{LM}$ .

### Effect of FWM on Soliton Transmission

In WDM systems, four wave mixing may occur when solitons from three different channels interact with each other. It can be viewed as a collision between three particles traveling with different velocities.

The phase matching condition for the FWM in case of solitons is given by

$$|\beta_2| \left( \frac{\Omega_{ch}}{T_o} \right)^2 = \frac{2\pi m}{L_A} \quad (3.8.1)$$

where  $m$  is an integer. In dispersion managed soliton systems, where the localized GVD is usually maintained high, the FWM effects are mostly negligible. They are prominent only in the loss managed soliton systems, where the lumped amplification scheme causes energy fluctuations to the soliton which can create a nonlinear index grating that can phase match the FWM process. The collision between three soliton pulses can sometimes cause a permanent shift in the frequency and the energy of the individual solitons. However, the FWM effects can be reduced by using high dispersion fibers.

### Amplifier Noise

Amplifiers, while compensating for the attenuation in fiber links by providing the necessary gain, also introduce some amplified spontaneous emission (ASE) noise into the pulses. Since this ASE noise has some spectral width to it, frequency fluctuations are

introduced in the pulses at each EDFA stage. Since the GVD varies with frequency, these frequency fluctuations result in a timing jitter of the soliton pulse in the bit slot. This timing jitter accumulates at each amplifier stage resulting in a considerable fluctuation of the pulse position in the bit slot, which can be quite degrading to the final performance of the system. Mathematically, the pulse position at the end of  $n^{\text{th}}$  EDFA is given by the following recursion relationship

$$q(z_n) = q(z_{n-1}) + \Omega(z_{n-1}) \int_{z_{n-1}}^{z_n} (z) dz + \delta q_n \quad (3.9.1)$$

where  $q(z_n)$  is the position of the pulse in the bit slot at the end of  $n^{\text{th}}$  EDFA and the integration extends from  $z_{n-1}$  to  $z_n$ .

ASE noise induces frequency fluctuations, energy fluctuations and position fluctuations as follows:

$$\frac{d\Omega}{dz} = \Sigma \delta\Omega_n \delta(z - z_n) \quad (3.9.2)$$

$$\frac{dE}{dz} = \Sigma \delta E_n \delta(z - z_n) \quad (3.9.3)$$

$$\frac{dq}{dz} = \beta_2 \Omega + \Sigma \delta q_n \delta(z - z_n) \quad (3.9.4)$$

where  $\delta\Omega_n$ ,  $\delta E_n$  and  $\delta q_n$  are the random frequency, energy and position changes induced by the ASE at the  $n^{\text{th}}$  amplifier at  $z_n$ . Here,  $E$ ,  $\Omega$  and  $q$  are given by

$$E = \int |B|^2 dt \quad (3.9.5)$$

$$\Omega = \frac{j}{2E} \int \left( B^* \frac{\partial B}{\partial t} - B \frac{\partial B^*}{\partial t} \right) dt \quad (3.9.6)$$

$$\text{and } q = (1/E) \int t |B|^2 dt \quad (3.9.7)$$

where  $B = \frac{A(\xi, \tau)}{\sqrt{P}}$  is the normalized envelope and the integrals extend from  $-\infty$  to  $+\infty$

Now, if  $z_a$  is the position of an amplifier in the fiber, then

$$\langle \delta B(z_a, t) \delta B(z_a, t') \rangle = S_{sp} \delta(t - t') \quad (3.9.8)$$

where  $S_{sp} = n_{sp} h\nu_o (G - 1)$  is the spectral density of the ASE noise and  $\langle E \rangle$  is the time average of E. Also,

$$B(z, t) = \text{asech} \left[ \frac{t - q}{T} - j\Omega(t - q) + j\phi \right] \quad (3.9.9)$$

for a constant dispersion fiber.

Solving the set of equations (3.9.2) through (3.9.4), we eventually get the variance in the position variation of the soliton pulse for a loss-managed system as

$$\sigma_t^2 = S_{sp} \frac{T_o^2}{3E_s} \left[ N_A + \left( \frac{1}{6} \right) (N_A - 1) (2N_A - 1) d^2 \right] \quad (3.9.10)$$

where  $E_s = \frac{2f_{LM} |\beta_2|}{\gamma T_o}$

### Calculating the Q Factor for Solitons

As we have seen in the preceding sections, the inter channel collisions and the ASE noise introduced by the EDFAs induce a timing jitter in the solitons propagating in a WDM system. This timing jitter causes the soliton to fluctuate in its bit slot and may eventually push it to an adjacent bit slot which causes an error in decoding at the receiver end. This timing jitter is the dominating factor that decides the BER of the soliton based systems. Hence, unlike the non-soliton systems, the quality factor of a soliton system is

determined by the variance of the soliton position in the bit slot, which has two main origins:

- inter channel two-soliton collisions, and
- the ASE noise induced by the amplifiers at each of the EDFA stage.

The Q factor for a soliton system is thus given by

$$Q = 10 \log_{10} \left( \frac{T_b}{\sigma_{ASE}} + \sigma_{XPM} \right) \quad (3.10.1)$$

where  $\sigma_{ASE}$  and  $\sigma_{XPM}$  are the standard deviations corresponding to the pulse fluctuations due to the ASE noise from the EDFAs and position fluctuations caused by the XPM effect. In addition to the timing jitter, the ASE noise also causes energy fluctuations as we have seen which slightly distort the shape of the solitons. The amplitude fluctuations are also caused by the residual unbalanced distortion in the lumped amplification scheme of a loss managed soliton system. We anticipate that the error caused by these amplitude fluctuations at the receiver is negligible as compared to that caused by the timing jitter in a soliton system. A soliton has the property of regaining its shape after a XPM induced collision if the conditions are favorable, as mentioned earlier and hence is not affected by the amplitude fluctuations as much as by the timing jitter.

#### Comparison of Q factor of a Link for Non-Soliton and Soliton Based System

One of the main objectives of our study is to get the best performance out of a fiber system, which is already in place and where the basic fiber components cannot be changed. Hence, we assume typical values for the fiber related parameters and for amplifier spacing, while we try out various values for pulse-related parameters to find the

best combination of values. We also compare the Q factors calculated by assuming solitons and non-soliton pulses like the Gaussian pulse. In these simulations both the SPM and XPM effects are considered along with the linear GVD effect and attenuation in the fiber.

### Results and Discussions

Simulations are run using PHOTOSS software for various values of GVD,  $F_{ch}$ , NF,  $L_A$  and bit rates for both solitons and non-solitons and the results are compared with the theory. In all the simulations, unless specified otherwise, the following values are assumed which are quite typical of the existing fiber networks:

$\alpha = 0.19\text{dB/km}$ ,  $F_{ch} = 75\text{GHz}$ ,  $L_A = 50\text{kms}$ ,  $q_0 = 5$ ,  $\gamma = 2\text{W}^{-1}\text{km}^{-1}$ . The non-soliton pulses used here are Gaussian pulses. The input power is selected in such a way as to satisfy equation (3.4.1).

- Variation with the noise figure (NF) of the EDFAs

The noise figure (NF) of the EDFAs assumed in the fiber span is varied from 2dB to 10dB with the GVD parameter  $\beta_2$  assumed to be  $-0.4\text{ps}^2/\text{km}$  and the bit rate to be 5Gbps. The Q factors of both solitons and non-solitons are plotted as shown in figure (3.3) below

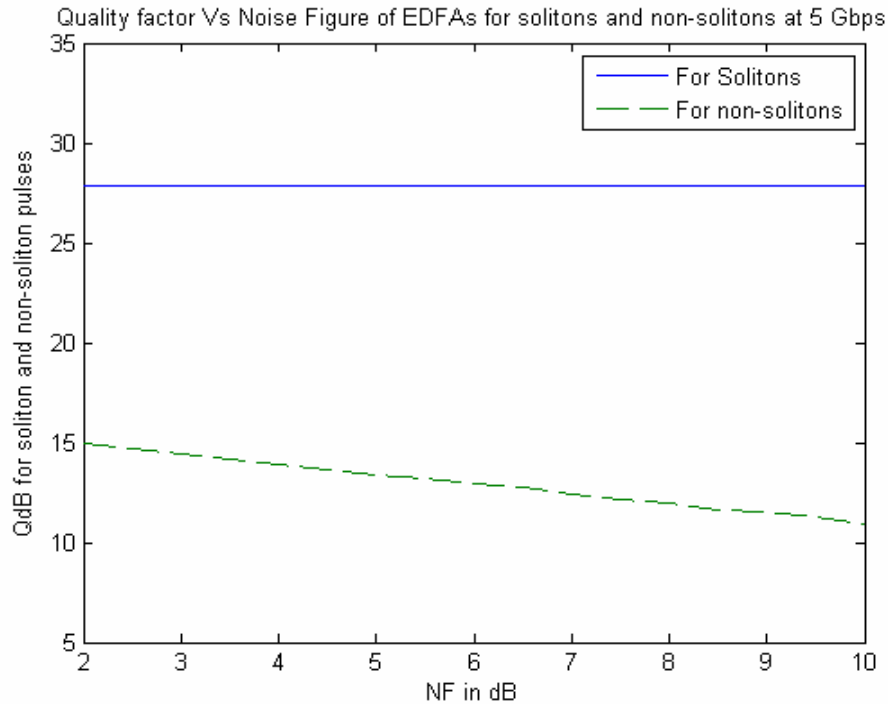


Figure 3.3 Result showing the Q factor Vs NF of EDFAs for solitons and non-solitons at 5Gbps and  $\beta_2 = -0.4 \text{ ps}^2/\text{km}$

Figure (3.3) shows that the solitons perform much better than the non-soliton pulses at 5Gbps. There is a significant difference of around 12dB between the Q factors for a noise figure of about 5dB which is the most commonly encountered value. Here we have assumed only one other channel that interacts with the test channel. But in WDM systems there are many such channels that propagate together and interact with each other which will decrease the overall Q factor.

The above result also shows that there is not much variation in the soliton Q factor with changes in NF, whereas the non-soliton Q factor decreases more rapidly with the NF. This may be due to the dominance of the XPM effect on the solitons for the parameters assumed.

- Dependence of Q factor on the number of spans

For  $\beta_2 = -0.8\text{ps}^2/\text{km}$ , the Q factor is plotted against the number of spans for both solitons and Gaussian pulses as shown below:

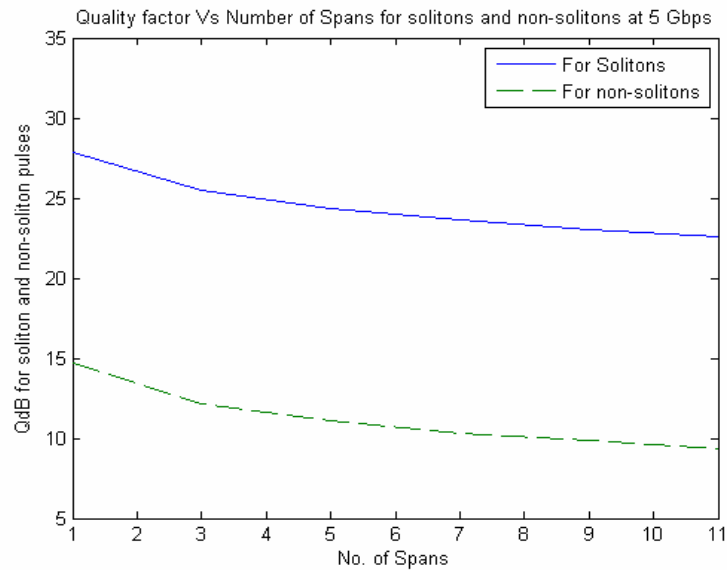


Figure 3.4 Result showing the Q factor Vs No. of Spans for solitons and non-solitons at 5Gbps and  $\beta_2 = -0.8 \text{ ps}^2/\text{km}$

From figure (3.4), it is clear that at an increased GVD solitons perform much better than non-solitons for any number of spans. Also, here, Q factors of both solitons and non-solitons seem to decrease by the same extent with the number of spans.

- Variation of the Q factor with Amplifier spacing,  $L_A$  and bit rate

As the amplifier spacing increases, the Q factor of both the solitons and non-solitons drops down rapidly as shown in the figure (3.5) below:



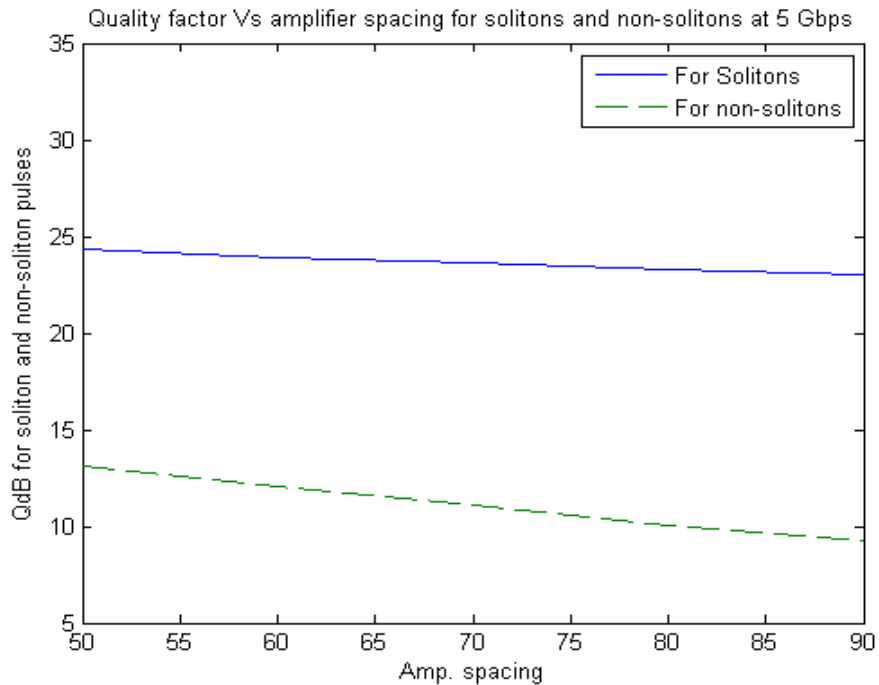


Figure 3.5 Result showing the Q factor Vs  $L_A$  for solitons and non-solitons at 5Gbps

After a certain value of the amplifier spacing, loss-management will no longer be effective and the solitons start losing their shape, resulting in errors at the receiver. So, even if the timing jitter is small enough, the EDFAs placed very far apart will distort the solitons and become a source of error in the system. Hence, the Q factor calculated from the timing jitter alone will no longer reflect the true BER

Now the bit rate of the system is changed, first to 7.5 Gbps and then to 10 Gbps and the Q factors are plotted against  $L_a$  in each case. The results for bit rates 7.5 and 10 Gbps are shown below in figures (3.6a) and (3.6b) respectively.

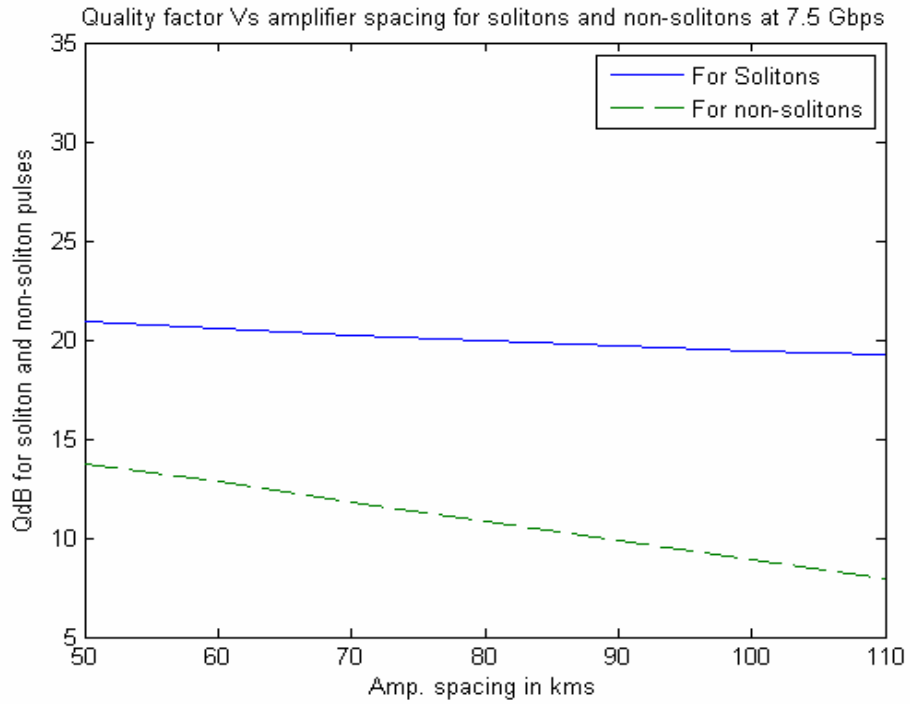


Figure 3.6a Result showing the Q factor Vs  $L_A$  for solitons and non-solitons at 7.5Gbps

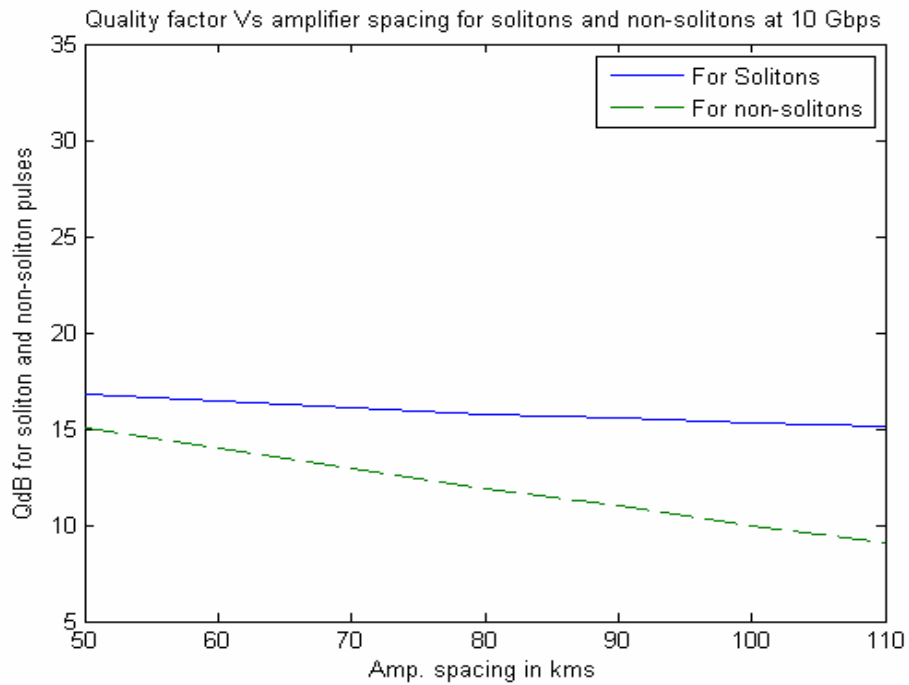


Figure 3.6b Result showing the Q factor Vs  $L_A$  for solitons and non-solitons at 10Gbps

As can be seen from these figures, the difference between the Q factors of the solitons and non-solitons decreases considerably with increased bit rates. This is because an increase in the bit rate decreases the bit-period and hence the soliton pulse can easily move into the adjacent bit slot for a given amount of jitter. Hence the timing jitter plays an important role at higher bit rates and the solitons perform worse than non-solitons after the bit rate crosses a certain value, even when other conditions for soliton propagation are maintained. A close look at the figures (3.6a) and (3.6b) shows that the Q factor for non-solitons has actually increased, instead of decreased, by about 1dB with increase in the bit rate from 7.5Gbps to 10Gbps. This is because, when the bit rate of the system is increased, the value of the parameter N, given by the equation (3.4.1), deviates from 1. Hence, the power of the soliton pulse is increased to maintain the value of N at 1. Consequently, the power of the non-soliton pulse is also increased to the same value in order to maintain consistency. Thus the Q factor of the non-soliton system is increased by 1dB since the input power has a greater effect on the Q factor of a non-soliton system than its bit rate.

- Q factor versus channel spacing

The channel spacing plays an important role in the Q factor calculations of a soliton in optical WDM systems even at low power levels since it strongly affects the timing jitter independent of the power of the pulse. Simulations were run in PHOTOSS for different values of the channel spacing in a two-channel system for different spans and is plotted in figure (3.7) as shown below:

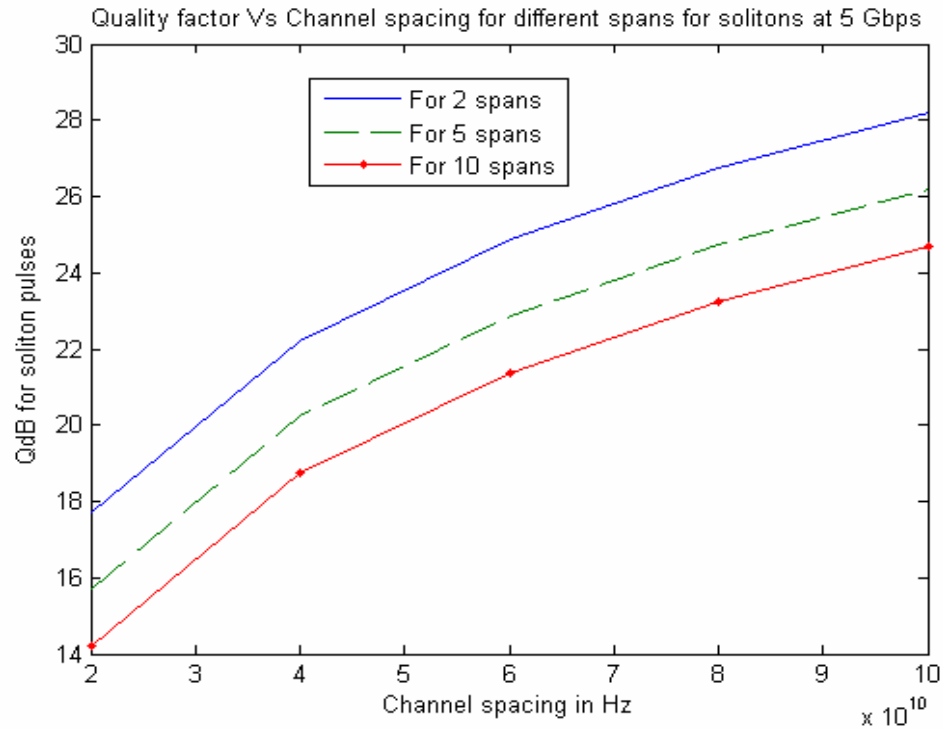


Figure 3.7 Result showing the Q factor Vs  $F_{ch}$  for solitons for different spans

The figure (3.7) shows that the Q factor increases rapidly with an increase in the channel spacing. The channel spacing that is usually considered while designing a WDM optical network is about 75GHz but can be as low as 50GHz. Observe that the drop in the Q factor for a specific channel spacing, say 75GHz, from 2 spans to 10 spans is just 4dB. Hence the signal can travel 1000-1500 (corresponding to about 20-30 spans) kms without a huge drop in the Q factor with respect to the timing jitter.

All the above results suggest that the present day fiber infrastructure can be carefully used to propagate solitons with considerably higher performance than the conventional systems. But till now we have only seen the results for a point-to-point links which do not represent the complete picture since there are other routing issues involved

in a network that are affected by the Q factor. Hence it is desirable to examine how the Q factor affects these other issues. Hence, in the next chapter we will deal with the end-to-end Q factor of a WDM network.

## CHAPTER 4

## Q FACTOR OF A WDM NETWORK

In the preceding chapters we have seen how the Q factor depends on the selection of the pulse profiles and other pulse-related and fiber related parameters like the dispersion constant and the attenuation constant of the fiber. However, until now we have only considered point-to-point links. A WDM system typically consists of many such links interconnected in various ways depending upon the topology of the network. A network can have different topologies including bus, star, ring or mesh. In case of a WDM network, the mesh topology is widely used.

Impact of Q factor on Routing and Wavelength Assignment

In any network with many nodes, there are several paths that can be taken by a signal originating from its source node to its destination node, which may be separated by many intermediate nodes. For routing the traffic through several intermediate nodes, various algorithms have been developed which take into account several factors such as the shortest possible distance, blocking probability etc, before selecting the best possible route[22][23]. And in the case of WDM networks, there is an additional problem of wavelength assignment along with the routing assignment for which many routing and wavelength assignment algorithms (RWAs) have been developed[21]. In all-optical WDM networks, a single wavelength is used from the source to the destination, unless wavelength converters are incorporated. Typically, these network layer RWA algorithms do not take into account the physical layer impairments on the signal[21]. But in WDM

optical networks, as pointed out in chapter 1, the linear and nonlinear impairments play a vital role in the selection of the best route from source to destination. The best route decided by a network layer RWA algorithm may not meet the requirements of the signal quality decided by the physical layer impairments[21]. For instance, in a random wavelength assignment[23], wavelengths are assigned randomly from the available set of wavelengths. If the XPM effect is maximum for the selected set of wavelengths along the path, then the quality of the signal is greatly affected, which is detrimental to the performance of the system.

Hence an ideal RWA algorithm should take into account the quality factor of the light path while assigning wavelengths and paths to a signal from source to destination. For doing this, we need to have a mathematical model to calculate the end-to-end Q factors between any two nodes in the network.

Considerable research has been reported regarding the use of solitons and other pulse shapes in long-haul point-to-point systems[15][17][18]. In this chapter, we analyze the performance of soliton pulses in WDM optical networks that may have mesh topology without disturbing the existing fiber infrastructure. We anticipate that in future all-optical networks, routing and wavelength assignments will be carried out dynamically to choose the best route for a connection request[22]. We explore the feasibility of using soliton pulses rather than conventional pulse shapes, to minimize physical layer impairments and achieve optimal end-to-end performance. We consider a part of a long-haul, WDM network and calculate the Q factor between two points in this network and compare the results with that of a non-soliton pulse shape. We also compare and contrast

the Q factor variations for different wavelength allocations by plotting the Q factor values for the same end-to-end link with a different set of channels. The theoretical results are compared against simulations run on network models using PHOTOSS software[24].

One of the main objectives of this research is to explore ways of improving the capacity of the WDM optical network without changing the infrastructure of the existing networks. This is particularly important in the case of soliton systems which normally require considerable changes in the design of the network which increases the cost and makes the implementation practically difficult. As discussed in chapter 3, for maintaining the shape of soliton pulses and for achieving high bit rates, many fiber dependent parameters and network parameters such as the amplifier spacing need to be changed. However, solitons can also be maintained by controlling pulse-related parameters like the width of the pulse. Since most of the existing optical networks employ constant dispersion fibers, dispersion management is not possible and the only way to overcome the attenuation effects is to control the pulse-parameters by treating them as loss-managed systems.

#### Quality Factor for a Loss-Managed Soliton System

We have seen that in a loss-managed soliton system, there will be a series of amplifiers placed between two nodes of a network to boost the power level of the pulse so that the soliton shape is maintained. For a given amplifier spacing, we can always control the pulse-related parameters so that the balance between the dispersion and the phase modulation components is not significantly affected before the signal is amplified again. This process of maintaining the pulse shape by a series of amplifiers placed at



regular intervals is called ‘‘Lumped Amplification’’. The NLSE for such a lumped amplification process is given by

$$j \frac{\partial u}{\partial \xi} + \left(\frac{1}{2}\right) \frac{\partial^2 u}{\partial r^2} + |u|^2 u = -\left(\frac{j}{2}\right) \Gamma u + \left(\frac{j}{2}\right) g(\xi) L_D u \quad (4.2.1)$$

and the Quality factor of a loss-managed soliton system is given by

$$Q = \frac{T_o}{\sigma t^2} \quad (4.2.2)$$

where  $T_o$  is the pulse width and  $\sigma t^2$  is the variance of the timing jitter due to the amplifier noise, which, in case of loss-managed solitons is given by

$$\sigma_t^2 = S_{sp} \frac{T_o^2}{3E_s} \left[ N_A + N_A (N_A - 1) (2N_A - 1) \frac{d^2}{6} \right] \quad (4.2.3)$$

where  $S_{sp}$  is the spectral density of the ASE noise at the amplifiers,  $E_s$  is the input soliton energy,  $N_A$  is the total number of amplifiers in the link and  $d$  is the normalized dispersion parameter.

### Sample Network for Calculating the End-to-End Q Factor

For analyzing the performance of soliton systems through Q factor calculations, we consider a part of a large backbone WDM network, shown in fig. (4a) having many nodes interconnected in a random way, with multiple wavelengths (channels) along each link, and each link has multiple EDFAs. The amplifier spacing,  $L_A$  is indicated for each link.

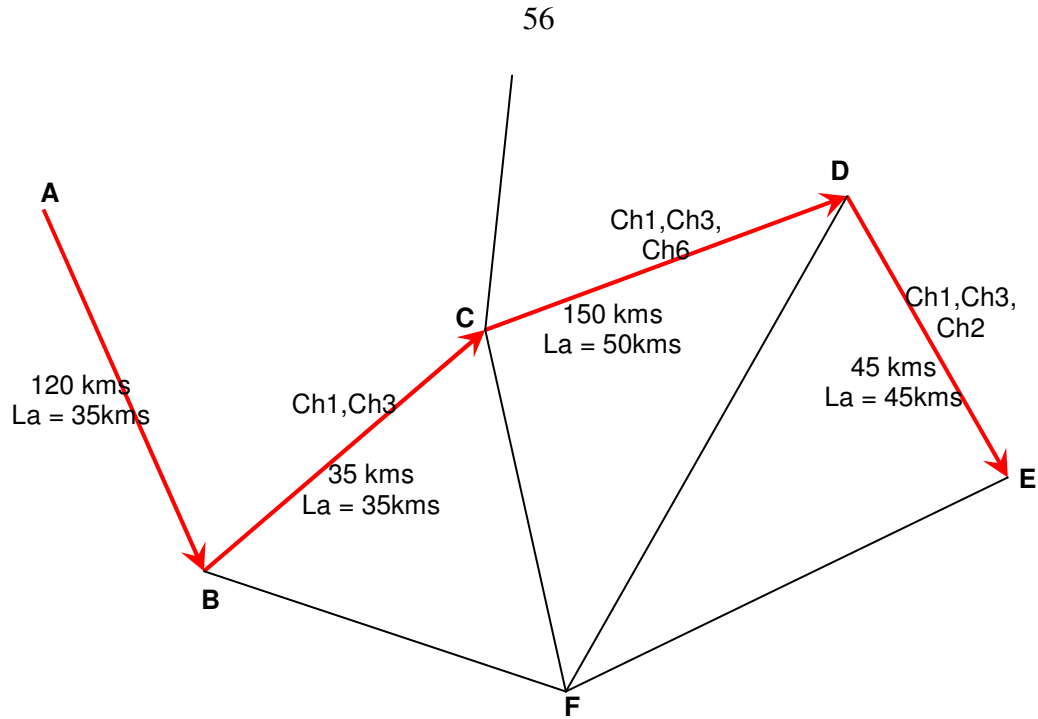


Figure 4a Optical signal routed through path 1 in the sample WDM network assumed to be a part of a large backbone network.

Suppose we have to transmit a signal from node A to node E in the network.

There are several routes possible for the signal to reach its destination. The challenge here is to route the signal through the path which has the highest quality factor. For doing this, the routers at each node should have a global knowledge of the surrounding nodes. As an example, node B should know how the signal gets effected if it is routed through node F or through node C. We will study how the quality factor is affected by routing decisions using the sample network shown in figure 4.

Let us consider three possible paths for the signal to reach destination node E from source node A. The paths and the channels encountered in each link of the three paths are shown in the figures (4a), (4b) and (4c). The center frequency of the signal under consideration is 193.1THz. We call it channel 3. In the first link between A and B,

only channel 3 travels for all the three paths. In path 1, the signal is routed through A-B-C-D-E where it travels through some distance with channels 1,6 and 2. Similarly in path 2, the signal is routed through A-B-F-D-E where the signal travels with channels 2, 7, 5 and 1. Finally, in path 3 as shown in figure (4c), the signal is transmitted through nodes A-B-C-F-E, where it counters channels 1, 4, 5 and 7.

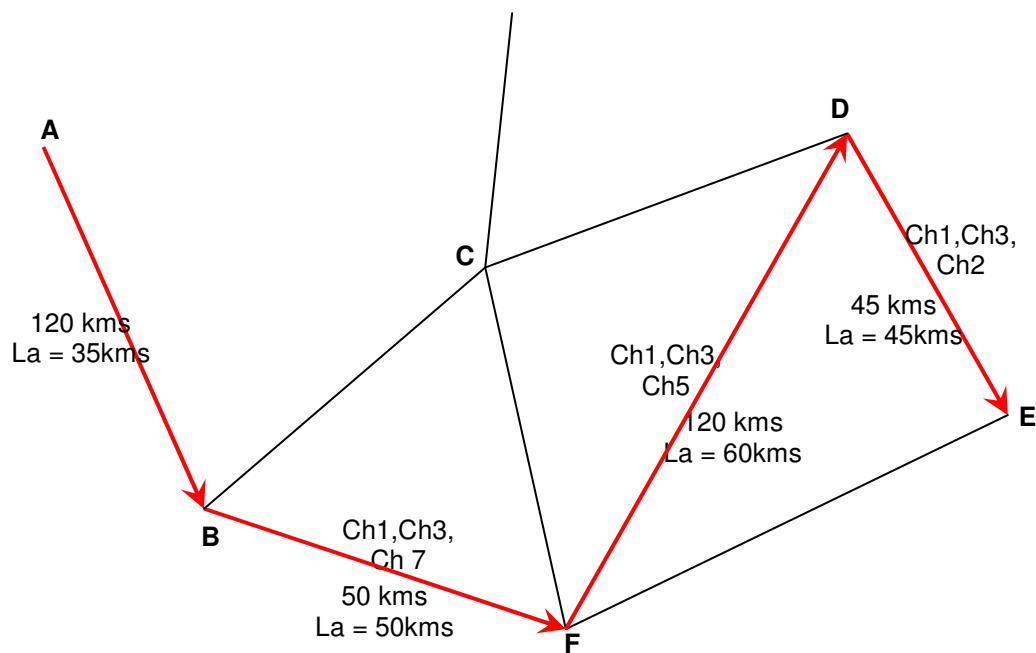


Figure 4b Optical signal routed through path 2 in the sample WDM network

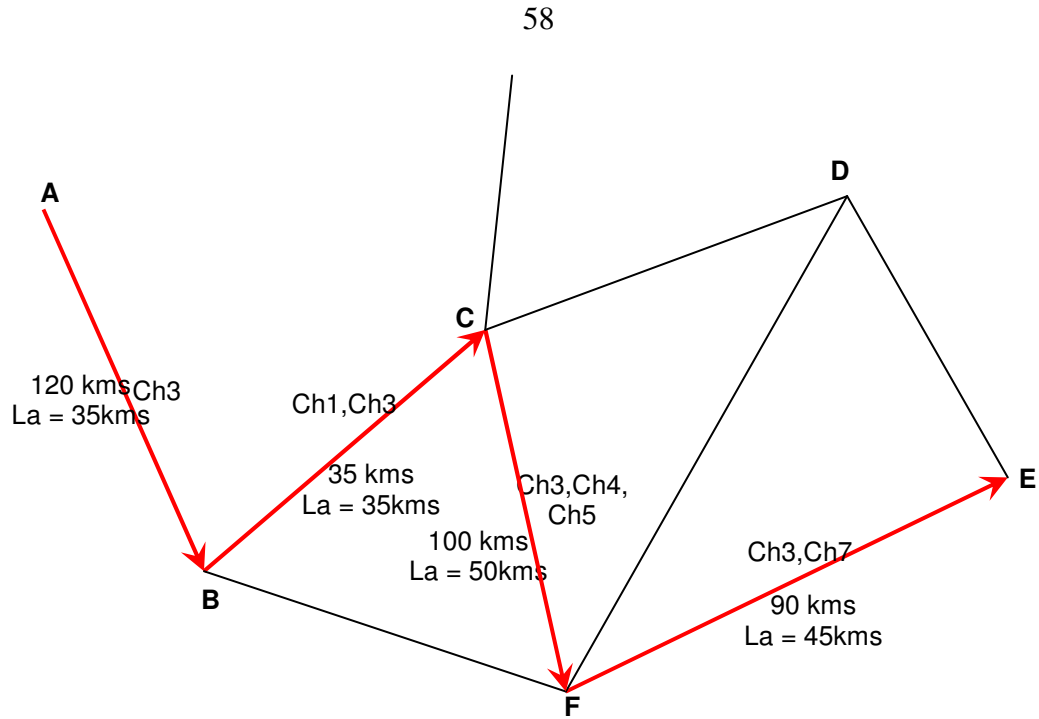


Figure 4c Optical signal routed through path 3 in the sample WDM network

The quality factors are calculated for the three paths and plotted against the noise figure (NF) of the EDFAs present throughout the network. We assume that all the amplifiers have the same NF and that there is no additional gain provided by them other than compensating for the fiber losses in each span. We also assume a constant dispersion profile for the fiber throughout the network with a second order GVD of  $-0.5\text{ps}^2/\text{km}$ , an attenuation constant of  $0.19\text{dB}/\text{km}$ , a nonlinear coefficient of  $2\text{W}^{-1}\text{km}^{-1}$ , pulse width parameter  $T_0$  of  $20\text{ps}$  and a bit rate of about  $5\text{Gbps}$ . The power of the pulse has to be maintained at  $0.625\text{mW}$  for the fundamental soliton to sustain in the fiber. The frequencies of the channels from 1 through 7 are  $193\text{THz}$ ,  $193.05\text{THz}$ ,  $193.1\text{THz}$ ,  $193.15\text{THz}$ ,  $193.2\text{THz}$ ,  $193.25\text{THz}$  and  $193.3\text{THz}$  with a channel spacing of  $50\text{GHz}$ .

### Results and Conclusions

The simulations are run for both soliton pulses (that is, for pulses with a sech profile) and non-soliton pulses (cosine squared pulses) and the Noise Figure of the EDFAs was varied over a range including typical values of 4-5dB. Figures (4d) through (4f) show the individual plots for the corresponding paths for both soliton and non-soliton cases.

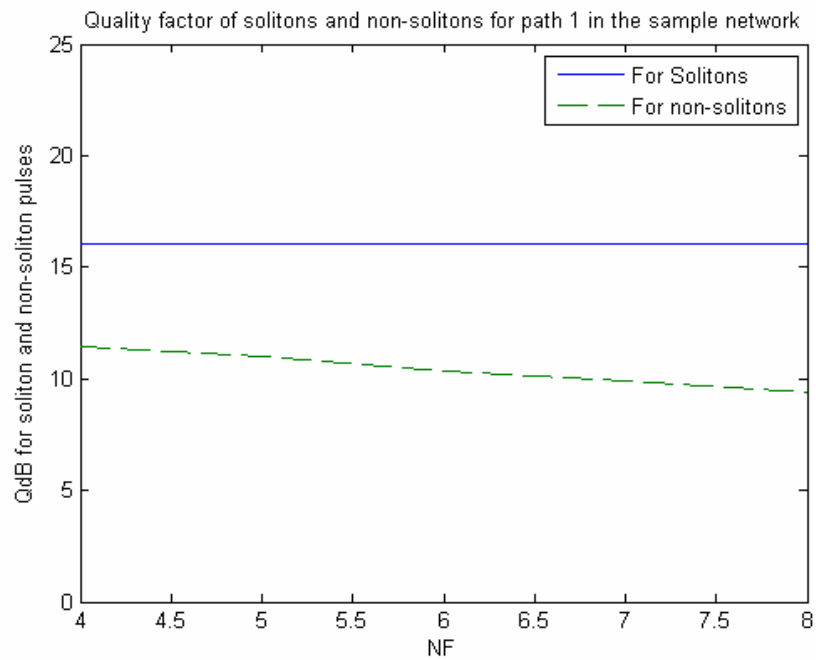


Figure 4d Result showing the variation of Q with NF of the EDFAs for path 1 of the sample network for both solitons and non-solitons

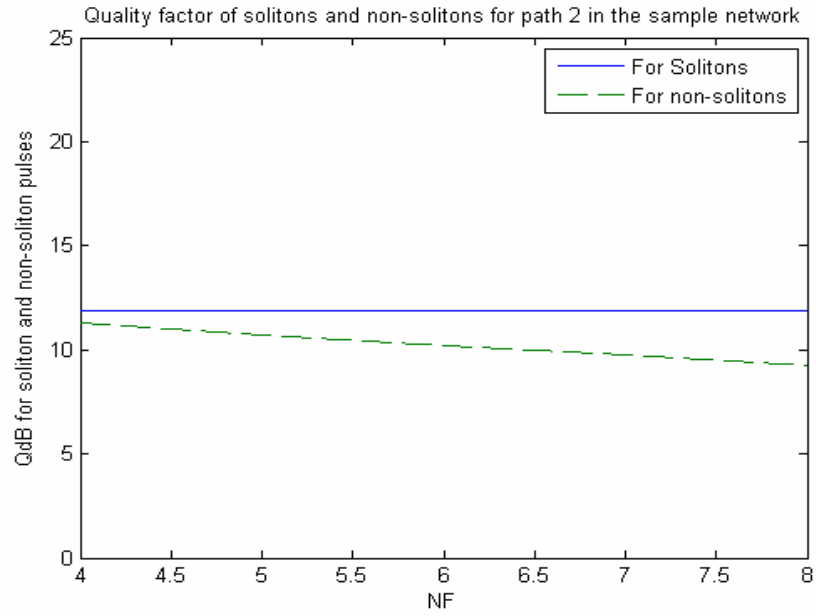


Figure 4e Result showing the variation of Q with NF of the EDFAs for path 2 of the sample network for both solitons and non-solitons

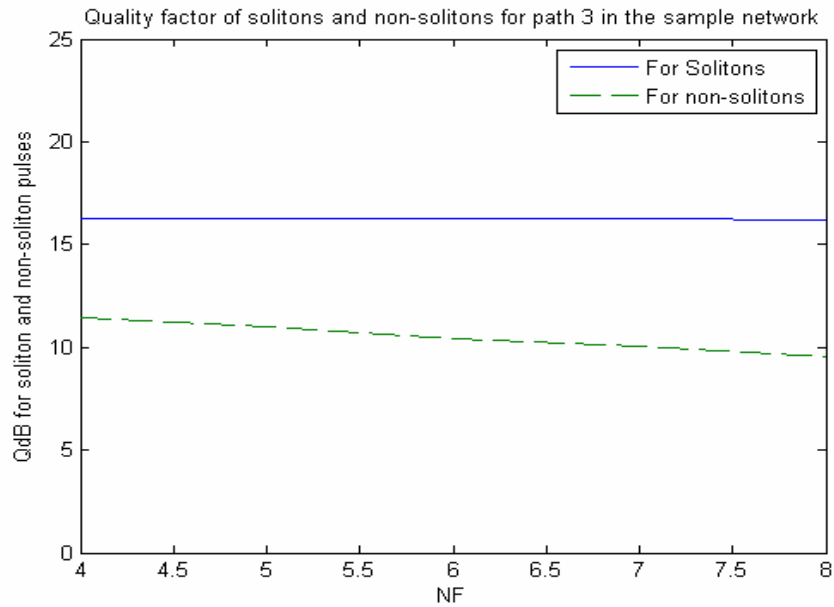


Figure 4f Result showing the variation of Q with NF of the EDFAs for path 3 of the sample network for both solitons and non-solitons

From these results, it is apparent that solitons perform much better in the conditions which are typical of the present day WDM networks for bit rates of about 5Gbps. The difference between the Q factors of solitons and non-solitons is quite significant. Also, the Q factor of the non-soliton pulses are really affected by the increase in the noise figure as can be seen from the plots while solitons hold up well against the raise in the NF of the EDFAs. One reason for this might be the low input power that we used (of about 0.625mW). This value of input power is used to make the parameter N unity required for maintaining the soliton shape, which is why the Q factor of the solitons is not much affected by the amplifier ASE noise.

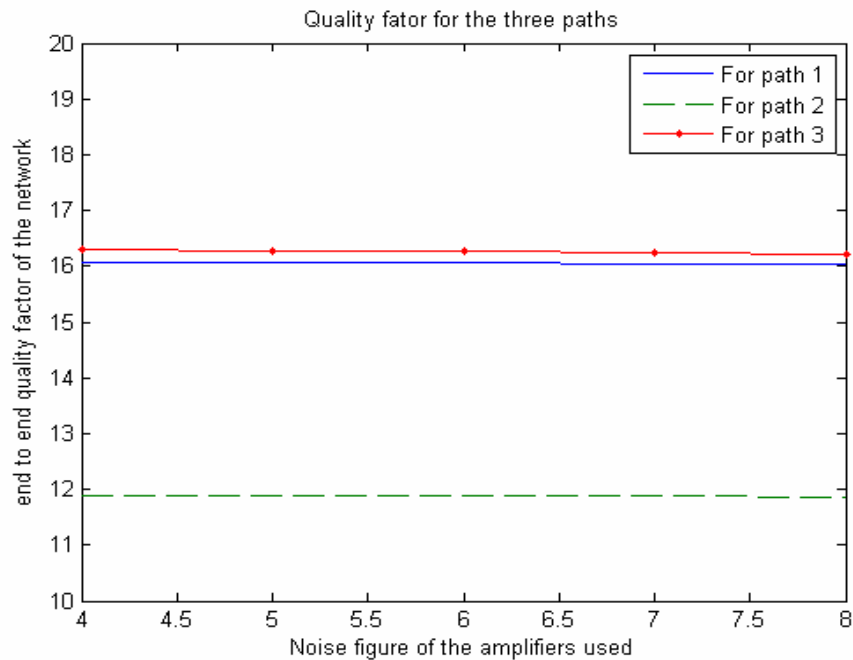


Figure 4g Result showing a comparison of Q factors of the three paths in the network

Figure (4g) shows a comparison of the three quality factors for soliton pulses. The plot shows that the path that gives best performance is the path 3 where the signal is routed through A-B-C-F-E, while the least performing path is path 2. And the difference between path 2 and the other paths seems to be quite large which is not obvious given the fact that the physical distance traveled while taking path 2 is the least among the three paths (335kms). Hence, it is apparent that for soliton-based WDM systems, the effect of the co-propagating channels and the amplifier spacings between the links should be seriously considered before deciding on the best route.

The effect of the co-propagating channels can be more clearly demonstrated by considering the case where one more channel is added to one of the paths, in this case path 3. Suppose, at the node F, channel 2 instead of going to D, goes to E as shown in the figure (4h) below:

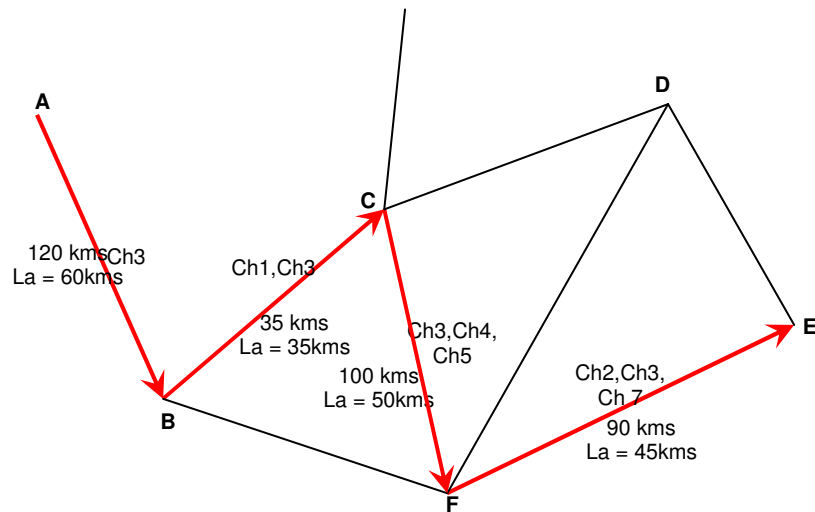


Figure 4h Optical signal routed through path 3 with Channel 2 added between nodes F to E in the sample WDM network



The results for the new assumption are shown in figure (4i) below:

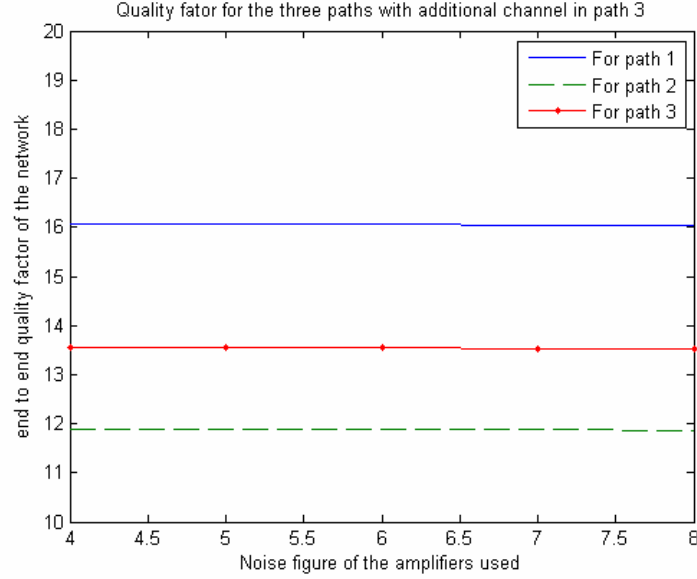


Figure 4i Result showing a comparison of Q factors of the three paths in the network

Results show a considerable fall in the Q factor for path 3 and it is no longer the best performing path. This is due to the additional effect of the adjacent channel (channel 2) which interacts with the channel 3 strongly. Hence an ideal routing algorithm must be globally aware of the surrounding links in order to make the best decision possible.

To avoid cross-phase modulation effects on the soliton pulses, the pulse parameters and the fiber parameters must satisfy the following condition:

$$N_{ch} < \frac{T_s L_D}{T_o \Omega_{ch} L_A} \quad (4.4.1)$$

where  $N_{ch}$  denotes the number of channels,  $T_s$  is the pulse-width parameter,  $L_D$  is the dispersion length,  $\Omega_{ch}$  is the channel spacing and  $L_A$  is the amplifier spacing. This constraint indicates that the XPM and FWM effects can be completely eliminated by

restricting the number of channels and the adjacent channel separation in the network, which in turn restricts the throughput.

There are many such constraints or conditions, which when satisfied, improve the performance of the soliton system, including modifications to the existing infrastructure. But there are many economic considerations that need to be taken into account before trying to implement them. There is always a tradeoff in the industry between the improvement in the performance of a system that can be achieved by a technology and the cost that goes into its implementation. This is one of the most important factors that decide the practical feasibility of any technology and hence need to be given due consideration in an engineering analysis. This is the topic of our next chapter.

## CHAPTER 5

## SYSTEM CONSIDERATIONS AND OTHER CHALLENGES

While discussing the performance efficiency of a technology and the improvement it brings to the existing ones, it is equally, if not more, important to discuss the feasibility of its implementation, which is decided by a combination of economic and system considerations. In this chapter, we will deal with some of these issues and challenges involved in the effective implementation of soliton technology.

In a given network, there is always a trade off existing between the improvement achieved by a particular technology and the cost of its implementation. In case of soliton based systems, as discussed in chapter 3, performance of an optical network can be considerably enhanced if the physical layer of the network is changed which imposes an economic barrier in the implementation of soliton technology. This barrier can be crossed if the improvement in the performance is large enough for a given application in an optical system. As pointed out in chapter 1, the performance of any communication system is measured in terms of its BER which depends on the SNR of the system, which in turn depends on the Q factor of the system. Suppose the decision variable at the receiver is Gaussian. Then the relation between the Q factor and the BER of the system is given by equation (1.2.6). In this case, the BER corresponding to a Q factor of  $5dB$  is  $7.85 \times 10^{-4}$  whereas the BER corresponding to a Q factor of  $6dB$  comes out to be  $3.43 \times 10^{-5}$ . Hence, the BER improves by about a factor of 10 for  $1dB$  improvement in the Q factor at  $5dB$  for a Gaussian decision variable. Further, at a Q factor of  $7dB$ , the BER improves by a factor of 1000 for the same  $1dB$  improvement in the Q factor. Hence

the improvement in the performance of the system also depends on the operating value of the Q factor. However, the decision variable cannot be approximated to be Gaussian in most of the cases for fiber systems and hence is of limited use in the field of optical communications[6].

### Dispersion Managed Solitons

In chapter 3, while discussing the design issues, we listed several conditions that need to be satisfied and design changes that need to be made for an effective implementation of a soliton system. The major design modification involves the implementation of dispersion-managed soliton system.

The dispersion managed soliton system requires use of dispersion decreasing fibers (DDF), dispersion compensated fibers (DCF) and several other fibers with varying dispersion profiles, which are not commonly used in conventional optical systems[1]. The dispersion management technique has shown promising results in improving the quality of an optical network and proves to be more efficient than the standard soliton approach when used in WDM systems[25]. The dispersion management technique uses strong dispersion maps wherein the dispersion profile of the fiber link is maintained in such a way as to constantly balance the power dependent nonlinearity with the dispersion and hence maintain the shape of the soliton. Recent work has shown that use of the dispersion management technique can enable the transmission of eight 20Gbps channels over 10,000 Kms[26]. Another experiment in 2000 has shown that bit rates close to 1Tbps can be achieved with solitons by transmitting 25 channels at 40Gbps over 1500 Kms using periodic dispersion maps[27].

In spite of these improvements that can be achieved with solitons using dispersion management, the cost and complexities of its installation is limiting the extent of its usage. Studies are going on to further improve the transmission capacity and new dispersion profiles are being investigated that can improve the error rate of the system.

Hence, it is desirable to achieve better performance with the existing infrastructure without any design changes, by controlling the pulse parameters. Results from chapter 4 show that considerable improvement to the BER can be achieved by the use of soliton pulses instead of the conventional NRZ systems at reasonably high bit rates.

### Soliton Chirp

Another minor, yet important factor considered in the design issues is the chirped nature of the pulses emitted out of the lasers used for WDM communication. We have already discussed how detrimental chirp can be to a soliton pulse traveling in a nonlinear fiber. Hence, even when the dispersion management technique is not used, the chirped nature of the light source in an optical WDM system needs to be reduced. Research is active in this direction as well. Recent papers show that mode-locked semiconductor lasers and synchronously diode-pumped tunable fiber Raman sources of subpico second solitons both produce nearly chirp-free pico-second pulses and can replace the conventional gain switching technique used to produce pico-second pulses in fiber systems[28]. However a more cost efficient approach would be the use of compact erbium doped fiber lasers which have the additional advantage of stability and ease of use[28].

### Soliton Transmitter

The use of solitons, which are a Return-to-Zero (RZ) pulses, requires that the conventional optical transmitter, which is a non-return-to-zero pulse transmitter, be replaced by an RZ transmitter which increases the complexity of the system for both the standard and dispersion-managed soliton systems. A typical optical transmitter which is capable of transmitting both NRZ and RZ pulses is shown below.

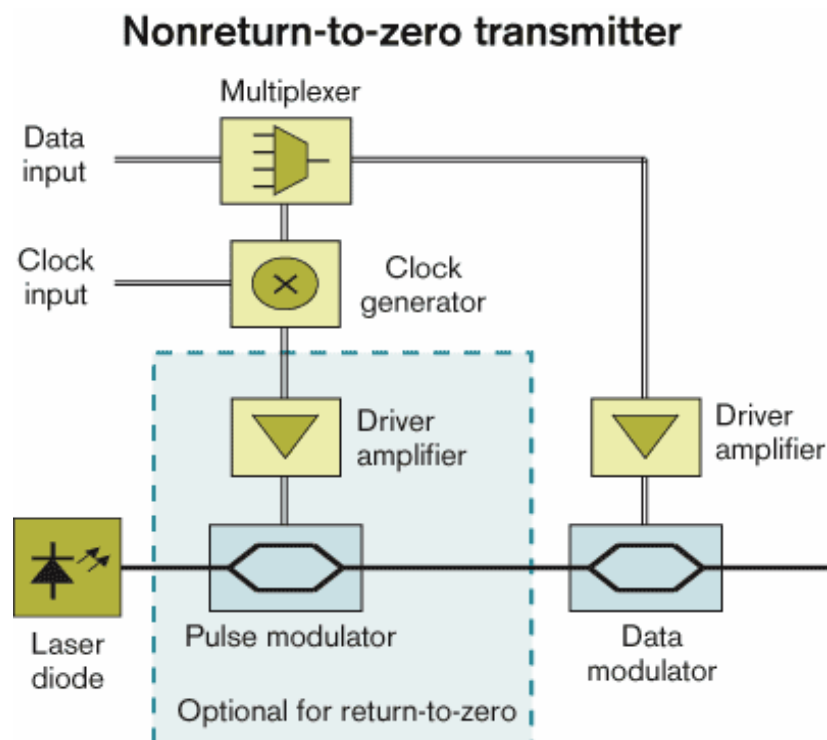


Figure 5 Figure showing a typical optical transmitter[29]

As can be seen from the figure, several input data with low bit rates are multiplexed to give a high bit rate output in the electrical domain, which is then amplified before coupling with the optical signal using the data modulator. The laser diode generates a

continuous wave signal and is inputted to the data modulator directly for generating NRZ optical pulses. Hence the central block in figure (5) saying “optional for return-to-zero” may be absent in case of NRZ pulses. But for transmitting RZ pulses, the input to the data modulator should be discrete pulses instead of a continuous wave. Hence a pulse modulator needs to be introduced to produce RZ soliton pulses just before the data modulator as shown in figure (5) which increases the cost of the system considerably[29].

### Optical Switch Induced Impairments

Apart from the various linear and nonlinear impairment effects that were addressed in the preceding chapters, all-optical systems also suffer from some impairments induced by Semiconductor Optical Amplifier (SOA) switching nodes, typically used in conventional WDM optical systems. A non-ideal switching node made up of several SOAs typically introduces noise, crosstalk and signal distortion due to amplifier saturation. The SOA-based switching nodes can also introduce chirp in the outgoing pulses which again distorts the soliton shape in the case of soliton-based systems. These impairments impose a limit on the maximum size and throughput of the Tune And Select (TAS) nodes[30] in the TAS architecture used in the WDM optical systems. These impairments can be compensated by using large and fast Optical Burst Switching (OBS) core nodes with TAS node architecture. This again requires a new optical layer to be introduced by upgrading the present WDM optical systems to support optical burst mode transmission.

Current reports indicate that, assuming dispersion management, soliton systems show better performance than the conventional NRZ pulses for a single channel system

[31]. In case of a WDM system, it has been shown that solitons perform better than the NRZ pulses for bit rates of up to 40Gbps. At 40Gbps, however, as the dispersion effects start to dominate over the nonlinearities, NRZ pulses show better performance than the RZ-soliton pulses[31].



## CHAPTER 6

## CONCLUSIONS AND FUTURE SCOPE

While a good deal of research is going on in the field of optical communications, there is still a lot of progress to be made as compared to the other areas of the communications field like the wireless communications. There is a lot of optical bandwidth that can be utilized, while, at the moment we are only able to use about 1-10 THz. There is a lot of research presently going on in WDM systems and soliton-based systems are being analyzed for their ability to achieve high bit rates[25][31].

In this thesis, we have mainly focused on the performance comparison of soliton-based and non-soliton optical systems in a typical WDM optical network. We have also examined an alternate method of approximation to arrive at the solution of the NLSE numerically for non-soliton pulses.

Results shown in figure (2.1) show that an RP based approach give more accurate results than the standard SSF method in solving the NLSE numerically. Some papers have reported that for less number of power series terms, RP method even performs faster than the SSF method[11][10] which makes it a more useful technique in the analytical calculation of Q factor.

Results in chapter 3 show the comparison of Q factors of solitons and non-solitons for various fiber and system parameters like the noise figure of the EDFAs, number of spans, amplifier spacing, channel spacing and bit rates. The Q factors of the soliton based systems are about 10dB greater than that of the non-soliton systems. The results further show that the effect of variation of system parameters is much larger for

non-soliton systems as compared to that of the soliton systems (figure (3.3), figure (3.5), figure (3.6a) and figure (3.6b)) with the exception of variation in the bit rate.

As can be seen from figures (3.5), (3.6a) and (3.6b), the Q factor of soliton systems decreases rapidly and comes very close to the Q factor values of non-solitons. The difference in the Q factors between the soliton and non-soliton systems reduces from over 13dB at 5Gbps to about 4dB at 10Gbps for an amplifier spacing of 70kms. If we further increase the bit rate to about 20Gbps, the Q factor curve for soliton systems comes below the curve for the non-soliton systems. There is a cross over of the Q factor curves of soliton and non-soliton systems at around 17Gbps for the typical system parameters assumed in this thesis. There is a lot of research going on aimed at understanding where this cross-over of the Q factor curves occurs[31]. Some papers have reported a cross-over at bit rates as high as 40Gbps[31]. The reason for such a cross-over can be easily understood given the fact that the Q factor of a soliton mainly depends on the timing jitter in the bit position of the soliton in its bit-slot. As the bit rate of the channel increases, the soliton pulse can easily shift into its adjacent bit-slots and hence the Q factor decreases rapidly. This imposes a limit on the overall capacity of a WDM system since the channel spacing cannot go too low. Hence, further research in the direction of analyzing the cross-over bit rate of soliton and non-soliton systems is required to establish the ideal technology for very high capacity WDM systems.

We have also demonstrated that the soliton systems give better performance as compared to the non-soliton systems in WDM networks with mesh topology. Results from chapter 4 show that, in a mesh based WDM network, the wavelength assignment has a considerable

effect on the end-to-end Q factor of a particular path (figures (4g) and (4i)). Hence there is a need to have an intelligent centralized RWA algorithm for the WDM systems in order to get best performance out of the system[2].

Finally, recent research in solitons is aimed mostly at achieving higher bit rates with low BER by changing the fiber links and other system parameters (like the dispersion managed soliton systems) which gives better results than using the existing infrastructure[1].

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APPENDIX A

BASIC MATLAB CODE FOR ANALYTICAL CALCULATION OF Q FACTOR FOR  
SOLITONS

```

Qdbf = [];Qdb = 0;
%for b=[-0.5:0.1:-0.1 -0.04 -0.02 0]; %[-21.68:10:-1.68 0 1:10:41
61:30:171] %0.5
for Fch = 20e9:20e9:100e9%spn = 1:2:11%Fdb=2:0.5:10%La = [50:10:110]
    %Fch = 75e9;%20e9:20e9:100e9
    Fdb = 4.5;
    La = 50;
    spn = 10;%10;%1:2:11

u1=[];v1=[];u2=[];l=0;
% Po=2*10^-3;
%b=0.1;

%Assumed Parameters:
    b = -0.8;
    %Fdb = 4;
    F = 10^(Fdb/10);
    To=20e-12;C=0;
    r=2;
    Ts = 1.763*To;
    Lt = spn*La;
    adb=0.19;
    ae = 10^(adb/10); %a=0.1*adb/log10(exp(1));
    q = 5;
    h = 6.626e-34;
    freq = 193.1e12;
    %Fch = 75e9;

%Derivables
Na = floor(Lt/La);
Ld = To^2/abs(b);
Tb = 2*q*To; B = 1/(2*q*To);
Po = abs(b)*1e-24/(r*To^2);
aldb = adb*Ld;
ale=10^(aldb/10);
G = ae^La;
fLM = G*log(G)/(G-1);
% dt=4095;
% t=-16*T/5:(32*T/5)/dt:16*T/5;

Lcoll = 0.28*Tb/(q*abs(b)*1e-24*Fch);
Ncoll = Lt/(2*Lcoll);

Ssp = F/2*h*freq*(G-1);
Es = 2*fLM*abs(b)*1e-21/(r*To);
%Es = 2*fLM*abs(b)/(r*To);
d = b*La*1e-24/To^2;
%d = b*La/To;
var = Ssp*To^2/(3*Es)*(Na+Na*(Na-1)*(2*Na-
1)*d^2/6)+Ncoll*(1/(pi^2*To*Fch^2))^2;

```



```
%var = (sqrt(Ssp*To^2/(3*Es)*(Na+Na*(Na-1)*(2*Na-1)*d^2/6)))^2;  
Q = Tb/sqrt(var);  
Qdb = 10*log10(Q);  
  
Qdbf = [Qdbf Qdb];  
K=4096;  
t=linspace(0,32*To/5,K);  
%u=sqrt(P)*exp(-(1+j*C)/2.*(t/T).^2);  
uo=sqrt(Po*fLM)*sech(t).*exp(j*0/2);  
end  
R = [20e9:20e9:100e9];%[50:10:110]  
%R = [-0.5:0.1:-0.1 -0.04 -0.02 0];  
figure(2)  
plot(R,Qdbf)
```