NANOCOMPOSITES: A STUDY OF THEORETICAL MICROMECHANICAL
BEHAVIOR USING FINITE ELEMENT ANALYSIS

by

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Eric Carlton Milliren

May 2009
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The following parameters refer to parameters utilized in the ABAQUS finite element models and appear in results, figures, and tables:

(XYZ) Global coordinate system with origin at absolute center of cylindrical fiber, Z-direction coincides with fiber axis

(1-2-3) Equivalent coordinate directions to (XYZ) respectively

(x,y,z) Points or distances from origin of model created

z’ Equal to z minus L, or the distance from the free end of the fiber

L Half length of the fiber, such that the fiber is of length 2L and ranges from –L to L on the Z-axis

L_f Equal to L

L_m Half length of matrix, such that the embedded fiber model is of length 2L_m and ranges from -L_m to L_m on the Z-axis

L_cap Equal to L_m minus L_f, or the length of matrix cap

d_c Diameter of conventional fiber

d_n Diameter of nanofiber

σ’ Nominal applied stress, defined as E_f multiplied by ε_m.

E_f Young’s modulus of elasticity for fiber material

E_m Young’s modulus of elasticity for matrix material

ε_f Average strain of fiber

ε_m Average strain of matrix

σ or σ_{ii} Normal stress where i=1, 2, 3 referring to coordinate direction

S_{ii} Equal to σ_{ii}

τ or τ_{ij} Shear stress where i=1,2,3 and j=1, 2, 3 referring to coordinate direction

S_{ij} Equal to τ_{ij}

α Angle of fiber orientation measured from Z-axis

θ Polar angle measured from positive XZ-plane

β or β_i Constituent and geometry parameter, specific to individual shear lag model, where i=Cox, Nairn
ABSTRACT

Current research in nanotechnology has produced an increasing number of possibilities for advanced materials. Among those materials with potential advanced mechanical properties are fiber-reinforced composite laminates that utilize nanoscale fiber diameters. Through a combination of studying classic micromechanical models and modern computer-aided finite element analysis (FEA), the advantages for utilizing these nanofibers in advanced structural applications, such as space mirror backings, was investigated. The approach for modeling these composite structures was that of a Representative Volume Element (RVE). Using the program ABAQUS/CAE, a RVE was created with the goals of accurately comparing to the shear lag theory, effectively incorporating “interphase” zones that bond the constituents, and demonstrating effects of down-scaling fiber diameter.

In this thesis, the progression of the ABAQUS model is thoroughly covered as it developed into a verified model correlating with the shear lag theory. The model produced was capable of utilizing interphase if desired, and was capable of off-axis loading scenarios. A MathCAD program was written in order to employ the published theoretical techniques, which were then compared to the FEA results for verification. The FEA model was found to work well in conjunction with the theory explored using MathCAD, after which the nanofiber FEA model showed some clear advantages over a conventional-sized model, specifically an increase in strength of the composite RVE. Finally, it was determined that the interfacial bonding strength plays a large role in the structure of the interphase zone, and thus the overall strength of the composite.
INTRODUCTION AND BACKGROUND

Introduction to Study

In a society and time as demanding as the current one, engineers and scientists find themselves busier than ever. All over the globe, people are searching for answers to help improve transportation, eliminate waste, quicken communications, advance safety, upgrade infrastructures, find cures, and computerize anything that can be computerized. These are merely a few general categories in which government, corporate, and private projects are all aiming to tackle. Today, pure specialization in one field of study has become almost obsolete, as many of today’s technologies and innovations incorporate knowledge attained from many sources and fields. Mechanical engineering specifically has always been known for its multidisciplinary nature. It spans areas such as structural mechanics, fluid mechanics, heat transfer/HVAC, materials science, electrical engineering, computer science/programming, and more. Such is the characterization of this study. This project focuses on some classical structural mechanics and materials engineering employing the latest in computer-based mechanical analysis for the further advancement and comprehension of nanotechnology.

Nanotechnology describes the study, development, and integration of materials, structures, and systems on the order of magnitude of nanometers. The length scales generally range between one one-millionth (10E-9 m) and one-hundred one-millionths (10E-7 m) of a meter (1). In Figure 1, an ant can be seen carrying an electronic chip, small enough to be considered a “nanochip.”
Nanotechnology itself is extremely multifaceted. Its applications extend far beyond classical engineering such as mechanical, civil, and electrical, but pertain to chemistry, biology, and medical professions as well. As stated before, this study aims to enhance nanotechnology specifically as it pertains to structural mechanics and materials engineering.

New materials are being synthesized every day to be utilized for various applications. Some examples of material groups are metals, polymers, ceramics, epoxies, and rubbers. Combinations of these materials can result in one of two main outcomes. The combined materials can be uniformly distributed, resulting in a single new homogeneous material, or purposely organized in the form of a composite. In composite materials, macroscopically there are separate constituents with individual physical and chemical properties, but they behave as one structure with its own completely different material properties (3). This has many advantages as well as disadvantages. In the case
of a fiber-reinforced laminate (defined momentarily), as will generally be the case with this study, a main advantage is a high strength-to-weight ratio. Other advantages include high stiffness-to-weight ratios and favorable thermo-electric properties. The disadvantage, as is commonly found, is that strength is direction-dependent, and the creation of the material must then be strictly tailored to its specific purpose. These features make composites a great choice in applications such as airplanes and wind turbines, as can be seen in Figure 2.

![Figure 2: Images of structures containing composite materials: (a) Airplane (4); (b) Wind turbine (5).](image)

As mentioned, fiber-reinforced laminates were the focal point of this project. Here, some terminology must be clarified. Simply put, the term “fiber” is used for the directional reinforcing constituent, and the term “matrix” is commonly used for the binding or filler constituent, that each make up the composite material. Now, the bonding between the two constituents is a common topic of discussion. The term
“interface” can suffice as a description of the surface area in which the two constituents meet, and is commonly used in most literature. However, it is an assumption of simplified geometry that the two constituents would bond as a perfect surface, when in reality there is a finite volumetric zone in which the molecules must blend in order to create the bond. This finite volume zone is referred to as the “interphase.” This interphase zone, when explored, is commonly a small shell of material surrounding the fiber, which contains its own individual material properties. The material properties for the individual constituents are primarily considered elastic isotropic and include simply Young’s modulus of elasticity and Poisson’s ratio.

The term “nanocomposites” refers to any type of composite material with constituents whose geometric shapes and dimensional properties fall into the scale of nanometers as described previously. Many different types of nanocomposites exist already, and are exhaustively being researched for understanding and potential applications. Several types of polymer based nanocomposites include carbon nanotubes, nanoplatelets, nanoclays, and nanofibers (6). For this study, nanofiber-reinforced composites were the specific nanomaterial of interest, but all are being looked at with great expectations. For example, the future of tire technology may lie in the introduction of carbon nanotubes into rubber compounds to increase durability while maintaining grip (7). A picture of carbon nanotube structures can be seen in Figure 3.
To study the nanofiber-based nanocomposites, a Representative Volume Element (RVE) approach was utilized. This standard technique was chosen as a means of exploring the micromechanical behavior of a unidirectional, fiber-reinforced composite laminate by manner of creating a three-dimensional (3-D) model. Few other studies, if any, were found that produced satisfactory comprehensive three-dimensional RVE models, because most micromechanical analyses simplify down to one or two-dimensional cases. To further explain the concept of the RVE, Rosen defined a Representative Volume Element as a subregion of a specimen for which strain and stress averages are the same for all similar subregions of the specimen (9). Essentially then, the RVE is the most simplified microscopic (or smaller) model that represents the material macroscopically.
Similar in concept to the RVE is the concept/technique of a “unit cell.” A unit cell can be employed to produce a microscopic model that is representative of a material’s macroscopic structure, in much the same way a RVE can. In fact, it is possible by definition for a model to be both a RVE and a unit cell. However, a model can be a RVE without being a unit cell, and vice versa. The difference is the unit cell’s dependence on geometric symmetry that creates a repeatable structure. A unit cell must be symmetrical such that multiple cells placed side by side for any number of dimensions will eventually build the actual structure of the material. The unit cell can be thought of as a building block in a sense, where someone could stack enough of them together and then produce the desired structure. The RVE on the other hand, need not be symmetrical for “building” purposes. RVE’s require that their structure be only simple enough to correctly represent the behavior of the greater structure. For example, some statistical analyses of unidirectional composites have been performed in order to determine the minimum number of fibers required in a RVE to account for the variations in fiber diameter, variations in fiber spacing, and probability of surface defects (10). RVE’s from these analyses contain many fibers of different diameters, randomly spaced and certainly not in a repeatable, symmetrical shape.

On the topic of surface defects, an important concept is that of the “ineffective fiber length” (11). This concept is based on the fact that failure of a fiber during tensile loading is most often caused by surface defects. After a fiber break, the fiber can no longer sustain the load it originally endured, and so must transfer it to the nearest fiber. The length of fiber needed to transfer 90 percent of the original load to the next nearest
fiber is considered the ineffective fiber length. As a rule of thumb, this normally amounts to about ten fiber diameters. The idea of ineffective fiber length for transferring the stress between fibers is associated with the theory of “shear lag.” This is discussed further in the Literature Review section, but in-depth coverage of the concept of ineffective fiber length is outside the scope of the present study.

Finally, the 3-D RVE model was developed in a Finite Element Analysis (FEA) program. Current technology has made FEA a powerful tool, especially with the further advancement of Computer Aided Drafting (CAD) -like capabilities. For those unfamiliar with FEA, the method can predict stresses and strains of a loaded specimen by systematically dividing (meshing) a part into many smaller elements, defining individual propagation equations between the elements (at the nodes), and finally simultaneously solving the systems of equations based on the fact that there are equal numbers of unknowns to numbers of equations. The program chosen for this study was ABAQUS/CAE. Although the ABAQUS program has been in use for some time, the CAE application is more recent. CAE allows a GUI (Graphics User Interface) approach to building the model, rather than creating a word processor-based batch file that requires extensive knowledge of program-specific commands that are different for each program. This not only eases the learning curve of a new program, but also gives advanced visual aid simulating current drafting programs like Pro-Engineer or Solid-Works. ABAQUS is a common industry standard and is very capable of three-dimensional solid part creation.
Collaborations

This project was part of a joint effort between several institutions. The sponsor was the Air Force Research Laboratory (AFRL), which has graciously provided the student funding for graduate research at Montana State University. AFRL is the Air Force's only organization wholly dedicated to leading the discovery, development, and integration of warfighting technologies for our air, space and cyberspace forces (12). AFRL is known for its projects in aerospace applications such as solar sails, space mirrors, space structures, propulsion, and vehicles. AFRL has worked in cooperation with many other national, international, and federal organizations, including NASA, DoD, and DARPA. As such, AFRL intends to use the findings of this project to improve technology in the aeronautics and space industries. An immediate means of production of nanocomposites includes strengthening backing structures for novel space mirrors.

This project, is being performed in cooperation with South Dakota School of Mines and Technology (SDSMT). The investigator at SDSMT is Dr. Hao Fong. Dr. Fong is responsible for the production and testing of nanofibers that are the foundation of these nanocomposite materials. SDSMT has devised a start-of-the-art electrospinning process that creates continuous nanofibers for several material variations. A schematic of the process is shown in Figures 2 and 3.
Figure 4: Schematic of nanofiber electrospinning process employed by SDSMT.

Figure 5: Schematic of nanofiber electrospinning process employed by SDSMT.
As stated before, several material options are possible for creation of nanofibers using the specified electrospinning process. The foremost materials include glass (SiO$_2$), titania (TiO$_2$), titania aluminum alloys, carbon, polymer, and nano-IPN. The process produces a single, continuous nanofiber loosely aligned along the rotational direction. It is also possible to produce hollow fibers for some materials such as the titania fibers. In particular, the carbon fibers caught the interest for this study due to their promising quality in structural homogeneity and lack of surface defects, which as mentioned before, determines the mechanical strength of the fiber.

Comparatively, the average diameter of a “conventional” carbon fiber, that is to say a carbon fiber that is of current standard, ranges from about 5-8 micrometers (μm), while the carbon nanofibers being produced contain average diameters of up to two orders of magnitude less than this (50-80 nm). While this hardly seems to fit into the defined nanoscale, it is a significant step in material development. In this paper, the word nanofiber may be loosely defined in this way, but refers to a fiber that is likewise significantly smaller in diameter than that of a conventional fiber. Size comparisons of nanofibers can be observed in Figure 6.

![Figure 6: Images of nanofibers provided by SDSMT.](image-url)
Problem Statement

It is the essence of engineering to establish that anything can be improved. Even a simple conclusion could revolutionize what was at one time the cutting-edge of technology. With the understanding of the subject fields, background, and production capabilities previously discussed, the problem can be stated as follows: on the premise that South Dakota School of Mines and Technology can produce nanofibers, use finite element analysis software to investigate advantages of utilizing nanofibers in current fiber-reinforced composite laminates.

There were several specific purposes associated with performing this analysis. The first important purpose was to find any advantages that scaling fiber diameter would have on the mechanical behavior of the composite. This directly related the notion for utilization of nanofibers in composites, such that the advantages were desired to be observed from stress comparisons of a conventional fiber model and a nanofiber model.

A second purpose was to determine the role of “interphase” within the composite. The interfacial bonding zone between the constituents has long been a topic of discussion and study. It was imperative to find a way to include it in the model and observe its effects on stress outcomes.

The third and final specific purpose was to utilize suitable RVE’s in ABAQUS/CAE finite element software. The application of FEA not only would serve as a testament to the program itself, but also serve as a means to satisfy the long-sought problem of developing an accurate micromechanical model for a composite material.
Specifically stated, there were two goals desired to be accomplished. One goal was to produce a viable FEA model. To be considered viable or realistic, three objectives were devised. First, the model would be required to give relevant comparisons to shear lag models and published results. Second, the model needed to be able to handle scenarios with and without the inclusion of an interphase region. Third and lastly, the model would be required to allow for on-axis and off-axis loading conditions.

Having accomplished the first goal, the second goal would be to produce a correlating model utilizing fibers with smaller diameter. This would be vital to answering the original problem statement. Definitive results from this second model would then affect the course of action for all other collaborative parties.

Inspection of Geometry

A simple diagram can help to visualize the hypothesis of advocating smaller fiber diameters in fiber reinforced composite laminates. It can be visually shown that a composite utilizing multiple small fibers can create equal fiber volume (equal cross sectional area given a uniform depth dimension) to a composite using a single larger fiber, yet produce an increase in circumferential surface area (see Figure 7). Note that the representation shown in Figure 7(b) is not necessarily an accurate configuration for the fibers.
Figure 7: (a) Representation of a conventional composite unit cell showing a single unidirectional fiber with diameter $d_c$ (5-8 μm typical). (b) Representation of a modified nanofiber composite unit cell composed of many unidirectional fibers (total of 37) with diameters $d_n$, and the total nanofiber cross-sectional area equal to that of the conventional fiber.

If we accept the postulate that strength is a function of surface area, since fiber loading occurs through the fiber surface (13), it can be mathematically shown that the nanofiber composite contains significantly more surface area over the conventional composite at no cost to volume fraction. Given that the two cells possess the same depth dimension, the two fiber scenarios have the following definitions for cross sectional area:

\[ A_{x,c} = \frac{\pi}{4} \cdot d_c^2 \]  
\[ A_{x,n} = \frac{\pi}{4} \cdot d_n^2 \]

where $A_x$ denotes cross-sectional area, subscripts with $n$ refer to the nanofiber composite, subscripts with $c$ refer to the conventional composite, and $d$ is the fiber diameter. Now,
the cross-sectional area of the conventional composite can be acknowledged as some multiple, \( N \), of the individual nanofiber cross-sectional area as such:

\[
A_{x,c} = N \times A_{x,n}
\]  

(3)

In other words, the multiple \( N \) gives the number of nanofibers that produce the same cross-sectional area as the conventional fiber. In Figure 7, the variable \( N \) is equal to 37 nanofibers. Substituting in the definitions for \( A_{x,c} \) and \( A_{x,n} \) and simplifying results in the following relationship:

\[
\frac{d_c}{\sqrt{N}} = d_n
\]  

(4)

Next, in assessment of the surface areas, the following two relations define the total surface areas for each cell:

\[
A_{s,c} = \pi \times d_c \times L
\]  

(5)

\[
A_{s,n} = \pi \times d_n \times L \times N
\]  

(6)

In these definitions, \( A_s \) stands for the surface area with subscripts \( c \) representing that of a conventional fiber and \( n \) representing that of a nanofiber. The diameter is \( d_c \) for the conventional composite, \( d_n \) for the nanocomposite, and both of the composites can be given the same length, \( L \). Now, substituting the relationship found from assessing the cross-sectional areas into the definition of \( A_{s,n} \), the following proportionality is obtained:

\[
A_{s,n} = \pi \times \frac{d_c}{\sqrt{N}} \times L \times N
\]  

(7)

Note that this previous equation was rearranged to show that the definition of \( A_{s,c} \) is contained within the relationship. Making the substitution of the aforementioned definition yields the end result:
This relationship clearly shows that the surface area of fiber in the nanofiber cell will be greater than the surface area of fiber in the conventional cell by a factor of $N^{1/2}$. Therefore, if it can be shown that strength is indeed dependent upon surface area, then it is verified that a composite consisting of nanofibers will be stronger than a conventional composite prepared with the same volume fraction.
LITERATURE REVIEW

Aboudi Method of Cells

In a series of papers documented by Dr. Jacob Aboudi (14; 15), the overall behavior of composite materials was explored through micromechanical analysis. The performed analysis, while correlated with experimental data, was used to formulate a theory that could predict the response of elastic, thermoelastic, viscoelastic, and viscoplastic composites. The responses of interest included yield surfaces, strength envelopes, and fatigue failure curves.

One of the focal points of this theory was the determination of static strength of composite materials. Instead of treating fiber-reinforced material as a homogeneously anisotropic continuum observed macroscopically, it was desired to create a more effective model that incorporated the interaction of the fibers and matrix. The resulting micromechanical model could predict static and fatigue failure based on individual constituent properties as opposed to generalized bulk properties achieved almost entirely from experimentation.

The model used for the analysis was based upon a representative volume element (RVE) approach. Assuming a composite with regularly distributed fibers in a doubly periodic array, a representative cell could be used from the cross section that contained both fiber and matrix components. The cell was then divided into four subcells, all of rectangular geometry. Naturally, one subcell represented the fiber and the other three represented the matrix material partially surrounding the fiber. The fibers had a square
area ($h_1^2$) and were a distance $h_2$ apart. Thus, the overall area of the cell could be determined by squaring the sum of $h_1$ and $h_2$ or $(h_1+h_2)^2$. A convention was used for labeling the subcells such that $\beta, \gamma=1,2$. To clarify this, the fiber subcell was $(\beta \gamma)=(11)$ and the matrix subcells were $(\beta \gamma)=(12), (\beta \gamma)=(21)$, and $(\beta \gamma)=(22)$. Figure 8 shown below illustrates the representative cell.

Figure 8: Diagram of representative unit cell with $x_1$ direction normal to the $x_2$-$x_3$ plane.

In a plane stress scenario occurring in the $x_1$-$x_2$ plane, satisfying one of the following conditions could result in failure:

$$|S_{11}^{(11)}| = X_f^{(F)} \quad (12)$$

$$|S_{22}^{(\beta \gamma)}| = X_m^{(F)} \quad (13)$$

$$|S_{12}^{(\beta \gamma)}| = S_m^{(F)} \quad (14)$$
where \((βγ)\) was limited to matrix subcells: \((22), (12), (21)\). In these conditions, \(X^{(F)}\) denotes ultimate tensile strength, \(S^{(F)}\) denotes ultimate shear stress, \(S_{11}\) and \(S_{22}\) are the principal stresses in the \(x_1\) and \(x_2\) directions respectively, and the subscripts \(f\) and \(m\) pertain to the fiber and matrix constituents, respectively. These conditions created seven different stress equations to explore, which are discussed in the Results section.

**Theory of Shear Lag**

Background information on the mechanical theory of composites and the history of RVE usage was researched. This had several purposes, one of which was to further confirm that “the wheel was not being reinvented” so to speak, by conducting research on anything that had already been done. There were no RVE models found that resembled the Initial RVE model, explained in the next chapter. As explained in the Introduction, several works have shown that RVE’s require a minimum number of fibers, determined through complex statistical analysis, in order to properly represent the composite’s behavior. The statistical analyses were performed in order to account for variations in diameter over the length of fibers as well as variations in diameter between different fibers at any one cross-section. The result of researching these statistic-based RVE’s was that they were not helpful for this particular study because they created a homogenized material that did not include interphase, nor account for fiber-fiber interaction. Since uniform fiber diameter was a desired constraint, this analysis appeared unique.

A substantial purpose for background research was determining the link between fiber/matrix interaction and material strength. One area that caught attention was the theory of shear lag \((16; 17; 18)\). The micromechanical equations developed by Nairn
and the others were seen as a possibility for investigating the down-scaling of fiber size. The theory of shear lag predicts the behavior for stress transfer between elastic matrix and fiber constituents of a composite material. Shear lag was developed from the scenario of a fiber break, in which the fiber, no longer capable of sustaining load, must transfer the load to the matrix where it can propagate to another fiber. A diagram of a fiber break is provided as Figure 9 below.

![Diagram of fiber break within uniaxial composite laminate](image)

Figure 9: Diagram of fiber break within uniaxial composite laminate, showing theoretical stress distributions. The term $\sigma$ stands for the applied stress. Not drawn to scale.

This theory makes several important assumptions. First of all, the geometry is highly simplified, using a uniform cylindrical fiber perfectly bonded to a matrix filler.
material. Next, there is no load applied to the surface of the fiber because it is broken from the other end of fiber and no matrix connects the ends. All load transferred between the constituents is performed through shear stress. Therefore, at the point of fiber break there is no fiber stress seen in the axial normal direction, but the load is purely contained in shear stress at the interface in a plane containing the axial and radial directions.

Progressing down the length of the fiber, stress is recovered in the axial direction, having been transferred from the still-stressed matrix through shear. At far-field, the composite comes into equilibrium and both the matrix and fiber are then loaded purely in the axial direction. Finally, this axial stress is assumed to exhibit a uniform profile over the cross-section of the fiber. Figure 10, from Chon and Sun (19), shows the typical profiles in the axial direction for the interfacial shear stress and axial stress (alpha equals zero degrees for an on-axis loading case).
Figure 10: (a) Typical shear stress distribution from shear lag. (b) Typical axial stress distribution from shear lag. Figures from Chon and Sun (19).

The above figure comes from a publication utilizing an embedded fiber model, which will be further described in the proceeding section. The terms displayed in the above figure are defined as follows: for an origin placed in the center of the fiber, (XYZ) is the global (matrix) coordinate system and (xyz) is the rotated coordinate system. Hence, \( z \) is the axial distance along the fiber, starting at the origin, for the local (rotated) coordinate system. The half length of the fiber is \( l_f \). The term \( \sigma \) stands for the normal stress applied to the face of the embedding matrix, and is used to non-dimensionalize the interfacial shear and the axial stresses seen by the fiber, or \( \tau \) and \( \sigma_f \) respectively. Finally,
α is the fiber-orientation angle measured from the matrix Z-axis and θ is the polar angle measured from the x-z plane.

Cox (16) is credited with developing the shear lag method in 1952, while Rosen (11), Nairn (18), and others were soon to follow with their own amendments. For comparing the different details of shear lag theory between the several noted experts, a highly useful article by Goh, et al. (17) was found. The theory of shear lag gives a general model in the form of two equations that provide the axial stress (Equation 15) and interfacial shear stress (Equation 16) as follows:

\[
\sigma_f = \sigma' \left[ 1 - \frac{\cosh (\beta \cdot z)}{\cosh (\beta \cdot L)} \right] \quad (15)
\]

\[
\tau = \frac{\sigma' r_o}{2} \cdot \frac{\sinh (\beta \cdot z)}{\cosh (\beta \cdot L)} \quad (16)
\]

These equations each possess hyperbolic sinusoidal (or cosinusoidal) terms. The coefficient \(\sigma'\) is a nominal applied stress term defined as the product of the strain in the matrix \(\varepsilon_m\) and the elastic modulus of the fiber \(E_f\). The parameter \(r_o\) is the fiber radius, and \(z\) is the variable for distance along the fiber in the axial direction. \(L\) is again the fiber half-length. The coefficient, \(\beta\), is a constitutive term based on material properties and geometry parameters of the specimen in question. The main discrepancy between the different authors lies in their different definitions for the \(\beta\) coefficient. While the general model equations are displayed here, the equations for the different \(\beta\) terms are better observed in the MathCAD attachment found in Appendix B. The \(\beta\) term strongly determines the curvature of the hyperbolic term. The curvature is, in turn, related to the stress transfer rate and related to the ineffective fiber length. The greater the curvature,
the quicker the stress transfers and hence the smaller the ineffective fiber length. Again, the ineffective length is defined as the fiber length needed to regain 90 percent of the load from the matrix to the fiber after a break (11). Nairn (18) uses a 50 percent load transfer length, which although it does not fit the definition of ineffective length, was used in this study for comparisons sake, as will be seen in future sections.

While the increase in surface area will essentially increase the amount of matrix that will be supported by fiber reinforcements, an increase in the frequency of fibers will improve the strength of the composite from a statistical standpoint as well. With more fibers, the probability is decreased that a single flaw or surface defect will dominate a region during failure. In other words, the probability that the stress will transfer to another surface flaw is decreased. Also, the distance required for stress at a fiber break to lag through the matrix to the next fiber is lessened, therefore decreasing the ineffective fiber length. The lower the probability of flaws and these factors, the closer that the strength of the composite will then approach the bonding strength of the constituents, which is to say, far greater than the strength of either of the constituents individually.

**Embedded Fiber Model**

In addition to the shear lag equations being developed in MathCAD, an “embedded-fiber” model was created using ABAQUS/CAE to reproduce the scenario created by Chon and Sun (19) in particular for the nanofiber. They develop an analysis using shear lag theory that predicts the axial stress and interfacial shearing stress distribution along the length of a fiber. The fiber is only a short segment and is completely embedded in a matrix region. Figure 11 shows the RVE of Chon and Sun.
Several important parameters are present in Figure 11. Their analysis allows for off-axis loading, with two rotational degrees-of-freedom, although one was sufficient for the present study. The equations derived for the stresses appeared noticeably similar to the general shear lag equations given previously, with the exception of the terms included
to account for off-axis loading. Their equations for axial stress and interfacial shear stress appear as follows:

$$\sigma_f = \sigma' \cdot \cos^2(\alpha) \cdot \left[1 - \frac{\cosh (\beta \cdot z)}{\cosh (\beta \cdot L)}\right]$$  \hspace{1cm} (17)

$$\tau = Q \cdot \sinh (\beta \cdot z) - \sigma' \cdot \sin (\alpha) \cdot \cos (\alpha) \cdot \sin (\theta)$$  \hspace{1cm} (18)

Notice that the simple on-axis case is satisfied when the angles $\alpha$ and $\theta$ are set to zero, because the new terms drop out and appear exactly as the original general shear lag form. The term $Q$, not defined here, contains parameters including the constituent material properties, geometry factors, and appropriately includes the hyperbolic cosine term located in the denominator of Equation 16. It is not listed here because it becomes a lengthy chain of new parameters, but can be observed in its entirety in (19) or in Appendix B. These new models were added into MathCAD and could be compared to the Cox, Nairn, and ABAQUS models as is shown later.

**Shear Lag to F.E.A. Comparisons**

It was highly desired to compare our loading scenarios, boundary conditions, and model geometry to those proven effective by others who have used similar FEA techniques. Nairn (18) and some others provide the results of their FEA analyses as comparisons to their analytical models, but do not extensively explain the creation of their FEA codes. However, Xia, et al (20) approached the shear lag versus FEA problem utilizing symmetry of an axi-symmetric model, for which he provided extensive details. The fiber was not embedded in matrix but existed as a pure fiber/matrix concentric annulus. The mesh was composed of hexagonal elements in the cross-sectional plane,
which led to the finding that a 30 degree wedge of the annulus was the smallest possible reduction that could maintain the correct micromechanical behavior. The mesh also contained high density in the fiber and tapered off into the matrix. A similar approach will be seen in the following chapter.
MODELS

Theoretical Model

The theory of shear lag develops several equations for stress in the fiber of the unidirectional composite. This is done considering a cylindrical fiber of length $2L$ and radius $r_0$. In a radial coordinate system, $z$ will be defined as the variable distance along the direction of the fiber axis and $r$ is the variable distance in the radial (azimuthal) direction in the fiber. As stated in the previous chapter, the two equations of interest are for the normal stress seen in the direction of the fiber axis and the interfacial shear stress seen along the surface of the fiber. The general form of the axial stress and interfacial shear stress were previously given respectively as:

$$\sigma_f = \sigma' \left[ 1 - \frac{\cosh (\beta \cdot z)}{\cosh (\beta \cdot L)} \right]$$

$$\tau = \sigma' \frac{r_0}{2} \frac{\sinh (\beta \cdot z)}{\cosh (\beta \cdot L)}$$

The general model geometry is assumed to be a concentric annulus of fiber and matrix with equal length, but the embedded fiber clearly modifies this as was seen in Figure 11 of the Literature Review chapter. Then once again, the off-axis embedded fiber modifies the above equations as follows:

$$\sigma_f = \sigma' \cos^2(\alpha) \left[ 1 - \frac{\cosh (\beta \cdot z)}{\cosh (\beta \cdot L)} \right]$$

$$\tau = Q \sinh(\beta \cdot z) - \sigma' \sin(\alpha) \cos(\alpha) \sin(\theta)$$

These equations were put to use in a MathCAD file that is represented in its entirety in Appendix B. The MathCAD theoretical model is built with parameter inputs
that allow individual material properties between matrix and fiber, tailored volume fraction via fiber and matrix radii inputs, fiber half-length, a Nairn defined (18) nominal stress input (as discussed in the Literature Review chapter), and angles of axis orientation. Stress results could be found using β constants from Nairn, Cox, and Chon. The stress results from MathCAD were then exported into Microsoft Excel, where they could be compared to ABAQUS results.

Also produced in MathCAD were the 50 percent transfer lengths for the model under Nairn and Cox coefficients. Inputting the specific geometric and material parameters used by Nairn (18), the theoretical model was then set to compare to Nairn’s own published results from the same journal article. This was done as a validation of the MathCAD program. The results can be seen in the next chapter. Once this was verified, the appropriate parameters were changed to match those input into the ABAQUS model, described shortly, in order to verify the FEA was functioning properly. Once again, the outcomes of this test are displayed in the Results chapter, but first the ABAQUS model is explained in depth.

**Finite Element Analysis Models**

**ANSYS Initial (Square) RVE**

After examining Aboudi’s micromechanics model, attention was refocused on producing a working RVE model of a fiber-reinforced composite via finite element analysis programs. The first model was created using ANSYS. The initial RVE created appears as in Figure 12. This program was chosen based on an initial familiarity with the program, but this program proved difficult to use and was halted due to licensing issues.
Initial (Square) RVE

At this time, a second FEA model went into development using the program ABAQUS/CAE. Due to this program’s excellent 3-D solid part creation capabilities and clear usability of contact properties, considerably more progress was made in this model. This program was very efficient for testing several different loading scenarios and geometries. The Square RVE used in the ANSYS model was recreated in ABAQUS (see Figure 12). With this geometry, the fiber volume fraction examined was 38.5 percent. Contact between the fiber and matrix components was made rigid (interphase added later). The material properties were set for simple elasticity. This property setting included the Young’s modulus and Poisson’s ratio (see Table 1).
Table 1: Material properties and geometric dimensions of Square RVE model in Figures 12-15.

<table>
<thead>
<tr>
<th></th>
<th>Young's Modulus (Gpa)</th>
<th>Poisson's Ratio (unitless)</th>
<th>Inner Radius (μm)</th>
<th>Outer Radius (μm)</th>
<th>Depth (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
<td>150</td>
<td>0.24</td>
<td>NA</td>
<td>7</td>
<td>40</td>
</tr>
<tr>
<td>Matrix</td>
<td>80</td>
<td>0.4</td>
<td>NA*</td>
<td>NA*</td>
<td>40</td>
</tr>
<tr>
<td>Interphase</td>
<td>115</td>
<td>0.32</td>
<td>7</td>
<td>8</td>
<td>40</td>
</tr>
</tbody>
</table>

* Distance between fiber centers of 20 μm

The model was constrained such that the back surface (surface coincident with the principal XY-plane) maintained zero displacement in the direction of the extrusion (Z-direction, displayed as the 3-direction in Figure 12), and a corner on this same surface was pinned to prevent translation in the X- and Y-directions. Tensile loads were applied on the front surface (surface at the extruded length z=L) in the Z-direction (axial direction of fibers). Several different load scenarios were examined including uniform surface displacement, displacement of fiber only, displacement of matrix only, uniform surface pressure, surface pressure on fiber only, surface pressure on matrix only, and several others. Field outputs examined included Von Mises stress, maximum principal stress, and maximum principal strain. Figure 13 below gives an example of the results of a loading test in ABAQUS.
This model was much more successful than the initial attempt in ANSYS. However successful it was at the time though, several issues presented themselves as necessary improvements. The next step was to create a model that included the interphase region. Since the interphase zone is a combination of both fiber and matrix, elementary material properties given to the zone were an average of the individual constituent properties. The interphase was designed to be simply one element thick. The fibers retained the same diameters. Figure 14 shows the new geometry.
There were several concerns in producing these models that needed to be overcome. The Square RVE without interphase was originally constrained rigidly on the entire back surface, and this caused necking from any axial force or displacement that was applied to the front surface. This was changed such that the back surface was constrained in the axial direction while pinning one corner to prevent translation. Another concern was that it was found that the boundary conditions being used allowed the model to rotate around the axis of one of the quarter fibers. It was determined that due to meshing constraints and computer numerical limits, small shear stresses created at the boundaries of the fiber and matrix caused these small rotations. Since the interphase model contained more of these geometric boundaries, it produced slightly more shear stress such that it became noticeable. Figure 15 displays this clearly via deformed and undeformed shapes of the model with a pinned corner and symmetrically-constrained back surface.
Attempts to resolve this problem with rotation included creating a new boundary condition scenario. The back surface remained constrained in its normal direction. Then, an adjacent side was constrained in a direction normal to its plane, followed by similarly constraining one more side, adjacent to both other constrained surfaces. This appeared valid because an RVE can be assumed symmetric on all six sides. The result was a model that was still allowed to change in volume from Poisson effects, yet did not rotate, translate, or twist. Analysis done to compare the two boundary scenarios seemed to show little differences in stress results. However, distortion occurred on the unbound sides. Since it was desired for the model to contract naturally due to Poisson effects, it was not feasible to apply the same symmetry conditions to the remaining side surfaces. Surface node coupling attempts were initiated but were not completed because a decision was
made to take a new direction with the model in order to better compare with published results and better utilize symmetry.

**EMBEDDED FIBER**

A great leap toward creating a working finite element model came at the finding of the Embedded Fiber Model (19). This paved the way for creating a new geometry setup that employed the theory of shear lag and provided results with which to compare the FEA model. Having discussed the main points of this in the Literature Review, only those points that are necessary to understand the FEA model will be repeated. Figure 16 gives a schematic of what was created in ABAQUS/CAE as a first attempt for the Embedded Fiber.

![Diagram of Initial Embedded Fiber model](image)

**Figure 16**: General diagram of Initial Embedded Fiber model. Not shown to scale.

It is important to note that this diagram, and any following schematic diagrams, are not necessarily drawn to scale, but merely intended for visual clarity. Note that in Figure 16, there are two planar views shown. This is done to show that this model
The attempt consisted of the full cylindrical fiber completely embedded in a concentric matrix cylinder. What will be seen during the model progression is that symmetry was employed several times until the FEA model did not appear as in Figure 16, but still retained all the necessary properties of this model. Figure 17 shows another representation of the initial Embedded Fiber in an “exploded view” in the YZ plane.

![Diagram](image)

Figure 17: Exploded view of initial ABAQUS Embedded Fiber model. Not shown to scale.

Since the figures are not drawn to scale, the dimensions will now be explained. The initial Embedded Fiber Model was set up similar to the Square RVE model. The same elastic properties were used, including Young’s modulus and Poisson’s ratio. The fiber diameter is the same as that of the conventional RVE fiber, and the matrix diameter is three times the fiber diameter. The overall length of the matrix is twice that of the
fiber, therefore the matrix on each end of the fiber each amount to half a fiber overall length. This model did not contain the interphase zone yet.

Table 2: Material properties and geometric dimensions of Embedded Fiber model in Figures 16-19.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus (Gpa)</th>
<th>Poisson's Ratio (unitless)</th>
<th>Inner Radius (μm)</th>
<th>Outer Radius (μm)</th>
<th>Depth (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber</td>
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<tr>
<td>Matrix</td>
<td>80</td>
<td>0.4</td>
<td>8</td>
<td>21</td>
<td>40**</td>
</tr>
<tr>
<td>Interphase</td>
<td>115</td>
<td>0.32</td>
<td>7</td>
<td>8</td>
<td>22*</td>
</tr>
</tbody>
</table>

*Inner depth same as fiber depth.
**Inner depth same as interphase depth.

Having described the basic geometry, it can be noted again that the simplest scenario was created such that the fiber axis is concentric with the matrix axis. Chon and Sun (19) eventually gave results for the fiber being set at an angle within the matrix and this was later examined. In ABAQUS, the model was created such that there were two physical parts, although this is not accurately depicted in the exploded view of Figure 17. One part was designated as a matrix section and the other was designated as fiber section. The matrix section was designated with matrix material properties, and likewise, the fiber section was assigned fiber material properties. It is extremely important to note how ABAQUS makes these types of definitions because different options were utilized later on. With ABAQUS, after a material is created, it is assigned to a “section.” This section is then assigned to a “part” or a “partitioned” area of a part. For now, the models continued to be defined in the same fashion as this initial Embedded Fiber being described, in that a separate part was created for each material section.
When using multiple parts in an ABAQUS model, they require assembly before any loading, meshing, or testing can be performed. Consequently, the next step was to assemble the two parts. After assembling, it was required that some kind of contact constraints be applied to specify the interaction between the two parts. To accomplish this, “surface ties” were applied to any surfaces that were in contact between the different parts.

Next, boundary conditions were applied. The first boundary condition, as was applied in the Square RVE model, was a symmetry condition for the Z-direction on the back surface of the model (lying in the XY-plane). This condition, called ZSymm, allowed no translation in the Z-direction and no rotation about the X- or Y-axis for any nodal point on that surface. An additional boundary condition was added such that one point on that same rear surface was encastred, or restrained from translating or rotating in any direction.

Several loading scenarios were created for analysis. When one was being tested, the other loadings were simply suppressed in the model. It was possible to load the front matrix surface (XY-plane) with either a surface pressure or surface displacement. Following loading definitions, meshing was applied to the model. For this initial Embedded Fiber attempt, the meshing was not refined well, but was improved in later models. Finally, a job was created in order to run the analysis, and the model was complete. Figure 18 depicts the results of this first attempt at analyzing the Embedded Fiber in the simplest scenario (parallel axes).
Again, it was desired that results obtained from this ABAQUS model would match results from the work by Chon and Sun (19). Specifically, stress distributions along node paths were extensively created and analyzed from these models. An example is given here of a shear stress distribution along the top surface path (“Path-1”) as shown in Figure 19.
In depth analysis of these nodal path results will be discussed later. It was necessary to show these preliminary results to demonstrate the need for future changes to the model. Many measures were taken to increase the meshing effectiveness and decrease computational time, as will now be shown.

SSLIP

After interpretation of initial shear lag model results, several measures were taken. As can be observed from Figure 19, the distribution is very coarse and appears mostly linear. This did not match up with the distribution shown in Gibson (21), who provided a sample of the Cox (16) shear lag theory. It was decided that the model needed to have a finer mesh along the edge of the fiber. This led to the improvement of several aspects of the model, but not without obstacles. Most of these tribulations related to mesh definitions, computational limitations, and server options. However, the
improvements were accomplished. Firstly, the model was sliced in half (XY or 1-2 plane), utilizing symmetry to decrease computation time and hence allow increased meshing. This ABAQUS model was labeled the “Symmetric Shear Lag” model or SSL. The addition of interphase prompted the model to be relabeled Symmetric Shear Lag with Interphase or SSLIP. Figure 20 demonstrates the change in the model in exploded schematic form.

Figure 20: Diagram of Symmetric Shear Lag with Interphase (SSLIP). Not shown to scale.

Again, this model was created in ABAQUS using separate parts for each section. Specifically, the five parts were fiber, interphase shell, interphase cap, matrix shell, and matrix cap. Note as a result of the cross-sectional view, the schematic in Figure 20 displays two areas of shell for both the interphase and matrix. These two areas of matrix shell are the same part in the model and the two interphase shell areas are the same part
as well. Surface ties were put in place for all contacting surfaces. Boundary conditions were applied in the same fashion as before, with ZSymm on the back surface (XY-principal plane), and one point encastre at the node located at the origin. Meshing was improved so that the matrix contained fewer elements than the fiber. Since most analysis was performed on the fiber, this made sense to reduce the meshing in the matrix. High meshing in the matrix was unnecessary. Mesh elements were 3-D hex types and possessed quadratic geometric order (C3D20R). At this point in the model progression, the number of elements varied as the model was continually refined. Load scenarios were created, again resuming and suppressing each as necessary to test only one load at a time. An example of an analysis output appears as shown in Figure 21.

Figure 21: SSL, shown with full cross-section view, and quarter section of fiber. Results display max principal stress.
Note in Figure 21, that the Symmetric Shear Lag (SSL) model physically only contains half of the true model length of the Embedded Fiber Model. For display purposes only, Figure 21 shows SSL cut in the YZ plane, displaying half the volume of all constituents. In addition, Figure 21 displays a quarter of SSL’s volume of fiber. Again, interphase zones were added soon after. This can be viewed in Figure 22, again noting that the views are cross-sectioned for display purposes only.

The interphase can be observed in Figure 22. It is the thin (thickness of 1 mesh element), dark shell around the quarter section of fiber, and the thin (thickness of 1 mesh element), disc immediately appearing to the left of the fiber. The interphase disc is more readily apparent in Figure 23.
Figure 23: SSLIP, cross section view shown. Results display axial strain.

Again, the meshing in the fiber region became much finer than that of the matrix regions and even finer than previously. This was acceptable because the matrix contained a lot of volume relative to the fiber, such that the finer mesh added significantly to computation time and decreased memory. Also, it was unneeded to observe fine distributions in the matrix regions because they were nearly constant anyway. The meshing of the interphase was set to match the meshing of the inner matrix region.

The interphase zone was set with material property parameters separate from those of the matrix and fiber regions so that they could be allowed to vary. In fact, the shell of interphase around the fiber was separate from the disc of interphase in front of the fiber, so that each of them could be set to vary from each other. This became the key variable for most tests performed on the SSLIP models. Specifically, most attempts to
match the ABAQUS shear lag model with the Cox model (16) meant decreasing the elastic modulus of the interphase cap (disc) by several orders of magnitude. This basically forced all load transfer to occur on the interface between the interphase shell and fiber radial surface. For most of these tests, the interphase shell properties were set to those of the matrix, which in a way simulated not having an interphase shell region. The Poisson’s ratio of the interphase cap was set to that of the matrix also. A closer look at the top of the fiber surface after analysis of SSLIP provides the view seen in Figure 24(a), while a plot of the stress distribution along the topmost surface path on the fiber produces the graph in Figure 24(b).

![Figure 24](image)

**Figure 24**: Shear stress 2-3 on fiber in Symmetric Shear Lag model, shown with cross-section view. (a) View of fiber; (b) Stress distribution (shear 2-3) along topmost path on fiber surface.

**SSLIP2**

As seen in SSLIP, even with increased fiber mesh, there was “noise” in the shear stress distribution along the top of the fiber (see Figure 24b). These disturbances in the distribution(s) were thought to be “fiber end effects” or simply “end effects.” It became
highly desired to mitigate these end effects and the first step taken was to modify the
geometry of the interphase regions in SSLIP. Figure 25 displays an exploded schematic
of the second of the SSLIP models called “Symmetric Shear Lag model with Interphase
number 2” or “SSLIP2.”

Figure 25: Exploded diagram of SSLIP2. Not shown to scale.

Figure 25 can be compared with Figure 20 to reveal the contrasting geometry for
the interphase. Specifically, the interphase shell was made shorter in length in the Z-
direction such that it was the same length as the fiber. Subsequently, the interphase cap
was given a greater radius such that it would cover the top surfaces of the fiber and
interphase shell. The dimensions of the matrix cap, matrix shell, and fiber remained the
same as in SSLIP. Assembly was carried out and surface ties were applied
correspondingly. SSLIP2 retained all other definitions and parameters that SSLIP did,
including material properties, section assignments, boundary conditions, loads, and meshing.

**SSLIP3**

It was found from SSLIP2 that some noise still occurred near the end of the fiber shear distribution. Although it was considered an improvement, the end effects required further alleviation. The development of a third SSLIP model (“SSLIP3”) took the interphase geometry to a different level. Instead of overlapping the interphase shell and cap, each were designed to meet at a 45 degree angle, while again maintaining the same geometry for the fiber and matrix parts. Figure 26 gives a depiction of the changes.

![Figure 26: Exploded diagram of SSLIP3: Not shown to scale.](image-url)
SSLIPANN

The SSLIP model was modified one last time using substantial changes. Basically, the front half of SSLIP was cut off on the XY plane and removed from the model. This allowed the creation of only three parts, to be assembled in a purely annular fashion. The fiber was left the same length it had been in SSLIP, the matrix and interphase shells were given the same length as the fiber, and no interphase or matrix caps were included on the model. A schematic can be seen in Figure 27.

![Diagram of SSLIPAnn](image)

Figure 27: Exploded diagram of SSLIPAnn. Not shown to scale.

This was carried out for several reasons. First, it was an attempt to mitigate end effects again. Second, it was an attempt to avoid Poisson contraction effects. Lastly, it
was an attempt to transfer all load through shear. These ideas will be discussed in the Results.

**QSIP**

Another great transition is made between models here. This is important to note because most all important conclusions were made from the following four models, as will be discussed later. This success was possible due to a fundamental change in the design process of the models using ABAQUS. Also, Xia, et al. (20) provided a more comprehensive comparison of Finite Element Analysis and shear lag theory than had been found in any previously resources. Xia (20) determined that a 30 degree wedge of a cylindrical fiber concentric with a cylinder of matrix was the smallest allowable reduction of the model that could properly allocate symmetry. QSIP stands for “Quarter Section with Interphase” and was created with Xia’s model (20) as an inspiration. A quarter section was used first in order to reproduce something similar yet unique to the published model. Second, building the model this way permitted certain surfaces to receive ABAQUS’s built-in symmetry capabilities for boundary conditions, since a 30 degree wedge would have one surface not lying in a principal plane. Figure 28 shows two planar views of the 90 degree wedge model known as QSIP.
Again, the view of the YZ plane depicts that the model omits half of the axial length of the initial Embedded Fiber. The parameters defined in this figure are $L$ for the half-length of fiber and $L_{\text{cap}}$ for the length of the matrix cap. The material designations can be further explained in Figure 29.
Note that there is no interphase cap created in QSIP. This was done to simplify geometry near the end of the fiber, closest to the loading end, thus possibly mitigating end effects. Originally, QSIP did not have this matrix cap, and the addition of the cap will be explained in Results. Since Xia (20) did not utilize any kind of cap like this, this was also slightly experimental and helped to distinguish the QSIP model from already published models.

The fundamental design difference between QSIP and SSLIP was as follows: while SSLIP (and the initial RVE) was created using multiple parts in ABAQUS, QSIP was sketched using a single part. This part was then given volumetric “partitions” that allowed the part to stay intact while giving more options for section assignments. Material properties were created and assigned to material sections as in previous procedures and then these material sections were assigned to the volume partitions. What this did was eliminate the need for surface contact constraints, since only one “part instance” was required for assembly. This was a more natural way of creating a no-slip condition between materials because they always shared the same nodes without additional input to define. Arguably, the results should have been the same, but this was not the case and may be due to peculiarities in ABAQUS’s 3-D model creation.

Following part creation and assembly, boundary conditions were applied. The usual ZSymm was applied to the back surface where the initial Embedded Fiber Model would have inherently continued, symmetric to the XY-principal plane. Again, ZSymm provided zero translation in the Z-direction, and zero rotation about the X- or Y-axes for any point located on the applicable surface. Similarly, XSimm was a boundary condition
that allowed for zero translation in the X-direction and zero rotation about the Y- or Z-axis for any point on the applied surface. Furthermore, YSimm allowed zero translation in the Y-direction and zero rotation about the X- or Z-axes for any points on an applied surface. Consequently, XSymm was applied to the surface coincident with the YZ-plane and YSimm was applied to the side surface coincident with the XZ-plane. Now, symmetry could be maintained on all virtual surfaces created from cuts. Additionally, the corner point coincident with the principal origin was given an encastre condition (no translation or rotation) and the axis of the wedge was given an antisymmetry condition for the Z-direction, called ZASymm (no translation in X or Y and no rotation about Z-axis). These two boundary conditions were applied as precautions even though the junction of the symmetry constraints should have created the same outcomes.

Four loading scenarios were created and gave results that were analyzed, not discussed here. The scenarios included surface pressure and displacement conditions. Either type of load was either applied to the entire front face of the model, or to the outer (shell) face of the matrix. The most frequently utilized of these conditions was displacing the face of only the matrix shell, as this provided the best results. The goal of not loading the matrix cap was to not transfer load into the XY-face of the fiber nearest the loading end, an assumption of shear lag discussed previously.

Meshing was strategically employed such that the matrix did not contain an excessive number of elements and the fiber contained finer mesh in the Z-direction where it was important to analyze node paths. Also, the interphase region was given finer mesh to allow better transition for the load from matrix to fiber. Again, it was possible to
attribute different material properties to the interphase for different tests. For example, the interphase was at times set the same as matrix, essentially excluding it from the model, or it was sometimes set as an average between fiber and matrix constituents. Mesh elements were 3-D hex types and possessed quadratic geometric order (C3D20R). Model QSIP produced 18,144 of these elements. The mesh is readily apparent in Figure 30.

Figure 30: ABAQUS screen capture showing QSIP mesh.
HSIP

The model QSIP was very successful, but it had one flaw: it was not applicable for an off-axis loading case in the manner needed. There was too much assumed symmetry for a surface load to be directed anywhere other than in the Z-direction. For example, consider a load placed on the XY-plane in Figure 30, with components in the positive Y- and Z-directions. The load would still apply when reflected over the YZ-plane but would attain a negative Y component if reflected over the XZ-plane, indicating that an off-axis load cannot be modeled by quarter-symmetry. For this reason, the model was changed to a half-symmetry model, or half of the SSLIP model but without the interphase cap. This model was named “Half Section model with Interphase” or “HSIP.”

A schematic of HSIP is displayed in Figure 31.

![Schematic of HSIP](image)

Figure 31: General diagram of HSIP. Not shown to scale.

Thus the half section model was created similar to QSIP. Interphase was built in as a shell around the fiber but not as a cap in front of the fiber. A cap of matrix was
created on the free surface end (loading end) of the model where \( z = L \). The exploded diagram to explain the material composition is Figure 32.

![Exploded diagram of HSIP](image)

Figure 32: Exploded diagram of HSIP. Not shown to scale

Like QSIP, only one ABAQUS part was created for this model. It was then divided using partitions and material sections were assigned to their proper volume partitions and one assembly instance was created. Boundary conditions were applied although HSIP required one less condition than QSIP. ZSymm was applied on the back surface (XY principal plane), XSymm was applied on the side surface (YZ principal plane), encastré point was placed on the fiber surface coincident with the coordinate origin, and ZASymm was defined along the axis of the fiber (including the matrix cap).
A displacement load was applied upon the front surface of the matrix shell. Meshing was performed similar as before, coarse in the matrix, fine in the fiber with appropriate grading, and fine in the interphase. Once again, mesh elements were 3-D hex shaped (C3D20R) and possessed quadratic geometric order. There were a total of 26,233 elements for this model. A picture of the meshed part in ABAQUS can be seen in Figure 33.

Figure 33: ABAQUS screen capture showing HSIP mesh.
The final version of HSIP was renamed HSIPOFFAXIS after a small change was made in the boundary conditions to allow off-axis loading. The encastre point was relocated to the lowest point on the matrix shell rear face (XY principal plane). This was done to eliminate distortion in the axial normal stress distribution due to the traction forces applied in the off-axis case. For a load directed purely in the axial direction, this relocation was an insignificant change. Since this encastre point was originally applied merely to maintain static equilibrium in the model, it was not obligated to any one point. Therefore, it was considered acceptable to be placed in the far-field of the matrix where any nodal distortion would not negatively affect the analyzed node paths. For simplicity, this model will still be referred to as HSIP. An alternative schematic of the loading scenario is shown in Figure 34.
Figure 34: Schematic of off-axis loading.
Note in this diagram, another parameter is defined. The parameter $z'$ indicates the distance from the free end of the fiber which is equal to fiber half length $L$ minus principal distance $Z$.

**HSNF**

HSIP was the final and most important model in this study, save for one other that needed to be created. While HSIP was needed for direct comparison to the theoretical shear lag model, the final new model to be created needed to show the effects of scaling fiber size. In other words, the new model needed to be comparable to HSIP, since it was not comparable with any published results found. As was found from early tests on the Initial RVE, nothing was gained by simply changing the dimensions to a smaller order of magnitude. It became required that the model relating size effects should possess multiple fibers with smaller diameters. The key to this was maintaining the same cross-sectional area of fiber so that the volume fraction of fiber within the model was the same as HSIP while the surface area would be increased. This relation was proved in previous sections and so it emerged as the vital element to be implemented.

Working off the success of HSIP and bearing in mind the need for comparison, the new model was naturally obligated to be similar in structure to HSIP. If SSLIP was completely axi-symmetric, then HSIP was axis-symmetric for a rotation of 180 degrees. The new model, containing multiple fibers required to be cyclic-symmetric for a rotation of 180 degrees, since it could not be axis-symmetric. The result was a model with the half-cylinder shape of HSIP, a central fiber, and three other fibers in radial proximity of the central fiber. All four (partial) fibers contained the same diameters and maintained
equal spacing from each other. This model was allotted the name “Half Section model with Nano-Fibers” or “HSNF” for short. A schematic is visible in Figure 35.

The coordinate system was placed congruently with that of HSIP. The view of the XY plane in Figure 35 clearly shows the placement of the fibers. The central fiber’s axis is concentric with the matrix cylinder’s axis. The axes of all outer fibers occur in tangential fashion on the circular outer surface of the model, and run parallel to the central axis. The material breakdown can be viewed in Figure 36.
Figure 36: Exploded diagram of HSNF.

Noted on this model is that no interphase zones were created; however, this was marked as a possibility for future work. Also noted was the creation of multiple matrix caps. Again, only one part was sketched and extruded, followed by dividing into multiple volume partitions. The material sections were then applied to their appropriate partitions. For boundary conditions, ZSymm was applied to the back surface, XSymm to the side surface, ZAsymm was applied to all fiber axes, and the point encastre was placed on the lowest point of the back surface. The point encastre was placed there even though the point constituted fiber whereas in HSIP it was matrix. This was determined reasonable since this lower fiber would behave similarly to the upper fiber, therefore analysis of the upper fiber would suffice for both. A displacement load was applied to the front surface of the matrix material not including matrix caps. This meant that the shape of the load differed from HSIP, although geometrically it covered the same surface
area. However, the magnitude of the displacement was not set the same as that of HSIP, but rather it was set to a value that resulted with equality between total reaction forces on the $z = 0$ surfaces of both models, thus keeping the total load on both models the same. The displacement was initially directed in the axial direction. However, off-axis components were easy to apply and were investigated. The mesh was created in similar fashion to HSIP, such that the fibers utilized a gradient seeding, producing finer mesh near the loading end, and the matrix contained a coarser mesh than the fiber. Mesh elements were 3-D hex shaped and possessed quadratic geometric order. Again, 26,233 elements of C3D20R type were created. Once everything was in place, the simple single-step job was carried out and analyzed. Two views of the meshing are provided as Figure 37.

![ABAQUS screen captures showing HSNF mesh.](image)

The geometric differences between HSIP and HSNF are better viewed with the side-by-side comparison shown in Figure 38 below.
Figure 38: ABAQUS screen captures comparing model meshes: (a) View of HSIP mesh; (b) View of HSNF mesh.

A summary of the models with their reasons for modifying, boundary conditions, loadings, and pertinent material properties and dimensions can be seen in Table 3.

<table>
<thead>
<tr>
<th>Model</th>
<th>Impetus</th>
<th>BC's</th>
<th>Loading</th>
<th>$E_f$ and $v_f$</th>
<th>$E_m$ and $v_m$</th>
<th>Fiber Diam.</th>
<th>Fiber Vol. fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>Desired to create repeatable unit cell and unique analysis.</td>
<td>Symmetry on 3 adjacent (principal) faces, pinned point on origin, surface ties between constituents</td>
<td>Displacement or pressure load (tensile) on $z=L$ face of matrix, fibers, or both</td>
<td>150 GPa 0.24</td>
<td>80 GPa 0.40</td>
<td>14 units ($\mu$m presumably)</td>
<td>38%</td>
</tr>
</tbody>
</table>

Table 3: Summary of models.
<table>
<thead>
<tr>
<th>Initial Embedded Fiber</th>
<th>Comparable to published results and simplified geometry.</th>
<th>Symmetry on XY matrix face (z=-L), encastre point at center of matrix face (z=-L), constituent surface ties</th>
<th>Displacement or pressure load (tensile) on z=L+L_{cap} face of matrix</th>
<th>150 GPa 0.24</th>
<th>80 GPa 0.40</th>
<th>14 units (μm presumably)</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSLIP</td>
<td>Fixed numerical program errors and utilized symmetry for more efficient computation and meshing.</td>
<td>Symmetry on principal XY face of all constituents, encastre point at center of fiber (z=0), constituent surface ties</td>
<td>Displacement of full matrix face (z=L+L_{cap})</td>
<td>150 GPa 0.24</td>
<td>80 GPa 0.40</td>
<td>14 units (μm presumably)</td>
<td>11%</td>
</tr>
<tr>
<td>SSLIP2</td>
<td>Modified interphase zone for potentially cleaner shear stress distribution.</td>
<td>Symmetry on principal XY face of all constituents, encastre point at center of fiber (z=0), constituent surface ties</td>
<td>Displacement of full matrix face (z=L+L_{cap})</td>
<td>150 GPa 0.24</td>
<td>80 GPa 0.40</td>
<td>14 units (μm presumably)</td>
<td>11%</td>
</tr>
<tr>
<td>SSLIP3</td>
<td>Modified interphase zone for potentially cleaner shear stress distribution.</td>
<td>Symmetry on principal XY face of all constituents, encastre point at center of fiber (z=0), constituent surface ties</td>
<td>Displacement of full matrix face (z=L+L_{cap})</td>
<td>150 GPa 0.24</td>
<td>80 GPa 0.40</td>
<td>14 units (μm presumably)</td>
<td>11%</td>
</tr>
</tbody>
</table>
Table 3-Continued

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Symmetry on principal face of all constituents, encastre point at center of fiber (z=0), constituent surface ties</th>
<th>Displacement of matrix face (z=L)</th>
<th>150 GPa</th>
<th>80 GPa</th>
<th>14 units (μm presumably)</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSLIPAnn</td>
<td>Modified interphase zone for potentially cleaner shear stress distribution, and improved meshing capability.</td>
<td>Symmetry on principal face of all constituents, encastre point at center of fiber (z=0), constituent surface ties</td>
<td>Displacement of matrix face (z=L)</td>
<td>150 GPa 0.24</td>
<td>80 GPa 0.40</td>
<td>14 units (μm presumably)</td>
<td>11%</td>
</tr>
<tr>
<td>QSIP</td>
<td>Comparable to published 3-D model, improved meshing capability,</td>
<td>Symmetry on principal planes, antisymmetry on fiber axis, encastre point at z=0 of fiber axis</td>
<td>Displacement of matrix face (z=L+L_cap) excluding face of matrix cap</td>
<td>150 GPa 0.24</td>
<td>8 GPa 0.40</td>
<td>14 units (μm presumably)</td>
<td>11%</td>
</tr>
<tr>
<td>HSIP [OFFAXIS]</td>
<td>Decreased usage of symmetry based on need for off-axis loading.</td>
<td>Symmetry on XY and YZ faces, antisymmetry on fiber axis, encastre point at (0,-21,0)</td>
<td>Displacement of matrix face (z=L+L_cap) excluding face of matrix cap</td>
<td>150 GPa 0.24</td>
<td>8 GPa 0.40</td>
<td>14 units (μm presumably)</td>
<td>11%</td>
</tr>
<tr>
<td>HSNF</td>
<td>Utilized multiple fibers for investigation of surface area to strength.</td>
<td>Symmetry on XY and YZ faces, antisymmetry on fiber axes, encastre point at (0,-21,0)</td>
<td>Displacement of matrix face (z=L+L_cap) excluding faces of matrix caps</td>
<td>150 GPa 0.24</td>
<td>8 GPa 0.40</td>
<td>8.2 units (μm presumably)</td>
<td>11%</td>
</tr>
</tbody>
</table>
RESULTS

Aboudi Micromechanics

The primary purpose of our investigation into the Aboudi theory was to examine the effects of geometry on the stress states of composite materials. The objective of examining geometric effects was pursuing possible advantages of utilizing nanofibers within composite materials. For this reason and for simplicity sake, temperature and viscous effects were neglected from the systems of equations. The resulting systems of equations were very complex and were better suited for a computer program than application by hand. However, the equations were manipulated such that geometry effects could be observed without difficulty.

After expanding the systems of equations, all cases came to a similar finding. Since the expanded equations were too complex to fit elegantly on one page, one equation is presented here that illustrates the common result quite clearly (see Appendix A for a more complete presentation of equations). In the case of $S_{12}^{(βγ)}$, the matrix constituent of $(βγ)=(21)$ gives the following stress equation in its most basic form:

$$S_{12}^{(21)} = c_{44}^{(21)} \left( \frac{\partial W_2}{\partial x_1} + \phi_1^{(21)} \right) \quad (19)$$

In general for these systems, $c_{ij}^{(βγ)}$ represents one of an array of equations that contain material properties (i.e., Young’s modulus, Poisson’s ratio, shear modulus, etc.), the partial derivatives are strain terms, and $\phi_i^{(βγ)}$ represents one of another system of equations which include more material property, strain, and geometry terms. Definitions
for these terms can be found in Appendix A. Then, substitution of given definitions results in the following equation:

$$S_{12}^{(2i)} = c_{44}^{(2i)} * \left\{ \frac{\partial w_2}{\partial x_1} h_1 \frac{\partial w_1}{\partial x_2} + \frac{\partial w_1}{\partial x_1} h_2 \frac{\partial w_2}{\partial x_2} - h_1 * \left( \frac{\partial w_1}{\partial x_1} c_{44}^{(m)} h_1 + \frac{\partial w_2}{\partial x_2} c_{44}^{(m)} h_2 + \frac{\partial w_1}{\partial x_1} c_{44}^{(m)} h_2 - \frac{\partial w_2}{\partial x_2} c_{44}^{(f)} h_2 \right) \right\} \right\}$$

(20)

This equation shows the extent of geometry terms on the stress. However, the effects can be shown more clearly in the following representation of the equation:

$$S_{12}^{(2i)} = (...) * \left\{ \left( \frac{h_1}{h_2} \right) (...) + (...) - h_1 * \left( \frac{h_1}{h_2} \right) (...) \right\}$$

(21)

The (...) represents any quantity that is not directly geometry related. What can be identified from this equation is that all geometric dimensions appear in the form of a length ratio. From this, as well as the other six stress equations, it can be stated that the stress is a function of the ratio \(h_1/h_2\). This is important when considering the volume fraction of fiber to matrix. Consider that a proposed advantage to using nanofibers instead of conventional sized fibers is to increase strength while maintaining the same volume fraction. According to this micromechanics theory, an increase in strength cannot be obtained from scaling fiber size while maintaining the same fiber volume fraction. Volume fraction of fiber for this model is defined as follows:

$$V_f = \frac{h_1^2}{(h_1 + h_2)^2}$$

(22)
It can be seen that if the volume fraction is maintained while scaling $h_1$ down, this will directly affect the value of $h_2$. Consequently, the ratio $h_1/h_2$ in the micromechanics equations will remain the same and therefore not affect the strength at all.

Aboudi’s micromechanical formulation for composite materials does not directly suggest any advantage for utilizing nanofibers instead of conventional fibers. In other words, higher strength-to-weight ratios cannot be obtained from scaling fiber size down at constant volume fraction. However, the theory states that continuity of displacements and tractions were applied at the interfaces between the subcells and between neighboring cells on an average bases in order to satisfy equilibrium. In other words, perfect bonds were assumed between fiber and matrix. Aboudi also suggests that interfacial strength may need to be considered. It has been suggested through other studies that surface area ($A_s$) of fiber plays an important role in strength of the composite. An advantage to using nanofibers is an increase in surface area, which can help compensate for imperfect bonding. Figure 2 illustrates the concept of maintaining the same fiber cross-sectional area while increasing fiber surface area.

However, it is concluded that this strength advantage is not apparent in the Aboudi micromechanics theory. The assumptions made therein do not allow for any effects from surface area. Thus, another theory must be found to support the use of nanofibers in composite materials.
Theoretical Model Results

As described in the previous chapter, several different theoretical models were used for comparisons with each other, as well as with the FEA model which will be seen shortly. A table giving a brief overview of the models used and their significant results is provided as Table 4 below.

Table 4: Overview of theoretical model descriptors.

<table>
<thead>
<tr>
<th>Analysis Descriptor</th>
<th>Impetus</th>
<th>Information Gained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aboudi method of cells</td>
<td>Basic micromechanical behavior model, exhaustive mathematical analysis</td>
<td>Unrealistic geometry, not comparable with desired FEA models, no fiber scaling effects found from equation manipulation</td>
</tr>
<tr>
<td>Cox shear lag</td>
<td>Fundamental/pioneering model for shear lag theory, simple parameter definitions</td>
<td>Rarely produced results accurate with other theoretical and FEA models, consistently underpredicted all other models for stress results, resulted in high stress transfer lengths in all cases seen</td>
</tr>
<tr>
<td>Nairn shear lag</td>
<td>More recent than Cox model, more detailed parameter definitions</td>
<td>Produced useable results for comparison with FEA model, consistently overpredicted stress results of all other models, produced very accurate stress transfer lengths compared to FEA model and Nairn reference figures</td>
</tr>
<tr>
<td>Chon/Sun shear lag</td>
<td>More recent than Nairn model, implementation of 3-D model, more comprehensive with parameter options, further revised parameter definitions</td>
<td>Produced results accurate with FEA model, results were bound between Cox and Nairn models, stress transfer lengths appeared very accurate, scenarios for different loading angles produced verifiable results, 3-D model geometry compared well with FEA model geometry</td>
</tr>
</tbody>
</table>
As mentioned in the previous chapter, the first significant result desired was to match up the theoretical MathCAD (MC) model with published results to see that the program was set up correctly. Some changes were made that included converting sign conventions and deciphering parameter names between different authors’ work, but comparable results for 50 percent transfer lengths were obtained in the end. Using the same material and geometry parameters set by Nairn (18), Table 5 shows the results of this effort.

### Table 5: Comparison of 50% transfer lengths in terms of number of fiber diameters between Nairn (18) published values and MathCAD (MC) analyses at 11% fiber volume fraction.

<table>
<thead>
<tr>
<th>(E_f/E_m)</th>
<th>Ref-Nairn (est.)</th>
<th>Nairn (MC)</th>
<th>% Diff.</th>
<th>Cox (MC)</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of fiber diameters</td>
<td>Number of fiber diameters</td>
<td>Ref. to Nairn (MC)</td>
<td>Number of fiber diameters</td>
<td>Ref. to Cox (MC)</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.422</td>
<td>15.6</td>
<td>0.736</td>
<td>-47.2</td>
</tr>
<tr>
<td>10</td>
<td>0.67</td>
<td>0.645</td>
<td>3.7</td>
<td>1.207</td>
<td>-80.1</td>
</tr>
<tr>
<td>20</td>
<td>0.75</td>
<td>0.716</td>
<td>4.5</td>
<td>1.324</td>
<td>-76.5</td>
</tr>
<tr>
<td>200</td>
<td>0.9</td>
<td>0.806</td>
<td>10.4</td>
<td>1.449</td>
<td>-61.0</td>
</tr>
</tbody>
</table>

First, what can be seen from Table 5 is that different ratios of fiber elastic modulus to matrix elastic modulus were investigated. This is the same procedure followed by Nairn. Although Nairn experimented with different volume fractions at times, this study is interested in keeping the same volume fraction. To reiterate the purpose of this study, keeping the same volume fraction was desired to produce a composite of the same weight, however with increased strength as a result of optimizing other conditions such as geometry. Therefore, all tests performed utilized the same
volume fraction of fiber to matrix. That volume fraction was determined to be approximately 11 percent.

The reference values taken from Nairn’s publication were estimates from a figure using fiber volume fraction as its control variable, 50 percent transfer distance for the dependent variable, and the curves were specific to modulus ratios. Considering the size of the chart in the publication, to obtain MathCAD outputs for the Nairn setup within five percent of the estimated value was considered extremely reasonable. The reference chart did in fact include a curve for modulus ratio of 10 utilizing $\beta_{\text{cox}}$, but the line was off the chart for the volume fraction used, so a percent difference of 80 percent was not surprising. Also not surprising was that the negative sign, which shows the Cox coefficient consistently over-predicting the transfer length which is the same as is shown on the reference chart. The Nairn outputs from MathCAD consistently under-predict the transfer length, so all in all the results seemed very consistent.

The conditions used by Nairn are here noted along with any other assumptions. Fiber diameter was 1 mm, fiber length was 10 mm, matrix diameter of 3 mm was utilized to maintain 11 percent fiber volume fraction, matrix elastic modulus $E_m$ was 2500 MPa, matrix Poisson’s ratio was 0.333, fiber Poisson’s ratio was 0.25, and a nominal strain of 0.033 was assumed, as none was found to be given in the publication.

Next, having verified that the theoretical model was performing adequately, material dimensions and property values were set to those that had already been implemented in the FEA program. Fiber volume fraction remained at 11 percent, but the fiber diameter was set at 14 $\mu$m, then the matrix diameter was 42 $\mu$m, the fiber (half)
length was set to 25 μm, fiber modulus was set to 150 GPa, fiber Poisson’s ratio was 0.24, the matrix Poisson’s ratio was set to 0.4, and the same nominal strain of 0.033 was used. With this setup, 50 percent transfer length values were calculated and recorded from MathCAD and from the FEA model QSIP. The comparisons are given in Table 6.

<table>
<thead>
<tr>
<th>$E_f/E_m$</th>
<th>Nairn (MC)</th>
<th>Cox (MC)</th>
<th>% Diff.</th>
<th>ABAQUS</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fiber diameters</td>
<td>Number of fiber diameters</td>
<td>Nairn (MC) to Cox (MC)</td>
<td>Number of fiber diameters</td>
<td>Nairn (MC) to ABAQUS</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.364</td>
<td>0.460</td>
<td>-26.4</td>
<td>0.286</td>
<td>21.5</td>
</tr>
<tr>
<td>10</td>
<td>0.446</td>
<td>0.510</td>
<td>-14.3</td>
<td>0.446</td>
<td>-0.1</td>
</tr>
<tr>
<td>20</td>
<td>0.459</td>
<td>0.516</td>
<td>-12.4</td>
<td>0.454</td>
<td>1.2</td>
</tr>
<tr>
<td>200</td>
<td>0.474</td>
<td>0.522</td>
<td>-10.1</td>
<td>0.493</td>
<td>-4.0</td>
</tr>
</tbody>
</table>

Once again, it can be seen that the control variable observed is the modulus ratio (although Nairn manipulated the fiber modulus to produce the ratios, in ABAQUS the matrix modulus was manipulated to obtain the same values). Consistent with Table 5, the best percent differences occur in the elastic modulus ratios ranging of order 10. Surprisingly, the transfer lengths using Cox’s coefficient were significantly better when using these material parameters, although still not quite sufficient. The percent differences between the ABAQUS model and the theoretical model with $\beta_{Nairn}$ is very agreeable. Nairn reported to have the best results at a modulus ratio of approximately 19, and this agrees well with the theoretical MathCAD model and the ABAQUS model,
which gave confidence in the model for investigating on- and off-axis analysis with the FEA model.

**Finite Element Analysis Results**

**Early Model Results**

Some results of the earlier FEA models were touched on during the description of the model progression in an effort to explain the logic behind the changes and decisions made. These results will be reiterated here and further explained as necessary. Starting with the Initial RVE model, the main problem arose with constraining the sides so that they were allowed to contract in size, without losing their shape. Further work with this model could result in improvements including elimination of contact controls, material sections born from partitions rather than multiple part assemblies, and surface coupling conditions. It should also be noted that early attempts at creating nanoscale models were attempted using this geometry setup. The procedure included creating a model with the exact same geometry but with scaled-down dimensions. The lesson was learned, that this would not be sufficient to prove any kinds of scaling effects, because the results showed the same values of maximum principal stress. This could have been predicted using the outcome of the Aboudi micromechanical analysis. It was shown that the actual shapes of the constituents’ geometry would be required to change if any different results were to be found.
SSLIP Results

Here again the results are listed fairly briefly because the SSLIP models exhibited less than conclusive details, but more provided a learning experience. The Embedded Fiber model, although unrefined at the time, was an acceptable setup for an RVE but the computational time led to the conclusion that there was redundant volume being used in the model. SSLIP was the answer to this problem by reducing the volume in half. After refining the mesh to cover the fiber surface more thoroughly, results began to take the hyperbolic sinusoidal shapes that had been sought after. Two examples of ABAQUS screen captures are provided as Figure 39.

![SSLIP model results](image)

Figure 39: SSLIP model results, shown in cross section view: (a) Max principal stress; (b) Shear stress in 2-3 plane.

Several things need to be observed in Figure 39, bearing in mind that these are section views and that SSLIP was designed to contain a full cylinder with half the length of the Embedded Fiber model. The max principal stress is shown in Figure 39(a) and the
2-3 shear stress in Figure 39(b). Note that 38(a) displays an improved meshing from the older test run for 38(b). Additionally, Figure 39(b) provides a deformed representation of the fiber, which displays the interphase cap greatly expanding in volume. This was due to defining a significantly lower elastic modulus than the other constituents. This was explained previously (see Models chapter under SSLIP) but is reiterated because of its key importance to these analyses. Lowering the elastic modulus of the interphase cap was an attempt to maintain the shear lag assumption that no load was to be applied on the front surface of fiber, such that all loading was transferred through interfacial shear. Figure 24 (see Models chapter under SSLIP) displays some results of an SSLIP test that started to show the desired trend.

However, even through high mesh refinement the curve in Figure 24(b) would not smooth out like desired. After some random mistakes were found and corrected, the results seemed to worsen and the good results were not repeatable. This led to experimentation with the different interphase configurations, with the unfortunate result of not improving the model.

The most unconstructive effect on results though was not the noise in the data curve but the bizarre Poisson contraction effects that happened nearest the loading end of the fiber. The shear distribution curve would oftentimes dip down, even into the compressive zone for the nodes near the front of the fiber. Little explanation could be offered for this since the logical explanation would point that the tensile load should displace all nodes in the same direction as the node. This was the more prominent “end effect” on the fiber that had to be dealt with in order to produce a viable FEA model.
Two resolutions presented themselves at this time. The idea to eliminate the interphase cap and then not load the matrix in front of the fiber was the first solution. The other came as inspiration from Xia, et al. (20). That decision was to make the node connection between constituents more efficient such that obscure surface ties would no longer be needed.

**QSIP Results**

Upon creating QSIP and tailoring its new boundary conditions and section assignments, it was discovered that this was the most effective model thus far at producing the desired curves for shear stress along the top fiber nodal interface and axial stress along the center axis of the fiber. Figure 40 displays a view of the fiber alone and its shear distribution curve.

![Figure 40: QSIP results for shear stress 2-3: (a) View of fiber; (b) Nodal values on top interface.](image)
It was observed that the curve still produced a slight end effect on the front end of the fiber. This only occurred for the first node or two, and so it was concluded that a slight end effect was unavoidable. The model shown in Figure 39 utilized an interphase shell with matrix properties, thus effectively the model contained no interphase for this specific test. Also, the same test resulted in an axial stress distribution seen in Figure 41.

![Figure 41: QSIP results for axial stress 3-3: (a) Deformed view; (b) Nodal values along fiber axis.](image)

The deformed configuration in Figure 41 shows that only the outer matrix ring was loaded in tensile displacement. Also, this model contained a matrix end cap that is the foremost one-sixth of the inner partition. Some experimenting with different modulus ratios was performed for this model, particularly since it was the model being compared to MathCAD using 50 percent stress transfer lengths. Since Nairn mentioned (18) that a modulus ratio of about 19 was ideal for his published values, all tests run after this unless otherwise mentioned utilize 18.75 ratio of fiber elastic modulus (150 GPA) to matrix elastic modulus (8 GPA).
HSIP Results

After the decision was made to transform QSIP into HSIP, the transition was made and few differences needed to be accounted for before results were sought. Figure 42 shows a deformed configuration of HSIP displaying axial stress $S_{33}$.

![Figure 42: View of HSIP with deformed nodes, displaying axial stress $S_{33}$.

First, interpreting the results of HSIP meant comparing them to the previous model QSIP, which had thus far been the basis of comparison with the theoretical model. The results of HSIP and QSIP compared with theoretical models for axial stress can be observed in Figure 43.
Figure 43: Comparison of axial stresses for HSIP, QSIP, and theoretical values.

As can be observed, the models HSIP and QSIP produced normal stresses in the fiber axis in the z direction that overlap each other near perfectly. It was shown earlier that the 50 percent transfer lengths were nearly the same between QSIP and the Nairn based theoretical shear lag model. Here, the 50 percent transfer length is not readily visible, but since the stress that HSIP exhibits is nearly the same as that of QSIP, the 50 percent transfer length for model HSIP was sensibly assumed to be the same.

Also interesting to note was that the ABAQUS model results were bounded by the Nairn (above) and Cox (below) curves. This seemed to show that not only was the ABAQUS model producing acceptable stress results, but that there existed some merit in
both forms of the theoretical model coefficients. Next, Figure 44 displays the shear stress comparisons in the 2-3 direction on a top surface nodal path for the fiber.

![Graph of shear stress vs. fiber distance](image)

**Figure 44:** Comparison of shear stresses for HSIP, QSIP, and theoretical values.

It can be observed from this figure that HSIP and QSIP both overlap nearly exactly again, as well as appear bounded between the two theoretical models to a point. The end effects prevent the FEA shear from shooting above the Nairn result. The Nairn equation once again over-predicts the Cox equation due to the nature of the coefficient definition. It is important to note that the ABAQUS models display curvature that appears more reminiscent of the ideal curves displayed in Figure 10. An explanation can be offered here. Nairn states that using the shear lag method can give reasonable
estimates for stress transfer but does not work as well for low fiber volume fractions, does not work as well with displacement boundary conditions, and is probably too qualitative for calculations of shear stress (18). These issues can be addressed individually. First of all, an 11 percent volume fraction is probably approaching the lower limits of what could be considered useable volume fraction but it is still not too low. Second, displacement boundary conditions were used, but only because they produced smoother, more reasonable results than surface pressure (force) loads. Also, our RVE is not set up exactly like Nairn’s fiber/matrix annulus. And finally, the qualitative nature can be seen in the curvature but a little discrepancy in calculated results is reasonable due to the dissimilarity between the definitions of $\beta$ coefficients.

Now, moving on to the results for the off-axis loading, several different comparisons were made that gave conclusive outcomes. For previous results, most stress distributions produced by the FEA model were compared to the Nairn and Cox theoretical shear lag models. Off-axis loading was compared to the Chon and Sun version of shear lag model. Also, our FEA model was created based on the Chon and Sun (19) embedded fiber model, so it was appropriate to compare the two. However, at zero degree angle for load, all three theoretical models were compared to the HSIPOFFAXIS. Figure 45 gives the most basic comparison of axial stress along the fiber.
Figure 45: Comparisons of axial stress for off axis loads at $\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$.

Again, what is seen in Figure 45 are the theoretical curves for Nairn, Cox, and Chon/Sun and FEA results using HSIP, at various angles of displacement loading. The axial stress was normalized using a nominal applied stress which is described in the general shear lag model (17) as being the product of the fiber modulus and the nominal strain in the matrix. This was a very arbitrary value applied to all curves and thus does not change the nature of any of them relative to each other. It merely changes the scale of the dependent variable axis.
At $\alpha = 0^\circ$, the ABAQUS result fits the Chon/Sun curve much closer than it does either the Nairn or Cox curves. For the ABAQUS model and Chon/Sun model, different load angles were compared and superimposed on the same graph. Stress results were observed for load angles between 0 and 90 degrees at 15 degree increments, although Figure 45 omits the 15 and 75 degree cases to help reduce graph clutter.

It can be observed from the above figure that both the FEA model and theoretical model approach a flat stress distribution of zero as the angle increases. This matches the results found by Chon/Sun. It makes sense because a load completely perpendicular to the fiber should not produce any stress in the axial direction. When the load angle was 0 degrees, the axial direction was a principal direction (Z-direction) and so stress in the normal direction (Y-direction) was zero. Thus, load purely in the Y-direction makes this the new principal direction and the Z is normal and develops zero stress. At the different load angles between 0 and 90 degrees, the stress distributions show different slopes and different rates of change between the slopes. In other words, the Chon/Sun model overpredicts the FEA model until about 38 degrees, after which it underpredicts the FEA model. This phenomenon is apparent in Figure 46.
Figure 46: Comparison of off-axis angle vs. normalized axial stress at $z' = L$.

Figure 46 gives axial stress as a function of the load angle, alpha, instead of distance along the fiber in the Z-direction. Consequently, this data is taken at a single point from the Z-distribution which is the $z' = L$ end of the fiber (center of embedded fiber, furthest from the free face). This point was chosen because it is the point with the maximum axial stress seen and the stress is fully developed, meaning far away from any end effects.

As mentioned previously, Figure 46 gives a clear depiction of the relation of HSIP and Chon/Sun for different angles. Sun overpredicts the FEA model until about 38 degrees of load angle. Then, Sun underpredicts the FEA model, until they are equal again at alpha equal to 90 degrees. Chon/Sun provided a similar chart (19) and it was identical in behavior to the theoretical distribution based on their shear lag model found
in Figure 46. The FEA model has slightly different behavior, although maintains the same general trend. Oddly, the maximum axial stress slightly increased from 0 to 15 degrees. This was unexplainable but was considered negligible since the stress levels of Chon/Sun and FEA were so close and both occurred within the range of Nairn and Cox.

Next, the role of interphase was examined in the model. As stated before, the interphase zone had existed as a physical entity but it had been assigned matrix material properties for a more appropriate correlation with the shear lag model. When activated, the Young’s modulus and Poisson’s ratio of the interphase were set to the averages of the fiber and matrix constituents. For different load angles, HSIP with average interphase was compared to HSIP with no defined interphase in the next two figures. Figure 47 shows the axial stress of this comparison.

![Figure 47: Comparison of axial stress for off-axis loads on HSIP with and without interphase.](image-url)
What is seen in the previous figure is that the model, with average-valued interphase, correlates strongly with the no-interphase version. However, the average interphase model consistently overestimates the no-interphase model slightly for every load angle tested. This came as interesting news and led to a few major findings about the role of interphase. First of all, for the same boundary conditions and load, the average-interphase model attained a higher maximum axial stress. The interphase was hence shown to ease the transfer of stress between the constituents by bearing some residual stress in its own section. Also, for this model the end effects were virtually eliminated, as was apparent from the zero value of stress at the free end of the fiber ($z / L = 1$ end, nearest the loading). The graph also displayed a slightly decreased slope for the average interphase curves. This indicated a slight increase in the 50 percent transfer length. Figure 48 supports this change in a clearer fashion.

![Graph](image_url)

**Figure 48:** Comparison of axial stress on asymptotic scale for off-axis loads on HSIP with and without interphase.
For Figure 48, a modification was made to the stress axis, such that each axial stress distribution was normalized to its own maximum value, as opposed to normalizing to a universal value as done before. This placed each distribution on the same asymptote, and hence allowed the 50 percent stress transfer lengths to be read simply by finding the non-dimensional length corresponding to a non-dimensional stress of 0.5. The inclusion of interphase consistently added a small amount to the transfer length for all load angles. This again means that as the load in the fiber transitions from shear to normal stress, the change is made less abruptly by distribution of some load within the interphase entity at the cost of raising the maximum axial stress. Now to see some effects that interphase has on shear stress, Figure 49 is provided for different on- and off-axis cases.

Figure 49: Comparison of shear stress for off-axis loads on HSIP with and without interphase.
Important to note from the above graph is that the average-interphase model once again overpredicts the stress values for all load angles tested. Again, there are also greater fiber end effects occurring in the average-interphase model that cause drastic changes in slope. The average-interphase configuration causes the shear stress to transfer slower and more evenly along the fiber, as will again be apparent with Figure 50.

Figure 50: Comparison of shear stress on asymptotic scale for off-axis loads on HSIP with and without interphase.

Since the shear graphs displayed in Figures 49 and 50 reflect the same results as in Figures 47 and 48, a suggestion can be made about the role of interphase in a composite. Using an average of the material properties does not appear to suggest any
advantages in strength, but seems to increase the stress seen by the fibers. Since the ideal composite would approach a uni-axial strength determined by the bonding strength of the constituents, it may be more appropriate to treat the interphase with higher elastic properties than those of the constituents instead. This appears worthwhile as a potential future project.

**HSNF Results**

To reiterate the purpose of this study, the final FEA model was created as a scheme to prove that an increase in fiber surface area by decreasing fiber diameter results in an improvement in composite mechanical properties. As discussed previously, early attempts at rescaling the square-based RVE were inconclusive so the nanofiber FEA model was created to contain a greater number of fibers with equal diameters different than the previous model (HSIP). Upon loading HSNF with a displacement purely in the Z-direction and recalling that the magnitude was reduced in order to match reaction forces (total load) with HSIP, the results appeared very satisfactory. Figure 51 displays an undeformed view of the loaded HSNF along with the normal stress distribution given by an axial path in the central fiber.
As can be seen in the above figure, the normal stress maintains a hyperbolic form starting at near zero stress and increasing rapidly until approaching a plateau near the end. Similarly displayed is a picture of the fibers alone with the 2-3 shear stress and the corresponding interfacial distribution in Figure 52.
Figure 52b shows a strong, smooth curve with only minor end effects. Given this good outcome, the results of the HSNF distribution were compared with the results of the HSIP distribution. Since HSNF was not constructed with any interphase zones, for simplicity sake, all comparisons with HSIP were made with the no-interphase configuration. Figure 53 displays the asymptotic stress situation for the normal stress seen in the central fiber of HSNF compared with the single fiber of HSIP.

![Graph showing stress comparison](image)

Figure 53: Comparison of HSIP and HSNF axial stress on asymptotical scale for on and off-axis loads.

As can be seen in Figure 53, two different loads at $\alpha = 0, 45$ degrees are compared for the two different models, creating a total of four distributions. For both 0 and 45 degree angles, HSIP maintained nearly the same shape, as did HSNF for both angles respectively. However, there is significant difference in the behavior of HSNF relative to HSIP. Namely, both HSNF distributions approach their max stress value much quicker
than the HSIP distributions (however note that some end effects exist, such that the axial stress does not start at the origin). In any case, the model HSNF reaches a 50 percent transfer length in nearly half the distance as HSIP does. This was deemed a positive outcome (despite the small end effects). Figure 54 displays the axial stress of the same two models with non-normalized, absolute value axes, and the same trend is recognized.

![Graph comparing HSIP and HSNF axial stress](image)

**Figure 54**: Comparison of HSIP and HSNF axial stress with $\alpha = 0, 45$ degree loads.

Another observation of model success was that the HSNF model distribution attains a significantly lower maximum stress than HSIP does. Here again, the end effects were observed for HSNF as being high, and yet the distribution still ascended to a much lower stress state. This further supports crediting the behavior in Figure 53 for the improved 50 percent transfer lengths to be accurate. So, for the same fiber location
between models, the overall axial stress in the Z-direction was decreased with the modification addition of nanofibers.

Also notable in Figure 54 was the familiar trend that axial stress consistently decreased when the loading angle was increased to 45 degrees (only two angles were investigated since the embedded fiber model could not be well-correlated to this new multi-fiber model). In any case, this could be an idea for future work if so desired.

One other aspect of the HSNF model was desired to be checked. Since all observed stresses had come from the center fiber, it was desired to compare the outer fibers to the center fiber of HSNF. Due to symmetry, only one outer fiber was checked from the three, namely the fiber that appears below all other fibers from Figure 52, because this fiber produced a similar node path that could follow a top-surface interface of the fiber (for shear stress seen momentarily). The results of this investigation for the axial stress are displayed in Figure 55.
Figure 55: Comparison of HSIP and HSNF different fibers for axial stress with $\alpha = 0^\circ$.

For the previous graph, it can be observed that the bottom fiber of HSNF results in higher axial stress than the central fiber of HSNF and the single fiber of HSIP. Furthermore, the curve overshoots its asymptote and dampens back down. These effects were undesired but it was easily observed from the deformed model that unrealistic geometry deformation occurred at the surface of the matrix, primarily in proximity with the axes of the outer fibers. This was due to the antisymmetry boundary conditions imposed upon the outer fiber axes, because these nodes held place in the X- and Y-directions, while nodes in proximity of the antisymmetry nodes contracted from Poisson effects. It was out of the scope of this study to find a way to rectify this situation.

Next, the shear stresses in the YZ (2-3) plane were observed for the top interfacial fiber surfaces of the central fibers in HSIP and HSNF. Once again, two different angles,
$\alpha = 0$ and 45 degrees, were plotted for the two models. First using the asymptotic scales, Figure 56 displays these results.

![Graph showing shear stress for different angles](image)

**Figure 56:** Comparison of HSIP and HSNF shear stress on asymptotic scale with $\alpha = 0^\circ, 45^\circ$.

For this comparison, HSIP at both angles maintains a very close asymptotic nature. Additionally, the HSNF in the 45 degree case also matches closely with both HSIP curves. The only one of the four distributions that varies significantly is HSNF for zero degree displacement angle. For this curve, the shear stress transfers away slightly quicker than do the other three curves. This is another indication of decreased ineffective length. Moving on, a shear stress comparison displaying actual values is provided in Figure 57, which will lead to more useable results than the previous since the 50 percent transfer length was computed using the axial stress.
In this case, it can be seen that the HSNF clearly attains much lower shear stress across the distribution than HSIP. This proved very conclusively that the nanofiber model was advantageous over the single fiber model. The less shear stress seen by the fiber, the less it would need to transfer to another fiber. The increase in surface area truly benefited the stress allocation throughout the model. However, for the 45 degree displacement was negatively affected in the HSNF model. For HSNF, the shear curve of \( \alpha = 45 \) degrees could actually be seen to overpredict the curve of \( \alpha = 0 \) degrees for most of the curve and then converge on the same maximum stress near the beginning of the fiber (some slight end effects existed). This was unexplainable except that the HSNF model simply was not well suited for off-axis loading. Lack of possible constraints on
the outside surface of the model caused excessive distortion to the outer fibers which, in turn, transferred through to the central fiber. This distortion only seemed to hinder the case of the off-axis displacement though, so for the on-axis loading, the HSNF model was considered a great success.

Once again, it was desired to compare the central fiber of HSNF to another fiber from the same model. The same bottom fiber was used to represent the outer fibers, and the distributions were obtained from top surface paths on both. The 2-3 shear stress results are displayed in Figure 58.

Figure 58: Comparison of HSIP and HSNF different fibers for 2-3 shear stress with $\alpha = 0^\circ$. 
From Figure 58, it is observable that the two different fibers in HSNF see almost exactly the same shear stress. This was great news because it showed that the load distributed well throughout the matrix and deformation was of minimal effect. It was also important that the bottom-fiber results stay below the HSIP curve, which is easily accomplished. Finally, we re-plot the previous and most significant results, but only for the $\alpha = 0$ degrees on-axis loading in Figures 59 and 60.

![Graph showing comparison of HSIP and HSNF for axial stress with $\alpha = 0^\circ$ and no interphase.](image)

Figure 59: Comparison of HSIP and HSNF for axial stress with $\alpha = 0^\circ$ and no interphase.
Figure 60: Comparison of HSIP and HSNF for shear stress with $\alpha = 0^\circ$ and no interphase.
CONCLUSIONS AND FUTURE WORK

Conclusions

In summary, the project was labeled a success in accomplishing the goals set forth and conforming to the problem statement made. The use of nanofibers in modern fiber-reinforced laminates was thoroughly investigated, not without allowing room for future work, but thoroughly nonetheless. Clear advantages in the mechanical behavior of nanocomposites were unveiled. The main advantage that was indicated was an increase in mechanical strength of the composite in the direction of the fiber axes. This was due to several factors. The first factor was an increase in surface area of fiber, producing more interface for the fiber to bond and reinforce the matrix filler material. The other advantages were uncovered through literature research, showing that the ineffective fiber length is decreased with the utilization of numerous small fibers, the shear lag is relieved with closer proximity of fibers, and also that the statistical probability of damage due to fiber surface flaws would decrease.

The role of interphase was also explored during the course of this study. It was found that the interphase region, when set to an average of the two connecting constituents, would serve to even out the stress over the length of the fiber distribution path, with the end result of allowing slightly higher maximum stresses at the end of the fiber appropriate to the type of stress being observed. Since the composite’s strength would ideally approach the constituent bonding strength based on proper fabrication and lack of fiber surface flaws, the interphase should not necessarily be assumed to be an average of the two constituents, referring to material properties, but possibly have even
greater elastic modulus and Poisson’s ratio than considered. In other words, the interphase zone may distribute load differently depending on the material properties it is assigned.

Several observations were made when experimenting with the theoretical models in MathCAD. The model set forth by Nairn (18) appeared to always overpredict both shear and axial stresses against other theoretical models and the FEA models. Contrarily, the Cox model (16) appeared to always underpredict the stresses compared to other theoretical models and the FEA models. The theoretical model defined by Chon and Sun (19) was bound between the Cox and Nairn models for all scenarios examined (using zero degree loading angle).

In this study, FEA software was extensively employed to model suitable Representative Volume Elements for composite micromechanical behavior. The RVEs chosen correlated with published versions for comparison sake but were created with uniqueness to avoid infringement on others’ works and to achieve a pioneering example. The FEA models produced results most similar to the Chon and Sun theoretical model for shear lag. The stresses observed in ABAQUS were numerically the closest to Chon and Sun outputs from MathCAD which again were bound between the Cox and Nairn results.

The descriptor of choice used to quantify the transfer of load from shear to axial was the 50 percent transfer length. This was chosen in order to compare with published results from Nairn (18) which served to first validate the theoretical model created in MathCAD and subsequently verify the FEA model to the MathCAD model. An increase or decrease in the 50 percent transfer length implied an increase or decrease respectively.
in the ineffective fiber length. Since Rosen (11) defined the ineffective fiber length as the length of fiber needed to transfer 90 percent of the load from shear to axial stress, Nairn’s 50 percent transfer length was simply a modification of the same technique. Therefore, the results of 50 percent transfer length were interpreted as direct relations to the ineffective length. The results of 50 percent transfer length computations were very satisfactory for both the MathCAD model and FEA model. For fiber to matrix modulus ratios on the order of 10, transfer lengths were found to be less than five percent different between the Nairn reference and Nairn model created in MathCAD. The transfer lengths compared between the Nairn theoretical model and the FEA model HSIP for the same modulus ratios were determined to be less than two percent different. This meant that the FEA models and Nairn model were qualitatively similar, and both models proved to display the best results at similar modulus ratios. Furthermore, the 50 percent transfer lengths were observed when interphase was included in the model. It was found that the transfer lengths were increased when interphase was included, which advocates further study into different material property assignments of the interphase zone.

Off-axis loading scenarios were observed for angled displacements between 0 and 90 degrees. Some minor end effects were seen, but the stresses behaved appropriately and correlated with Chon and Sun results. At zero degree loading angle (fully loaded in the axial or Z-direction), the FEA model and Chon/Sun models resulted in similar axial stresses. For 90 loading angle, both models exhibited zero axial stress across the distributions. This was desired because loading occurring perpendicular to the axial direction would realistically produce zero axial stress. Accordingly, loading angles
between 0 and 90 degrees resulted in distributions that gradually changed appropriate to
the angle given, in which cases the FEA and Chon/Sun models still saw strong
correlation with each other.

Model HSNF was created to observe effects of scaling fiber size in a RVE. Although
the fibers were not proportional to “nanofibers” in relation to HSIP’s “conventional fiber size,” the scaling effects were still apparent due to a change in fiber
surface area. The increase of interfacial surface area, while applying the same load,
resulted in less shear stress seen by the fiber. Less shear stress meant that less load
needed to be transferred to axial stress. The decrease in axial stress seen by the nanofiber
model thus demonstrated a stronger uni-axial composite in tension, for the same volume
fraction as was designed in the conventional model.

The FEA program of choice was ABAQUS/CAE, which proved to have a
multitude of options and capabilities, sure to provide a learning experience for any who
use it. The FEA models created were deemed successes for their realistic result outputs
in many scenarios performed. Model HSIP was proficient at on- and off-axis loading,
compared very well to published shear lag theories, and incorporated an interphase zone
joining the fiber and matrix constituents if activated. Model HSNF was successful in
proving advantages for scaling fiber size, utilizing on-axis loading as the primary
function. The off-axis case was not well supported for this model but was tested anyway
resulting with few usable conclusions. Also, interphase was not applied in the HSNF
model because it was undesirable to explore at this time. Pending positive results from
South Dakota School of Mines and Technology, as well as any other collaborative
institutions, the Air Force Research Laboratory would be well advised to take advantage of further exploring and implementing nanofibers for any applicable aeronautical and space technologies.

**Future Work**

Several ideas have been gathered over the course of this study for future tasks concerning the modeling of nanocomposites. First of all, it is possible to further research interphase properties and effects, and explore more options within a Finite Element Analysis based RVE. The idea that the bonding strength between the constituents being far greater than the individual constituent strengths leads to the possibility that the interphase could be set up with higher elastic material properties than those used in this study. Also, the width of the interphase regions, whether shell or cap, were set at a single value for all models that utilized interphase and it would be interesting to see the effects of smaller or larger zones. Even gradient interphase properties could be another possibility.

Also, further exploration with the nanofiber model HSNF could be accomplished. In this study, it was used purely as a mode for exploring strength advantages for on-axis loading only. Advanced symmetry conditions and boundary constraints could lead to a much more effective model at checking off-axis loading.

Although this project utilized background information from several different theories, the most prominently used theory was that of shear lag. For the future, more could be done to associate the ineffective fiber length to the interphase region.


8. (Swiss National Science Foundation) Retrieved from Nanoscale Science: http://www.nccr-nano.org/nccr/media/gallery/gallery_01/gallery_01_03/index_html_print


APPENDIX A

ABOUDI MICROMECHANICS MANIPULATIONS
The definitions of the material property terms are as follows:

\[
\begin{align*}
c_{11} &= E_A + 4\kappa \nu_A^2, \\
c_{12} &= 2\kappa \nu_A, \\
c_{22} &= \kappa + \frac{E_T}{2(1 + \nu_T)}, \\
c_{23} &= \kappa - \frac{E_T}{2(1 + \nu_T)}, \\
c_{44} &= G_A, \\
c_{66} &= 0.5(c_{22} - c_{23}),
\end{align*}
\]

where \(E\) is Young’s modulus, \(\nu\) is Poisson’s ratio, \(G\) is shear modulus, and the subscripts \(A\) and \(T\) denote axial and transverse directions, respectively. The term \(\kappa\) is defined as follows:

\[
\kappa = 0.25 E_A / [0.5(1 - \nu_T)(E_A / E_T) - \nu_A^2]
\]

Also, the average strain is defined as follows:

\[
\overline{\varepsilon_{ij}} = 0.5 \left( \frac{\partial w_i}{\partial x_j} + \frac{\partial w_j}{\partial x_i} \right).
\]

An example of another stress equation in expanded form is given. This is shown as an example that all geometry terms appear as a ratio again, and also for the sake of showing the complexity of this micromechanics system. For the case of \(S_{11}^{(11)}\), the basic equation appears as follows:

\[
S_{11}^{(11)} = c_{11}^{(11)} \overline{\varepsilon_{11}} + c_{12}^{(11)} (\phi_2^{(11)} + \psi_3^{(11)}) - \Gamma_1^{(11)} * \Delta T
\]

where \(\Delta T\) is the change in temperature and \(\Gamma_1^{(\text{by})}\) is another set of constants based on material properties such as coefficient of thermal expansion, Young’s modulus, Poisson’s ratio, and shear modulus. Since temperature effects were neglected, these definitions
have not been provided. Given the definitions of \( \phi_i^{(B)} \) and \( \psi_i^{(B)} \), the equation expands to the following form:

\[
S^{(1)}_{11} = c^{(1)}_{11} * \varepsilon_{11} + c^{(1)}_{12} * \{[c^{(m)}_{22} c_{23} \frac{h_1}{h_2} + c^{(f)}_{22} c_{23} \frac{h_1}{h_2} + 1 + c^{(m)}_{23} \frac{h_1}{h_2} - c^{(m)}_{23} \frac{h_1}{h_2} c^{(f)}_{22} c_{23} \frac{h_1}{h_2} + 1])
\]

\[
- c^{(m)}_{22} c_{23} \frac{h_1}{h_2} - c^{(m)}_{22} c_{23} \frac{h_1}{h_2} * [c^{(m)}_{22} \varepsilon_{22} (1 + \frac{h_2}{h_1}) + c^{(m)}_{23} \varepsilon_{33} (\frac{h_1}{h_2} + 1)]
\]

\[
+ [c^{(m)}_{22} c_{23} (1 + \frac{h_2}{h_1}) + c^{(m)}_{23} \frac{h_1}{h_2} - c^{(m)}_{22} c_{23} (\frac{h_1}{h_2} + 2) - c^{(m)}_{22} (1 + \frac{h_2}{h_1})] \]

\[
+ c^{(m)}_{23} (1 + \frac{h_2}{h_1}) + c^{(m)}_{23} \frac{h_1}{h_2} - c^{(m)}_{23} c^{(f)}_{22} c_{23} \frac{h_1}{h_2} + c^{(m)}_{22} c_{23} \frac{h_1}{h_2} - c^{(m)}_{22} (1 + \frac{h_2}{h_1}) - c^{(m)}_{23} \frac{h_1}{h_2} (1 + \frac{h_2}{h_1})
\]

\[
- c^{(m)}_{23} (\frac{h_1}{h_2} + 2) - c^{(m)}_{22} c^{(f)}_{22} (1 + \frac{h_2}{h_1}) * [(c^{(m)}_{12} - c^{(f)}_{12}) \varepsilon_{11} + c^{(m)}_{22} \varepsilon_{22} (1 + \frac{h_2}{h_1}) + c^{(m)}_{23} \varepsilon_{33} (\frac{h_1}{h_2} + 1)]
\]

\[
+ [c^{(m)}_{22} c_{23} (1 + \frac{h_2}{h_1}) + c^{(m)}_{23} \frac{h_1}{h_2} c^{(f)}_{22} c_{23} \frac{h_1}{h_2} + c^{(m)}_{23} \frac{h_1}{h_2} - c^{(m)}_{22} c^{(f)}_{22} c_{23} \frac{h_1}{h_2} (1 + \frac{h_2}{h_1})
\]

\[
- c^{(m)}_{22} c_{23} \frac{h_1}{h_1} + c^{(m)}_{22} c_{23} \frac{h_1}{h_1} * [c^{(m)}_{23} \varepsilon_{22} (\frac{h_1}{h_2} + 1) + c^{(m)}_{23} \varepsilon_{33} (1 + \frac{h_2}{h_1})]
\]

\[
L c^{(m)}_{22} c_{23} (1 + \frac{h_2}{h_1}) - c^{(m)}_{22} \frac{h_2}{h_1} c^{(f)}_{22} c_{23} \frac{h_2}{h_1} + c^{(m)}_{23} (\frac{h_1}{h_2} + 2) - c^{(m)}_{22} c^{(f)}_{22} c_{23} \frac{h_1}{h_2} + c^{(m)}_{23} \frac{h_1}{h_2} + c^{(f)}_{22} \frac{h_1}{h_2} - c^{(f)}_{23} \frac{h_1}{h_2}
\]

\[
+ 2c^{(m)}_{22} c_{23} (1 + \frac{h_2}{h_1}) (c^{(m)}_{22} \frac{h_1}{h_2} + c^{(f)}_{23} \frac{h_2}{h_1}) + 2c^{(f)}_{23} + 32 \frac{h_1}{h_2}
\]

This equation can be modified using the convention that (…) represents anything not directly related to geometry. This quantity appears as follows:
\[ S_{11}^{(11)} = (\ldots) + (\ldots) \times \left\{ \left[ (\ldots) \frac{h_1}{h_2} + 1 \right] + (\ldots) \frac{h_1}{h_2} - (\ldots) \frac{h_1}{h_2} - (\ldots) \left( \frac{h_1}{h_2} + 1 \right) - (\ldots) \frac{h_2}{h_1} \right\} \]

\[ \times \left[ (\ldots) \left( 1 + \frac{h_2}{h_1} \right) + (\ldots) \left( \frac{h_1}{h_2} + 1 \right) \right] \]

\[ + [ (\ldots) \left( 1 + \frac{h_2}{h_1} \right) + (\ldots) \frac{h_1}{h_2} - (\ldots) \frac{h_2}{h_1} + 2 \right] - (\ldots) \left( 1 + \frac{h_2}{h_1} \right)^2 + (\ldots) \left( 1 + \frac{h_2}{h_1} \right)^2 + (\ldots) \left( \frac{h_2}{h_1} \right)^2 - (\ldots) \left( \frac{h_2}{h_1} \right)^2 \]

\[ \times \left[ (\ldots) + (\ldots) \left( \frac{h_2}{h_1} + 1 \right) + (\ldots) \left( \frac{h_2}{h_1} \right) \right] \]

\[ + [ (\ldots) \left( 1 + \frac{h_2}{h_1} \right) + (\ldots) \frac{h_2}{h_1} + (\ldots) \left( 1 + \frac{h_2}{h_1} \right) - (\ldots) \left( \frac{h_1}{h_2} + 2 \right) - (\ldots) \left( 1 + \frac{h_2}{h_1} \right)^2 \]

\[ \times \left[ (\ldots) + (\ldots) \left( 1 + \frac{h_2}{h_1} \right) + (\ldots) \left( \frac{h_1}{h_2} + 1 \right) \right] \]

\[ + [ (\ldots) \left( 1 + \frac{h_2}{h_1} \right) + (\ldots) \frac{h_2}{h_1} - (\ldots) \frac{h_2}{h_1} - (\ldots) \frac{h_2}{h_1} + (\ldots) \left( 1 + \frac{h_2}{h_1} \right) - (\ldots) \left( \frac{h_2}{h_1} \right) \]

\[ \times \left[ (\ldots) \left( \frac{h_1}{h_2} + 1 \right) + (\ldots) \frac{h_1}{h_2} \right] \}

\[ \frac{1}{(\ldots) \left( 1 + \frac{h_2}{h_1} \right)^2 - (\ldots) \left( 1 + \frac{h_2}{h_1} \right)^2 \left( \frac{h_1}{h_2} + (\ldots) \right)^2 - (\ldots) \left( \frac{h_1}{h_2} + (\ldots) \right)^2 \]

\[ - (\ldots) \left( 1 + \frac{h_2}{h_1} \right) \left( \frac{h_1}{h_2} + (\ldots) \right) + (\ldots) \left[ (\ldots) \left( \frac{h_1}{h_2} + (\ldots) \right) + (\ldots) \right]^2 \]

The conclusion is that all lengths appear as a ratio again. This is the case with all seven equations derived from the three original failure conditions.
APPENDIX B

MATHCAD MODEL
Shear Lag Model

Material Properties

\[ \nu_f := 0.24 \quad E_f := 150 \times 10^9 \text{ Pa} \]

\[ \nu_m := 0.4 \quad E_m := 80 \times 10^8 \text{ Pa} \]

\[ G_f := \frac{E_f}{2(1 + \nu_f)} \quad G_f = 6.048 \times 10^{10} \text{ Pa} \]

\[ G_m := \frac{E_m}{2(1 + \nu_m)} \quad G_m = 2.857 \times 10^9 \text{ Pa} \]

\[ \lambda := \frac{E_f}{E_m} \]

\[ \lambda = 18.75 \]

Dimensional Properties

\[ r_o := 7 \times 10^{-6} \text{ m} \quad r_c := 21 \times 10^{-6} \text{ m} \quad L_z := 25 \times 10^{-6} \text{ m} \]

\[ s_r := 2 \cdot r_c \quad V_f := \frac{r_o^2}{r_c^2} \]

\[ V_m := \frac{r_c^2 - r_o^2}{r_c^2} \]

\[ s_r = 4.2 \times 10^{-5} \text{ m} \quad V_f = 0.111 \quad V_m = 0.889 \]

Miscellaneous Conditions

\[ \epsilon_m := 0.033 \quad z := -25 \times 10^{-6} \text{ m}, -24.5 \times 10^{-6} \text{ m}, 0 \text{ m} \quad z_{\text{max}} := -25 \times 10^{-6} \text{ m} \]

Constants (Nairn)

\[ \beta_n := \sqrt{\frac{2}{r_o^2 E_f E_m}} \left[ \frac{E_f V_f + E_m V_m}{4-G_f} \right] \left[ \frac{1}{2-G_m} \right] \left[ \frac{1}{V_m} \ln \left( \frac{1}{V_f} \right) - 1 - \left( \frac{V_m}{2} \right) \right] \]

\[ \beta_n = 6.639 \times 10^4 \frac{1}{\text{m}} \]
\[ \sigma_{\text{prime}} := E_f \varepsilon_m \quad \sigma_{\text{prime}} = 4.95 \times 10^9 \text{ Pa} \]

\[ \tau_n(z) := -\sigma_{\text{prime}} \frac{r_o \beta_n}{2} \cdot \frac{\sinh \beta_n z}{\cosh \beta_n L z} \]

Check: \[ \tau_n(0) = 0 \text{ Pa} \quad \tau_n \bigg|_{z_{\text{max}}} = 1.07 \times 10^9 \text{ Pa} \]

Constants (Cox)

\[ \beta_c := \frac{1}{r_o} \sqrt{\frac{G_m}{2 E_f \ln \left( \frac{s_r}{r_o} \right)}} \quad \beta_c = 2.083 \times 10^4 \frac{1}{\text{m}} \]
\[ \tau_c(z) := -\sigma_{\text{prime}} \cdot \frac{r_0 \beta_c}{2} \cdot \frac{\sinh |\beta_c z|}{\cosh |\beta_c L - z|} \]

Unit Check: 
\[ \tau_c(0) = 0 \text{ Pa} \quad \tau_c |_{z_{\text{max}}} = 1.726 \times 10^8 \text{ Pa} \]

Compare:
Exponential Estimate

\[ z_1 := -25 \times 10^{-6}, -24.5 \times 10^{-6}, 0 \]

\[ y_1|z_1| := 0.01 \exp \left( -1 \times 10^6 \cdot z_1 \right) \]
Axial Stresses

Miscellaneous Conditions

Constants (Nairn)

\[ \beta_n = 6.639 \times 10^4 \frac{1}{m} \]

\[ \sigma_{fn}(z) := \sigma_{\text{prime}} \left( 1 - \frac{\cosh |\beta_n \cdot z|}{\cosh |\beta_n \cdot Lz|} \right) \]

Check:

\[ \sigma_{fn}(0) = 3.133 \times 10^9 \text{ Pa} \]

\[ \sigma_{fn}\Big|_{z_{\text{max}}} = 0 \text{ Pa} \]
Constants (Cox)

\[
\beta_c = 2.083 \times 10^4 \frac{1}{\text{m}}
\]

\[
\sigma_{fc}(z) := \sigma_{\text{prime}} \left( 1 - \frac{\cosh \beta_c z}{\cosh \beta_c L_z} \right)
\]

Unit Check:

\[\sigma_{fc}(0) = 6.029 \times 10^8 \text{ Pa} \quad \sigma_{fc}|_{z_{\text{max}}} = 0 \text{ Pa} \]

\[
\begin{array}{c|c|c}
\sigma_{fc}(z) & 0 & 2.439 \times 10^7 \\
& 4.825 \times 10^7 & 7.157 \times 10^7 \\
& \cdots & \cdots \\
\sigma_{\text{fn}}(z) & 0 & 1.502 \times 10^8 \\
& 2.95 \times 10^8 & 4.347 \times 10^8 \\
& \cdots & \cdots \\
\end{array}
\]
Compare:

\[
\begin{align*}
\sigma_{fc}(z) &:= \frac{\sigma_{fc}(0)}{2} \quad \text{hff}_c := \frac{\sigma_{fc}(0)}{2} \\
hff_c &= 3.014 \times 10^8 \text{Pa} & \text{hff}_n &= 1.567 \times 10^9 \text{Pa} \\
z_{hc} := \left( \frac{1}{\beta_c} \right) \cdot \text{acosh} \left[ \left( 1 - \frac{\text{hff}_c}{\sigma_{\text{prime}}} \right) \cdot \text{cosh} \left( \beta_c \cdot L_z \right) \right] & z_{hc} &= 1.778 \times 10^{-5} \text{ m} \\
z_{hn} := \left( \frac{1}{\beta_n} \right) \cdot \text{acosh} \left[ \left( 1 - \frac{\text{hff}_n}{\sigma_{\text{prime}}} \right) \cdot \text{cosh} \left( \beta_n \cdot L_z \right) \right] & z_{hn} &= 1.858 \times 10^{-5} \text{ m} \\
z_{\text{ctrans}} := \frac{L_z - z_{hc}}{2 \cdot r_o} & z_{\text{ntrans}} := \frac{L_z - z_{hn}}{2 \cdot r_o} \\
z_{\text{ctrans}} &= 0.516 & z_{\text{ntrans}} &= 0.459
\end{align*}
\]
Off-Axis Conditions (Chon, Sun)

\[ \alpha := 45\text{deg} \quad \theta := 0\text{deg} \]

Dimensional Parameters

\[ L_0 := 30 \times 10^{-6} \text{ m} \]

\[ a_f := \frac{L_z}{2r_o} \]

\[ V_{fl} := \frac{r_o^2}{r_c^2} \cdot \frac{L_z}{L_o} \]

\[ R_o := \frac{|L_o - L_z|}{L_z} \]

\[ a_f = 1.786 \quad V_{fl} = 0.093 \quad R_o = 0.2 \]

Material Parameters

\[ k_m := \frac{E_m}{3\left(1 - 2\nu_m\right)} \quad k_f := \frac{E_f}{3\left(1 - 2\nu_f\right)} \]

\[ K_{23} := k_m + \frac{G_m}{3} + \frac{V_{fl}}{k_f - k_m + \frac{G_f - G_m}{3} + \frac{1 - V_{fl}}{k_m + \frac{G_f - G_m}{3}}} \]

\[ v_1 := \left[1 - V_{fl}\right] + R_o \cdot \nu_m + V_{fl} + R_o \cdot \frac{V_{fl}E_m + R_o \nu_m E_f}{R_o E_f + E_m} \]

\[ \mu_{23} := \left[1 + \frac{V_{fl}}{G_m + \left(\frac{7G_m}{3}\right) \cdot \left[1 - V_{fl}\right]} \right] \cdot G_m \]

\[ \mu_{12} := \left[\frac{G_f}{G_f} + V_{fl} + G_m \cdot \left[1 - V_{fl}\right] \right] \cdot G_f \]
\[ E_{11} := \frac{V_{fl} E_f}{1 + \left( \frac{E_f R_o}{E_m} \right)} + \left[ 1 - \frac{V_{fl}}{1 + R_o} \right] E_m \]

\[ \mu_1 := \frac{3}{8} E_{11} + \frac{\mu_{12}}{2} + \frac{(3 + 2v_1 + 3v_1^2) \cdot \mu_{23} K_{23}}{2|\mu_{23} + K_{23}|} \]

\[ \mu_2 := \frac{1}{8} E_{11} - \frac{\mu_{12}}{2} + \frac{(1 + 6v_1 + v_1^2) \cdot \mu_{23} K_{23}}{2|\mu_{23} + K_{23}|} \]

\[ E_a := \frac{1}{\mu_1} \left( \mu_1^2 - \mu_2^2 \right) \quad E_a = 1.001 \times 10^{10} \text{ Pa} \]

\[ \eta := \frac{a_f}{L_z} \cdot \left[ \frac{2 \cdot G_m}{E_f} \right]^{0.5} \cdot \left[ \frac{1}{\sqrt{V_{fl} + 1 + R_o}} - 1 \right] \]

\[ Q_o := \frac{\cos(\alpha) \cdot G_m^2}{E_a \cdot \left( 2 \cdot \frac{G_m}{E_f} \right) \cdot \left[ \frac{1}{\sqrt{V_{fl} + 1 + R_o}} - 1 \right] \cdot \cosh(\eta - L_z)} \]

**Stresses**

\[ \tau_{off}(z) := -Q_o \cos(\alpha) \cdot \sinh(\eta z) + \sigma_{prime} \sin(\alpha) \cdot \sin(\alpha) \cdot \cos(\alpha) \cdot \sin(\theta) \]

\[ \sigma_{off}(z) := \sigma_{prime} \cos(\alpha) \cdot \frac{E_f}{E_a} \left( 1 - \frac{\cosh(\eta z)}{\cosh(\eta - L_z)} \right) \]
Unit Check:

\[ \sigma_{\text{off}}(0) = 1.099 \times 10^9 \text{ Pa} \quad \sigma_{\text{off}} \left| z_{\text{max}} \right| = 0 \text{ Pa} \]

Unit Check:

\[ \tau_{\text{off}}(0) = 0 \text{ Pa} \quad \tau_{\text{off}} \left| z_{\text{max}} \right| = 2.241 \times 10^9 \text{ Pa} \]
\[ \sigma_{\text{off}(z)} = \begin{array}{c|c|c}
0 & \text{Pa} & 2.241 \times 10^9 \\
4.373 \times 10^7 & \text{Pa} & 2.195 \times 10^9 \\
8.656 \times 10^7 & \text{Pa} & 2.149 \times 10^9 \\
1.285 \times 10^8 & \text{Pa} & 2.104 \times 10^9 \\
\ldots & \ldots & \ldots 
\end{array} \]
APPENDIX C

ABAQUS INPUT FILES (TRUNCATED)
*Heading
** Job name: Job-pullHSIPOFFAXIS Model name: Model-HSIPOFFAXIS
** Generated by: Abaqus/CAE Version 6.8-2
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Main

*Node
 1, 0., 0., 0.
 2, 7., 0., 0.
 3, 0., 0., 0.
 4, 0., 7., 0.
 5, 0., 0., 10.
 6, 7., 0., 10.
 7, 0., 0., 10.
 8, 14., 0., 10.
 9, 0., 14., 10.
10, 0., 0., 0.

112332, 0., 6.70833302, 6.

*Element, type=C3D20R
 1, 1764, 1765, 11349, 11229, 79, 80, 1660, 1659, 28732, 28731, 28730, 28729, 28733, 28734, 28735, 28736, 28738, 28737, 28739, 28740
 2, 1765, 1766, 11350, 11349, 80, 81, 1661, 1660, 28743, 28742, 28741, 28731, 28744, 28745, 28746, 28734, 28737, 28747, 28748, 28739

26233, 2778, 368, 11116, 28252, 2763, 367, 11160, 28368, 36627, 111451, 111422, 111450, 36396, 112332, 112260, 112331, 36617, 36631, 112268, 112267

*Nset, nset=_PickedSet13, internal
 1, 2, 3, 4, 5, 6, 16, 18, 23, 26, 34, 35, 36, 37, 38, 39

112327, 112328, 112329, 112330, 112331, 112332

*Elset, elset=_PickedSet13, internal
 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16
26215, 26216, 26217, 26218, 26219, 26220, 26221
26222, 26223, 26224, 26225, 26226, 26227, 26228, 26229, 26230,
26231, 26232, 26233
*Nset, nset=_PickedSet14, internal
  2,  3,  5,  6,  7, 10, 13, 14, 15, 16, 17, 18, 19, 22, 23, 26

24561, 24562, 24563
*Nset, nset=_PickedSet15, internal
  7,  8,  9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24

110299, 110300, 110301, 110302
*Elset, elset=_PickedSet15, internal
  771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783,
  784, 785, 786

25516, 25517, 25518, 25519, 25520, 25521, 25522
  25523,

** Section: Section-Fiber
*Solid Section, elset=_PickedSet13, material=Fiber

** Section: Section-Matrix
*Solid Section, elset=_PickedSet15, material=Matrix

** Section: Section-Interphase
*Solid Section, elset=_PickedSet14, material=Interphase

*End Part
**
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Main-1, part=Main
*End Instance

**
*Nset, nset=_PickedSet5, internal, instance=Main-1
1,
*Nset, nset=_PickedSet10, internal, instance=Main-1
20, 21, 53, 54, 55, 56, 64, 73, 77, 387, 388, 389, 390, 391, 392, 393
108129, 108133, 108134, 108138, 108139, 108143, 108144, 108147,
.
.
.
111428, 111430, 111432, 111434

*Elset, elset=_PickedSet11, internal, instance=Main-1
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.
.
.
25854, 25855, 25856, 25857, 25858

*Nset, nset=_PickedSet12, internal, instance=Main-1
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.
.
.
112262, 112268, 112317, 112322, 112327, 112332

*Elset, elset=_PickedSet12, internal, instance=Main-1
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 61, 62, 63, 64
.
.
.
26148, 26153, 26213, 26218, 26223, 26228, 26233

*Nset, nset=_PickedSet13, internal, instance=Main-1
20, 21, 53, 54, 55, 56, 64, 73, 77, 387, 388, 389, 390, 391, 392, 393
.
.
.
95843, 95850, 95857, 95864

*Elset, elset=_PickedSet13, internal, instance=Main-1
6085, 6086, 6087, 6088, 6089, 6090, 6091, 6092, 6093, 6094, 6095, 6096, 6097, 6098, 6099, 6100
.
.
.
20102, 20108, 20114, 20120

*Nset, nset=_PickedSet14, internal, instance=Main-1
68,

*Nset, nset=Set-1, instance=Main-1
  1,  2,  3,  7,  8,  9, 10, 17, 18, 65, 66, 67, 68, 79, 80, 81
.
.
.
111428, 111430, 111432, 111434

*Elset, elset=Set-1, instance=Main-1
  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16
.
.
.
25854, 25855, 25856, 25857, 25858

*Elset, elset=_Surf-1_S2, internal, instance=Main-1
  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16
.
.
.
25858,

*Elset, elset=_Surf-1_S1, internal, instance=Main-1
  2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382
.
.
.
24560, 24561, 24562, 24563

*Surface, type=ELEMENT, name=Surf-1
_Surf-1_S2, S2
_Surf-1_S1, S1
*End Assembly
**
** MATERIALS **
*Material, name=Fiber
*Elastic
150000., 0.24
*Material, name=Interphase
*Elastic
79000., 0.32
*Material, name=Matrix
*Elastic
8000., 0.4

** BOUNDARY CONDITIONS **
**
** Name: BC-BackBottomPointEncastre Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet14, ENCASTRE
** Name: BC-BackSurfaceZSymm Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet11, ZSYMM
** Name: BC-SideSurfaceXSymm Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet12, XSYMM

** STEP: Step-1 **
*Step, name=Step-1
*Static
1., 1., 1e-05, 1.

** BOUNDARY CONDITIONS **
**
** Name: Displacement-MatrixStraight Type: Displacement/Rotation
*Boundary
_PickedSet10, 3, 3, 1.

** OUTPUT REQUESTS **
*Restart, write, frequency=0

** FIELD OUTPUT: F-Output-1 **
*Output, field, variable=PRESELECT

** HISTORY OUTPUT: H-Output-1 **
*Output, history, variable=PRESELECT
*End Step
** Job name: Job-pullHSNF Model name: Model-HSNF
** Generated by: Abaqus/CAE Version 6.8-2
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** PARTS
**
*Part, name=Part-Main
*Node

1, 20.599762, -4.08041763, 15.
2, 21., 0., 15.
3, 21., 0., 20.
5, 16.8999996, 0., 15.
6, 16.8999996, 0., 20.
7, 0., -16.8999996, 15.
8, 4.08041763, -20.599762, 15.
9, 4.08041763, -20.599762, 20.
10, 0., -16.8999996, 20.

203171, 20.227705, -3.47425175, 7.5
203172, 20.6933384, -3.57571483, 7.5

*Element, type=C3D20R

1, 1898, 1899, 15848, 15750, 74, 75, 1803, 1802, 51944, 51943, 51942, 51941, 51946, 51947, 51948, 51950, 51949, 51951, 51952
2, 1899, 1900, 15849, 15848, 75, 76, 1804, 1803, 51955, 51954, 51953, 51943, 51956, 51957, 51958, 51946, 51949, 51959, 51960, 51951

47472, 51404, 15679, 15672, 51300, 51940, 15728, 15721, 51485, 201920, 201921, 201754, 201919, 203158, 203166, 202990, 203165, 203171, 203172, 202998, 202997

*Nset, nset=_PickedSet22, internal

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16
203143, 203144, 203145, 203146, 203147, 203148, 203149, 203150, 203151, 203152, 203153, 203154, 203155, 203156, 203157, 203158, 203159, 203160, 203161, 203162, 203163, 203164, 203165, 203166, 203167, 203168, 203169, 203170, 203171, 203172

*Elset, elset=_PickedSet22, internal
   1,    2,    3,    4,    5,    6,    7,    8,    9,   10,   11,   12,   13,   14,   15,   16
   .
   .
   .
   47459, 47460, 47461, 47462, 47463, 47464, 47465, 47466, 47467, 47468, 47469, 47470, 47471, 47472

*Nset, nset=_PickedSet23, internal
   1,    4,    5,    6,    7,    8,    9,   10,   13,   14,   15,   16,   19,   22,   23,   24
   .
   .
   .
   192496, 192497, 192498, 192499, 192500, 192501, 192502, 192503, 192504, 192505, 192506, 192507

*Elset, elset=_PickedSet23, internal
   43814, 43815, 43816, 43817, 43818, 43819, 43820, 43821, 43822, 43823, 43824, 43825, 43826, 43827

*Nset, nset=Set-1
   57,    58,    59,    60,    61,    62,    63,    64,  65,  66,  67,  68,  69,  70,  1383,  1384
   .
   .
   .
   201775, 201778, 201779, 201782, 201785, 201786, 201789, 201792, 201793, 201796, 201798

*Elset, elset=Set-1
   32638, 32639, 32640, 32641, 32642, 32643, 32644, 32645, 32646, 32647, 32648, 32649, 32650, 32651, 32652, 32653
   .
   .
   .
*Elset, elset= Surf-1_S2, internal
  32638, 32639, 32640, 32641, 32642, 32643, 32644, 32645, 32646,
  32647, 32648, 32649, 32650, 32651, 32652, 32653
*Surface, type=ELEMENT, name=Surf-1_Surf-1_S2, S2
** Section: Section-Fiber
*Solid Section, elset=_PickedSet22, material=Fiber

** Section: Section-Matrix
*Solid Section, elset=_PickedSet23, material=Matrix

*End Part
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Part=Main-1, part=Part=Main
*End Instance
**
*Nset, nset=_PickedSet4, internal, instance=Part=Main-1
  57,  58,  59,  60,  61,  62,  63,  64,
  65,  66,  67,  68,  69,  70,  1383, 1384
  .
  .
  .
  .

201775, 201778, 201779, 201782, 201785, 201786, 201789
  201792, 201793, 201796, 201798

*Elset, elset=_PickedSet4, internal, instance=Part=Main-1
  32638, 32639, 32640, 32641, 32642, 32643, 32644, 32645, 32646,
  32647, 32648, 32649, 32650, 32651, 32652, 32653
  .
  .
  .
  .

46692, 46909, 46910, 46911, 46912, 46913, 46914, 46915, 46916,
  46917, 46918, 46919, 46920

*Nset, nset=_PickedSet5, internal, instance=Part=Main-1
*Nset, nset=_PickedSet6, internal, instance=Part-Main-1
  60,

*Nset, nset=_PickedSet7, internal, instance=Part-Main-1
  43, 44, 47, 48, 50, 51, 52, 53, 55, 56, 1076, 1077, 1078, 1079, 1080, 1081

*Elset, elset=_PickedSet7, internal, instance=Part-Main-1
  153816, 153820, 153824, 153828, 153832, 153836, 153840, 153844, 153848, 153852, 153856, 153860, 153864

*Nset, nset=_PickedSet8, internal, instance=Part-Main-1
  27249, 27253, 27257, 27261, 27265, 27269, 27273, 27277, 27281, 27285,

*Elset, elset=_PickedSet8, internal, instance=Part-Main-1
  31649, 31653, 31657, 31661, 31665, 31669, 31673, 31677

*Nset, nset=_PickedSet9, internal, instance=Part-Main-1
  17, 18, 35, 49, 69, 277, 278, 279, 280, 281, 282, 283, 777, 778, 779, 780

*Elset, elset=_PickedSet9, internal, instance=Part-Main-1

197129, 197237, 197345, 197453, 197561, 197777
197867,

*Elset, elset=_PickedSet9, internal, instance=Part-Main-1
1158, 1194, 1230, 1266, 1297, 1298, 1299, 1300, 10066, 10072, 10078, 10084, 10090, 10096, 10102, 10108
10114, 10385, 10386, 10387, 10388, 10389, 10390, 10391, 10392, 25739, 25745, 25751, 25757, 25878, 25879, 25880
25881, 44946, 44982, 45018, 45054, 45090, 45126, 45162, 45390, 45426, 45462, 45498, 45534, 45570, 45606, 45642

*Nset, nset=_PickedSet10, internal, instance=Part-Main-1
11, 12, 32, 46, 70, 206, 207, 208, 209, 210, 211, 212, 704, 705, 706, 707

199889, 199997, 200105, 200213, 200321, 200429, 200537
200627,

*Elset, elset=_PickedSet10, internal, instance=Part-Main-1
678, 714, 750, 786, 817, 818, 819, 820, 9046, 9052, 9058, 9064, 9070, 9076, 9082, 9088
9094, 9365, 9366, 9367, 9368, 9369, 9370, 9371, 9372, 25299, 25265, 25271, 25277, 25398, 25399, 25400
25401, 45846, 45882, 45918, 45954, 45990, 46026, 46062, 46290, 46326, 46362, 46398, 46434, 46470, 46506, 46542

*Nset, nset=_PickedSet11, internal, instance=Part-Main-1
3, 31, 45, 57, 96, 97, 98, 99, 100, 101, 102, 672, 673, 674, 675

158545, 158553, 158561, 158569, 158577, 158585, 158593
158600,

*Elset, elset=_PickedSet11, internal, instance=Part-Main-1
34, 60, 66, 72, 193, 194, 195, 196, 7433, 7434, 7435, 7436, 7678, 7684, 7690, 7696

33161, 46662, 46698, 46734, 46770, 46806, 46842, 46878, 47094, 47100, 47106, 47112, 47118, 47124, 47130, 47136

*End Assembly
**
** MATERIALS
**
*Material, name=Fiber
*Elastic  
150000.0, 0.24
*Material, name=Matrix
*Elastic  
8000.0, 0.4
**
** BOUNDARY CONDITIONS
**
** Name: BC-BackBottomPointEncastre Type: Symmetry/Antisymmetry/Encastre  
*Boundary  
_PickedSet6, ENCASTRE
** Name: BC-BackSurfaceZSymm Type: Symmetry/Antisymmetry/Encastre  
*Boundary  
_PickedSet4, ZSYM
** Name: BC-BottomFiberAxisZASymm Type: Symmetry/Antisymmetry/Encastre  
*Boundary  
_PickedSet10, ZASYMM
** Name: BC-RightFiberAxisZASymm Type: Symmetry/Antisymmetry/Encastre  
*Boundary  
_PickedSet11, ZASYMM
** Name: BC-SideSurfaceXSymm Type: Symmetry/Antisymmetry/Encastre  
*Boundary  
_PickedSet8, XSYM
** Name: BC-TopFiberAxisZASymm Type: Symmetry/Antisymmetry/Encastre  
*Boundary  
_PickedSet9, ZASYMM
**
**------------------------------------------------------------------
**
** STEP: Step-1
**
*Step, name=Step-1  
*Static  
1.0, 1.0, 1e-05, 1.0
**
** BOUNDARY CONDITIONS
**
** Name: Displacement-Matrix Type: Displacement/Rotation  
*Boundary  
_PickedSet7, 3, 3, 0.392677
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step