A STUDY OF THE DEVELOPMENT OF TECHNOLOGICAL PEDAGOGICAL
CONTENT KNOWLEDGE (TPACK) IN PRE-SERVICE SECONDARY
MATHEMATICS TEACHERS

by

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Rejoice Mudzimiri

July 2012
DEDICATION

This dissertation is dedicated to my loving parents, Janet and Ronald. Mom and dad, thank you for your words of encouragement and for your prayers. I will always love you!
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Thank you Lord for seeing me through my six years of graduate school at Montana State University!

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ABSTRACT

The Technological Pedagogical Content Knowledge (TPACK) framework is a relatively new construct that offers a useful perspective from which to understand the development of pre-service teachers’ abilities in teaching with technology. It is grounded in the understanding that if teachers are to effectively integrate technology into their instruction, they need to integrate their knowledge of content, pedagogy and technology, instead of viewing these components as separate entities. Current efforts to develop TPACK in pre-service teachers have tended to focus on experiences in either a methods course or an educational technology course. It is admittedly difficult to adequately address technology, pedagogy and content in a single college course. Therefore, this study proposes using three courses that are offered in collaboration, a mathematics teaching methods course, a technology-intensive content-rich mathematical modeling course, and a practicum course, to study the development of connections between technology, content and pedagogy. For this multiple case study, TPACK changes in five pre-service teachers were tracked during a period of about 15 weeks. Data were collected using a TPACK survey, teaching philosophy statements, lesson plans, student teaching episodes, and weekly instructor meeting notes. A detailed analysis of the results demonstrates that the development of pre-service teachers’ mathematics TPACK is complex, and there are a number of factors that are at play, such as the pre-service teachers’ prior experiences with technology, their mathematical backgrounds and their beliefs about the use of technology in mathematics instruction. Assessing the development of TPACK in pre-service teachers is complicated by the fact that the available model for mathematics teachers’ TPACK was developed using observations of in-service mathematics teachers.
CHAPTER 1
THE PROBLEM

Introduction

Despite the reported availability of technology as a tool for learning mathematics and recommendations from both state standards (e.g. California State Mathematics Content Standards, 2009; Montana State Mathematics Standards adopted by the Montana Office of Public Instruction, 2010) and national standards (NCTM, 2000; AMTE, 2006; Common Core State Standards, 2010), available research evidence has revealed that in-service mathematics teachers sparingly use technology (U.S. Department of Education, 2007). For those who do, the uses tend to be very basic. Lack of appropriate professional development has been posited as one explanation for the modest use of technology in mathematics instruction. According to the U. S. Department of Education (2007), it is difficult for in-service teachers to access and/or benefit from professional development due to lack of time to take technology-related training, the abbreviated nature of many professional development offerings, insufficient opportunities for immediate and frequent practice of what is learned, and the paucity of follow-up and advanced training on technology use. Therefore, embedding technology training in pre-service coursework seems a likely avenue for helping future teachers become proficient users of technology in mathematics teaching and learning.
Technology use in mathematics can be divided into two categories, productivity use and cognitive use. Productivity use includes word processors, spreadsheets, databases and multimedia and presentation software that are used to enhance learning. Cognitive use mainly involves the use of technology in doing mathematics, hence the term cognitive technology. According to Pea (1987), “cognitive technology is any medium that helps transcend the limitations of the mind (e.g. attention to goals, short-term memory span) in thinking, learning, and problem solving activities” (p. 91). An example of cognitive use is using a spreadsheet to explore a pattern and then make conjectures which otherwise would be challenging to do mentally. Note that the spreadsheet can be used as both a productivity tool and a cognitive tool. How the technological tool is used depends on the user’s purpose (Drijvers & Trouche, 2008). For instance, dynamic geometry programs were developed as a tool for inquiry (cognitive use) but can also be used by educators to create a static picture to paste into an exam or worksheet (productive use). Teaching mathematics with technology typically capitalizes on both uses, but this research will focus primarily on the use of cognitive technologies.

Technology in Mathematics Teaching and Learning

The functionality of handheld calculators has progressed from a basic four-function type to models with computer algebra systems (CAS) that can perform symbolic manipulations. Mathematics has benefited perhaps more than any other school subject
from this advancement in technology. According to Alagic (2003), appropriate use of
calculators can enrich the teaching and learning of mathematics because calculators “
empower students with a level of mathematical power they cannot achieve without
technology and, if used appropriately, have a great potential for stimulating higher order
thinking when freed from the mechanics of calculating” (p. 394).

Parallel to this development in calculators is the advancement in dynamic
gometry software. Cabri geometry, then Geometer’s Sketchpad, and now GeoGebra
have each offered increasing levels of sophistication in dynamic representation of
gometric objects. As computing power continues to increase, these developments will
continue. At the same time, technology will continue to revolutionize many jobs and
substantially increase mathematical skills needed across the workforce (Conference
Board of the Mathematical Sciences (CBMS), 2001). There is a current call to prepare
high school students who are able to succeed in workforce training programs and are
ready for college coursework (Common Core State Standards, 2010). This makes it
imperative that teacher preparation programs prepare teachers to integrate technology
into their instruction.

National Standards Recommendations
for Technology Use in Mathematics

National standards supporting the use of technology for the learning and teaching
of mathematics exist for both K-12 students and K-12 classroom teachers’ preparation.
These are discussed in the next two sections.
Recommended Standards for K-12 Students’ Use of Technology

The National Council of Teachers of Mathematics (2000) asserted that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (p. 24). In addition, the Association of Mathematics Teacher Educators (AMTE, 2006) “recognizes that technology has become an essential tool for doing mathematics in today’s world, and thus it is essential for the teaching and learning of mathematics” (p. 1). The recently released Common Core State Standards, which have been adopted by 45 states as of June 2012, outline eight Standards for Mathematical Practice. While technology is not an explicit standard, the fifth Standard states that (emphasis added):

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include such things as pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. Proficient students are familiar enough with tools appropriate for their grade or course to make sound decisions about when each might be helpful, realizing the limitations of the tools and the output that they generate. For example, mathematically proficient high school students are able to apply their understanding of limits of technology output to interpret graphs of functions and approximate solutions generated using a graphing calculator. They detect possible errors by using mathematical understanding and estimation strategically. When making mathematical models, they know that technology can enable them to vary assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to solve problems. They are also able to use technological tools to explore and deepen their understanding of concepts (p. 5).

By describing the mathematically proficient student as being able to use technology to “explore consequences and compare predictions” or “explore and deepen their
understanding of concepts,” the implication is that teachers are using these tools for inquiry-based activities in their classrooms.

Various states have also made recommendations for the use of technology in mathematics. For instance, the state of California requires the use of technology in mathematics instruction. The California State Mathematics Content Standards adopted in 1997 specify that “technology should be used to promote mathematics learning. Technology can help promote students’ understanding of mathematical concepts, quantitative reasoning, and achievement when used as a tool for solving problems, testing conjectures, accessing data, and verifying solutions” (California Department of Education 2009, p. 9). One of the principles guiding the Montana State Mathematics Standards is that “technology is integral to learning mathematics” (p. 8). The Montana Standards for Mathematics adopted by the Montana Office of Public Instruction, 2010 described a mathematically proficient student as one who “consistently uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems” (p. 11).

Recommended Standards for K-12 Teachers’ Technology Use Preparation

Standards calling for students to use technology are matched by a call for teachers to be prepared to use it. For instance, the NCTM (2007) posited that “if teachers are to learn how to create a positive environment that promotes collaborative problem solving, incorporates technology in a meaningful way, invites intellectual exploration, and supports student thinking, they themselves must experience learning in such an
environment” (p. 119). Similarly, the Association of Mathematics Teacher Educators (2006) advocated for enhancing the preparation of mathematics teachers in their Technology Position Statement: “mathematics teacher preparation programs must ensure that all mathematics teachers and teacher candidates have opportunities to acquire the knowledge and experiences needed to incorporate technology in the context of teaching and learning mathematics” (AMTE, 2006). The National Council for Accreditation of Teacher Education (NCATE, 2003) designed and adopted technology standards to prepare teachers to utilize technology. Specifically, they outlined that the accredited institutions should “prepare candidates who can integrate technology into instruction to enhance student learning” (p.3).

**In-Service Teachers’ Use of Technology**

Even with the availability of technology and recommendations from both national and state standards, research evidence shows that in-service mathematics teachers sparingly use technology (Artigue, 2007; Lagrange. 2007; U.S. Department of Education 2008). The U.S. Department of Education (2007), using data collected for the 2005 National Assessment of Educational Progress (NAEP), reported that only about 10% of fourth and eighth graders were in classrooms in which teachers used technology at least once a week to present mathematics concepts. In addition, more than 30% of the students were in mathematics classes that did not make use of computers at all. This observed non-use of technology is not only confined to the United States. Thomas, Bosley, Hong, and Santos (2008) reported the non-use of calculators among teachers in New Zealand,
though the teachers in their study generally believed that there were benefits in using calculators in teaching mathematics.

Although technology can be used as a productivity tool and as a cognitive tool, there has been a reported tendency among teachers to use technology more as a productivity tool than as a cognitive tool. The U.S. Department of Education (2007) administered a survey to teachers in classrooms where fourth- and eighth-grade students took the 2005 NAEP mathematics assessment about their instructional practices in 2004–05. Data from this survey indicate that over 50% of the teachers used technology to develop materials, prepare lessons, and/or create tests and quizzes. Forty percent of the teachers reported using technology while delivering lessons, while more than half the students reported using technology in school to conduct research, to practice or review classroom content, to take tests or quizzes, or to produce assignments. In addition, the U.S. Department of Education (2008) showed that very few teachers are using technology to support advanced instructional practices, such as inquiry (3%) and solving real-world problems (3%), with their students on a weekly basis. In addition, Artigue (2007) also noted that technology is not being used for exploration and inquiry, yet it is these strategies that enhance conceptual understanding in mathematics. Lagrange (2007) pointed out that “a study of textbooks and practices shows that only graphical and numerical capabilities have actually been used in the classroom” (p. 70).
Obstacles to Teachers’ Use of Technology in Mathematics Teaching and Learning

Possible explanations for the reported teachers’ modest use of technology in mathematics are numerous. Teachers’ beliefs have been reported to have a significant impact on the teachers’ use of technology in mathematics instruction. Walen, Williams and Garner (2003) investigated the relationship between elementary pre-service teachers’ use of calculators in college mathematics courses and their views of appropriate use of calculators in elementary school classrooms. The pre-service teachers had positive experiences using calculators in their mathematics classes. In contrast to the positive experience, the teachers indicated that calculators should be used only after students have learned the mathematics. Thus, changing mathematics teachers’ instructional practices towards the integration of technology also entails changing their beliefs about technology use, since existing beliefs may stand in the way of their success.

Additional explanations for the reported teachers’ sparing use of technology in mathematics teaching include lack of access to hardware and lack of relevant professional development (Thomas, Bosley, Hong & Santos 2006; Goos & Bennison 2008). For instance, the U.S. Department of Education (2007) reported that in 2004 - 05, almost half of America’s students were in classrooms where teachers lacked access to district- or school-provided professional development on the use of computers for mathematics instruction.
Technology Training in Teacher Preparation

The reported in-service teachers’ non-use of technology in mathematics teaching and learning and the lack of time to take technology-related professional development suggest that a more fruitful strategy might be embedding technology training in pre-service teacher preparation. This would give future teachers time to explore, to get acquainted with, and to become proficient users of technology in mathematics instruction. However, available evidence suggests that pre-service teachers are not being adequately prepared to integrate technology in their classrooms.

The Panel on Educational Technology (2000) found that “few colleges of education adequately prepare their graduates to use information technologies in their teaching” (p.7). In addition, the panel also noted that although pre-service instruction in the use of technology was required by 22 states, the courses used to satisfy such requirements typically provided no actual experience in using computers to teach and imparted little knowledge of available software. This Panel on Educational Technology was organized in April 1995 under the supervision of the President’s Committee of Advisors on Science and Technology (PCAST) to provide independent advice to the President on matters related to the application of various technologies to K-12 education in the United States. Its findings and recommendations are based on a (non-exhaustive) review of the research literature and on written submissions and private White House briefings from a number of academic and industrial researchers, practicing educators, software developers, governmental agencies, and professional and industry organizations involved in various ways with the application of technology to education.
Neiss (2001) reported teacher training programs as typically providing a course about technology “perhaps with a unit or discussion on teaching with technology and a requirement that students (pre-service teachers) design lessons to teach with the technologies” (p. 103).

Garofalo, Drier, Harper, Timmerman and Shockey (2000) categorized approaches taken by teacher educators to integrate technology into teacher training programs according to the end user of the technology as teacher educator, pre-service teacher and student. When teacher educators are the end users they primarily use technology to present information or demonstrate explorations. In cases where the pre-service teacher is the end user, the focus tends to be on productivity tools for word processing, grade and record keeping, presentations and and/or subject-specific software and websites for presentations, lectures, lessons and assessments. Finally, teacher education programs focus on guiding pre-service teachers in appropriate uses of technology particularly on preparing pre-service teachers to use technology in such a way that their own students will use technology for explorations and problem solving. In order to adequately prepare teachers to teach with technology, the best approach may be guiding pre-service teachers in appropriate uses of technology, such as using technology for exploration and problem solving.

In addition to using the right approach, the pre-service teachers also need the right experiences, such as learning mathematics with technology. Researchers (Niess, Browning, Driskell, Johnston & Harrington, 2009) have observed that while some of the pre-service teachers’ content courses use digital technologies, they often are not taught in
ways that challenge traditional instructional, curricular and assessment notions. Moreover, they are not engaged in learning mathematics in ways that model appropriate experiences learning mathematics with technology. Yet, according to Grandgenett (2008), there are expectations for these mathematics teachers that upon graduation, the teacher education program would have prepared them to “design creative and effective learning activities that take full advantage of educational technology” (p. 145). Teachers have been observed to teach the way they are taught. Therefore, if they are not taught in ways that model appropriate use of technology, they will not be able to use technology appropriately in their mathematics teaching. For instance, Morrison and Jeffs (2005) provided pre-service teachers an opportunity to learn with technology and the results indicated that a positive experience with technology during their pre-service training influenced the pre-service teacher’s decision to use technology in their future classroom. This supports the need for pre-service teachers to be exposed to the right experiences.

The TPACK Framework

Recent efforts towards preparing pre-service teachers to integrate technology in their mathematics teaching can be described as haphazard. This can be attributed to the lack of a common guiding framework on the appropriate teacher knowledge. The Technological Pedagogical Content Knowledge (TPACK) framework offers a useful guide for pre-service teachers’ technology training. Several researchers have conjectured that good teaching requires an understanding of how technology relates to pedagogy and content (Margerum-Leys & Marx, 2002; Lundeberg, Bergland, Klyczek, & Hoffman,
2003; Neiss, 2005). However, Koehler and Mishra (2005) were the first to articulate the relationships between content, pedagogy and technology. They envisioned a new knowledge framework (TPACK) resulting from a complex interplay between content, pedagogy, and technology. This TPACK framework builds on Shulman’s (1996) Pedagogical Content Knowledge (PCK), according to Koehler and Mishra (2008), to add a new component, technology, to the two interacting components of pedagogy and content. It is grounded on an understanding that if teachers are to effectively integrate technology into their instruction then they need to integrate their knowledge of content, pedagogy and technology instead of viewing these components as separate entities.

Figure 1 illustrates that TPACK is formed by the intersection between all the three components.

Figure 1: Technological Pedagogical Content Knowledge (TPACK) (http://tpack.org)
As can be seen from Figure 1, the interaction among the three components; content, pedagogy and technology bring about new interactions or knowledge domains; Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), Shulman’s Pedagogical Content Knowledge (PCK) and all three taken together, Technological Pedagogical Content Knowledge (TPACK). These relationships or interactions are important in the TPACK framework. Thus, taken independently, the TPACK framework has seven components, namely: Content Knowledge (CK), Technological Knowledge (TK), Pedagogical Knowledge (PK), Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), Pedagogical Content Knowledge (PCK), and Technological Pedagogical Content Knowledge (TPACK).

Components of TPACK

*Content Knowledge*, shown in Figure 1 as one of the main regions, is knowledge about the subject matter that is to be taught or learned. *Pedagogical Knowledge*, also shown as another main region of the Venn diagram, refers to knowledge about student learning, classroom management, lesson plan development and implementation, and student evaluation. *Technological Knowledge*, another outer region, is difficult to define since it is in a state of flux (Koehler & Mishra 2008). Koehler and Mishra (2008) aligned the definition of technological knowledge to that of fluency of information technology (FITness) proposed by the Committee of Information Technology Literacy of the National Research Council in 1999. In this regard TK was defined as being able to accomplish a variety of different tasks using technology and being able to develop different ways of accomplishing a given task. *Pedagogical Content Knowledge* is
consistent with and similar to Shulman’s PCK, which is knowledge of pedagogy that is applicable to the teaching of specific content. According to Koehler and Mishra (2008), it involves

an awareness of common misconceptions and ways of looking at them, the importance of forging links and connections between different content ideas, students’ prior knowledge, alternative teaching strategies, and flexibility that comes from exploring alternative ways of looking at the same idea or problem (p. 14).

Technological Content Knowledge, according to Koehler and Mishra (2008), refers to understanding of the manner in which technology and content influence and constrain one another. Technological Pedagogical Knowledge is an understanding of how teaching and learning changes when particular technologies are used. This, therefore, includes knowing the pedagogical affordances and constraints of technological tools and the contexts within which they function. Technological Pedagogical Content Knowledge is an understanding that emerges from an interaction of content, pedagogy and technology knowledge.

The TPACK Framework in Mathematics

The TPACK description above seems to be generic, but TPACK is discipline or subject specific. This means that TPACK in mathematics may not be the same as TPACK in science or social studies. With a focus on the intersection of technology, pedagogy and content, Neiss (2005) described TPACK in mathematics teachers as the knowledge and beliefs teachers demonstrate, consistent with:
• An overarching conception about the purposes for incorporating technology in teaching mathematics;

• Knowledge of students’ understandings, thinking, and learning of mathematics with technology;

• Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics;

• Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies.

These aspects seem to be consistent with Mishra and Koehler’s (2006) description of TPACK in general as an understanding of the representations of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students’ prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones (p. 1029).

Four themes that seem to stand out from both descriptions are technology, pedagogy, content and students. Since the focus of this research study is to develop TPACK in pre-service secondary mathematics teachers, Neiss’s description of TPACK in mathematics teachers shall be taken as the basic description of TPACK in mathematics.
The Technological Pedagogical Content Knowledge (TPACK) framework (Mishra & Koehler, 2006), a relatively new construct, offers a useful perspective from which to develop pre-service teachers’ abilities in teaching with technology. It is grounded in the understanding that if teachers are to effectively integrate technology into their instruction, they need to integrate their knowledge of content, pedagogy and technology instead of viewing these components as separate entities.

Current efforts to develop TPACK in pre-service teachers have tended to focus on experiences in either a methods course or an educational technology course. For instance, Shin et al. (2009) used an intense educational technology course sequence to help change in-service teachers’ understanding of TPACK. The results of a paired t-test suggested that in-service teachers’ knowledge of technology improved while their knowledge of both content and pedagogy did not improve in general. Cavin (2007) used a technology course to help pre-service teachers develop TPACK through microteaching lesson study. This context placed more focus on technology and pedagogy than on content.

From the cited research evidence, it is admittedly difficult to adequately address technology, pedagogy and content in a single college course. Therefore, this study proposed using three courses that are offered in collaboration: a mathematics teaching methods course, a technology-intensive content-rich mathematical modeling course, and a practicum course, to develop rich connections between technology, content and pedagogy. The methods course focused on Technological Pedagogical Knowledge (TPK), the mathematical modeling class focused on Technological Content Knowledge
(TCK) and the practicum was used to reinforce TPACK as a coherent and connected body of knowledge.

**Research Questions**

The main goal of this study was to understand the development of TPACK in pre-service secondary mathematics teachers who were enrolled concurrently in three courses (methods, modeling and practicum) that were taught in collaboration. Offering three courses in collaboration was intended to adequately address the three knowledge components (content, technology, and pedagogy) that form TPACK. The research questions that this research study sought to answer were:

1. How do pre-service teachers’ conceptions about the purposes for incorporating technology in teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

2. How does pre-service teachers’ knowledge of students’ understandings, thinking, and learning of mathematics with technology emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

3. How does pre-service teachers’ knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?
4. How does pre-service teachers’ knowledge of instructional strategies and representations for teaching and learning mathematics with technologies emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

The use of the word ‘how’ in the research questions, does is not intended to imply a cause-and-effect relationship. Instead the focus of this study was on identifying and describing stages of development in the pre-service teachers.

Significance of the Study

The Common Core State Standards’ (2010) fifth Standard for Mathematical Practice describes, in part, mathematically proficient students as those who are “able to use technological tools to explore and deepen the understanding of concepts” (p.7). Although this is a goal for K-12 students it is consequently a goal for teachers, since they need to be able to help K-12 students attain mathematical proficiency. The Common Core State Standards do not provide or suggest strategies that can help prepare teachers for such an undertaking. This study, therefore, has the potential to inform mathematics teacher educators on the kinds of experiences needed by pre-service teachers in order to be able to help their future students explore and deepen their understanding of mathematical concepts using technology. Consequently, it also has the potential to provide information on the development of pre-service mathematics teachers’ TPACK, the kind of knowledge that teachers need in order to teach with technology.
By focusing on Neiss’s (2005) definition of TPACK, this study seeks to illuminate this definition and suggest strategies that can be used to help pre-service teachers develop the kind of TPACK envisioned by Neiss (2005).

This study, by studying a combination of three courses (methods, modeling and practicum) may offer a powerful and comprehensive strategy for developing TPACK in pre-service mathematics teachers because this combination of courses is available in many institutions as part of their teacher preparation program. The teacher preparation programs need not have a capstone modeling course like the university that provides the research site for this study. However, they may have other capstone courses, as recommended in The Mathematical Education of Teachers (CBMS, 2001). In cases where the three courses are not offered concurrently, this study gives insight into some of the activities that might be used to make connections between such courses. For instance, when taking the technology-intensive course, a pre-service teacher might be asked to keep a journal of their misconceptions and struggles as they learned the material and to keep this journal until they take the methods course. These journal entries could then be used to orchestrate discussions on students’ misconceptions and struggles when learning mathematics with technology.
CHAPTER 2

REVIEW OF THE LITERATURE

Introduction

The main goal of this study was to understand the development of TPACK in pre-service secondary mathematics teachers who were enrolled concurrently in three courses (methods, modeling and practicum) that were taught in collaboration. This review of literature has three main purposes: (a) to illuminate the definition of mathematics teachers’ TPACK provided by Neiss (2005), (b) to suggest strategies that help mathematics secondary pre-service teachers develop TPACK as envisioned by Neiss (2005), and (c) describe the development of TPACK in mathematics teachers. This review of literature is therefore organized into three main sections:

- Mathematics teachers’ TPACK
- Strategies that help pre-service teachers develop TPACK
- The development of TPACK.

For clarity, the word student will be used to refer to K-12 students and undergraduate students will be referred to as pre-service teachers.

Mathematics Teachers’ TPACK

With a focus on the intersection of technology, pedagogy and content, Neiss (2005) described mathematics teachers’ TPACK as the knowledge and beliefs teachers demonstrate consistent with:
• An overarching conception about the purposes for incorporating technology in teaching mathematics;

• Knowledge of students’ understandings, thinking, and learning of mathematics with technology;

• Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics;

• Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies.

These aspects seem to be consistent with Mishra and Koehler’s (2006) description of TPACK in general as:

an understanding of the representations of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students’ prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones (p. 1029).

Four themes that seem to stand out from both descriptions are technology, pedagogy, content and students. Since the focus of this research study is to develop TPACK in pre-service secondary mathematics teachers, Neiss’s description of TPACK in mathematics teachers shall be taken as the basic description of TPACK in mathematics in this research study. This will be used to organize the rest of this section.
Purposes for Incorporating Technology in Teaching Mathematics

The Common Core State Standards’ (2010) fifth Standard for Mathematical Practice describes, in part, mathematically proficient students as those who are “able to use technological tools to explore and deepen the understanding of concepts” (p.7). Thus the main purpose for incorporating technology in mathematics teaching and learning is for exploration and ultimately deepening understanding of mathematical concepts. Exploration with technology in order to deepen understanding of mathematical concepts can be done in three main ways:

- Linked multiple representations (Davis & Maher, 1997; Kaput, 1992; Kaput, Noss & Hoyles 2002);
- Generalizations (Heid & Blume 2007) and;
- Parameterization (Drijvers 2003).

Linked Multiple Representations. As stated in the NCTM’s Principles and Standards for School Mathematics (2000), mathematics teachers should consistently “help students learn to use representations flexibly and appropriately by encouraging them as they create and use representations to support their thinking and communication” (p.284). More importantly, students should be able to exhibit representational fluency. Sandoval, Bell, Coleman, Enyedy and Suthers (2000) described representational fluency as:

...being able to interpret and construct various disciplinary representations, and being able to move between representations appropriately. This includes knowing what particular representations are able to illustrate or explain, and to be able to use representations as
justifications for other claims. This also includes an ability to link multiple representations in meaningful ways (p. 6). The importance of representational fluency is in its capacity to enhance mathematical understanding. Research has provided evidence that students learn concepts more readily when they experience the concepts across different forms of representation (Kaput, Noss & Hoyles, 2002). According to Rider (2007), the three types of representations that should be used in mathematics instruction are tabular, graphical and symbolic.

In an experimental study with 128 seventh and eighth graders, Brenner et al. (1997) found that students who received instruction that emphasized (a) representing math problems in multiple formats, (b) meaningful contexts, and (c) collaborative learning scored significantly higher on relevant tests. The study also showed that the representation-based instruction yielded the same achievement gains for Spanish-speaking students as for English-speaking students. Another large scale study with 95 educators and 1,621 students investigated SimCal, a curriculum whose approach emphasize multiple representations (among other things). The study provided strong evidence of the effectiveness of multiple representations in enhancing mathematical thinking (Rochelle et al., 2007).

Generalizations. Technology is effective as a learning tool for generating many instances from which generalizations can be made. For this reason technology has been used to support generalizations (Heid & Blume, 2007). Researchers have suggested that spreadsheets can support students in developing an understanding of variables. According to Sutherland and Rojano (1993, p. 380), “a spreadsheet helps students explore, express
and formalize their informal ideas”. Wilson, Ainley and Bills (2004) reported on *The Purposeful Algebraic Project*, whose aim was to explore the potential of spreadsheets as tools in the introduction to algebra and algebraic thinking. As part of the project, six tasks for students, amounting to about 12 hours of work, were assigned over the course of a year. Data collected included videos of students working, actual students’ work, interview scripts and field notes. Results revealed that spreadsheet features, such as focus on calculations use of notation, and feedback, acted as scaffolds for students and they helped keep the students in check in terms of their arithmetic procedures and their verbal attempts to work with mathematical relationships. It was therefore concluded that spreadsheets (or technological tools in general) can help students formalize their generalizations.

It should be noted that different technological tools support different generalization strategies. For instance, CAS supports the search for algebraic invariants; graphic calculators support recognition of a family of functions, and spreadsheets support generalizations stated recursively (Heid & Blume, 2007).

**Parameterization.** This strategy is related to generalization in that when students describe relationships in terms of parameters, generalizations might be involved. The concept of parameter is often considered difficult for students to understand because the roles of a parameter may change during the problem solving process (Drijvers, 2003). Drijvers identified four different roles that a parameter might take which are similar to the roles of ordinary variables, namely: a placeholder, a changing quantity, a generalizer and an unknown.
Drijvers (2003) reported on a design research study whose main aim was to help students develop an insight into the concept of parameter. The study had three cycles, each with three phases (preliminary, teaching experiment and retrospective analysis). One hundred ten students and 100 lessons were involved. Main data sources were observation of student behaviors and interview scripts. Results showed that the role of parameter as a placeholder was understood by students. Moreover, CAS aided students’ conceptualization of a parameter as a changing quantity by allowing changing parameter values one by one. It also facilitated the conceptualization of the parameter as a generalizer by allowing repetition of similar procedures for different parameter values. However, it did not contribute to the conceptualization of a parameter as an unknown.

Students’ Understanding, Thinking, and Learning of Mathematics with Technology

Knowledge of students’ understandings, thinking, and learning of mathematics with technology is important when teachers consider technology integration. This is a result of the fact that teaching with technology is different from a non-technology teaching environment. For instance, Pierce (2005) pointed out that CAS (technology in general) assists with routines but does not take over the role of mathematical thinking. When technology is involved in the processes of formulating and solving mathematical problems and then interpreting results or output, symbol sense is required. According to Fey (1990):

Even if machines take over the bulk of computation, it remains important for users of these machines to plan correct operations and to interpret results intelligently. Planning calculations requires sound understating of the meaning of operations – of the characteristics of actions that
corresponds to various arithmetic operations. Interpretation of results requires judgment about the likelihood that the machine output is correct or that an error may have been made in data entry, choice of operations, or machine performance (p. 79).

Since symbol sense is a broad concept, Pierce (2005) concentrated on a part of symbol sense required to monitor progress towards the solution of a mathematically formulated problem known as algebraic insight.

Figure 2: A model for problem solving showing the places of symbol sense and algebraic insights adapted from Pierce and Stacey (2002)

Figure 2 shows a model of problem solving and the places of symbol sense and algebraic insight. Algebraic insight is between the formulated mathematical problem and the solution. It involves checking that mathematical expressions have been entered correctly into CAS and that the output at each stage makes sense. In a study meant to explore the use of CAS as a didactical tool for coaxing the emergence of reasoning about equivalence of algebraic expressions, Kieran and Saldanha (2005) reported that “the fragility of students’ emerging knowledge of equivalence was being exposed by their difficulty with interpreting some of the CAS displays” (p. 3-199).
The first demand on teacher knowledge is that teachers know the mathematics they teach. Although much work remains to be done to specify “mathematical knowledge for teaching” (Hill, Sleep, Lewis & Ball 2007, p.151), it is certainly true that teachers at least need to know the mathematics they expect their students to learn. Beyond this they need to know much more mathematics. For instance, it is helpful and important to know what comes after the content they are teaching; what a student might learn next; what mathematics precedes or is a prerequisite for the topic being studied; and how what they teach relates to the real world.

Beyond all this, teachers need to know the curricula that integrate technology in mathematics teaching and learning. There has been a lot of effort towards the integration of technology in learning and teaching mathematics through the inclusion of technology sections or activities in textbooks. An example is the Center for Mathematics Education (CME) project, a National Science Foundation (NSF) funded high school program that is organized around Algebra 1, Algebra 2 and Precalculus. The main goal of the CME project is “to help students acquire a deeper understanding of mathematics” (EDC 2009, p. T3). It is guided by six principles among which is “using technology to sharpen, not to replace, thinking” (http://www2.edc.org/cme/principles.html). The CME project authors view technology as a tool for building new ideas and as a medium for developing mathematical habits of mind. Thus, together with pointing out the technology required on the relevant sections throughout the textbooks, such as CAS, Function Modeling
Language, Geometry Software, Spreadsheet Software in Algebra 2, and Graphing Calculator in Algebra 1, they also provide a TI-Nspire Technology handbook.

Apart from curriculum, there are also curricular materials that integrate technology. The NCTM, for example, provides two valuable resources, the NCTM E-Standards (http://nctm.org/standards/content.aspx?id=16909) and the Illuminations website (http://illuminations.nctm.org/). E-Standards are the online version of the paper-based NCTM Principles and Standards for Learning School Mathematics (NCTM, 2000), but on the Internet, teachers can jump directly from the text document to appropriate examples.

Another example is The Math Forum (http://mathforum.org/mathTools/index.html) which provides material, activities, person-to-person interactions, and educational products and services. It was designed to encourage communication by providing a number of mailing lists, discussion areas, and ‘ask-an-expert’ services. It is a project of Drexel University, partially funded by NSF, with the goal to create a community digital library that supports the use and development of software for mathematics education.

Texas Instruments also provides activities that teachers can download and use with their students in class (http://education.ti.com/educationportal/activityexchange). Another example is the Virtual Laboratories in Probability and Statistics, http://www.math.uah.edu/stat/index.xhtml which provides an integrated set of components that includes expository text, applets, data sets, biographical sketches, and an
object library. It is a project that is partially supported by the University of Alabama in Huntsville and by both NSF and the Mathematical Association of America.

Strategies and Representations for Teaching and Learning Mathematics with Technologies

Kutzler (2003) provided four pedagogical uses of CAS which can be generalized for all mathematics specific technologies. These are trivialization, experimentation, visualization and concentration.

**Trivialization.** The introduction of technology in mathematics teaching has trivialized some mathematical processes (Kutzler, 2003). Before the advent of technology, assessment items had to be carefully chosen so that both the intermediate and final result is “nice.” That is, answers had to be integers, simple fractions, or simple radicals. This was meant to ensure that students would not have to spend most of their time performing arithmetic operations. Technology has trivialized the performance of arithmetic operations since these can be done in an instant. The same is true for graphs and algebraic computation regardless of how complicated they might appear. This trivialization enables teachers to tackle more complex and realistic problems in order to exploit the power of technology.

**Experimentation.** Kutzler (2003) believes that the reason that so many students are at odds with mathematics may be related to their lack of experimentation, since a substantial part of knowledge acquisition occurs during experimentation. Experimentation is similar to or can be taken to be the same as inquiry, which is common
in science. According to the National Science Education Standards (National Research Council, 1996), student inquiry encompasses a range of activities: observing, formulating questions, and devising ways to answer those questions. Students generate answers by analyzing data or secondary-source information they have collected from experiments, books and other media, and experts. They generate new knowledge and test the reliability of that knowledge by explaining and justifying their work to themselves and to others. Experimentation that is performed with paper and pencil is both time-consuming and prone to error. The characteristics of inquiry-based instruction are closely aligned with the implementation of high-level mathematical tasks. According to Stein and Smith (1998), mathematics classrooms that engage students in high-level mathematical tasks require students to use complex and non-algorithmic thinking…; require students to explore and understand the nature of mathematical concepts, processes, or relationships; demand self-monitoring or self-regulation of one’s own cognitive processes; require students to access relevant knowledge and experience and make appropriate use of them…; require considerable cognitive effort and involve some level of anxiety for the student because of the unpredictable nature of the solution process required (p. 271).

**Visualization.** According to Kutzler (2003), visualization is “the illustration of an object, fact, or process” (p. 62). The results of this illustration can be graphic, numeric or algebraic. However, the term is primarily used for graphic illustrations of algebraic objects, numeric objects, or facts describing either the process of illustration or the result of the illustration process. Visualization is mainly used to acquire the competence of changing between representations.
Concentration. Kutzler (2003) compared the teaching and learning of mathematics to building a house. Thus, each mathematics topic that we teach forms a story or floor of the house. As such, the first floor must be complete before we can build the second floor. Similarly, learning any new material in mathematics requires mastery of previously learned materials. If a student has not mastered previously learned material can the student still learn new material? Using an example provided by Kutzler: Solving for $x$ in the equation $5x - 6 = 2x + 5$ by applying a sequence of equivalent transformations a student is told to transform the equation into the form "$x" =$. A student may do the following:

Figure 3: An Example of a Student’s Work

<table>
<thead>
<tr>
<th>$(5x - 6 = 2x + 5) - 2x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>To get; $3x - 6 = 15$</td>
</tr>
<tr>
<td>Applying another equivalent transformation, +6; $3x - 6 = 15$</td>
</tr>
<tr>
<td>To get; $3x = 21$</td>
</tr>
<tr>
<td>Then lastly the student chooses -3 and solves it the following way $3x = 21$</td>
</tr>
<tr>
<td>To get; $x = 18$</td>
</tr>
</tbody>
</table>

The student whose work is in Figure 3 chose an incorrect equivalence transformation and also simplified the algebraic expression wrong. In this case the choice of an equivalence transformation is the new skill that the student has to learn and the simplification of algebraic expressions is a previously learned skill that the student is supposed to have mastered by now. The student’s inability to simplify $3x - 3$ got in the way of learning
how to choose the correct equivalence expression. However, if technology had been used, the Figure 4 would have been the result.

Figure 4: Solving $5x-6=2x+5$ using TI-Nspire CAS

By getting $3x - 3 = 18$ the student would have known that he or she hadn’t gotten the equation in the “$x=$” form. Therefore, technology can be used to help learn new or higher-level skills without the hindrances of misconceptions from previously learned materials. Going back to Kutzler’s building a house analogy: technology fills in the gaps on the first floor so that the second floor can be built.

Strategies that Help Pre-service Secondary Mathematics Teachers Develop TPACK

Guidelines on the strategies and activities that help pre-service teachers can be found in the AMTE Position Statement titled “Preparing Teachers to use Technology to Enhance the Learning of Mathematics.” According to this position statement, the AMTE
encourages mathematics teacher educators to provide opportunities for pre-service mathematics teachers to strengthen their knowledge of how to integrate technology into the teaching and learning of mathematics through experiences that:

- Allow teacher candidates to explore and learn mathematics using technology in ways that build confidence and understanding of the technology and mathematics.
- Model appropriate uses of a variety of established and new applications of technology as tools to develop a deep understanding of mathematics in varied contexts.
- Help teacher candidates make informed decisions about appropriate and effective uses of technology in the teaching and learning of mathematics.
- Provide opportunities for teacher candidates to develop and practice teaching lessons that take advantage of the ability of technology to enrich and enhance the learning of mathematics. 

This framework will be used as an organizational structure for this section of the literature review.

Exploring and Learning Mathematics with Technology

If pre-service teachers do not learn mathematics with technology, they may fail to see the value in integrating technology in mathematics, let alone learn how to use technology in mathematics teaching and learning. Teachers are inclined to teach the way they were taught. The reason why many practicing mathematics teachers do not use technology in mathematics instruction might be related to the fact that they did not engage in similar learning experiences during their own education either in grades K-12 or during their teacher preparation programs. Thus as part of this research, pre-service teachers will learn mathematics with technology.
Available research evidence seems to point towards the fact that pre-service teachers are not afforded an opportunity to learn mathematics with technology (Neiss, 2001; Garofalo, Drier, Harper, Timmerman & Shockey, 2000). Neiss (2001) reported teacher training programs as typically providing a course about technology “perhaps with a unit or discussion on teaching with technology and a requirement that students (pre-service teachers) design lessons to teach with the technologies” (p. 103).

Garofalo et al. (2000) categorized approaches taken by teacher educators to integrate technology into teacher training programs according to the end user of the technology as teacher educator, pre-service teacher and student. In the case of the teacher educator, the teacher educator is the primary user of technology as they use technology to present information or demonstrate explorations. In cases where the pre-service teacher is the primary user, focus tends to be on productivity tools for word processing, grade and record keeping, presentations and and/or subject-specific software and websites for presentations lectures, lessons and assessments. While these are necessary and valuable skills, they are insufficient to prepare teachers to integrate technology into their instruction. Finally, teacher education programs preparing pre-service teachers to use technology in such a way that their own students will use technology for explorations and problem solving focus on guiding pre-service teachers in appropriate uses of technology. This would be a more effective way to prepare pre-service teachers for technology integration in instruction but is often missing.
Model Appropriate Uses of Technology

In addition to allowing pre-service teachers to learn mathematics with technology, mathematics teacher educators should model appropriate uses of technology. Teachers and teacher candidates have often reported that teacher educators advocate teaching with technology but do not do so themselves (Hardy, 2003). This is especially unfortunate because observing instructors who effectively teach with technology can enhance both in-service and pre-service teachers’ attitudes and confidence concerning teaching with technology, and thereby enhance their ability to teach in a similar manner (Pope, Hare & Howard, 2002; Mills & Tincher, 2003; Buckenmeyer & Freitas, 2007).

Teachers acquire behaviors and beliefs about teaching through experiencing school for seventeen or more years in a 30,000 hour apprenticeship-by-observation (Lortie, 1975). Thus, as far as teaching mathematics with technology is concerned, teacher educators should be actively engaged in modeling appropriate uses of a variety of established and new applications of technology.

In addition to having teacher educators model appropriate uses of technology in mathematics teaching and learning, video recordings of exemplary teaching can be also used. For instance, Merkley and Jacobi (1993) investigated the effects of using video tapes in helping pre-service teachers develop new ideas through observation and interpretation of videos of exemplary teaching. The participants, 100 elementary pre-service teachers, were randomly placed in three different settings: an instructional video setting, live telecast and a discussion group. Participants in the instructional video setting observed two instructional tapes and then described the teaching behaviors in the video.
tape. In the live cast setting, the participants observed live telecast of teaching from various elementary classrooms along with a teacher educator who responded to their questions and provided comments. In the last setting, the participants simply discussed issues associated with teaching and assessed a written example of teaching. Data analysis revealed that the pre-service teachers in the live cast setting developed better knowledge of teaching behavior, observation skills and analysis than those in the other two settings.

Making Informed Decisions about Technology Use

Pre-service teachers need help with making informed decisions about appropriate and effective uses of technology. Battey, Kafai, and Franke (2005) studied pre-service teachers’ criteria for evaluating mathematical software (in this case, rational number software at the elementary level). They found that most pre-service teachers focused on surface features, such as clear directions, rather than focusing on the content or pedagogical features afforded by the technology tools.

Zbiek (2005) described a lesson sequence for her technology-focused methods class that helped pre-service teachers realize the power of technology in mathematics teaching and learning. The sequence started with the pre-service teachers being given a task which can be solved using a common high school or college geometry theorem. Some pre-service teachers solved it but others were not able to. The class then considered ways in which secondary school students who are given this problem might use technology. Initially most pre-service teachers suggested using Sketchpad to check arithmetic done by hand or for initial computation. Still some pre-service teachers did not see the point. Textbooks used in schools the pre-service teachers attended or visited and
books that predated the lives of most of the pre-service teachers were brought in to guide discussion on when students might first see the theorem in question and how the books presented it. Though the books varied, they commonly presented a set of three theorems; chord-chord power theorem, tangent-secant power theorem and secant-secant power theorem. A discussion ensued on the similarities and differences among these theorems. The pre-service teachers noted the presence in all three theorems of four segment lengths, the equating of two products of segment lengths and the importance of some point of intersection. Tasked to brainstorm how they can use GSP to help high school students see these mathematics connections, most groups suggested having one GSP file to illustrate each of the three theorems. The class was further challenged to consider how the dynamic nature of GSP may be used to get at the essential connections among the three theorems. Eventually someone noticed that the point of intersection is the point they want to see move and they set out to create a computer file that allows the user to drag the intersection point.

This done, the pre-service teachers were then asked to develop two lesson ideas. According to Zbiek (2005, p. 296) “a lesson idea is a description of the general flow of a lesson but far less detail than a lesson plan.” The goal of the first one was to have secondary students know and be able to use a formula to solve problems similar to their initial task. The goal of the second was to use the file or applet to help students understand the proofs of theorems. Follow-up discussion included how college mathematics content arises in high school textbook and also connecting lessons to NCTM and state standards. The pre-service teachers enrolled in this course were reported to have
“moved from seeing technology only as a way to check answers to seeing technology as a venue for illustrating mathematical principles, for exemplifying different aspects of mathematics concepts, and illuminating mathematical connections” (Zbiek, 2005, p. 301). In addition, they also moved from seeing technology as “another thing to teach” or “another thing to learn” to a tool to help students develop a deep understanding of concepts.

In another study, Niess (2005) required pre-service mathematics teachers enrolled in her course to locate over 60 technology resources for teaching mathematics. The pre-service teachers were then required to align the concept or skill taught using technology to the appropriate mathematics and technology standards (state and national), thereby forcing them to reflect upon how and why the technology tools should be used in mathematics instruction. The result of this assignment was a “consistent focus in considering the curriculum from a standards base” (p. 522). The goal of this exercise was not to integrate technology for technology’s sake. Rather, the pre-service teachers considered not only the features of the technology tools themselves, but also the goals of the instruction supported by technology.

Grandgenett, Harris & Hoffer (2011) suggested Learning Activity Types Taxonomy to help teachers develop and use TPACK in ways that attend to particular demands of different subject matter domains. TPACK-related learning activity types are basically lists of content based activities and the compatible technology. This is intended to help teachers become aware of the full range of possible curriculum-based learning activity options and the different ways that technologies support each of them. This can
help teachers effectively select among, customize, and combine activity types that are well matched to both students’ learning needs and preferences and classroom contextual realities such as computer access and class time available for learning activity work.

Since TPACK is subject specific, the activity types that are subject specific are still being developed and refined and are available for perusal at http://activitytypes.wmwikis.net/.

Table 2 is an example of Mathematics Learning Activity Types (Grandgenett, Harris & Hoffer, 2011) that have been developed so far. Seven genres of mathematics activity types (consider, practice, interpret, produce, apply, evaluate, and create) were conceptualized based on the NCTM’s process standards. Table 2 shows activities that fall under the “apply” category.

Table 1: An Example of the “apply” Activity Types (Grandgenett et al., 2011)

<table>
<thead>
<tr>
<th>Activity Type</th>
<th>Brief Description</th>
<th>Example Technologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose a Strategy</td>
<td>The student reviews or selects a mathematics related strategy for a particular context or application.</td>
<td>Online help sites (WebMath, Math Forum), Inspire Data, dynamic geometry/algebra software (Geometry Expressions), Mathematica, MathCAD</td>
</tr>
<tr>
<td>Take a Test</td>
<td>The student applies their mathematical knowledge within the context of a testing environment, such as with computer-assisted testing software.</td>
<td>Test-taking software, Blackboard, survey software, student response systems</td>
</tr>
<tr>
<td>Apply a Representation</td>
<td>The student applies a mathematical representation to a real life situation (Table, formula, chart, diagram, graph, picture, model, animation, etc.).</td>
<td>Spreadsheet, robotics, graphing calculator, computer-aided laboratories, virtual manipulatives (algebra tiles)</td>
</tr>
</tbody>
</table>
Although these learning activity types have essentially been developed for in-service teachers, their nature enables them to be used by pre-service teachers as they design or learn to design their own lessons.

Opportunities to Develop and Practice Teaching Lessons

According to Silk, Higashi, Shoop, and Schunn (2010, p. 21), “teaching mathematics in a technology classroom requires more than simply using mathematics with technology. It requires designing the lesson to focus, motivate, and highlight the mathematics in a meaningful way.” Thus pre-service teachers need to be prepared to design such lessons. Currently two approaches have been reported to foster the development of TPACK in teachers: learning technology by design (Mishra & Koehler 2006), and the use of learning activity types (Harris, Mishra, & Koehler 2009). Learning technology-by-design involves having the teachers participate in the curriculum design process. Emphasis is placed on learning by doing. This affords the teachers the opportunity to transcend the passive learner role and to take responsibility of their own learning. For instance, in order to give teachers additional insight into educational psychology and educational technology, Mishra and Koehler (2006) had teachers work in groups to make idea-based videos to communicate an important educational idea. In another instance, participants were expected to both learn interactive web-based technology and also generate abstract knowledge designing educational technology. As such, participants were involved in re-designing existing web site and web resources. The
emphasis on design was to ensure that participants would not spend time researching the topic but instead would focus on key issues related to technology, pedagogy and content.

Faced with limited availability of field placements for pre-service teachers and lack of sufficient preparation to focus attention on mathematics teaching and learning in production ways, Fernandez (2005) developed Microteaching Lesson Study (MLS). Fernandez (2008) extended this further by describing ways of incorporating MLS in mathematics teacher education. The five-week long sequence began with MLS groups researching and planning their lessons followed by teaching and revision of lesson plans. Finally there was analysis and revisions for the re-teaching. Lessons were video-taped for analysis and discussion during the analytic and revision phases. The groups completed a written reflective report of the MLS experience for submission as part of the course requirements. In addition, they also submitted individual analysis of group collaboration. Results revealed that the experience enabled the pre-service teachers to develop knowledge and use of mathematics teaching practices aligned with recent reforms and theories. In addition, it was also observed that MLS helped pre-service teachers learn to create and implement lessons that are more student-centered and engage students in tasks that are worthwhile. They also learned the value of collaboration.

Matthews, Hlas and Finken (2009) incorporated four-column (Figure 5) lesson planning into lesson study as a way of encouraging pre-service teachers to be more student-centered and to value collaboration. According to Matthews et al. (2009), the traditional U.S. lesson plan format which consists of one column, is sequential and focuses on the teacher’s actions for the lesson. Instead, a four-column plan uses both the
vertical and the horizontal dimension since items are arranged vertically in sequential order and are synchronized horizontally. Developing a four-column plan requires predicting students’ responses and assessing students’ understanding. The four-columns are organized horizontally as follows:

Figure 5: A Blank Four-column Lesson Plan Suggested by Matthews, Hlas & Finken (2009)

<table>
<thead>
<tr>
<th>Overall Goal:</th>
<th>Material:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steps of the Lesson:</strong> Learning Activities and Key question</td>
<td><strong>Expected Student Reactions or Responses</strong></td>
</tr>
</tbody>
</table>

Using the Fernandez (2005) adaptation, which streamlined the lesson study format for methods classes, pre-service teachers developed goal-driven lessons, used peer observation for lesson evaluation, and conducted post-lesson debriefing which led to informed revisions. In a similar lesson study adaptation, they incorporated cooperatively written four-column lesson plans as part of the practicum and methods course content. Surveys conducted at the end of the course, follow-up surveys conducted six months later, comments from pre-service teachers’ student teaching supervisors, and the pre-service teachers’ journal entries from the methods class were used to examine the impact of the lesson study adaptation. The pre-service teachers reported that the four-column planning and lesson study helped them to become more student centered in their
approach to teaching, planning and reflection. They also reported that the lesson study process helped them to recognize the value of collaboration.

An important part of developing and practicing lessons is the post reflection. Good teachers are reflective teachers. It has become common to use video tapes of pre-service teachers’ lessons and micro-lessons with peers for self-assessment and as a basis for discussion. According to Staudt and Fuqua (2000), in order for pre-service teachers to effectively assess their own teaching, accurate data must be collected through the use of videotapes. They argued that videotapes of teaching episodes foster self-assessment and reflection since they can be stopped, reversed, and fast forwarded thereby allowing pre-service teachers to focus on specific aspects of their lessons.

There is, however, the need for structure in the video analysis, otherwise the pre-service teachers may fail to notice or learn what they are supposed to learn (Staudt & Fuqua, 2000). In a study by Sherin and van Es (2005), six pre-service teachers working towards certification in secondary mathematics or science participated in three hour-long video sessions in which they used software Video Analysis Support Tool (VAST) to analyze their own and others’ teaching. The pre-service teachers were prompted to analyze three aspects: a) student thinking, b) the teacher’s role and c) classroom discourse. A month before and a month after the VAST sessions, the pre-service teachers wrote narrative essays in which they discussed a videotaped lesson of their own teaching. Data for the study included two essays from each of the six teachers who took part in the VAST sessions and two from each of the six pre-service teachers who did not. Results showed a change in what the pre-service teachers who participated in the VAST sessions
noticed. Initially pre-service teachers from both groups (VAST & non-VAST) identified all the events in the video segments as noteworthy. However, in the second essays, VAST participants became more discriminatory and organized their essays around significant aspects of teaching.

In another study, Bulgar (2007) used supported video exemplars to foster the professional development of pre-service teachers. The video vignettes were part of a Virtual Learning Community (VLC) website that was designed with Mathstore software. The Mathstore software integrates support directly into the video observations by allowing comments and questions to be embedded into the videos. Thirty-four pre-service teachers who were enrolled in two sections of Methods of Teaching Mathematics in Elementary School were given an opportunity to observe video vignettes of 2nd grade mathematics teaching. They worked in groups of three or four to examine and reflect on the vignettes, discuss their observations and answer four questions that were embed in the videos. Data for this study were the ten groups’ final written products. Results of the study indicated that the pre-service teachers were able to gain a deeper and insightful understanding of the four aspects that were under investigation (affect, mathematical representations, and classroom discourse and student engagement).

**The Development of Mathematic Teachers’ TPACK**

Neiss, Sadri and Lee (2007) proposed a model for the development of TPACK in mathematics teachers. The model was informed by Everett Rogers’ (1995) model of innovation-decision process and observations, over four years, of teachers learning about
spreadsheets and how to integrate them as learning tools in their mathematics classroom. It, therefore, emerged from these observations that mathematics teachers progressed through a five-stage developmental process when learning to integrate a particular technology in their classroom:

1. **Recognizing** (knowledge), where teachers are able to use the technology and recognize the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.

2. **Accepting** (persuasion), where teachers form a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology.

3. **Adapting** (decision), where teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with an appropriate technology.

4. **Exploring** (implementation), where teachers actively integrate teaching and learning of mathematics with an appropriate technology.

5. **Advancing** (confirmation), where teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology.

(Neiss, Sadri & Lee, 2009)

Based on these levels the AMTE’s Technology Committee created a visual description of the TPACK model, as depicted in Figure 6. Although the model may seem to be depicting linear progression toward TPACK, Neiss, Ronau et al. (2009) pointed out that the transition from one level to another does not display a regular, consistently increasing pattern.
Inherent in the TPACK development model in Figure 6 is the assumption that teachers already have PCK which might not be the case with some pre-service teachers. However, for pre-service teachers in this study, who are four months away from student teaching, it would be reasonable to assume that they have some form of PCK from all the courses that they have taken throughout their teacher preparation training. However, application of this model at pre-service level should be done with caution and I will elaborate on this in chapter 5.

Summary

The position statement from AMTE (2006) provides a comprehensive list of strategies that can help pre-service teachers develop TPACK. Pre-service teachers need opportunities to learn and explore mathematics with technology. Neiss (2005, p. 509)
pointed out that “learning the subject matter with technology is different from learning to teach the subject matter with technology.” Therefore, pre-service teachers also need opportunities to design and practice lessons that integrate technology, a strategy that Mishra and Koehler (2006) referred to as learning technology-by-design. The Learning Activity Types Taxonomy can also be used to inform or enhance the design process. Lastly, it is equally important for mathematics teacher educators to model good teaching practices with technology since teachers have been reported to teach the way they were taught. The AMTE’s position statement was used to guide the methodology of this study. The model for the development of mathematics teachers’ TPACK provided a basis for tracking the development of pre-service mathematics teachers’ TPACK.
CHAPTER 3

RESEARCH DESIGN AND METHODS

Introduction

The main goal of this study was to understand the development of TPACK in pre-service secondary mathematics teachers who are enrolled concurrently in three courses (methods, modeling and practicum) which are taught in collaboration by three different instructors. The four questions that this research study sought to answer were:

1. How do pre-service teachers’ conceptions about the purposes for incorporating technology in teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

2. How does pre-service teachers’ knowledge of students’ understandings, thinking, and learning of mathematics with technology emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

3. How does pre-service teachers’ knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

4. How does pre-service teachers’ knowledge of instructional strategies and representations for teaching and learning mathematics with technologies
emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

Since the study focused on the development of TPACK in pre-service teachers in a real context and the researcher had little control over the pre-service teachers’ experiences, this study was a case study. In order to capture maximum variation, the pre-services teachers were treated as individual cases and a cross case analysis was conducted. This research study, like all research studies may be influenced by the researcher. Because the researcher is the data collection instrument in qualitative studies, it is particularly important to understand the researcher’s subjective stance toward the topic. As such, this section is devoted to the researcher’s background and values in order to help expose any biases that the researcher might have in this research study and to also convince the reader about the researcher’s ability to carry out this study.

The Researcher

The researcher taught high school mathematics for two years before completing a masters degree in science education majoring in mathematics. This research study is in partial fulfillment of the requirements of a Doctor of Philosophy in Mathematics with a Mathematics Education specialization. Prior to this study, the researcher completed some other research projects. For her master’s thesis she carried out a developmental study in which she used Java script to create classroom-ready applets for teaching transformation geometry concepts in Zimbabwean schools. Data for this study was collected using surveys, classroom observations and interviews. As a graduate student at the current
university the researcher completed a mini-qualitative research project studying students’ perspectives on their use of a computer algebra system (the TI-Voyage 200) in a college algebra course. She also conducted a descriptive and correlational study investigating the beliefs and practices of university mathematics instructors regarding calculator use in mathematics teaching and learning. Lastly, the researcher conducted a pilot study which informed the design of this study.

As a researcher I acknowledge the fact that I have a bias towards the use of technology in mathematics teaching and learning although I never had a chance to learn mathematics with technology myself. This bias developed while I was in graduate school in Zimbabwe. After teaching mathematics in a high school for two years I enrolled for a master of science education in mathematics degree. As a master’s student in Zimbabwe, I was required to take two Java programming courses and another course on mathematical software. This, together with reading available literature, opened up a new world in mathematics that I didn’t know existed. The only time that I had used any piece of technology in mathematics before this was in high school statistics because we had a lot of numbers to “crunch.” For my master’s thesis I developed Java applets for teaching transformation geometry concepts in Zimbabwe because I had learned about the difference that technology could make in a mathematics lesson. My goal was to develop something that was customized to the local syllabus so that teachers could use it with little or no technology training.

When I moved to the U.S, I began to teach mathematics courses at the university. In the first class I taught, pre-calculus, I was expected to use the TI-83 to demonstrate
concepts. I was very new to this technology and struggled to make it through the semester. I learned a lot during this time and taught the class two more times, becoming proficient at using the TI-83 in this way. However, I have never stopped wondering whether I could have taught with technology if it wasn’t imposed on me. Will a high school mathematics teacher take such a risk without proper training?

By associating with my advisor I also learned about the power of CAS calculators especially from Texas Instruments such as the TI-89, TI-92, Voyage 200, and the TI-Nspire CAS. Not only can these calculators perform symbolic manipulations, they also have dynamic geometry software on them. Instead of trying to create my own applets I decided to focus on learning to use the available resources. I attended a TI-Nspire summer institute in which I learned a bit about using the TI-Nspire CAS. I have observed the modeling instructor teach with the TI-Nspire for three semesters. The first time I was contemplating my study, the second time I tried to pilot my study idea with only the modeling course, and the third time I was doing a pilot study with all three courses (methods, modeling, and practicum). I have also taken time to learn how to use the TI-Nspire on my own and have occasionally used it to demonstrate concepts in the classes that I teach. However, I have not had a chance to teach a class in which I make extensive use of the TI-Nspire CAS.

As far as TPACK is concerned, I know about it through literature review. In addition I have attended two Societies for Information Technology and Teacher Education (SITE) conferences in which there were a lot of TPACK sessions. I presented at the first conference and also presided over one session. This gave me a chance to meet
and discuss about TPACK with other seasoned researchers such as Margret Neiss. To me, this enhanced my understanding of TPACK and TPACK related research.

My high school teaching experience although from Zimbabwe coupled with what I have learned about the U.S. high school curriculum as part of my doctoral courses helps me evaluate a high school teaching episode. Although TPACK is somewhat an ill-defined concept, I have a good sense of what it means to have TPACK and therefore able to evaluate either lesson plans or teaching.

The Research Study

This study sought to understand the development of TPACK in five pre-service secondary mathematics teachers who were enrolled concurrently in three courses (methods, modeling and practicum) that were taught in collaboration by three different instructors. To this end, TPACK changes in pre-service teachers were tracked during a period of about 15 weeks. Each individual pre-service teacher was treated as a single case making this study a multiple case study. Mathematics teachers’ TPACK was described by Neiss (2005) as having four components:

- An overarching conception about the purposes for incorporating technology in teaching mathematics.
- Knowledge of students’ understandings, thinking, and learning of mathematics with technology.
- Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics.
Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies

Thus, tracking changes in TPACK entail tracking the changes in the four components of mathematics teachers’ TPACK.

Participants

The participants of this study were five pre-service secondary mathematics teachers who were concurrently enrolled in three courses (methods, modeling and practicum) during the fall 2011 semester at this north-western university. At this university, pre-service teachers typically enroll in all three courses during their senior year in the semester before their student teaching. Occasionally, though, there are pre-service teachers who enroll in only a subset of the courses. In the pre-service teacher preparation program at this university, only two mathematics content courses for pre-service teachers (Modern Geometry and Mathematical Modeling) make extensive use of technology. Geometry is usually completed before the modeling course, but some pre-service teachers take the modeling class early. As a result some of the pre-service teachers in the modeling course may not have any prior experience with a technology-intensive mathematics content course.

Sampling

The researcher had no control over pre-service teachers’ enrollment in the three courses (methods, modeling and practicum). It is common for pre-service teachers to
enroll in all three courses at the same time, though it is not a requirement. As such, there are pre-service teachers who enroll in one or two of the courses and not the other(s). Although the researcher had no control over the pre-service teachers’ enrollment in the three courses, purposive sampling and, in particular, *criterion sampling*, was used to select participants for this study. This sampling technique involves selecting cases that meet a certain predetermined important criterion (Patton 2002). For instance, Tarr et al. (2008, pp. 252) selected schools for their study on the impact of middle-grades’ mathematics curricula and the classroom learning environment on student achievement based on three criteria: “(a) number of years since adoption of current mathematics textbook series, (b) organizational structure of the middle school, and (c) size of the community in which the school was located.” The main criterion for this study was enrollment in all the three courses (modeling, methods and practicum). This was meant to exclude those pre-service teachers who were enrolled in one or two of the courses and not all three. Participation in this study was voluntary. At the beginning of the semester, the participants signed a consent form (Appendix C) which specified this and also the fact that they could decline to participate at any point during the study.

**The Courses**

To demonstrate that this study took place in an environment that supports the development of TPACK among pre-service teachers enrolled in the three courses, the next three sections are devoted to describing the courses in which the study participants were enrolled.
The Mathematical Modeling Course

The modeling course is a three-credit senior capstone course that is offered every fall semester. The main purpose of this senior capstone course is to help pre-service teachers make insightful connections between the advanced mathematics they learned in college and the high school mathematics they will teach (CBMS, 2001). In order to make connections between college and high school mathematics clear, most of the problems presented in this course are high school level, taken from different high school materials including the Navigation Series for grades 9 – 12. The pre-service teachers are also introduced to the Center for Mathematics Education (CME) project, a National Science Foundation (NSF) funded high school program that is organized around Algebra 1, Algebra 2, and Pre-calculus and integrates technology throughout. The main goal of the CME project is “to help students acquire a deep understanding of mathematics” (EDC 2009, p. T3). It is guided by six principles among which is “using technology to sharpen, not to replace, thinking” (http://www2.edc.org/cme/principles.html). The CME project authors view technology as a tool for building new ideas and as a medium for developing mathematical habits of mind. Thus, together with pointing out the technology required on the relevant sections throughout the textbooks, such as CAS, Function Modeling Language, Geometry Software, Spreadsheet Software in Algebra 2, and Graphing calculator in Algebra 1, they also provide a TI-Nspire Technology handbook.

Why mathematical modeling? According to Blum and Ferri (2009), “mathematical models and modeling are everywhere around us, often in connection with powerful technological tools, preparing students for responsible citizenship and for
participation in societal developments requires them to build up modeling competency’’ (p. 47). In addition, the Common Core State Standards’ fourth Standard for Mathematical Practice specifies that mathematically proficient students should be able to model with mathematics.

The course instructor, using recommendations from both the standards and research, developed the following goals for the course to:

- familiarize pre-service teachers with technological tools for mathematical modeling in the classroom
- help pre-service teachers develop a better understanding of some standard mathematical models
- help pre-service teachers develop their own mathematical modeling habits of mind
- provide pre-service teachers with modeling activities, technologies, and materials that they can use in the secondary classroom; and
- help pre-service teachers connect their mathematical, technological, and didactical perspectives as they enter the teaching profession (Appendix F)

The course is divided into three main units, namely:

1. Statistics and Probability: Simulation Modeling
2. Sequences, Functions, Difference Equations and Curve Fitting
3. Measurement, Data Gathering, Motion Simulation, Dynamic Systems

The main technology for this course is the TI-Nspire CAS. However, the course begins by introducing the pre-service teachers to online simulations. This is done for two main reasons: (1) to allow pre-service teachers time to purchase their TI-Nspire CAS calculators and, (2) to have pre-service teachers become familiar with how simulators work before they start building their own.

The class meets twice a week on Tuesdays and Thursdays for 75 minutes since this a three credit hour course (one credit hour is 50 minutes long). Each lesson typically
begins with a mathematical investigation that provides opportunities for the pre-service teachers to work together to solve problems, connect mathematical ideas, and expand their understanding of high school mathematics content. An example of such an investigation is provided in Appendix H. The instructor also takes this time to model appropriate teaching strategies with technology and to expose the pre-service teachers to activity-centered learning, allowing them to participate and lead. The pre-service teachers usually demonstrate, using a computer that is projected to the white board.

Besides the weekly assigned homework, there are three other types of assessments: end of unit projects and unit exams, papers and ERMO summaries. Unit projects are a summative form of assessment and are used to assess the pre-service teachers’ a) mastery of both the content and the technology and b) ability to use technology appropriately for the chosen content. These unit projects take different forms such as PowerPoint presentations and student activities. For instance, after the simulation unit, the unit project involved pre-service teachers creating a PowerPoint presentation for a hypothetical group of in-service teachers who were described as new to both technology and the Common Core State Standards. The technology was supposed to support the pre-service teachers’ interpretation of the Common Core State Standards. For the curve fitting unit, the pre-service teachers wrote two student activities/lessons that help students understand the relationship between the coefficients and degree of a polynomial and its associated sequences. Initial projects are turned in for the instructor’s feedback. This usually exposes some pre-services misconceptions. A feedback session is
held between the instructor and all the individual pre-service teachers. The pre-service teachers are then required to redo their projects and then submit them for a grade.

The Methods Course

This course is a three-credit hour methods class that examines curricular issues, learning theories, teaching strategies, instructional materials, and assessment procedures for teaching secondary school mathematics. The course is designed to enable pre-service teachers to become effective professional decision makers in the context of standards-based teaching. Since the course is focused on standards-based teaching, the pre-service teachers, given a set of standards, should be able to:

- Understand what students are to learn and the contextual factors that influence their learning
- Create an assessment plan
- Select teaching strategies
- Implement the strategies
- Implement the assessment plan
- Assess the effectiveness of their teaching (Course Syllabus in Appendix E)

The course also considers five aspects of a professional mathematics educator’s work: teaching strategies, mathematical access, tools for teaching and learning, research skills and the profession of teaching. For teaching strategies, the pre-service teachers are required to observe, discuss, analyze and practice various teaching skills and strategies which are consistent with the goals for 9-12 mathematics education as defined by both the state and national standards. For mathematical access, the pre-service teachers examine issues related to social justice, gender and multiculturalism and the teaching of mathematics through reading selected literature sources. They are then required to formulate a plan for ensuring mathematical access among all students. For the tools for
teaching and learning mathematics, the course has a particular focus on the integration of technology, pedagogy and mathematical content knowledge and therefore the pre-service teachers are required to plan and practice lessons that integrate the technology, pedagogy and mathematical content knowledge. The pre-service teachers are also required to read about and practice using other tools such as manipulatives. To enhance the pre-service teachers’ research skills, the pre-service teachers are required to search for classroom activities and lessons that make use of both technology and other tools such as manipulatives and for research articles that have a direct impact in their mathematics classroom. Lastly, for the profession of teaching, the pre-service teachers are required to read articles relating to the professionalism including the role and nature of professional organizations.

The class meets twice a week, on Mondays and Wednesdays for 75 minutes. The instructor creates an agenda for each class such as the one in Appendix I. This helps focus each lesson. The class is made up of whole class discussions, instructor exposition, and group work and student presentations. The whole class discussions are usually teacher-led and are centered on issues in mathematics education. These discussions stem from readings assigned by the instructor, ideas that have been researched by the pre-service teachers, and pre-service teachers’ field experience observations. Sometimes the discussions take place on line. Instead of whole class discussions, the instructor occasionally presents the material pausing where necessary to either respond to or ask questions. Group work is also assigned at times. Student presentations mainly involve the pre-service teachers presenting their lessons.
Course assessments include written reports (and discussions); lesson plans (and presentations); field experience/reading response journals; and a teacher candidate work sample. The pre-service teachers are assessed on their ability to synthesize the readings through written reports. Using the lesson plan template in Appendix K, the pre-service teachers write at least four lesson plans in a semester. Six areas that are assessed in the lesson plans are activity selection and attention to standards, lesson goals and objectives, prerequisites, assessment procedures, lesson procedure and grammar, style and presentation. For activity selection and attention to standards, the focus is on the appropriateness of the technology choice and the identification and analysis of relevant standards. For lesson goals and objectives, the objectives should be both clearly stated and measurable. On prerequisites, the given prerequisites should be complete and they should enable the stated objectives to be accomplished. Assessment procedures should clearly correspond to objectives. For the lesson procedure, the written description should clearly describe how students are actively engaged in learning and the anticipated student responses. Lastly, the presentation has to be neat and without errors.

The pre-service teachers are required to keep participate in online discussions based on their practicum field experience. Lastly, the pre-service teachers are required to create a teacher candidate work sample that demonstrates their capacity to fulfill the course goals. A teacher candidate work sample is an instructional sequence in which the pre-service teachers are supposed to show evidence of their understanding of important mathematics for school students and of appropriate pedagogy and technological pedagogy in creating environments conducive to students’ learning of mathematics. The
sample also includes a statement of their teaching philosophy and other documents that provide evidence of their development as teaching professionals. This work sample is meant to prepare the pre-service teachers for student teaching.

The Practicum Course

This course is designed to provide an introduction to the professional practice environment of public school teaching at the secondary level and also to prepare pre-service teachers for their student teaching experience. Topics covered include: the structure of public school administration at national, state and district levels; the nature of school funding; issues of diversity; classroom management; the nature of teacher licensure; Indian Education For All (IEFA); engaging parents and community members, as appropriate, with the school community; working with other school professionals; school law; sexual harassment in public schools; Response To Intervention (RTI); and applying data-driven decision-making to the instructional process. (Appendix G)

This course has two components: the on-campus weekly seminar and the field experience that are supervised by two different instructors. Each component complements the other although the field experience is also used to complement the methods course. The field experience usually begins several weeks after the beginning of the semester. During fall 2011 the field experience began during 5 weeks into the semester. Pre-service teachers are supposed to complete no less than 60 hours in the classroom with their assigned cooperating teachers. The cooperating teachers must have at least three years of teaching experience. The 60 hours include class observations, teaching at least two lessons and any other tasks assigned by the cooperating teachers.
The university requires the pre-service teachers to teach at least three lessons, one that is observed by a peer and two that are observed and video-taped by the field supervisor. After each observed lesson, the field supervisor (who is also the researcher) meets with both the pre-service teacher and the cooperating teacher to discuss the lesson. The pre-service teachers are then required to reflect on their lessons using the videotapes.

For the practicum field experience, the pre-service teachers are assessed in four areas: content, diversity, pedagogy, assessment and reflection. For the content area, the pre-service teachers are assessed on their ability to demonstrate understating of central concepts, tools of inquiry, structure of the subject and knowledge of content standards. For diversity, the pre-service teachers are assessed on their ability to demonstrate effective classroom management and knowledge of relationship building strategies with school colleagues, families and agencies in the larger community to in order to support student learning and well-being. For pedagogy, the pre-service teachers are assessed on their ability to demonstrate research-based pedagogy including appropriate use of technologies. For assessment, the pre-service teachers are assessed on their ability to design instructional material and assessment that promote and measure student learning. Lastly, for reflection, they are assessed on their ability to reflect on student learning, teaching practice, and on identifying areas of future growth.

The Context of the Study

Since the three courses that were part of this study were offered in collaboration, this section is a description of both how the courses were coordinated and the content
covered in each course during the course of the study. Figure 7 helps illuminate the collaboration by showing the focus of each course during the semester and the timing of the lesson designs that formed an important part of this study. I will refer to Figure 7 constantly in the following paragraphs.

The following subsections are divided by weeks and these weeks were determined by the timing of the lesson designs and implementation, that is, the first two weeks before the first lesson design and implementation and the rest of the intervals were determined by the lesson design and implementation cycles. As part of the coordination, the instructors of the methods and modeling courses met with the researcher every Friday throughout the semester. Discussions centered on the previous week’s activities and assessments results and plans for the upcoming week. The meetings helped the researcher keep abreast with flow of the study. The researcher audio taped and kept personal notes of all the weekly meetings.
Figure 7: A Timeline Showing the Focus of the Three Courses during the Semester and the Timing of the Lesson Designs

<table>
<thead>
<tr>
<th>Time</th>
<th>1 - 2 weeks</th>
<th>3 - 4 weeks</th>
<th>5 - 6 weeks</th>
<th>7 - 8 weeks</th>
<th>9-10 weeks</th>
<th>11 - 12 weeks</th>
<th>13 - 14 weeks</th>
<th>15 – 16 weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Methods Course</strong></td>
<td>Understanding what students are to learn and contextual factors</td>
<td>Create assessments</td>
<td>Selecting teaching strategies</td>
<td>Implementing teaching strategies and assessment plan</td>
<td>Assessing the effectiveness of your teaching</td>
<td>Final exam</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Practicum Course</strong></td>
<td>21st Century Learners Student engagement</td>
<td>Reflection Project-Based Learning</td>
<td>Practicum field experience</td>
<td>Classroom management</td>
<td>Differentiation Strategies to ensure student learning</td>
<td>Assessment &amp; Evaluation In defense of homework</td>
<td>Measuring Student Learning Integrating Science &amp; Math</td>
<td>Thanksgiving</td>
</tr>
<tr>
<td><strong>Modeling Course</strong></td>
<td>Statistics and probability : Simulation modeling</td>
<td>Sequences, functions, difference equations and curve fitting</td>
<td>Measurement, data gathering, motion simulations, dynamical systems</td>
<td>Final exam</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Lessons</strong></td>
<td>First group lesson design and implementation</td>
<td>Second group lesson design and implementation</td>
<td>First individual lesson design and implementation</td>
<td>Second individual lesson design and implementation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Weeks 1 & 2

The methods and modeling instructors met with the researcher Friday before the semester started to synchronize their syllabuses and also plan for the major data collection activities, that is, the lesson plan design and implementation, administration of the TPACK survey and teaching philosophy statements. Although the researcher was the field supervisor for the practicum students, she was not responsible for the practicum syllabus. For this reason the practicum syllabus was not available for synchronizing with the other two syllabuses.

During the first week, the modeling courses introduced the pre-service teachers to, “Statistics and probability: Simulation modeling.” This involved pre-service teachers working with online simulations. Since the pre-service teachers’ first lesson group design involved designing a lesson for an 8th grade Algebra class, the second week was spent focusing on solving high school problems using the TI-Nspire CAS calculator.

The methods course introduced the pre-service teachers to; “understanding what students are to learn and contextual factors.” This included an introduction to the NCTM and the Common Core State Standards, Hands on Equations, research in mathematics education, equity issues in mathematics education and the big ideas in high school mathematics. Readings for the two weeks included among others the Common Core State Standards for Mathematical Practice, Chapters 1 and 2 of the NCTM Standards and an journal article entitled Doing it Right: Strategies for Implementing Technology.

The practicum course, introduced the pre-service teachers to; the 21st learner, student engagement, reflection and project-based learning. The vision of the 21st teachers
was presented as, “adapting and evolving, teachers digitally empower diverse learners to connect, communicate, collaborate, and create in an interactive technology-rich environment.” Although this practicum syllabus was not synchronized with the other two syllabuses, this introduction helped tie together the three courses.

As part of the study, the pre-service teachers signed a consent form (Appendix C) agreeing to participate in the study on the first day of the methods course. They were then asked to complete the TPACK survey online by the next class day. The pre-service teachers wrote their teaching philosophy statements during the second week of classes.

**Weeks 3, 4 & 5**

For these three weeks the modeling course focused on simulation modeling with the TI-Nspire CAS CX calculators. The practicum course focused on Reflection during the 3rd week, then Project-based Learning in the 4th week and Classroom Management in the 5th week. Four of the five study participants began their practicum field experience in the fifth week.

To enhance the pre-service teachers’ ability to design lessons that integrate technology, the pre-service teachers were involved in two collaborative group lesson design tasks in a manner similar to the model proposed by Groth, Spickler, Bergner, and Bardzell (2009). At the beginning of the 3rd week, an 8th grade middle school algebra teacher who was known to the modeling course instructor to use technology in her teaching was invited to the methods class to talk about her class, her curriculum and the technology that was available in her classroom. Using this information, the groups were given the following assignment:
In your assigned groups, design a lesson that could be taught in Ida’s (not her real name) 8th grade algebra class. Focus in on the curriculum, big ideas, unit, and lesson sequence to identify a lesson. The lesson should demonstrate the use of technology as an appropriate tool for enhancing student learning.

Both groups created lessons on “solving linear equations in one variable with variables on both sides.” The two groups presented their lesson ideas in the methods class for feedback. For implementation, the lessons were refined and given to the middle school teacher to choose the one she wanted to teach. She taught the lesson in her 8th grade classroom. According to Lewis (2002), it is ideal to have all the collaborative group lesson design participants being present during implementation. However, it was hard to schedule a time when all the pre-service teachers were available. This lesson was therefore videotaped by the methods instructor. The video was played during the methods class to facilitate a classroom discussion on the implemented lesson. Both the methods and the modeling instructors and the researcher participated in the class discussion on the implemented lesson. The pre-service teachers wrote individual reflections on how the lesson went.

Weeks 6, 7 & 8

The modeling course ended the focus on simulation modeling during the 6th week. This culminated in an end of unit project. The course then introduced the pre-service teachers to the second unit: *Sequences, Functions, Difference Equations and Curve Fitting.* The practicum course focused on: *Teaching Blunders, Differentiation and Strategies to Ensure Student Learning.* The methods course introduced the pre-service teachers to *Teaching Strategies.* Beginning the 6th week, the pre-service teachers started
having online discussions on the methods D2L shell about their practicum field experience. The 6th week’s discussion was on Contextual Factors, the 7th week focused on Equity while the 8th week focused on Assessment.

At the beginning of the 6th week an AP statistics and calculus high school teacher who was also known by the modeling course instructor to use technology in his teaching was invited to the methods class to talk about his statistics class, his curriculum and the technology available for use in his classroom. Using this information and maintaining the groups from the first group lesson design and implementation, the pre-service teachers were given exactly the same assignment as they were for the first group lesson design and implementation. The presented their lessons for feedback before having them implemented by the AP statistics teacher. For implementation, the two lessons were given to the high school teacher to choose one he would want to teach but he decided to teach both because he liked both of them. The pre-service teachers watched the video of the lesson implementation and wrote individual reflections on their group lessons.

Weeks 9, 10 & 11

The modeling course spent the three weeks focusing on Sequences, Functions, Difference Equations and Curve Fitting. The pre-service teachers had an end of unit project on difference equations. As part of the project, the pre-service teachers were asked to write two student activities/lessons that help students understand the relationship between the coefficients and degree of a polynomial and its associated difference sequences. The practicum course focused on Assessment and Evaluation during the 9th week, In Defense of Homework during the 10th week and Measuring Student Learning.
during the 11th week. The methods course focused on Selecting Teaching Strategies during the 9th week and then Implementing Teaching Strategies and Assessment during the 11th and 12th weeks. The practicum field experience based online discussions for the 9th week focused on Manipulatives, the 10th week focused on Classroom Discourse and the 11th week focused on Curriculum.

At the beginning of the 9th week, the method instructor asked the pre-service teachers to design individually lessons that incorporate social justice. Incorporating social justice in a lesson means designing a lesson that addresses equity or fairness issues. The methods instructor added that ideal lesson as being in a social justice context and incorporating meaningful use of technology. For the implementation of this lesson, the pre-service teachers presented their lessons in the methods class first, got some feedback from, revised the lessons and then taught the lessons in their practicum classrooms. I observed and videotaped the lessons. After each lesson I held a post conference discussion with each of the pre-service teachers to discussion how the lessons went. The pre-service teachers then got a videotape of their teaching and used it to write a post-lesson reflection. The whole teaching cycle (lesson planning, micro-teaching, lesson revision, teaching, post lesson discussion) lasted for about three weeks.

Weeks 12, 13, 14, & 15

For the entire four weeks the modeling course focused on Measurement, Data Gathering, Motion Simulations and Dynamical Systems. The practicum course focused on Integrating Science and Mathematics and the other three weeks were spent working with the pre-service teachers on their Teacher Work Sample. During the 12th week the
methods course focused on *Implementing Teaching Strategies and Assessment Plans* and then focused on *Assessing the Effectiveness of your Teaching* during the other three weeks. The practicum field experience based online discussion focused on *Classroom Management* during the 12th week and on *Technology* during the 13th week. This marked the end of the online discussions.

At the beginning of the 12th week the methods instructor assigned the pre-service teachers their final individual lesson designs. The pre-service teachers were asked to design and teach in their practicum classrooms lessons that integrated technology. The implementation of these lessons was exactly the same as the first individual lessons. This teaching cycle lasted for four weeks because of the thanksgiving break in between.

As part of this study, the pre-service teachers wrote teaching philosophy statements, completed the TPACK survey, and scheduled an exit interview (Appendix B) during the last two weeks of school. At the end of each exit interview, the participants were given $25 bookstore certificates for agreeing to participate in the study.

**Summary of the Context of Study**

The pre-service secondary mathematics teachers learned mathematics with technology in the modeling course. This was meant to help them have an understanding and appreciation of the possibilities of teaching and learning mathematics with technology. Besides teaching mathematics with technology, the modeling instructor modeled appropriate uses of technology in mathematics teaching and learning such as multiple linked representations, generalizations and parameterization. The methods course helped the pre-service teachers design lessons that integrate technology. The first
two lessons were implemented by two in-service teachers and the other two were implemented by the pre-service teachers themselves in their practicum classrooms.

Data Collection

Various forms of data were collected in order to help answer the research questions and these are a TPACK survey, teaching philosophy statements, lesson plans, materials, teaching, weekly meeting notes and exit interviews transcripts. The following is a description of these five data sources. For the TPACK survey a description of how the instrument was developed will also be offered.

TPACK Survey

A TPACK survey in Appendix A is made up of open-ended and Likert-type questions. The open-ended questions were developed by the researcher using Neiss’s (2005) definition of mathematics teachers’ TPACK. To enhance the validity of the questions, the questions were given to an outside researcher who has been working on TPACK and the following suggestions were given:

What do you think are the purposes of incorporating technology in teaching mathematics?

Instead the following was suggested:

What does the phrase “teaching mathematics with technology” mean to you? Describe this in the context of a high school class.

What is technology good for in terms of learning and teaching mathematics?

Instead the following was suggested:
a. Describe and compare a good use of technology in teaching mathematics to a trivial or unproductive use of technology in teaching mathematics.

b. Describe what it means to learn mathematics with technology. What understandings and thinking should students have in order to learn mathematics with technology?

Instead the following was suggested:

If technology is used, what should students know in order to learn mathematics with technology?

The Likert-type questions were adapted from Schmidt et al. (2009). Schmidt et al. (2009) developed a survey instrument to gather data on elementary pre-service teachers’ self-assessments of their understanding of the seven components of the TPACK framework (CK, PK, TK, PCK, TCK, TPK and TPACK). Schmidt et al. (2009) adapted some of the items from other surveys found in the literature that addressed the TPACK constructs and also wrote some additional items. All items were revised in an iterative process among the research group and then sent out for expert content validity analysis. The final instrument had 47 items for measuring pre-service teachers’ understanding of the seven knowledge components of TPACK - 7 TK items, 12 CK items, 7 PK items, 4 PCK items, 4 TCK items, 5 TPK items, and 8 TPACK items. The instrument also included nine items addressing demographic information, 11 items addressing faculty and PreK-6 teacher models of TPACK, and three open-ended questions. The internal consistency reliability (coefficient alpha) ranged from .75 to .92 for the TPACK
components. For this study only items that are relevant to mathematics were included. Items that referred to other content areas (content, social studies, English) were dropped.

The survey was administered online after the first day of class and on the last day of class using the methods’ course D2L online shell. The main purpose of this TPACK survey was to help evaluate the pre-service teachers’ self-reported level of TPACK both at the beginning and at the end of the semester.

**Teaching Philosophy Statements**

In the methods course, the pre-service teachers were asked to write their statement of teaching at the beginning of the semester. This statement was supposed to include the role played by technology in mathematics instruction. The pre-service teachers wrote this essay again at the end of the semester to enable comparison. The purposes of the teaching philosophy statements were to help describe the individual cases and to identify any changes in their understanding of teaching mathematics with technology, if any.

**Lesson Plans, Materials, Teaching Episodes**

The lesson plans and lesson materials that the teachers wrote and the teaching episodes were a major source of data for this study. The lesson plans together with the teaching helped shed light on the pre-service teachers’ pedagogical and technological choices. The pre-service teachers wrote two lessons in groups and two others individually. Since some of the pre-service teachers had not taken a methods course before this one, they did not have any experience in lesson plan writing. Writing two group lessons was meant to ensure that the pre-service teachers were at the same page in
terms of lesson plan writing. Also, by writing lesson plans that were to be implemented by an in-service teacher, the pre-service teachers were forced to write well-detailed lesson plans that could easily be followed by the in-service teachers. The group lessons were taught by two in-service teachers whereas the pre-service teachers taught their individual lessons.

At the end of each lesson design and implementation cycle (design, microteaching, revision and implementation), the pre-service teachers wrote individual post reflection notes. For the first group lesson, they watched the video in class and had a discussion on the lesson before writing their own reflections. For the second group lesson, the video was posted online and the pre-service teachers watched the video on their own and wrote their individual reflections. For the two individual lessons, the pre-service teachers watched their personal videos and wrote their personal reflections. The following questions guided the reflections:

- What influenced your technology choice?
- What influenced your pedagogical choice?
- What worked well in your lesson?
- What can be improved in your lesson?

The Lesson Plan Template To help the pre-service teachers create good lesson plans, a lesson plan template was provided. The following paragraph describes the template.

The lesson plan template had six main sections: learning objectives and assessments, standards, prior knowledge, materials and technology, procedure or
sequencing of the lesson, and differentiation strategies. The learning objectives and assessments were right next to each other so that the pre-service teachers would always remember to match the lesson objectives with the assessments. The procedure or sequencing of the lesson section had four columns as shown in Figure 8 as suggested by Matthews, Hlas & Finken (2009). The primary goals of the four-column format were to draw the pre-service teachers’ attention to matching instructional activities with the students’ anticipated thinking and activities and to have them consider the cognitive demands of the activities they assigned to their students. Figure 8 shows the four columns and also provides descriptions of each column.

Figure 8: The Four-column Format of the Procedure or Sequencing of the Lesson Section

<table>
<thead>
<tr>
<th>Teacher Activity</th>
<th>Anticipated Student Thinking and Activities</th>
<th>Analysis about the Cognitive Demand of Activities, Instructional Choices, and Curricular Materials</th>
<th>Materials, Strategies, other Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explanations/Definition for Column Headings</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. **Teacher Activity** – Give a description of the teacher activity for each step. Also include any questions or examples that you will use.
2. **Anticipated Student Thinking and Activities** – Describe how you expect students to react to any of the activities and also include any student activities for each step.
3. **Analysis about the Cognitive Demand of Activities, Instructional Choices, and Curricular Materials** – Consider the cognitive demand of the activity. Is it a high cognitive demand or a low cognitive demand? How did you decide on the instructional strategy? Describe the curricular materials?
4. **Materials, Strategies, other Notes** – What materials are needed to complete the task or activity? Calculators, notebook, white board, computer etc?
Weekly Meeting Notes

Recall that the instructors of the methods and modeling courses met with the researcher every Friday throughout the semester. Discussions during these meetings centered on the previous week’s activities and assessments results and plans for the upcoming week. The meetings helped the researcher keep abreast with flow of the study. The researcher audio taped and kept personal notes of all the weekly meetings and these provided another important data source. This data was intended to shed light on the collaboration itself and corroborate the pre-service teachers’ reported reasons for TPACK changes. It also helped clarify the experiences and performances of the individual cases/pre-service teachers.

Exit Interviews

At the end of the semester, the researcher conducted 30-minute structured exit interviews with the research participants. An interview protocol (Appendix B) was used. These exit interviews were meant to get a sense of a) what the pre-service teachers know about teaching mathematics with technology and b) which of the three courses impacted the pre-service teachers most. Table 2 is a summary of these data types, when it was collected and the purpose for collecting the data.
Table 2: Data Type, Time of Collection and Purpose of the Data Collection

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Time</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPACK Survey</td>
<td>After the first day of class</td>
<td>• To evaluate the pre-service teachers self-reported initial TPACK level</td>
</tr>
<tr>
<td>Philosophy Teaching Statement</td>
<td>After first day of class</td>
<td>• To evaluate the pre-service teachers’ self-reported initial TPACK level</td>
</tr>
<tr>
<td>Group Lesson 1</td>
<td>3rd to 5th week of class</td>
<td>• To help pre-service teachers practice writing well-detailed lesson plans</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• To help the pre-service teachers gain lesson writing experience</td>
</tr>
<tr>
<td>Group Lesson 2</td>
<td>6th to 8th week of class</td>
<td>• To help pre-service teachers practice writing well-detailed lesson plans</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• To help the pre-service teachers gain lesson writing experience</td>
</tr>
<tr>
<td>Individual Lesson 1</td>
<td>9th to 11th week of class</td>
<td>• To assess/identify individual TPACK levels</td>
</tr>
<tr>
<td>Individual Lesson 2</td>
<td>12th to 15th week of class</td>
<td>• To assess/identify individual TPACK levels</td>
</tr>
<tr>
<td>TPACK Survey</td>
<td>Last Day of Class</td>
<td>• To identify changes in TPACK if any</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• To account for the changes in TPACK</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• To evaluate the pre-service teachers self-reported TPACK level</td>
</tr>
<tr>
<td>Philosophy Teaching Statement</td>
<td>Last week of class</td>
<td>• Identify changes in the pre-service teachers’ TPACK if any</td>
</tr>
<tr>
<td>Weekly meeting notes</td>
<td>At the end of every week</td>
<td>• To help clarify experiences and performances of the individual cases</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• To shed light on the collaboration</td>
</tr>
<tr>
<td>Exit Interviews</td>
<td>During the finals week</td>
<td>• To get a sense of what the pre-service teachers know about teaching mathematics with technology</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• To also get a sense of which of the three courses impacted the pre-service teachers most</td>
</tr>
</tbody>
</table>
How Data was Analyzed

Data was analyzed continuously from the beginning of the semester to the end of the semester. The data for each participant/case were analyzed separately and then a cross case analysis was done for each data source. In the following section, the analysis of each data type (TPACK survey, teaching philosophy statements, lesson designs and implementation, weekly meeting notes and exit interviews) is considered in turn.

The TPACK Survey

The TPACK survey shown in Appendix A has two distinct parts, an open-ended part and a Likert part. These were analyzed separately as described in the following subsections. The open-ended questions were analyzed using a qualitative coding system. The Likert part was analyzed using a method suggested by Schimdt et al. (2009).

Coding the Open-ended TPACK Survey Questions The coding of the data was based on qualitative methods described by Corbin and Strauss (2008). I used internal (data-driven) codes to analyze the open-ended questions. The responses were coded through multiple iterations of: a) developing codes, b) applying the codes c) searching for inconsistencies, d) redefining the codes and e) re-applying the codes. To develop the codes, I first compiled the responses on a single table as show in Table 3 so that I could easily read across the responses.
Table 3: A Sample of TPACK Survey Responses on an Open-ended Question

<table>
<thead>
<tr>
<th>Question</th>
<th>Eddy</th>
<th>Heidi</th>
<th>Ethan</th>
<th>Ally</th>
<th>Doug</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1. What does the phrase &quot;teaching mathematics with technology&quot; mean to you? Describe this in the context of a high school class</td>
<td>Teaching mathematics with technology means using computer based programs such as Sketchpad or Geogebra in lessons. It can also mean using tools such as a Smartboard or an ELMO camera</td>
<td>Both teachers and students using technology to enhance student learning. Presentation tools such as ELMO, smart board, adaptors to let calculator screen be seen by class. Learning tools like online applets, graphing calculators, simulation software. Technology should SUPPORT learning, not be an end in itself - has to be thoughtfully integrated into the learning environment</td>
<td>Using a computer projector or smartboard to show concepts to the class that animation can help illustrate. Allowing students to use computers to do research or run simulations to help answer questions.</td>
<td>Change the old way of teaching; with piece of paper pen and pen. Teaching with technology is the best way to differentiate the classroom based on each student need.</td>
<td>Teaching mathematics with technology means using technology as a teaching strategy to enhance the learners understanding of a topic.</td>
</tr>
</tbody>
</table>
I read across the table once to get a sense of the students’ responses and then read again to identify the themes that emerge from the participants’ explanations. For the first question: *What does the phrase “teaching mathematics with technology” mean to you? Describe this in the context of a high school class,* I first noticed that the participants identified two main things, that is, the users of the technology and the technology. The identification of the users was either explicit, for instance, “teachers and students” in Heidi’s response or implicit, for instance, Doug’s response that, “using technology as a teaching strategy,” implies that the teacher is the user of the technology. For the technology, some participants chose to identify both general technologies such as Smartboards and mathematics-specific technologies such as graphing calculators. Eddy was able to distinguish between hardware and software by listing these separately and Heidi divided technology into two, presentation tools and learning tools. Another theme or category that emerged was the idea of complexity that is involved when technology is integrated into the classroom. This emerged from Heidi’s statement that, “Technology should SUPPORT learning, not be an end in itself - has to be thoughtfully integrated into the learning environment.” This then formed my first three categories: a) Users (teacher or students), b) Technology (hardware or software), and c) Awareness of the complexity.

As a second step in the cycle of iterations, I then applied the above categories to the participants’ responses in Table 3. However, I could not capture Ally and Doug’s responses using these three categories. Ally’s reference to using technology to “differentiate” and Doug’s response that it “means using technology as a teaching strategy to enhance the learners understanding of a topics,” seemed to point to technology
at a pedagogical tool. Also, Heidi’s distinction between presentation tools and learning tools also added another dimension worth exploring because it could not be effectively captured by the second category on technology. In addition, Ethan’s statement on, “Using a computer projector or SmartBoard to show concepts to the class that animation can help illustrate,” also suggest using technology as a presentation tool. This made me redefine the codes or categories as follows: a) Users (teachers and/or students), a) the role of technology (presentation or learning), and c) technology as a pedagogical tool. I then applied these codes to the responses.

Analyzing the Likert-type Questions For the Likert part, the responses for both the pre- and the post survey were analyzed using guidelines provided by Schmidt et al. (2009). Each item response was scored with a value of 1 assigned to strongly disagree, incrementally to 5 for strongly agree. For each TPACK component (TK, CK, PK, TCK, TPK, PCK, TPACK) the pre-service teachers’ responses was averaged and tabulated and bar graphs were plotted for each pre-service teacher showing their pre- and post survey averages on each TPACK component. This was meant to allow for comparison between pre- and post survey responses.

Philosophy Statements of Teaching

The teaching philosophy statements were analyzed using the same coding system as the one used for the open-ended TPACK questions. Like the open-ended TPACK survey questions I used internal (data-driven) codes. Using this method I coded the teaching philosophy statements through multiple iterations of: a) developing codes, b)
applying the codes c) searching for inconsistencies, d) redefining the codes and finally e) re-applying the codes.

**Lesson Designs and Implementation**

A rubric adapted from Lyublinskaya and Tournaki (2011) in Appendix D was used to analyze both lesson designs and implementation. The rubric is a matrix with levels of TPACK being matched against the four TPACK components suggested by Neiss (2005). Each TPACK component has two performance indicators for each level of TPACK. For scoring, each performance indicator is assigned a value of .5 (half) if met thus, the possible range of scores for each TPACK component is 0 – 5, where the component score can be an integer (both performance indicators are met) or half-integer (one out of two performance indicators are met). The score is assigned for each component independently. In order to achieve a particular level of TPACK, teachers must meet both indicators of that level for each component. Thus, the teacher’s TPACK level is determined by the lowest score across all four components. For instance, if a teacher scores 3.5, 4, 4.5, and 4 on the four components of TPACK, the TPACK level of the teacher is *Adapting*. Although in three out of four components the teacher met both performance indicators of *Exploring* level (score of 4 or 4.5), the lowest score is 3.5, and thus this teacher is still in transition from *Adapting* to *Exploring*.

**Weekly Meeting Notes and Exit Interviews**

Weekly meeting notes and exit interviews were used to support or confirm observed pre-service teachers behavior. For instance, one of the pre-service teachers did
not integrate technology in her first lesson and it emerged during the weekly meetings that she was struggling with both the content and the technology in the modeling course.

How the Research Question was Answered

In order to answer each of the four research questions, I identified each case’s initial level of TPACK, and then consecutive levels throughout the semester, in order to identify changes in TPACK if any. To do so I used the results from the pre-service teachers’ lesson design and implementation. I also used evidence from the open-ended survey questions, the teaching philosophy statements and the exit interviews to illuminate the observations from the lesson design and implementation.

Validity and Reliability

Like any research, there is need to address issues of validity and reliability. In qualitative research, however, validity and reliability take on new terms. The processes of dependability, trustworthiness and transferability are fairly comparable to the processes of reliability, validity and generalizability in the quantitative paradigm respectively according to Hoefl (1997) and Moschkovich and Brenner (2000). The following sections describe how issues related with trustworthiness, dependability, and transferability were addressed in this research study.
Trustworthiness

Trustworthiness refers to the extent to which one can believe in research findings. Since this is a qualitative case study, there are ways of addressing trustworthiness that are unique to case studies such as:

- Use of multiple sources of evidence
- Creating a case study database
- Maintaining a chain of evidence

These will be described first. However, there are other ways of addressing trustworthiness in qualitative research such as prolonged engagement, carrying out a pilot study, participant observation and these will be discussed as other ways on enhancing trustworthiness of this research study.

**Use Multiple Sources of Evidence.** Patton (2002) cited four different kinds of triangulation, that is, methods triangulation, triangulation of sources, analysts’ triangulation and theory triangulation. However, Yin (2004) cites triangulation of sources as a way of enhancing case study evidence. I used documents, observations, the TPACK survey, teaching philosophy statements and the exit interviews to corroborate my findings. I also compared my multiple cases and examined negative cases.

**Creating a Case Study Database.** Another way I enhanced the trustworthiness of my research study was through creating a case study database that is available for independent inspection. This database had all the data that I collected. I stored it as a folder on my desktop and can easily be copied to another computer for inspection. The
database includes, the lesson plans, videos of teaching episodes, TPACK survey responses, teaching philosophy statements, weekly meeting audiotapes and notes, the three courses’ syllabuses and materials and both the exit interview audiotapes and transcripts.

**Maintaining a Chain of Evidence.** Maintaining a chain of evidence is intended to allow an external observer to follow the derivation of any evidence from initial research questions and ultimately to the case study conclusions. In addition, the observer should be able to trace the steps in either direction. In order to enable this:

- In my result analysis section I made sufficient citation to the relevant portions of the case study database.
- My database reveals the actual evidence and also indicates the circumstances under which the evidence was collected such as time and place of an interview.

**Other Ways of Enhancing Trustworthiness of this Study.** For this study, a pilot study was conducted prior to the actual study. Results of the pilot study we used to refine the methodology. Also, being a participant-observer in this research study enabled me to gain access to the participants’ information and also to perceive things from the pre-service teachers’ viewpoints. My being a course assistant in all the three courses helped with developing trusting relationships with the participants thereby eliminating any distortions in the data that might occur because of my presence. In this case my presence was just a normal presence. In addition, I did my observations persistently (prolonged engagement), that is from the first lesson of the semester up to the last day of the
semester in both the methods and the modeling courses, so that I can provide depth to my observations.

Transferability

Transferability is the degree to which the findings a researcher can apply or transfer beyond the bounds of the research. Patton (2002) referred to transferability as extrapolation. According to Patton (2002), “extrapolations are modest speculations on the likely applications of the findings to other situations under similar, but not identical, conditions.” As a way of trying to enhance transferability of my research I tried to provide a dense or thick description of the context and dynamics within the context of the study. Zbiek and Hollebrands (2008) also echoed the need for research reports on teaching mathematics with technology to provide detailed description of both the teachers and the teaching. Transferability can also be enhanced by purposely sampling, according to Moschkovich and Brenner (2000), and hence by specifying my purposeful sampling technique, which in this case is criterion sampling, I enhanced the transferability of this research study.
CHAPTER 4

RESULTS

Introduction

This research study is a multiple case study in which five pre-service teachers who were enrolled in three courses (methods, practicum and modeling) are treated as individual cases. This chapter presents the results of the analysis of data for the four research questions. Data for the four research questions were collected from five sources:

a) Lesson plans designed by the pre-service teachers, both individually and in groups.

b) A TPACK survey with both open-ended and Likert-type questions.

c) Teaching philosophy statements written by the pre-service teachers.

d) Weekly meeting notes with modeling and methods instructors.

e) An exit interview.

Data for questions 1, 2, 3 and 4 were collected using lesson plans and teaching episodes, open-ended questions on a TPACK survey, teaching philosophy statements and an exit interview. The weekly meeting notes provided additional data to enhance case descriptions. Since this is a case study, I will describe the five individual cases, the lessons developed by the participants (pre-service teachers) and then the results that answer the research questions. Recall that the pre-service teachers experienced a coordinated methods-mathematics-practicum curriculum designed to elicit the specific
domains of TPACK. This chapter summarizes data collected from those subjects regarding four aspects of instruction with technology:

1. Overarching conceptions about the purposes for incorporating technology in teaching mathematics
2. Knowledge of students’ understandings, thinking, and learning of mathematics with technology
3. Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics

The Cases

Ally, Doug, Heidi, Eddy and Ethan (all pseudonyms used to maintain confidentiality) were the five individual cases for this study. They were pre-service secondary mathematics teachers who were concurrently enrolled in three courses: Methods for Teaching 9-12 Mathematics, Mathematical Modeling for Teachers, and Practicum. The secondary certification in the state where this study takes place certifies a teacher to teach mathematics in grades 5 – 12, and as a result the pre-service teachers in the program take two methods courses, one for middle school and another one for high school mathematics teaching. Ideally, the pre-service teachers take the middle school methods course before taking the high school methods course. However, this is not required, and it is therefore not uncommon to have pre-service teachers take the high
school methods course first. For this group, only two of the participants (Heidi and Eddy) had taken the middle school methods course during the spring 2011 semester. Because the pre-service teachers are certified to teach grades 5 – 12 they can complete their field experience in either middle or a high school. For their practicum field experience, Ally, Heidi and Doug were placed in middle schools, while Eddy and Ethan were placed in high schools.

None of the participants in the study was a traditional undergraduate mathematics teaching major: all of them have a double major, a second degree, or prior work experience. They also had very diverse prior experience with technology, ranging from having only used technology in the geometry course to having years of experience as in computer programming. The following is a description of each of the cases.

Ally

Ally had the least amount of experience with technology as compared to the other participants. Her only stated experience with learning mathematics with technology was through a geometry course she took at this north-western university during the spring 2011 semester. She had just bought her first calculator, a TI-83. Her limited experience with technology could have resulted in her struggling with the technology a little longer than others in the modeling course, as pointed out by the modeling instructor during one of our weekly meetings. Ally was one of the non-traditional students because she had an associate degree in Civil Engineering and Mathematics and was studying for a double major in mathematics education and engineering. Besides struggling with English because she was not a native speaker, Ally struggled with the mathematics they were
focusing on in the modeling course, according to information provided by the modeling instructor during a weekly meeting. In the methods course, the participants, together with the other students who were enrolled in the course, were required to make presentations. According to the methods instructor and my own observations of the classroom, Ally did not ever volunteer to go first. She told me during one of our conversations that her classmates made her “nervous.” Until half-way through the semester, Ally seemed to not have enough time to finish her course assignments. For instance, in the methods course it was common for Ally to say that she was not ready, or to present something that showed evidence of lack of preparedness. One such incident involved Ally presenting her first individual lesson in the methods class. She defined the slope as “run over rise” and made some other mistakes which clearly indicated that she had not taken time to read and prepare for this microteaching assignment. She worked part-time as a mathematics tutor. Her plan was to eventually study for a masters’ degree and teach at a community college.

Doug

Doug was a double major in mathematics and physics education; however, he identified himself more with physics than with mathematics. For instance, when responding to a question on how his professors model teaching with technology on the TPACK survey, he cited physics professors both on the pre- and the post survey. Also, when he wrote his introductory letter for his field placement, he wrote about his love for physics, yet the letter was addressed to a mathematics teacher. He was a laid-back individual who, according to the methods instructor during one of our weekly meetings, did not seem to put more effort in his work than the minimum. He rarely turned in his
assignments on time and rarely participated in class unless called upon. His stated plan was to become an effective mathematics and physics high school teacher. However, his cooperating teacher complained about his lack of enthusiasm during his field placement. He did not have any prior teaching experience, and this was his first time taking a methods of teaching mathematics course. He had prior experience with technology from his high school calculator use, an educational psychology class, the geometry course and his physics courses. He was assigned to an 8th grade class for his field placement.

Eddy

Eddy had a double major in Civil Engineering and mathematics education. He was an enthusiastic learner, as evidenced by his participation during class and had a pattern of asking for a lot of feedback on his teaching from both the methods instructor and myself as his field supervisor. The modeling instructor commended Eddy for making tremendous gains in terms of technology skills, as evidenced by the kind of assignments he produced for the modeling course. He had taken a middle school methods of teaching mathematics course during the spring 2011 semester. He was one of the two pre-service teachers who were placed in a high school for his practicum field experience. He was placed in an algebra class, and his cooperating teacher used technology in her teaching. His prior experience with technology was in the geometry course taken at this northwestern university and some high school calculator use. His plan was to get a teaching job locally.
Ethan

Ethan had the most prior experience with technology. He had a degree in engineering, worked in that field for several years and then worked as a web programmer. He described that he quit the programming job because it involved sitting at a computer eight hours a day. He then enrolled at this north-western university for a second undergraduate degree in mathematics education, which he recently subsumed under a master’s program. He was an above-average student and was working on his high school mathematics teaching certification. He was also teaching a college algebra course as part of his graduate teaching assistantship. He was the other pre-service teacher who was placed in a high school. He was placed in an AP Calculus class and an AP Statistics class. He participated in class but often did not provide a lot of written details with his assignments. According to the modeling instructor he always tried out new things with technology and would show it to him for either approval or disapproval.

Heidi

Heidi graduated three years prior to this study with a mathematics degree and was returning to school to get a teaching certificate. She was also getting K-12 art certification, since her other undergraduate degree and master’s work was in fine art. She was an above average student who regularly performed beyond the expectations and was very focused on her work. She had prior experience with technology from her previous mathematics degree and from having taken the geometry course at this university. She had taken the middle school methods of teaching mathematics course during the spring 2011 semester. She was a very detailed and thorough person.
Lessons Developed by the Participants

The participants developed four lessons as part of this study and these lessons were spread out during the semester, as illustrated in Figure 7 in chapter 3. The first two lessons were developed in groups and the pre-service teachers worked on the third and fourth lessons individually. For the group lessons, the pre-service teachers, including those who were not research participants, were divided into two groups. Ethan, Ally, and Eddy were grouped together and Heidi, Doug and two other pre-service teachers who were not participating in the study were grouped together. Figure 7 in Chapter 3 gives a timeline of when the lessons were designed and implemented, and it also shows the focus of each of the three courses throughout the semester. This is provided to help with describing the lessons in context and to help explain and justify any evidence of TPACK in the pre-service teachers. As shown in Figure 7 in Chapter 3, it took about three weeks to design and implement a lesson for both group and individual lessons. Once the lesson design task had been assigned, the pre-service teachers took time to work on it and present it to the class for feedback, and to make revisions before implementing it in a classroom. Because the most important source of data for this research were the lesson plans, I will provide details about each of the lesson plan assignments.

First Group Lesson Design and Implementation

During the first two weeks, the methods instructor introduced the pre-service teachers to What Students are to Learn and Contextual Factors that Influence this Learning by focusing on the NCTM and CCSS standards, big ideas, equity and
technology. At the same time, the practicum course introduced the pre-service teachers to 21st Century Learners, with an emphasis on the need for teachers to create an interactive technology-rich environment and student engagement. The modeling course introduced the pre-service teachers to Simulation Modeling during the first week and high school algebra during the second week. During the 3rd week of the semester, the seven (five study participants and two study non-participants) pre-service teachers who were enrolled in the methods course were divided into two groups, Team A (Ally, Eddy and Ethan) and Team B (Doug, Heidi and two study non-participants). An 8th grade middle school algebra teacher who was known to the methods course instructor to use technology in her teaching was invited to the methods class to talk about her class, her curriculum and the technology that was available in her classroom. Using this information, the groups were given the following assignment:

In your assigned groups, design a lesson that could be taught in Ida’s (not her real name) 8th grade algebra class. Focus in on the curriculum, big ideas, unit, and lesson sequence to identify a lesson. The lesson should demonstrate the use of technology as an appropriate tool for enhancing student learning.

Both groups created lessons on “solving linear equations in one variable with variables on both sides.” The two groups presented their lesson ideas in the methods class for feedback.

Prior to implementation, the teams refined the lessons. These refined lessons were given to the middle school teacher so that she could choose the one she wanted to teach. She chose Heidi and Doug’s (Team B) because their lesson was more detailed, and the instructions were easy to follow as compared to Team A’s lesson. She taught the lesson
in her 8th grade classroom. This lesson was videotaped by the methods instructor. The video was played during the methods class to facilitate a classroom discussion on the implemented lesson. The following are brief descriptions of the two lessons. Team A’s first lesson design was not implemented. The description of the lesson is based on the lesson plan and as a result makes use of future tenses.

**Team A’s Lesson Design.** Team A’s lesson on “equations with variables on both sides” had two objectives:

- Students will be able to solve equations with variables on both sides.
- Students will be able to identify equations that are identities or have no solution.

The lesson starts with a warm up segment, during which students are to work on two problems from the previous day’s lesson on equations with variables on one side. This is to be followed by giving the students a new problem with variables on both sides, letting them work on it on their own and having a volunteer student demonstrate the solution process. The teacher then is to use the pan balance tool from the NCTM Illuminations website using the projector to show a graphical representation and solution of the problem. Finally the teacher is to distribute a worksheet with two word problems to give the students an opportunity to model situations using linear equations with variables on each side.

**Team B’s Lesson Design.** Team B’s lesson on “solving equations with variables on both sides” had three objectives:
• Students will be able to use two or more steps to solve linear equations
• Students will be able to solve equations of the form $ax + b = cx + d$.
• Students will be able to use the TI-83 to connect the table, graph, and equation

The lesson begins with a warm-up exercise with two problems from the previous lesson on equations with variables on one side of the equation. The teacher then demonstrated how to solve an equation with variables on both sides where there is a solution and how to find the solution graphically using the TI-83. Students are then paired and given a worksheet to complete. The teacher’s role at this point is to circulate among students to help with questions they might have. The class would then be brought together 10 minutes before the end of the period to go over the solutions.

Second Group Lesson Design and Implementation

As shown in Figure 7 in Chapter 3, by the 6th week the practicum course had introduced the pre-service teachers to 21st Century Learners, Student Engagement, Reflection, Project Based Learning and Classroom Management. The methods course had introduced the pre-service teachers to What Students are to Learn and the Contextual Factors that Influence this Learning, which included the use of technology and creating assessments. The modeling course had introduced the pre-service teachers to Simulation Modeling. At the beginning of the 6th week, an AP statistics and calculus high school teacher who was also known by the modeling course instructor to use technology in his teaching was invited to the methods class to talk about his statistics class, his curriculum and the technology available for use in his classroom. Using this information and maintaining the teams from the first group lesson design and implementation, the pre-
service teachers were given the same assignment as they were for the first group lesson design and implementation, but for a lesson to be implemented in AP Statistics.

For implementation, the two lessons were given to the high school teacher to choose one to teach, but he decided to teach both because he liked both of them. Team B’s lesson (Cautions about Regression and Correlation) was taught first and then Team A’s lesson (Linear Regression, Outliers and Influential Points). Both lessons were videotaped and the videos were posted online for the pre-service teachers to watch and then write a one-page post-lesson reflection.

**Team A’s Lesson Design.** Team A developed a lesson on “linear regression, outliers and influential points.” This lesson’s objectives were that students will be able to:

- Describe what conditions must be met for a point to be considered an outlier
- Describe what properties of a point make it influential
- Identify outliers and influential points on a scatter plot

The lesson started with answering students’ questions from the previous day to correct any students’ misconceptions and to also make sure that the students are ready for the day’s lesson. Using a TI-Nspire, the teacher then showed the students a scatter plot of some data on which he plotted the regression line and the residual plot. The teacher then virtually dragged the outlier to show that moving it does not have a large influence on the regression line. Switching to the influential point example and instead of dragging the point, the teacher deleted the point to show that deleting the point has a large effect on the regression line. The lesson ends with students working in pairs on a worksheet involving
finding outliers and influential points using a TI-83 calculator and the teacher walking around and offering help where it is needed.

**Team B’s Lesson Design.** The objectives for Team B’s lesson on “cautions about regression and correlation” were that students will be able to:

- Determine if linear progression is appropriate to use on a set of data for predictive purposes
- Evaluate using the residual plot to decide if the linear regression line is a good fit for the data

The lesson starts by having the teacher review the meaning of correlation. The teacher distributed a worksheet which had three different data sets for which the students are to calculate the regression equation, correlation coefficient, and coefficient of determination, and draw a residual plot for each data set. The students are to decide whether they could use the regression equation for making predictions. This is followed by another worksheet with the same data sets as worksheet 1. However, this time around the students are to draw a scatter plot that included the regression line and determine whether they can use their regression line for future predictions. The lesson ends with a classroom discussion on what students discovered about the dangers of using correlation to predict into the future and the usefulness of the residual plot. Homework is assigned to help students practice what they learned.
Participants’ First Individual  
Lesson Design and Implementation

By the 9th week of the semester the pre-service teachers had been in their practicum classrooms for about four weeks, giving them enough time to get acquainted with both their cooperating teacher and the students. The pre-service teachers had also been exposed to a lot of classroom issues in both the methods and the practicum courses. The two group lesson design assignments had also given them some experience in lesson planning. Combined experiences from the three courses rendered the pre-service teachers ready to design and teach their own lessons.

For the first individual lesson design the methods instructor asked the pre-service teachers to design a lesson that incorporated social justice. Incorporating social justice in a lesson means designing a lesson that addresses equity or fairness issues and attending to the contextual relevance of the material for the students. The methods instructor added that the ideal lesson was one in which the lesson context is a social justice context and the teaching strategies incorporate meaningful use of technology.

For the implementation of this lesson, the pre-service teachers presented their lessons in the methods class first, got some feedback, revised the lessons and then taught the lessons in their practicum classrooms. I observed and videotaped the lessons. After each lesson I held a post conference discussion with each of the pre-service teachers to discuss the lessons. The pre-service teachers then got a videotape of their teaching and used it to write a post-lesson reflection. The whole process from the initial assignment to the written lesson reflections lasted about three weeks. The following is a description of each of the lessons that the pre-service teachers designed and implemented.
Ally’s Lesson: Slope Formula. Ally’s 8th grade lesson’s objectives were that:

- Students will be able to find the slope of a line using the slope formula
- Students will be able to find the slope of a line from a graph, two points, a table, and equation of a line

The lesson started with a warm-up exercise where the students had to graph a line using the x and y intercepts. Ally defined the slope as “rate of change in y over a unit change in x” and gave the students the formula for finding the slope given any two points. Ally demonstrated how to find the slope of a line for four different situations a) given an equation of a line, b) given two points, c) given a table, and d) given a graph. Students were given a worksheet to complete, after which student volunteers showed their work on the board. After this Ally assigned another worksheet on which the students had to calculate the school rate of change of enrollment and predict future enrollment. Ally did not address any social justice issue in the lesson, nor did she integrate technology in her lesson. The school enrollment problem she assigned was intended to be relevant to the students but did not address any equity or fairness issue.

Doug’s Lesson: Ratio and Proportion. The objectives of Doug’s 8th grade lesson were that students will be able to:

- Solve for a variable in the numerator and denominator of a ratio in a proportion using cross multiplication or other methods
- Create a proportional length from a ratio and given distance

Doug’s lesson began with going over homework and giving his students a pre-test to assess their prior knowledge. He then introduced ratios, went over several problems
and then moved to proportions. He concluded by giving his students an activity he created based on a conversation he heard from the students. Doug had heard some of the students complain about the how far their lockers were from their classroom as compared to the other students. In trying to help the students understand the complexity of solving this problem, he created an activity where the students developed a scaled model of the hallway and then calculated the distances. By addressing a fairness issue, Doug incorporated a social justice issue in his lesson. However, he did not use any technology despite the instructions given by the methods instructor about an ideal lesson.

Heidi’s Lesson: Connecting Percents, Fractions and Decimals. Heidi’s 6th grade lesson’s objectives were that students will be able to:

- Convert between fractions, decimals, and percents
- Evaluate statistics given by media sources

The lesson began with the students writing each of the following as a percent: \(\frac{15}{40}, \frac{12}{50}, \frac{60}{150}\) and then going over these problems as a class. After a discussion on how surveys can be false, Heidi gave her students a worksheet with poverty rates in Montana from three different sources. The students had to respond by putting the three statistics in order from the least to the greatest and then respond to these two questions: Why are the poverty rates for Montana reported by these sources different? What might affect statistical data? The students worked in pairs on the task while Heidi circulated through the room. With about 15 minutes left in the lesson, Heidi had students share their answers by bringing their work to the ELMO. She then had students talk about what they discovered, what
they learned about strategies to convert between different representations, and why the numbers were different. By using poverty data for the state of Montana with students at a school where more than half of the students get free lunch, Heidi was addressing the social justice requirement of the lesson. Heidi addressed the technology requirement of the lesson by having students use calculators for computation.

**Eddy’s Lesson: Graphing Linear Equations (9th grade).** The objectives of the lesson were that students will be able to:

- Identify and write linear equations and intercepts.
- Graph linear equations.
- Model real-world situation with linear equations.

Eddy began by reviewing the last unit on “solving linear equations in one variable.” He introduced a word problem about the school’s team fundraising efforts. He solved the problem with the class and graphed the solution. Eddy introduced the x and y intercepts using the graph. Eddy asked his students to graph three lines on graph paper. He graphed the line on the SmartBoard with the help of the students. To wrap-up the lesson Eddy gave the students two equations and asked them whether these were equations of lines and whether they were in standard form. Eddy used the sports team’s fundraising problem to satisfy the social justice requirement of the lesson. He used the SmartBoard’s graphing tool to satisfy the technology requirement of the lesson.

**Ethan’s Lesson: Finding Derivatives Related to Harmonic Motion.** Ethan’s lesson objectives (11th/12th grade) were that students will be able to:
• Write a sine or cosine function that fits a set of $x, y$ data points
• Explain why the derivative of $\sin(2x) = 2 \times \cos(2x)$
• Point out where maximum velocity occurs given a graph of position

Ethan began his lesson by asking his students if they knew what “harmonic motion” meant. He asked them if they knew what “simple harmonic motion” meant. He explained that they would be considering simple harmonic motion because they did not yet have the calculus tools to find the derivatives of the formula. Ethan read the introduction of the activity worksheet, drew two pictures of the situation and had a short discussion with the class about the pictures and the activity. He asked the students to get in groups of four and handed out the worksheet on which the students had to find an equation that fits the data using TI-83 calculators. Ethan and the cooperating teacher walked around helping students. With ten minutes of class time left, Ethan led a whole-class discussion to get some consensus on the right answers. He ran out of time before he could really get to the final conclusion. Ethan’s activity worksheet satisfied that social justice aspect of the lesson. The student’s use of TI-83s to fit a curve satisfied the technology requirement of the lesson.

Second Individual Lesson Design and Implementation

By the 12th week of the semester, the practicum course was wrapping up in terms of introducing the pre-service teachers to issues in teaching. The modeling course had started the last unit on Measurement, Data Gathering, Motion Simulations and Dynamical Systems. The methods class was wrapping up the unit on Implementing Teaching Strategies and Assessment Plans. As a final teaching assignment, the pre-
service teachers were asked to design and teach in their practicum classrooms lessons that integrate technology. The implementation of these lessons followed the same schedule as the first individual lessons.

Ally’s Lesson: Solving Systems of Equations by Substitution. Ally’s (8th grade) lesson objective was that “students will be able to solve systems of linear equations in two variables by substitution.”

Ally started the lesson by reminding the students about the definition and the solutions of systems of equations and giving the students three warm-up examples to solve either by hand or using a TI-83 calculator. Using a PowerPoint presentation, Ally introduced the substitution method and demonstrated this method using two problems. Ally gave the students a worksheet that involved solving systems of equation by substitution. The students had an option of either working in groups or individually and they were free to move around. Ally circulated in the room and helped those who were struggling or answered students’ questions. She collected the students’ written worksheets at the end of class. Ally’s use of the PowerPoint presentation was her way of satisfying the technology requirement of the lesson.

Doug’s Lesson: Writing the Function as a Rule. Doug’s (8th grade) lesson objectives were that students will be able to:

- Create a function rule from a table of ordered pairs
- Create a function rule from the graph of a linear function
The lesson started with going over homework problems. Doug introduced writing a function rule from a table by explaining to students that they were not given the rules explicitly, instead they have to come up with the rules from the tables. To motivate the lesson Doug gave the students a function and asked them to create a table of ordered pairs for the given function. He then gave his students three different tables and asked them to come up with the rules. Finally Doug gave his students a time-temperature table for which a function rule could not easily be found. Doug then asked the student to take their calculators out and he demonstrated step-by-step how to fit a simple linear regression function on the data using the TI-83 calculator. He ended the lesson by handing out homework sheets. Doug’s use of the TI-83 calculator to fit a simple linear regression equation was his way of satisfying the technology requirement of the lesson assignment, although the middle school curriculum does not require the students to fit a simple linear regression.

Heidi’s Lesson: Multiplying Fractions. Heidi’s (6th grade) lesson on multiplying fractions had two objectives, that is, students will be able to:

- Use the area models to represent the product of two fractions, each less than one
- Multiply fractions less than one using the “part of a part” approach to fraction multiplication

Heidi started the lesson by going over some homework problems. Heidi demonstrated how to multiply fractions on the board using an area model. She then divided the students into groups of four and gave each group a Sharpie, a white board and a sheet of paper. Since Heidi had one computer in the classroom, each group would take turns to solve a
different multiplication problem using an online fraction multiplication applet. The rest of the groups would use the area model to solve the problem and write their answers on the white board. While one group was on the computer, Heidi walked around helping the groups that were using the area model. To wrap up the lesson, Heidi asked the students to work on two problems from their textbook and gave them a choice of working individually or with a partner. Heidi’s use of the online applet was her way of satisfying the technology integration requirement of the lesson.

Eddy’s Lesson: Graphing Linear Equations Using Slope-intercept Form. Eddy’s (9th grade) lesson objectives were that students will be able to:

- Write and graph linear equations in slope-intercept form
- Model data using the slope-intercept form of a linear equation

Eddy started by reviewing the previous unit. He gave his students a worksheet with spaces on the edges for the students to take notes. Eddy extended on the previous day’s lesson by starting with an equation for direct variation, then showed the students a TI-Nspire demonstration designed to engage the students in understanding the slope and y-intercept of the line as parameters. He then introduced the concept of slope intercept form of a linear equation and asked his students to graph four different linear equations on graph paper. Eddy graphed the equations using the SmartBoard’s graphing tool. Eddy concluded by introducing an application problem he created in class that used students’ iPods as the context. Using the TI-Nspire to make changes to an equation and using the
Smartboard’s graphing tool were Eddy’s ways of satisfying the technology integration requirement of the lesson assignment.

**Ethan’s Lesson: Simulation.** Ethan’s (11th/12th grade) lesson objectives were that students will be able to:

- Use the *randint* function in their calculator to simulate involving a binomial random variable.
- Analyze a situation involving a binomial event using simulation and make a decision about the best course of action.

Ethan started the lesson by handing out the worksheet and instructing the students to read the paragraph without discussing it with their neighbors. After a short review, Ethan had the students work on the first part of the worksheet on which they were supposed to simulate five flights when 15 tickets are sold and count how many people don’t show up for each flight using their TI-83 calculators. The students then had to do the same thing for the flights when 16 and 17 tickets were sold, respectively. In the meantime, Ethan created three tables on the board one each of the 15, 16 and 17-ticket selling policies. After the students were done with the simulations they recorded their results in tables on the board. He discussed with the students how no-shows translate to empty seats for the different selling policies. The students used the class results to compute the frequency, probability and empty seats for each of the 15, 16, 17-ticket policies. To conclude the lesson, Ethan asked for a consensus on the average number of empty seats and coupons issued for each policy. The students’ use of TI-83 calculators to simulate flights satisfied the technology integration requirement of the lesson assignment.
Results for the Research Questions

To answer the four research questions, I used a modified version of the TPACK lesson plan rubric developed by Lyublinskaya and Tournaki (2011) to analyze the participants’ lesson design and implementation described in the preceding section. Recall from the description in Chapter 3 that the rubric is a matrix with levels of TPACK being matched against the four TPACK domains suggested by Neiss (2005). Each TPACK domain has two performance indicators for each level of TPACK (Recognizing, Accepting, Adapting, Exploring and Advancing). For scoring purposes, each performance indicator is assigned a value of 0.5 if met, thus the possible range of scores for each TPACK domain is 0 to 5, where the domain score can be an integer (both performance indicators are met) or multiple of one-half (one out of two performance indicators are met). The score is assigned for each domain independently.

Since the four research questions are based on the four TPACK domains, I only used part of the rubric that focuses on the TPACK domain in question. Since this is a case study, I did a cross-case analysis for each question by contrasting the individual cases. I used data from the TPACK survey, the participants’ teaching philosophy statements and the participants’ exit interviews to enrich the picture of a pre-service teacher struggling with TPACK.
Results for Question 1 from the
Lesson Designs and Implementation

Research Question 1: How do pre-service teachers’ conceptions about the purposes for incorporating technology in teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific domains of TPACK?

Since the pre-service teachers participated in four lesson developments, I will present the results of each lesson development separately and then consolidate the results to enable a cross-case analysis. Table 4 provides part of the rubric that was used to analyze the lesson plans.

First Group Lesson Design and Implementation. Using the rubric in Table 4, Team A’s lesson (Ally, Eddy and Ethan) was at Accepting: Level 2, and Team B’s lesson (Heidi and Doug and two non-participants) was at Recognizing: Level 1, for this particular TPACK domain. Team A’s lesson was at Accepting: Level 2 because they first had the teacher introduce solving equations with variables on both sides without the online applet from the NCTM’s Illuminations website, and then they had the teacher solve two new problems with the applet. On the other hand, Team B had the teacher demonstrate solving equations with variables on both sides without using the TI-83s and then solve exactly the same problems with the TI-83s. Thus, the teaching of new material was done without the technology.
Table 4: Part of the Lesson Plan Rubric Showing the Indicators at each Level of Development for the Overarching Conception about the Purposes for Incorporating Technology in Teaching Mathematics TPACK Domain

<table>
<thead>
<tr>
<th>Recognizing (1)</th>
<th>Accepting (2)</th>
<th>Adapting (3)</th>
<th>Exploring (4)</th>
<th>Advancing (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>An overarching conception about the purposes for incorporating technology in teaching subject matter topics.</strong></td>
<td>□ Technology is used for practice only, and all learning of new ideas done through the teacher mostly without technology</td>
<td>□ Larger part of technology use is for demonstrations, which include presenting new knowledge</td>
<td>□ Teacher is using technology in a way that is new and different from teaching without this technology (dynamic nature, linked representations) and used for learning new knowledge by students</td>
<td>□ Larger part of technology use is by students who explore and experiment with it for new knowledge and for practice</td>
</tr>
<tr>
<td>□ Technology activities do not include inquiry tasks. Technology procedures concentrate on drills and practice only.</td>
<td>□ Technology activities do not include inquiry tasks. Technology procedures concentrate on teacher demonstration and practice.</td>
<td>□ Technology activities include inquiry tasks. Technology procedures concentrate on mathematical tasks with connections and doing mathematics – and on inquiry activities that use or develop connections (especially between multiple representations).</td>
<td>□ Technology activities include inquiry tasks. Technology procedures concentrate on mathematical tasks with connections and doing mathematics – and on inquiry activities that use or develop mathematical knowledge of high value; connections (especially between multiple representations) and strategic knowledge.</td>
<td>□ Technology tasks provide students with deeper conceptual understanding of mathematics and its processes.</td>
</tr>
</tbody>
</table>

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Second Group Lesson Design and Implementation. Using the rubric in Table 4, Team A’s lesson was at Adapting: Level 3 because the strategy of deleting and dragging points in the graphics window was a new way of using technology. Asking students to notice the influence on the regression line of deleting and dragging points as well as the strategy of discussing whether an outlier can be an influential point are indicators of an inquiry task. Using the same rubric, Team B’s lesson was partly at Adapting: Level 3 and at Exploring: Level 4, and hence their score was 3.5. The lesson was at Exploring: Level 4 in the sense that most of the technology use was by the students as they progressed through the worksheet. Although the activities were inquiry-based, connections were not developed using multiple linked representation, thus not satisfying the second indicator for being at Exploring: Level 4.

First Individual Lesson Design and Implementation. Ally and Doug did not use any technology when they taught their first individual lessons and thus to classify their non-use of technology I scored their lessons at 0. When Heidi taught her lesson “connecting percents, fractions and decimals,” she purposely had numbers that were not easy to convert between fractions, decimals and percents so that her students had to use calculators for computation. Using the rubric, her lesson for this TPACK domain was at Recognizing: Level 1 since she taught the new materials for the day without any form of technology and the technology tasks did not involve any inquiry. Eddy’s first lesson was partly at Accepting: Level 1. Eddy used an electronic graphing tool to demonstrate graphing a line, but he had his students graph their lines on graph paper. Ethan’s lesson
was partly at Exploring: Level 4 and partly at Adapting: Level 3. It was at the Exploring Level because the students were the main users of the technology as they tried to find an equation that fit the data. The lesson was partly at the Adapting Level because the tasks involving inquiry connections were not made using either the dynamic features or multiple linked representations.

Second Individual Lesson Design and Implementation. When Ally taught her second lesson on “solving systems of equations by substitution” she used a PowerPoint presentation to present the lesson material to the students. In addition, she had students solve the systems using their graphing calculators as a way of linking the new material to the previous lesson and also as a way of checking answers. Using the PowerPoint is a productive use of technology rather than a cognitive use, and therefore this use could not be classified within the rubric. However, the use of graphing calculators by the students was at Recognizing: Level 1 because the new content for the day was not taught using technology. Doug showed the students how to fit a simple linear regression using a TI-83, however, the technology task was not inquiry-based, and as a result I classified his lesson at being at Accepting: Level 2. Heidi’s technology use for her lesson on fraction multiplication was also at Accepting: Level 2, since she demonstrated how to multiply fractions with the online applet and then had her students take turns to use the applet.

Eddy’s use of technology in his second lesson was at Adapting: Level 3, because of his use of linked representations. Although he was the main user of the technology, the technology activities included some inquiry. Ethan’s lesson on simulation was at Exploring: Level 4 because the students were the main users of the technology in
simulating data. In addition, Ethan’s technology activities concentrated on making mathematical connections.

Cross Case Analysis of the Results for Question 1

To enable a cross case analysis as suggested by Yin (2008), I compiled and graphed the participants’ scores (See Figure 9.) Comparing the scores showed that the second group lessons (red bars) had better TPACK lesson rubric scores that the first group lessons (blue bars). Team B (Heidi, Doug and two other pre-service teachers who were not participating in the research) had a bigger difference between the first and the second group lessons than Team A. There was also a noticeable difference in TPACK lesson rubric scores between the first and the second individual lessons, with the second individual lessons having higher scores than the first individual lessons. In Figure 9, Ally and Doug did not use technology in their first lesson hence the absence of the green bars above their names.

Figure 9: A graph showing the Participant’s Score on the Conceptions about the Purposes for Incorporating Technology in Teaching Mathematics
Reading across the participants’ TPACK survey’s open-ended responses, teaching philosophy statements and exit surveys, revealed two distinct categories. The first category involves a general consistency between the lesson design and implementation performance other data sources such as the TPACK survey open-ended responses and the teaching philosophy statements. The other category involves a contrast between lesson plan design and implementation performance and the other sources of evidence such as the TPACK survey and teaching philosophy statements. The contrast is made apparent when a pre-service teacher’s teaching does not match their description of what it means to teach mathematics with technology and their description of the role of technology in their teaching philosophy statements. I will refer to these categories as I describe each participant.

Ally falls in the first category because her lesson design and implementation performance was consistent with her TPACK survey responses and her teaching philosophy statement. She had a 0 (no green bar in Figure 9) on the first individual lesson and a 1 (purple bar in Figure 9) on the second individual lesson. This performance seems to be supported by evidence from the TPACK survey on which she failed to articulate what teaching mathematics with technology meant. When asked at the beginning of the semester what the phrase teaching mathematics with technology meant to her Ally’s response was, “change the old way of teaching; with piece of paper and pen. Teaching with technology is the best way to differentiate the classroom based on each student need.” In follow-up questions, I asked Ally to describe and compare a good use of technology in teaching mathematics to a trivial or unproductive use of technology in
teaching mathematics. Ally’s response was “using technology helps students learn in an abstract and concrete way. Students will be able to link math problems with the real life problems. Technology is the best tools for problem solving.” The final prompt asked Ally to describe what it meant to learn mathematics with technology, and she described this as “learning math with technology means choosing the shortest path to a solution and having tons of ways to get to the solutions. Technology makes it easier for students to check progress and work.”

Ally’s first response seems to incline more towards technology as a pedagogical tool because of her use of the word “differentiate,” but when pressed further to compare a good use to a trivial use, instead of expanding on how a teacher can differentiate with technology, Ally went a different route. Her use of words like “differentiate,” “abstract,” “concrete” and “problem-solving” gave me the impression that she knew all the pedagogical terms but did not know how to articulate these in a technological context. Ally did not identify the technology users explicitly, but her reference to differentiation implies the teacher as the user. The idea that technology can either be used as a presentation tool or a learning tool was missing from her responses.

An analysis of Ally’s teaching philosophy statement that she wrote at the beginning of the semester revealed that those words (differentiate, abstract, and concrete) form part of her belief about teaching. About differentiation, as part of her teaching philosophy, Ally wrote that

Teachers should come up with new ways of learning and should differentiate the classroom based on each student’s needs and interests, because students learn what they want to learn, not what the teachers’
lesson plans states…….. The best way to differentiate the classroom based on each student’s need is to use technology.

Similarly, the words “abstract” and “concrete” also feature in her teaching philosophy statement, making them an important part of her philosophy of teaching mathematics.

The following are the instances when Ally used the words “abstract” and “concrete” in her teaching philosophy statement at the beginning of the semester:

Students develop their abstract thinking along with concrete thinking when they do math. They are able to see the world differently and make sense of it… Thinking abstract might be clear for me but not for a high school student. Students want to hear concrete answer; answer that does not contain complex formula. Being able to explain math in a concrete way is not always easy that is why I am taking math modeling…. Having a high math level does not necessarily require geometry. The more abstract math gets the less useful geometry is.

In the above statements from Ally’s teaching philosophy statement, Ally seems to have a good grasp of what it means for something to be either “abstract” or “concrete.” By using these words several times, she makes it seem like something she strongly believes in.

Ally’s post-survey responses showed some improvement in her understanding of teaching mathematics with technology. At the end of the semester, when asked what the phrase “teaching mathematics with technology” meant to her, Ally responded that:

Teaching mathematics with technology helps students understand math easier and better. Implementing technology in the classroom enhances learning, saves time for the teacher, and motivates students. Technology is an essential tool to use when having students explore math concepts, and trying to look for patterns. Students appreciate the value and the importance of doing Math if they have the opportunity to be creative to come up with new procedures that works better for them and not for the teacher.

Ally’s response clearly identifies both the teacher and the students as the users of the technology. She also seems to have a better understanding of the pedagogical role of
technology as evidenced by her statements that technology “helps students understand math,” “enhances learning,” “saves time for the teacher,” and “motivates students.” Although she does not explicitly identify mathematics-specific technologies, her statement that technological tools are used for exploring math concepts and also for looking for patterns suggests that she might be aware of them.

In support of the above, Ally’s teaching philosophy statement at the end of the semester, although very much like the first one but with some modifications, showed an improved understanding of the pedagogical role of technology. At the beginning of the semester Ally wrote that “they (referring to teachers) should learn how to use it (referring to technology) in order to be accepted by students along with improving the learning tactics.” However, at the end she made some slight modifications to that statement as follows, “they (referring to teachers) should learn when and how to improve learning tactics and motivate students.” The main differences between the pre and the post statements are italicized. At the beginning, Ally thought that teachers need to know only how to use technology but at the end of the semester, she included both “how” and “when.” Also, she added student engagement to her list of the advantages of using technology. She also went on further to add exploring with technology. The differences in the pre and post teaching philosophy statements, I believe, point toward growth in terms of her knowledge and understanding about teaching mathematics with technology during the semester.

Ally’s response during the exit interview also supports her later view about the role of technology in mathematics instruction. When asked during the exit interview
about her view of the role of technology in mathematics Ally responded that it was “really an essential teaching tool” and that she thinks that, “technology helps students explore, visualize and deeply understand math concepts.”

Doug falls in the second category where there is a contrast between the lesson design and implementation performance and the other sources of evidence. He had a 0 (no green bar in Figure 9) on his first individual lesson and a 2 (purple bar) on the second individual lesson. However, his performance seems to be in contrast with his TPACK survey responses at the beginning of the semester. When asked what the phrase teaching mathematics with technology meant to him, Doug responded that it meant “using technology as a teaching strategy to enhance the learners’ understanding of a topic.” Doug’s response seems to incline towards using technology as a pedagogical tool. In a follow up question, Doug was asked to describe and compare a good use of technology in teaching mathematics to a trivial or unproductive use of technology. He described a good use of technology as “using technology as a hands-on activity for the students to develop a better understanding of a topic” and a trivial use as using a “PowerPoint presentation to teach a topic rather than going through step-by-step by hand.” This response shows that Doug had a mature understanding of technology use in teaching mathematics because he was able to provide two distinct uses. For the final question, Doug was asked to describe what learning mathematics with technology meant and he responded that it meant “using computer software, electronic devices or other tools to supplement or enhance ones’ understanding of a topic being taught.” Doug’s use of the words “supplement” and
“enhance” shows that Doug is aware of the cognitive role that technology plays in learning mathematics.

Surprisingly, his philosophy statement does not make any reference to technology use. This observation is supported by a response that Doug gave during the exit interview that he doesn’t think that technology “play enough roles right now in schools.” During the same interview Doug also mentioned that he used technology to teach his lesson on “function as a rule” in his 8th grade practicum classroom because “it was a requirement of the methods class” but he did not think that it was effective.

However, Doug’s TPACK survey responses at the end of the semester showed growth in terms of his knowledge of what it means to teach mathematics with technology. When asked what the phrase “teaching mathematics with technology” meant to him, Doug responded that,

Technology can enhance student understanding and exploration of mathematical concepts in high school by offering alternate or advanced ways of representing a problem or data set. Typically with many applications/programs students can use on a computer or calculator such as Geogebra, NSpire Student Software, or other freeware, students can quickly change important parameters of problems to gain insight into the process or concept being studied rather than just studying it from different problems. This way, students can explore the underlying relationships or concepts quickly and effectively by letting the computer or calculator complete calculations or arithmetic of problems and are able to focus on these bigger concepts or relationships.

Doug’s response demonstrates a higher level of understanding in terms of teaching mathematics with technology and I have italicized the words or phrases that help make this conclusion. Not only does he know the purpose of teaching mathematics with technology such as enhancing student understanding and doing explorations, he is also
aware of examples of the technology in question such as Geogebra and the TI-Nspire software. Doug’s reference to the high school use of technology supports his non-use of technology in his middle school practicum classroom.

Heidi like Doug was in the second category because her lesson design and implementation performance did not match up with her TPACK survey responses and her teaching philosophy statement. Heidi seemed to have a clear understanding of what it means to teach mathematics with technology right from the first day of class. When responding to the question on what the phrase “teaching mathematics with technology” meant to her, Heidi noted that:

(It means) both teachers and students using technology to enhance student learning. Presentation tools such as ELMO, Smartboard, adaptors to let calculator screen be seen by class. Learning tools like online applets, graphing calculators, simulation software. Technology should SUPPORT learning, not be an end in itself - has to be thoughtfully integrated into the learning environment.

Heidi’s response demonstrated a higher level of maturity in terms of technology use in mathematics instruction. She was able to clearly identify both the teacher and the students as the users, she identified the technologies and even went on to distinguish them as either for presentation or for learning. Her statement that, “technology should SUPPORT learning, not be an end in itself - has to be thoughtfully integrated into the learning environment,” shows an awareness of the complexity involved when technology is integrated into instruction. However, her performance on the two individual lessons did not seem to match up to her response. She was at Recognizing: Level 1 for her first lesson in which she let her students use calculators to help with computing decimals,
fractions and percents. Her second lesson on multiplying fractions was at the Accepting: Level 2.

Heidi’s teaching philosophy statement at the end of the semester contradicts her performance. As part of her teaching statements, Heidi wrote that, “it is only through exploration that meaningful learning occurs…” It is apparent from her teaching statements that she believes that exploration is an important part of teaching mathematics and that technology enhances mathematical explorations. In contrast, however, her teaching during her practicum field experience was not in any way close to what she described in her teaching statement and the responses she gave on the TPACK survey.

This contradiction was illuminated by one of Heidi’s responses during the exit interview. In response to the question: What’s your view of the role of technology in mathematics Heidi responded that:

I’m torn sometimes because I feel that there is value to solving things by hand… I don’t know where to draw the line. Do we let them draw everything on the calculator? Do I want them to be able to graph things by hand? Where is that line? I think for me, I tend to come down more on the traditional side of things, I guess... but I definitely think that if used properly in the classroom, technology can be phenomenal for exploration and coming up with answers they could get to by hand, so it’s kinda this balancing act I guess that I always come to. I don’t know where the tipping point is. At some point in mathematics instruction I’m like they should be using their calculators but at some earlier point I’m like oh no they shouldn’t be using their calculators and I haven’t really come up with where that break point is for me.

The questions Heidi raises point to a conflict within herself on whether or not to use technology in her teaching although she is well aware of quite a number of the capabilities of technology because of her statement that “technology can be phenomenal for exploration.”
Eddy falls in the first category because there was consistence between his lesson design and implementation performance and his responses on the TPACK survey open-ended questions. He had a 1.5 on his first individual lesson and a 3 on the second individual lesson. Eddy started off with a very simplistic view of using technology to teach math. When responding to the TPACK survey response on what the phrase teaching mathematics with technology meant to him, Eddy wrote that, “Teaching mathematics with technology means using computer based programs such as Sketchpad or Geogebra in lessons. It can also mean using tools such as a Smartboard or an ELMO camera.” Eddy’s response only focuses on using technology and the types of technology. Although not explicit, he seemed to be aware of technology as either a tool for presentation or learning because he listed the technologies separately depending on whether they are used for presentation (Smartboard or ELMO) or for learning (Geogebra or Sketchpad). When pressed further to describe and compare a good use of technology to a trivial use of technology in teaching mathematics, Eddy’s response was that, “if it takes too much time to boot-up or use a Smartboard when the same goal could be accomplished by using the dry-erase board, then this would be unproductive.” Eddy’s concern was about time not so much of how technology was used. This showed that he had a limited view of the use of technology in teaching mathematics. However, he knew what it meant to learn mathematics with technology because of his response that “Learning with technology is when understanding is gained because of that use of technology.”
As part of his teaching philosophy statement at the beginning of the semester, Eddy wrote that,

Technology can often be valuable in making predictions and exploring consequences as well… Another important skill that students will consider is the available tools and technology when solving a math problem and the ability to use the tools sufficiently… The goal is that students are able to use tools such as graphing calculators or mathematical computer programs in a way that deepens their mathematical understanding.

The above statements from Eddy’s teaching philosophy shows that Eddy knew about the ways that technology can be used, that is, to make predictions and to explore consequences. He also knew about the main purpose of technology use in mathematics instruction, which is deepening students’ understanding. Eddy wrote exactly the same statements on the teaching philosophy statement he submitted at the end of the semester, suggesting no change in his view of the role of technology in mathematics instruction.

Like his philosophy statements, his TPACK survey response at the end of the semester to the initial question on what the phrase teaching mathematics with technology meant to him was the same as the one he gave at the beginning of the semester. It was only modified by the list of technologies, to which he added calculators and TI-Nspire, which was the new technology they used in the modeling course. His description and comparison of a good use to a trivial use of technology in teaching mathematics at the end of the semester was different from the one he gave at the beginning of the semester. Eddy wrote that

Using a Smartboard to graph linear equations could be useful, but using a PowerPoint to present all the steps of a problem could be trivial because it may be more useful to work through a problem line by line.
This response is not surprising because Eddy used the Smartboard to graph linear equations for the two individual lessons he taught in his practicum classroom as part of his study. It is possible that he found this experience valuable and hence made it part of the survey response at the end of the semester. During the exit interview, Eddy said that he thought that “the Smartboard is the coolest thing ever.” Also, Eddy’s description of what it means to learn mathematics with technology was different. Eddy wrote that “learning occurs when a technological tool enhances the content being taught or provides opportunities that were not available before.” This description given by Eddy provides reasons for teaching mathematics with technology and therefore does not describe what it means to learn mathematics with technology. Asked about his view of technology during the exit interview, Eddy responded that

I think it (technology) can enhance (teaching), like it can make it clear but I don’t think that it is going to teach anything new. You should be able to learn the same things without technology but technology can get you there and it might make it clear for everyone. You don’t have to have it, but it is nice. It is convenient.

The use of the word “enhance” shows that Eddy is aware of the advantage of using technology. However, his belief that you should be able to learn the same things without technology supports his limited use of technology during his practicum classroom. Eddy was the sole user of technology is his classroom. His students used graph paper to graph linear equations.

Ethan’s performance on the lesson design and implementation were in contrast with his responses on the TPACK survey open-ended questions and his teaching philosophy statement. Ethan progressed with each lesson design and implementation.
However, his responses on the TPACK survey did not match up with the level of maturity or understanding he displayed in the classroom. His responses were not articulated to a level that one would expect from someone who exhibited a better understanding of teaching with technology than the other participants. For instance, at the beginning of the semester when responding to the survey question on what the phrase “teaching mathematics with technology meant to him, Ethan responded that it meant “using a computer projector or Smartboard to show concepts to the class that animation can help illustrate. Allowing students to use computers to do research or run simulations to help answer questions.” Ethan’s response is not as mature as the response given by Heidi to the same question. Ethan identifies the technology users as both the teacher (because teachers are the ones who show concepts to the class) and the student. His response also demonstrates that he is aware of the role of technology as a presentation tool (when used to show concepts) and also as a learning tool (when used by students).

When prompted further to describe and compare a good use of technology in teaching mathematics to a trivial use of technology in teaching mathematics, Ethan gave the good use as “allowing students to use something like Geogebra to do geometric constructions that they can play and experiment with” and a trivial use as “using a projector or Smartboard to show students how to do something with geometry.” By being able to articulate a specific example from geometry his response showed that he had a good grasp of the phenomenon. Ethan also acknowledges that technology is “a huge help in making geometry lessons much faster” in his teaching philosophy statements. When asked to describe what it meant to learn mathematics with technology, Ethan responded
that it meant “using calculators and computers when it helps to develop a better understanding of a concept and avoiding them when they are an unnecessary distraction.” Not only was Ethan aware of when use technology to learn mathematics he also thought about the fact that technology is not always necessary to use.

Ethan’s teaching philosophy statements both at the beginning and at the end of the semester were not very different from each other. The following is Ethan’s description of the role of technology in mathematics teaching:

This (availability of technology) will provide a lot of opportunity for increased feedback and assessment. Teachers will be able to quickly see statistics on which problems the most students are having trouble with. Computers will help students do research *when solving word problems*. They already are a huge help in making geometry lessons much faster. They are already used to do many repetitions of a simulation in statistics and probability lessons. Student work stored in “the cloud” will eliminate huge backpacks and forgotten homework.

His statement shows that he has a global view of technology because he was thinking more in terms of its availability and some of the possible advantages of this. However, during the exit interview Ethan said that he doesn’t see himself as having “that much view one way or the other.” At the end of the semester Ethan wrote the same statement in his teaching philosophy statement except for the italicized phrase that he replaced with “modeling problems” and at the end he added that his “background in engineering and programming” will help him adapt to and use new technologies as they become available. The additional part indicates that Ethan was thinking of the role that he as a teacher with such a technology background would play. He was placing himself in the situation possibly identifying more with his profession than he did at the beginning of the semester.
Results for Question 2 from the Lesson Designs and Implementation

**Research Question 2:** How does pre-service teachers’ knowledge of students’ understandings, thinking, and learning of mathematics with technology emerge in as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific domains of TPACK?

To answer this question, I used part of the TPACK lesson plan rubric (see Table 5) that focuses on the pre-service teachers’ knowledge of students’ understandings, thinking, and learning of mathematics with technology. For the cross case analysis, I compared the cases and also used data from the TPACK survey, the participants’ teaching philosophy statements, and the participants’ exit interview to provide a rich description of the pre-service teachers’ knowledge of students’ understandings, thinking, and learning of mathematics with technology.
Table 5: Part of the Lesson Plan Rubric Showing the Indicators at each Level of Development for the Knowledge of Students’ Understandings, Thinking, and Learning in Subject Matter Topics with Technology TPACK Domain

<table>
<thead>
<tr>
<th>Knowledge of students’ understandings, thinking, and learning in subject matter topics with technology</th>
<th>Recognizing (1)</th>
<th>Accepting (2)</th>
<th>Adapting (3)</th>
<th>Exploring (4)</th>
<th>Advancing (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Technology is used primarily for student practice.</td>
<td>□ Technology is mostly used for teacher demonstrations or teacher-led student-follow work with technology; it is rarely used for students’ independent explorations.</td>
<td>□ Teacher focuses on students’ thinking of mathematics while students are using technology on their own – both for learning new knowledge and review of prior knowledge.</td>
<td>□ Technology teacher focuses on students’ mathematics conceptual understanding and serves as a guide of student learning with technology, not a director.</td>
<td>□ Teacher facilitates students’ high level thinking with technology (linked representations, reasoning and proofs).</td>
<td>□ A Technology document provides an environment for students to deliberately take mathematically meaningful actions on objects and to immediately see the mathematically meaningful consequences of those actions.</td>
</tr>
<tr>
<td>□ A Technology document does not present any new material, and only provides space for applications and drills.</td>
<td>□ A Technology document mirrors the structure of the textbook presentation of mathematics without active explorations.</td>
<td>□ A Technology document provides an environment for students to do mathematics with teacher guidance.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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The lesson plan rubric I used was developed for evaluating lessons that use TI-Nspire calculators. The TI-Nspire product line is a series of graphing calculators developed by Texas Instruments. The TI-Nspire has a user interface that is more similar to PCs than regular calculators, and it also handles documents in a similar way to PCs. As a result of this capability of the TI-Nspire, teachers can create documents with instructions and exercises for the students instead of creating the usual paper worksheet. Thus, the second indicator for this part of the rubric focused on the TI-Nspire document. However, the pre-service teachers were not in classes where students had access to the TI-Nspire and so they did not have to create a TI-Nspire document. Instead they had to either create a worksheet for students to use with the technology or give their students instructions either verbally or by writing them on the board. I considered this to be an equivalent of a TI-Nspire document. I replaced this word TI-Nspire with the word technology so that I can generalize to any type of technology that the pre-service teachers used. Therefore, the phrase technology document means either the teacher created worksheet or the teacher-directed instructions that accompany any technology use in the classroom.

First Group Lesson Design and Implementation. Team A’s (Ethan, Ally, Eddy) technology was intended to be used only by the teacher for demonstration without any independent student exploration. Using the rubric in Table 5, the lesson was partly at the Accepting: Level 2 because there was no worksheet or instructions to accompany the technology use. Team B’s (Heidi, Doug) lesson was at the Accepting: Level 2 because the teacher demonstrations of how to solve equations with variables on both sides were
followed by student practice, with instructions from the teacher and no independent explorations.

Second Group Lesson Design and Implementation. Team A’s (Ethan, Ally, Eddy) lesson had elements from both the Accepting: Level 2 and the Adapting: Level 3. The lesson was at Accepting: Level 2 in the sense that the teachers used the TI-Nspire to show the differences between an influential point and an outlier. The students’ use of the TI-83 calculators to find the regression line, residual plot and scatter plot indicated this lesson should be rated as partly at the Adapting: Level (3). Team B’s (Heidi, Doug) lesson was at the Adapting: Level 3 because the students used technology on their own as they worked through the worksheet. The teacher focused on students’ thinking by conducting whole-class discussions. The activity worksheet allowed the students to do mathematics with teacher guidance.

First Individual Lesson Design and Implementation. Ally and Doug did not use any technology in their first individual lessons and their scores were zeros for this TPACK domain. Heidi’s lesson was at the Recognizing: Level 1 because her students’ use of technology for computation was student practice. Eddy’s lesson was partly at the Accepting: Level 2 because he demonstrated graphing the equations using the Smartboard’s graphing tool but had his students graph on a graph paper. Also there was no technology document to go with the activity. Ethan’s lesson was at the Adapting: Level 3 because once Ethan had introduced the activity and handed out the worksheet he
had his students work on their own with their TI-83s, while he walked around to guide them as they worked.

Second Individual Lesson Design and Implementation. In her second lesson plan, because Ally had the students review the previous section on the graphical method and also had them check their answers using a graph, this strategy qualified the lesson to be at the Recognizing: Level 1. For his lesson on “writing function as a rule” Doug demonstrated how to fit a regression line to a data set as his students followed along on their calculators. This was partly at the Accepting: Level 2 because there was no worksheet or instructions to go along with the activity. Heidi’s lesson on “multiplying fractions” was partly at the Accepting: Level 2 because she demonstrated how to use the computer applet and had students take turns using the computer applet. There was no worksheet or instructions to go along with the activity. Eddy demonstrated how to graph on an electronic graph but he had his students graph on a graph paper. For that reason his lesson was partly at the Accepting: Level 2 For his second lesson, Ethan focused on students’ conceptual understanding of mathematics as he had them simulate the different ticket scenarios and guided them through the discussions in which he encouraged students to interpret their simulation results. Therefore, his lesson was at the Exploring: Level 4.

Cross Case Analysis of the Results on Question 2

For question 2 I compiled and graphed the participants’ scores as shown in Figure 10 to enable easy comparison. The scores show that the second group lessons had better
scores than the first group lessons. Specifically, both groups scored 1.5 (being between the Recognizing: Level 1 and Accepting: Level 2) on their first lessons, and then Team B (Heidi, Doug and two other pre-service teachers who were not participating in this study) moved up by 1.5, while Team A (Ally, Eddy and Ethan) moved up 0.5. Similarly, the second individual lessons had better scores than the first individual scores. Scores on the individual lessons ranged from 0 to 1.5, except for Ethan, who was at the Adapting: Level 3 on the first individual lesson and at the Exploring: Level 4 on the second individual lesson.

Figure 10: The Participants’ Score on the Knowledge of Students’ Understandings, Thinking and Learning of Mathematics with Technology

An analysis of the TPACK survey revealed three main areas that pre-service teachers believed students should understand in order to learn mathematics with technology, namely, operations, the fundamentals (the mathematics concepts), and the
purpose (why use technology). In the pre-survey, three of the participants (Eddy, Ethan and Ally) thought that knowing operations was important. For instance, Eddy noted that “students need to be familiar working with computers,” whereas Ethan identified “the basic operation of their calculator or computer so that learning the machine doesn’t distract them from learning the math.” I considered this as a low level thinking about technology, and I will expand on this thought in Chapter 5. Three other participants (Heidi, Ally and Doug) cited the fundamentals in response to what students should know when learning mathematics with technology. Only Heidi cited “purpose” as one of the things that students should know in order to learn mathematics with technology.

The post-survey responses showed little or no improvement in terms of thinking about the students’ understandings and thinking when learning mathematics with technology. The participants still valued knowing the operations as important when learning mathematics with technology. Heidi even stepped down from her initial response to the question on the pre-survey where she articulated the purpose, that is, the need for students to “understand why they are doing something with technology and also the fundamentals,” to simply noting that students, “need to know the basics of using technology.” From this observation I conjectured that pre-service teachers struggled with what they encountered in the classrooms during their field experiences, and I will elaborate on this in Chapter 5.

However, Doug’s response stood out:

They should know how to use the technology; syntax, operation, etc. Students should understand what it is that the technology is showing to them. They should be familiar with the process of how the technology
works so that it’s not just magic but rather performing a consistent, well-defined process.

This response is notable because it was at the level that I expected all the pre-service teachers to be at the end of the semester in terms of knowledge about their students’ understandings and thinking when learning mathematics with technology. This response coming from Doug, who for the most part had not really shown any insights about the other TPACK domains, showed that he held some TPACK. However, in his teaching philosophy statement he did not provide any more insight into what students should know when learning mathematics with technology.

Four of the five teaching philosophy statements, both at the beginning and at the end of the semester, did not show any evidence of the knowledge about students’ understandings, thinking and learning of mathematics with technology. The pre-service teachers seemed to focus on either what they can do with technology or what technology is capable of doing instead of focusing on their students. Only Eddy’s teaching philosophy statement, both at the beginning and at the end of the semester, showed evidence of thinking about what students should know when learning mathematics with technology. In both teaching philosophy statements (at the beginning and at the end of the semester), Eddy wrote:

Students should also be able to consider whether or not a tool is helpful in getting the correct solution, or if it just creates more of a burden. They should have enough understanding of the material to know if errors have been made in generating a mathematical model, or if a solution seems to make sense. The goal is that students are able to use tools such as graphing calculators or mathematical computer programs in a way that deepens their mathematical understanding.
From Eddy’s statement, he believes that students should be able to determine whether or not it is appropriate to use technology. Similar to Doug’s statement that “students should understand what it is that the technology is showing to them,” Eddy also thinks that students should have a deeper understanding of the mathematics in order to make sense of the solutions from technologies. However, during the exit interview, Eddy only emphasized the need for students to learn how to use the technology.

Results for Question 3 from the Lesson Designs and Implementation

Research Question 3: How does the pre-service teachers’ knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics emerge in as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific domains of TPACK?

To answer this question, I used part of the TPACK lesson plan rubric that focuses on the pre-service teachers’ knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics.
Table 6: The Original Part of the Lesson Plan Rubric Showing the Indicators at each Level of Development for the Knowledge of Curriculum and Curricular Materials that Integrate Technology in Learning and Teaching Mathematics TPACK Domain

<table>
<thead>
<tr>
<th>Knowledge of curriculum and curricular materials that integrate technology in learning and teaching subject matter topics</th>
<th>Recognizing (1)</th>
<th>Accepting (2)</th>
<th>Adapting (3)</th>
<th>Exploring (4)</th>
<th>Advancing (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Teacher does not use technology for learning mathematics.</td>
<td>□ Teacher uses standard approach to the curriculum topics with Technology being used as add-on.</td>
<td>□ The Technology is used as a replacement for non-technology based tasks in a traditional curriculum approach</td>
<td>□ Students are given problem solving tasks with technology and are asked to expand math ideas on the basis of technology explorations</td>
<td>□ Technology is strongly aligned with curriculum goals. Teacher chooses essential topics of school mathematics curricula and technology use is effective for the chosen curriculum topics.</td>
<td></td>
</tr>
<tr>
<td>□ Technology if used is not aligned with one or more curriculum goals.</td>
<td>□ Technology is partially aligned with one or more curriculum goals. Teacher has difficulty in identifying topics in mathematics curriculum for including technology as tool.</td>
<td>□ Technology is aligned with one or more curriculum goals. Teacher chooses topics from school mathematics curricula; however, technology use is not always appropriate for the chosen curriculum topics.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6 is the original part of the rubric for the section on the knowledge of curriculum and curricular materials that integrates technology in learning and teaching mathematics, only with the terms “TI-Nspire” replaced by “technology.” However, when I tried using this part of the rubric to assess the participants’ Knowledge of Curriculum and Curricular Materials that Integrate Technology in Learning and Teaching Subject Matter Topics, I could not assess the participants’ abilities to identify topics from the mathematics curriculum for including technology as specified for Accepting, Adapting, Exploring, and Advancing levels. This was because during a practicum experience, pre-service teachers are not necessarily exposed to the whole curriculum. Instead, they gain experience with a few topics that are part of the curriculum. I therefore modified the rubric so that I could assess the alignment of their technology use to the lesson objectives and also the appropriateness of the technology use for the chosen topic. Table 7 shows the revised rubric that I used to assess the participants’ lessons for their knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics. I will further discuss the use of the rubric to assess this aspect of TPACK in Chapter 5.
Table 7: The Revised Part of the Lesson Plan Rubric Showing the Indicators at each Level of Development for the Knowledge of Curriculum and Curricular Materials that Integrate Technology in Learning and Teaching Subject Matter Topics TPACK Domain

<table>
<thead>
<tr>
<th>Knowledge of curriculum and curricular materials that integrate technology in learning and teaching subject matter topics</th>
<th>Recognizing (1)</th>
<th>Accepting (2)</th>
<th>Adapting (3)</th>
<th>Exploring (4)</th>
<th>Advancing (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>☐ Teacher does not use technology for learning mathematics.</td>
<td>☐ Teacher uses standard approach to the curriculum topics with Technology being used as add-on.</td>
<td>☐ The Technology is used as a replacement for non-technology based tasks in a traditional curriculum approach.</td>
<td>☐ Students are given problem solving tasks with technology and are asked to expand math ideas on the basis of technology explorations.</td>
<td>☐ Teacher uses technology in a fully constructive way, including tasks for development of higher level thinking and deepening understanding of mathematics concepts.</td>
<td></td>
</tr>
<tr>
<td>☐ Technology is not aligned with the lesson objectives.</td>
<td>☐ Technology use is partially aligned with one or more the lesson objectives. Technology use is not appropriate for the chosen topic.</td>
<td>☐ Technology use is aligned with one or more lesson objectives. Technology use may not be appropriate for the chosen topic.</td>
<td>☐ Technology use is aligned with lesson objectives. Technology use is appropriate for the chosen topic.</td>
<td>☐ Technology use is strongly aligned with lesson objectives. Technology use is appropriate and effective for the chosen topic.</td>
<td></td>
</tr>
</tbody>
</table>

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First Group Lesson Design and Implementation. Team A’s lesson was partly at the Accepting: Level 2 and Adapting: Level 3. The lesson was partly at the Accepting: Level 2 because the teacher had to solve an equation with variables on both sides first and then use the pan balance tool to solve similar problems. This was a traditional approach to the topic and technology was just an add-on. However, this lesson could not be fully at the Accepting: Level 2 because the technology use was aligned (instead of being partially aligned) with the two lesson objectives that:

- Students will be able to solve equations with variables on both sides
- Students will be able to identify equations that are identities or have no solution

Also, using the pan balance tool was appropriate for the solving equations with variables on both sides. Therefore, for these two reasons, the lesson was partly at the Adapting Level.

For Team B’s lesson, technology was also an add-on since the teacher solved three different problems at first and then introduced using the TI-83 to solve the same problems graphically. For this reason, the lesson was at the Accepting: Level 2. The lesson objectives were:

- Students will be able to use two or more steps to solve linear equations
- Students will be able to solve equations of the form $ax + b = cx + d$
- Students will be able to use the TI-83 to connect the table, graph, and equation
The technology use was aligned with these lesson objectives and the use of the TI-83 was appropriate for solving equations with variables on both sides. Therefore, Team B’s lesson score was 2.5.

Second Group Lesson Design and Implementation. Team A’s lesson on linear regression, outliers and influential points was at the Exploring: Level 4 for this TPACK domain. This was a result of a number of reasons. First, the students had to solve some problems with technology. For instance, the students were asked to identify outliers and influential points by looking at their scatter plots along with their residual plot. Secondly, the dragging and deleting of points by the teacher to help distinguish outliers from influential points was a way of exploring with technology. Students were not asked to do this because this cannot be done on TI-83 calculators and they did not have the TI-Nspire calculators. Thirdly, the objectives of this lesson were that students should be able to:

- Describe what conditions must be met for a point to be considered an outlier
- Describe what properties of a point make it influential
- Identify outliers and influential points on a scatter plot

These objectives were aligned with the technology use in the sense that the activities that the class engaged in with the technology were directed at achieving these objectives. Lastly, the technology use was appropriate for the chosen topics.

Team B’s lesson on “caution about regression and correlation” was at the Exploring: Level 4 because the lesson involved students working on two different worksheets to help them discover the dangers of using correlation to predict the future
and the usefulness of the residual plots. Moreover, the class discussion was centered on what students discovered through their work. In addition, there was alignment between the technology use and the lesson’s objectives that students will be able to:

- Determine if the linear regressions is appropriate to use on a set of data for predictive purposes
- Evaluate using the residual plot to decide if the linear regression line is a good fit for the data

Like Team A’s lesson, technology use was appropriate for the chosen topic.

**First Individual Lesson Design and Implementation.** Ally and Doug did not use technology in their first lessons and their scores for this TPACK domain were zeros. Although Heidi approached her lesson without technology she allowed her student to use calculators to help with computations. Heidi’s two objectives for the lesson were that student would be able to:

- Convert between fractions, decimals, and percents
- Evaluate statistics given by media sources

Technology use was partially aligned with first objective because converting between fractions, decimals and percents involves more than using a calculator for computations. Her non-use of technology in the learning of mathematics was at the Recognizing: Level 1 and the partial alignment between technology use and lesson objectives as being at the Accepting: Level 2. Heidi’s lesson was therefore between the Recognizing: Level 1 and Accepting: Level 2.
The graphing tool that Eddy used was more like graphing on graph paper because he had to select the points to graph (just as one would do with graph paper) and then the graphing tool helped with connecting the point with a straight line. Eddy’s lesson was at the Accepting: Level 2 since he approached his lesson on graphing the same way you would without any technology except that he didn’t have to draw the grid on the board. Eddy’s lesson objectives were that students will be able to:

- Identify and write linear equations and intercepts
- Graph linear equations
- Model real-world situation with linear equations

Eddy wanted his students to be able to graph linear equation on graph paper and he used a graphing technology that allowed him to graph the same way you would graph on a graph paper. This use of technology would not help him achieve the other two objectives. Thus, his technology use was partially aligned with his lesson objectives. For this TPACK domain, Ethan’s lesson on “finding derivatives related to harmonic motion” was at the Exploring: Level 4 because the students had to work on their own to come up with an equation that fits the data. The following were his three lesson objectives:

Students will be able to:

- Write a sine or cosine function that fits a set of x,y data points
- Explain why the derivative of sin (2x) = 2*cos (2x)
- Point out where maximum velocity occurs given a graph of position
The use of technology in this lesson was aligned with all the three lesson objectives and the use of the TI-83 calculator was appropriate for the chosen topic.

**Second Individual Lesson Design and Implementation.** I did not consider Ally’s use of a PowerPoint presentation as using technology, because she used technology as a productivity tool and not as a cognitive tool. The focus of this study is on the cognitive uses of technology. However, I considered her idea of having students review solving systems of equations using graphing calculators as using technology. Throughout her lesson on the substitution method Ally had students check their answers using graphing calculators. Therefore her lesson was at the Recognizing: Level 1 for this TPACK domain. It was at the Accepting Level because she did not use technology for learning about the substitution method which was the focus of the lesson and also that resulted in her technology use not being aligned with her only lesson objective that:

- Students will be able to solve systems of linear equations in two variables by substitution.

Doug’s lesson on “the function as a rule” was *partly* at the Accepting: Level 2. First, he did not use technology for the most part of his lesson except towards the end when he showed the students how to fit a simple linear regression. Therefore, his use of technology was more of an add-on. Doug’s lesson objectives were:

Students will be able to:

- Create a function rule from a table of ordered pairs
- Create a function from a graph of a linear function
Although Doug’s use of technology aligned with his first objectives, his technology use was not appropriate for the chosen topic because the 8th grade curriculum does not include fitting a simple linear regression. Therefore his technology use could not be classified as being at the Adapting: Level 3.

Heidi introduced her lesson on multiplying fractions by using the brownie model (area model). She demonstrated how to use the area model to multiply two fractions on the board and then introduced the fraction multiplication applet. Her lesson was therefore, partly at the Accepting: Level 2 and partly at the Adapting: Level 3 because she used technology as an add-on. More so the technology use was aligned with her lesson two lesson objectives that:

Students should be able to:

- Use area models to represent the product of two fractions, each less than one
- Multiply fraction less than one using the “part of a part” approach to multiplication

In addition, the technology use was appropriate for the chosen topic.

For his second individual lesson, Eddy used the multiple-linked representation feature of the TI-Nspire (in this case an equation connected to a graph) to graph the equation of a line and then have students make guesses about the slope and the y-intercept. This lesson was partly at the Accepting: Level 2 and Adapting: Level 3 because he used technology to replace a usually non-technology bases task in a traditional curriculum. However, his technology use was partially aligned with his first lesson
objective making Eddy’s lesson to be ineligible for being fully at the Adapting: Level 3.

The two lesson objectives were:

Students will be able to:

- Write and graph linear equations in slope-intercept form
- Model data using the slope-intercept form of a linear equation

His technology use was partially aligned with his first objective only because it would not enable students to write linear equations in slope intercept form. Instead, it would only help them graph the equations.

Ethan’s lesson on simulation was at the Exploring: Level 4 because he gave his students a problem they had to solve with technology and then asked them to draw conclusions based on some explorations. In addition, Ethan’s use of technology was aligned with his lesson objectives that:

Students will be able to:

- Use the `randint` function in their calculator to simulate involving a binomial random variable
- Analyze a situation involving a binomial event using simulation and make a decision about the best course of action

Having the students use the TI-83 and 84 calculators was appropriate for the chosen topic.

Cross Case Analysis of Results for Question 3

For this TPACK domain, the pre-service teachers performed better overall than the other TPACK three TPACK domains, as can be seen in the graph in Figure 11. The
scores ranged from 0 to 4. For the first group lessons, both groups were between Accepting: Level 2 and Adapting Level 3 which was the highest starting level for all the four TPACK domains. Both groups progressed to Exploring: Level 4 on the second group lesson.

Figure 11: A Graph Showing the Participants’ Scores on the Knowledge of Curriculum and Curricular materials that Integrate Technology in Learning and Teaching Mathematics

For the individual lessons, Ethan started at the Exploring: Level 4 because he demonstrated a good mastery of both the technology and the content he was teaching. He was at the same level for the second individual lesson. Eddy started off at the Accepting: Level 2 and he progressed to a slightly higher level, which is a level between Accepting: Level 2 and Adapting: Level 3, for his second individual lesson. Heidi was at the level between Recognizing: Level 1 and Accepting: Level 2 and progressed to the level between Accepting: Level 2 and Adapting: Level 3 on her second individual lesson. Ally
and Doug moved from non-use of technology (0) to Recognizing: Level 1 and Accepting: Level 2, respectively.

To capture the pre-service teachers’ self perception about the knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics, I used two questions:

Question 1: Do you know of any curriculum and/or curricular materials that integrate technology in learning mathematics? If so, which ones?

Question 2: In terms of technology integration in mathematics teaching and learning: (a) What online resources are you aware of? (b) What textbook resources are you aware of? (Please number your answers (a) and (b))

The first question was more general than the second question. Considering the first question, at the beginning of the semester, none of the pre-service teachers knew about a specific curriculum that integrated technology, as shown in the following responses:

Eddy’s response to Question 1: I do not know specific names, but I have seen textbook software that is made to work directly on a Smartboard to supplement a lesson.

Heidi’s response to Question 1: Not really - Illuminations website from NCTM, but that isn’t a full curriculum, more extension/enhancements to a curriculum.

Ethan’s response to Question 1: I’m not familiar with any specifics. I know that lots of probability lessons involve using a computer to simulate many trial runs.

Ally’s Response to Question 1: Not really.

Doug’s Response to Question 1: Many topics in math have applications in computer science, or to be demonstrated through computer software. This
could be a spectrum of things from plotting a function on a graphing calculator to writing software to simulate a physical or mathematical problem. For example: creating software that uses parameters as input, manipulates them in the appropriate way, and then gives an output based on the input.

Although they did not know any specific curriculum, the responses given by Eddy, Heidi and Ethan seem to point to an awareness of curricular materials that integrate technology. Thus, for the second question, all the pre-service teachers were able to give examples of online resources such as Geogebra, mathforum.org, illuminations from NCTM, math.com and 4teachers.org. They did not have an awareness of textbook resources with Eddy and Ally citing Sketchpad and Modern Geometry respectively. However, although he could not be specific, Doug seemed to have awareness of what the question was asking since he write that, “most textbooks have special problems which require the use of software or a graphing calculator to show something.”

At the end of the semester, the pre-service teachers still did not know any curriculum that integrates technology in mathematics learning or textbook resources except for Doug. Doug’s response to the first question on curriculum and curricular materials that “most middle school and high school textbooks include calculator activities. Math curriculum at the college level often uses technology in teaching and learning” demonstrates some knowledge about curriculum that integrates technology. However, they were well aware of curricular materials that integrate technology with the list of resources being almost the same as the one they came up with at the beginning of the semester. The new additions were Math4Media cited by Heidi and CAS cited by Ally.
The teaching philosophy statements did not show any evidence of the pre-service teachers’ knowledge of curriculum and curricular materials that integrate technology. However, it was apparent from the philosophy statements that the pre-service teachers had a sense of the curriculum they are going to teach. The following are Heidi’s, Eddy’s and Ally’s statements from their philosophy statements that attest to this observation:

Heidi – At the high school level, students best to develop this ability through studying several topics. I feel that the Common Core State Standards (CCSS) provides a comprehensive listing, organizing mathematics into 1) number and quantity, 2) algebra, 3) functions, 4) modeling, 5) geometry and 6) statistics and probability. The balance between these six topics should be weighted more heavily towards the last five, since by high school students should have developed strong number skills. As stated by the NCTM, that is more properly the focus of earlier grades; the other standard areas becoming increasingly the emphasis of study as students grow more experienced.

Eddy - Understanding numbers and operations is crucial to mathematical thinking. Students need to be able to represent and find relationships between numbers and operations, which will lead to an understanding and ability to do computations fluently and efficiently while retaining a sense of meaning. Another important aspect of high school mathematics is for students to gain an understanding for patterns, relations, and functions. An example of students understanding functions is the ability to study and relate several different types such as linear, quadratic, and exponential functions and being able to identify the type of function from the graph or from the equation (NCTM, 2000).

Ally – They should improve the skills in algebra, analysis, statistics, probability, geometry.

Results for Question 4 from the Lesson Designs and Implementation

Research Question 4: How does pre-service teachers’ knowledge of instructional strategies and representations for teaching and learning mathematics with technologies...
emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific domains of TPACK?

To answer this question, I used part of the TPACK lesson plan rubric and the TPACK survey that focuses on the pre-service teachers’ knowledge of instructional strategies and representations for teaching and learning mathematics with technologies (See Table 8).

First Group Lesson Design and Implementation. Both group lessons were at the Recognizing: Level 1 for this TPACK domain because the teacher for both group lessons had to demonstrate to the students how to use the technology to do the math and the students simply had to follow along.

Second Group Lesson Design and Implementation. Team A’s lesson on linear regressions, outliers and influential points had both elements of being teacher-directed and being student centered. The student worked on their own to find the regression line, find the centroid, and come up with the scatter plot and the residual plot using their TI-83 calculators. On the other hand, the teacher had to demonstrate to the students the difference between an outlier and an influential point by dragging the points around and also by deleting the points. The dragging around and deletion of points is way beyond a traditional curriculum. The lesson was therefore at the Exploring: Level 4. Moreover, the accompanying worksheet promoted student reflection.
Table 8: Part of the Lesson Plan Rubric Showing the Indicators at each Level of Development for the Knowledge of Instructional Strategies and Representations for Teaching and Learning Subject Matter Topics with Technologies TPACK Domain

<table>
<thead>
<tr>
<th>Knowledge of instructional strategies and representations for teaching and learning subject matter topics with technologies</th>
<th>Recognizing (1)</th>
<th>Accepting (2)</th>
<th>Adapting (3)</th>
<th>Exploring (4)</th>
<th>Advancing (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Teacher focuses on how to use technology rather than how to explore math ideas, using teacher-directed lectures followed by student practice.</td>
<td>□ The instructions are teacher-led. Teacher structures lesson plan with limited student explorations with technology.</td>
<td>□ Teacher uses, deductive (teacher-directed) approach with more focus on mathematics discoveries with technology rather than inductive, traditional curriculum.</td>
<td>□ Teacher uses various instructional strategies (deductive and inductive) and focuses on students thinking about mathematics. Teacher’s use of technology is beyond traditional approaches to curricular topics.</td>
<td>□ Teacher focuses on students’ hands-on and experimentation of new mathematics ideas with technology, and focuses on conceptual development.</td>
<td>□ Technology document is built around learning objects and must explicitly promote student reflection – especially the posing of questions for sense-making. Technology document is built around learning objects and must explicitly promote student reflection – especially the posing of questions for sense-making and reasoning, including explanation and justification.</td>
</tr>
</tbody>
</table>
Team B’s lesson basically had students work on two sets of worksheets, the first one where three different data sets were given and the other one where the scatter plots were given instead of the data. I classified the lesson as being partly at the Exploring: Level 4. This was necessitated by the fact that the lesson had both instances of teacher-centeredness and student-centeredness during the lesson. Also, so the two activity worksheets promoted student reflection. However, having students make predictions with or without the scatter plots was a traditional approach to the curriculum. For that reason, the lesson could not be fully at the Exploring: Level 4 for this TPACK domain.

First Individual Lesson Design and Implementation. Ally and Doug did not use technology in their first individual lessons and as a result their scores were zeros (0) for this TPACK domain. In Heidi’s lesson on comparing decimals, fractions and percents technology wasn’t really part of her instruction. The calculators were meant to help students with their computations and as a result her score was also a zero (0) for this TPACK domain. Eddy’s score on his lesson on graphing linear equations was also a zero (0) because the graphing tool that he used required him to do all the work on the board including finding the points to plot. The graphing tool only helped him with drawing a straight line between two points. His students had to use graph paper. Removing the graphing tool would not have changed anything in Eddy’s lesson except for the fact that he would have had to draw grid lines on the board. Ethan’s lesson on finding derivatives related to harmonic motion was at the Adapting: Level 3 because the lesson was mainly teacher directed. Ethan helped his students figure out what they were supposed to find although there was an emphasis on mathematical discoveries.
Second Individual Lesson Design and Implementation. Ally’s score was a zero (0) because she used a PowerPoint presentation for her lesson on solving systems of equations by substitution. The students used the TI-83 graphing calculator to review the previous unit and also to check answers. When Doug demonstrated how to fit a data set with a regression line during his lesson on the function as a rule, his focus was on how to use the TI-83 to generate the linear regression and as such he had his students simply follow along. Therefore his lesson was partly at the Recognizing: Level 1 since he did not have any worksheet to provide students with any opportunities for drill and practice. Heidi’s lesson on multiplying fractions was partly at the Recognizing: Level 1 because she simply showed her students how to use the applet and then had them take turns using the applet. However, she did not have any technology document or worksheet to provide students with opportunities for drill and practice. Eddy’s second lesson was at the Accepting: Level 2 because he was the sole user of the technology and he led his students throughout the lesson on figuring out the slope and the y-intercept. His worksheet was more for note-taking than for allowing for student reflection. Ethan’s lesson was at the Exploring: Level 4 because his use of simulations was way beyond traditional approached to curriculum. His worksheet also allowed for student reflection on what the simulation results meant.

Cross Case Analysis of the Results for Question 4

Both groups started off at the Recognizing: Level 1 and then progressed to the Exploring: Level 4 in the case of Ally, Ethan and Eddy, and the level between Adapting: Level 3 and Exploring: Level 4 in the case of Heidi and Doug. For this TPACK domain,
the first individual lessons had the weakest performance in terms of TPACK, with 4 of the 5 participants scoring a 0. This domain is about the instructional decisions that the teachers make in terms of using technology to teach mathematics. Ally and Doug did not use technology at all in their lessons; Heidi had students use calculators to help with computations so the use of technology did not impact her classroom instruction in any way and Eddy’s lesson could have been taught exactly the same way without the Smartboard. However, Ethan’s lesson stood out because he had his students involved in some mathematical discoveries, with his help, and hence he was at the Adapting: Level 3.

For the second lesson, except for Ally, there was a general improvement in terms of using technology in mathematics instruction with Eddy having the highest improvement from non-use of technology (0) to being at the Accepting: Level 2. Doug and Heidi, because of their strategy of demonstrating and then having students practice, were partly at the Recognizing Level. Ethan moved a level up to be at the Exploring Level.

Figure 12: A Graph showing the Participants’ Scores on the Knowledge of Instructional Materials that Integrate Technology in Learning and Teaching Mathematics
The TPACK survey results seem to agree for the most part with the lesson performance. To address the pre-service teachers’ knowledge of instructional strategies that integrate technology in learning and teaching mathematics, the pre-service teachers were asked to respond to the following question: *Do you know of any instructional strategies and representations for teaching and learning mathematics with technology? If so which ones?* Except for Doug, all the other four participants responded either “no” or “not sure.” Doug’s response was that “technology does not replace strong, well thought out pedagogy. Technology can be useful in representing data or providing simulations of processes that are hard or impossible to demonstrate in the classroom.” Although Doug knew some representations such as simulations his response to the question shows that he does not know instructional strategies for teaching and learning mathematics with technology.

An analysis of the pre-service teachers’ teaching philosophy statements, both at the beginning and at the end of the semester, revealed that although the pre-service teachers viewed technology as an essential tool in mathematics instruction, their description of their teaching strategies did not integrate technology for the most part. They tended to focus on what technology can do. For instance, Ethan wrote that:

This (the availability of tablet computers) will provide a lot of opportunity for increased feedback and assessment. Teachers will be able to quickly see statistics on which problems the most students are having trouble with. Computers will help students do research when solving word problems. They already are a huge help in making geometry lessons much faster. They are already used to do many repetitions of an experiment in statistics and probability lessons. Student work stored in “the cloud” will eliminate huge backpacks and forgotten homework.
Heidi’s statement during the exit interview helps shed light on why the pre-service teachers tended to view their teaching as separate from technology integration:

…I don’t feel that I had strong models of using some of the new technologies especially in terms of my teachers using it to teach, here and elsewhere in general. It’s been difficult to see or even envision how it could be used since I haven’t seen it being used anyways. I don’t know if that makes sense. I guess what I am saying is that, it’s hard for me to look at a lesson that I’m going to be teaching to think about how to incorporate technology because I have never seen strategies of how, I didn’t know a lot of the stuff that this could do and so I would never think to do it because I don’t know what it can do

Eddy’s expressed a similar opinion when he said that, “…like getting comfortable teaching a certain curriculum I think will come before I am comfortable putting tech in teaching that curriculum.”

The Emergence of TPACK in Mathematics Pre-service Teachers

The results presented in the preceding section show differences in the pre-service teachers’ performance on each TPACK domain. TPACK-related research tends to focus on the aggregate TPACK. To determine the pre-service teachers overall TPACK levels, I consolidated the results from analysis of individual TPACK domains. These then, represented the pre-service teachers’ enacted TPACK. I contrasted their enacted TPACK with results from the Likert part of the survey, which I consider to represent the pre-service teachers’ self-reported TPACK.
Pre-service Teachers’ TPACK Levels from Lesson Designs and Implementation

To determine the TPACK levels, Lyublinskaya and Tournaki (2011) suggested using the lowest TPACK rubric score of all four domains to represent the TPACK level of an individual. Therefore, to determine the pre-service teachers’ TPACK levels for each lesson, I compared their scores across all four TPACK domains for each TPACK domain and then selected the lowest score to represent their level of TPACK development for each lesson development. These overall levels of TPACK development are at the bottom row of each of the Tables, 9, 10, 11, 12, and 13 and they are in bold. The four TPACK domains are abbreviated as follows: PPIT (Overarching Conceptions about the Purposes for Incorporating Technology in Mathematics Teaching); SUTL (Knowledge of Students’ Understandings, Thinking, and Learning of Mathematics with Technology); CCMI (Knowledge of Curriculum and Curricular Materials that Integrate Technology in Learning and Teaching Mathematics) and ISTR (Knowledge of Instructional Strategies and Representations for Teaching and Learning Mathematics with Technologies). For instance, Ally’s scores on the first individual lesson were 0, 0, 1, 0 and the lowest of these four scores is 0 and therefore her TPACK level on the first lesson was a zero that is not even at the Recognizing Level. The following are the TPACK levels for each of the participants.
Table 9: Ally’s TPACK levels (in bold) from comparing the TPACK scores for each TPACK domain for each lesson

<table>
<thead>
<tr>
<th></th>
<th>Group Lesson 1</th>
<th>Group Lesson 2</th>
<th>Individual Lesson 1</th>
<th>Individual Lesson 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPIT</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SUTL</td>
<td>1.5</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>CCMI</td>
<td>2.5</td>
<td>4</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ISRT</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TPACK level</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 10: Doug’s TPACK levels (in bold) from comparing the TPACK scores for each TPACK domain for each lesson

<table>
<thead>
<tr>
<th></th>
<th>Group Lesson 1</th>
<th>Group Lesson 2</th>
<th>Individual Lesson 1</th>
<th>Individual Lesson 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPIT</td>
<td>1</td>
<td>3.5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>SUTL</td>
<td>1.5</td>
<td>3</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>CCMI</td>
<td>2.5</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>ISRT</td>
<td>1</td>
<td>3.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>TPACK level</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 11: Heidi’s TPACK Levels (in bold) from comparing the TPACK scores for each TPACK domain for each lesson

<table>
<thead>
<tr>
<th></th>
<th>Group Lesson 1</th>
<th>Group Lesson 2</th>
<th>Individual Lesson 1</th>
<th>Individual Lesson 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPIT</td>
<td>1</td>
<td>3.5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>SUTL</td>
<td>1.5</td>
<td>3</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>CCMI</td>
<td>2.5</td>
<td>4</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>ISRT</td>
<td>1</td>
<td>3.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>TPACK level</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 12: Eddy’s TPACK Levels (in bold) from comparing the TPACK scores for each TPACK domain for each lesson

<table>
<thead>
<tr>
<th></th>
<th>Group Lesson 1</th>
<th>Group Lesson 2</th>
<th>Individual Lesson 1</th>
<th>Individual Lesson 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPIT</td>
<td>2</td>
<td>3</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>SUTL</td>
<td>1.5</td>
<td>2</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>CCMI</td>
<td>2.5</td>
<td>4</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>ISRT</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>TPACK level</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 13: Ethan’s TPACK Levels (in bold) from comparing the TPACK scores for each TPACK domain for each lesson

<table>
<thead>
<tr>
<th></th>
<th>Group Lesson 1</th>
<th>Group Lesson 2</th>
<th>Individual Lesson 1</th>
<th>Individual Lesson 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPIT</td>
<td>2</td>
<td>3</td>
<td>3.5</td>
<td>4</td>
</tr>
<tr>
<td>SUTL</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>CCMI</td>
<td>2.5</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>ISRT</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>TPACK level</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Combining the TPACK levels for each participant enables the participants to be compared. To enable a cross case analysis of the participants’ TPACK levels, I consolidated the participants’ TPACK level scores (in bold) from the above five Tables 9, 10, 11, 12, and 13 into one table and the TPACK levels were then graphed as shown in Figure 13. The graph shows that the pre-service teachers came in with a general acceptance of the use of technology since they were all the Recognizing: Level 1 on their first group lesson. There was a general improvement from the first lesson to the second group lesson. However, except for Ethan, the pre-service teachers had a decline in performance on their first individual lesson. This could point to a possible group effect. Ethan’s trend shows an improvement throughout, as he moved to the next level of
TPACK with each lesson development. Ally, on the other hand, did not make any improvement at all, as she remained at 0 for her two individual lessons. Eddy had the highest improvement between the two individual lessons as he moved from a 0 on the first lesson to a 1.5 with Heidi, Doug and Ethan having the least amount of improvement, that is, 0.5.

Figure 13: The Pre-service Teachers’ TPACK levels from the lessons

Pre-service Teachers TPACK Levels from the TPACK Survey

The Likert part of the TPACK survey provides a quantifiable measure of the pre-service teachers’ self-reported TPACK levels. Figure 14 shows bar graphs of the pre-service teachers’ TPACK scores. (Ethan’s score at the end of the semester was a 0 because he did not complete the TPACK survey at the end of the semester or after several follow-up reminders. After more than 2 months had passed, I recorded that data source as unavailable because of the amount of time that passed and because of the further methods courses that Ethan was taking, both of which could affect his responses.)
Figure 14: A Graph Showing the Pre-and Post-Test Pre-service Teachers’ Self-Reported TPACK Scores (without Ethan’s post-test score)

Pre-service Teachers’ Enacted TPACK vs Self-reported TPACK

Comparing Figures 13 and 14 reveals a mismatch between the pre-service teachers’ enacted TPACK (Figure 13) and their self-reported TPACK (Figure 14). Although my assessment of their TPACK from their lessons six weeks into the semester revealed that they were at 0, in the case of Ally, Doug, Heidi and Eddy, self-reported quantitative scores at the beginning of the semester showed that they self-reported a good understanding in terms of TPACK, with scores of 3.8, 4.4, 2.8, and 3.8 respectively. Like the TPACK scores from the lessons, TPACK scores from the Likert part of the survey also increased by the end of the semester, as shown in Figure 14. However, Doug’s score actually declined from 4.4 to 4.2, which is in contrast to his score on the enacted TPACK where he improved from a 0 to 0.5. I will analyze this mismatch further in chapter 5.
Summary of Results

The pre-service teachers’ performed better in group lessons than they did on the individual lessons. Particularly, the second group lessons they designed for the 11th/12th grade AP statistics class was better in terms of the use of technology for teaching the content than the first group lessons they designed for the 8th grade algebra class.

There was a mismatch between the participants’ enacted TPACK levels from the TPACK lesson rubric and the self-reported TPACK scores from the Likert questions on the TPACK survey. The following paragraphs summarize results for each participant.

The TPACK survey responses and the philosophy statements shows that Ally struggled with what it meant to teach mathematics with technology at the beginning of the semester. She did not integrate technology in the two individual lessons she designed for the 8th grade algebra class and hence her overall TPACK scores for both lessons were zeros. However, at the end of the semester, both her TPACK survey responses and her teaching philosophy statements showed some improvement in term of what teaching mathematics with technology meant.

Doug’s overall TPACK scores were 0 and 1 for the first and second individual lessons respectively. Doug seemed to have a mature understanding of what it meant to teach mathematics with technology at the beginning of the semester. He, however, did not seem to think that technology has a lot of use in the right now and hence he did not use technology in his first lesson, he used technology in his second lesson because “it was a requirement of the methods class” but he didn’t think that “it was effective.”
possibly for this reason that his philosophy statement did not make reference to technology use in mathematics.

Heidi’s overall TPACK scores for her individual lessons were 0 and 0.5 for the first and second lesson respectively, showing a slight improvement in TPACK scores. Heidi, like Doug, had a mature understanding of what it meant to teach mathematics with technology. However, she was a firm believer of being able to do something by hand first before you use any technology. She did not use technology to teach in her first lesson; instead she had her students use calculators for their computations. Her second lesson mirrored her belief in the sense that she demonstrated how to multiply fractions using the area model by hand and then introduced the fraction applet for multiplying fractions using the area model.

Eddy made tremendous progress in terms of teaching mathematics with technology. His overall TPACK scores were 0 and 1.5 on the first and second individual lessons respectively. He had the most gain as compared to the other participants. Eddy’s response to the TPACK survey questions shows that he started with a simplistic view of the role of technology in mathematics teaching. Eddy believed that you should be able to learn the same things without technology and as a result during his practicum teaching he was the sole user of technology. He had his students graph linear equations on a graph paper.

Ethan performed better than the other participants on both the first and the second individual lessons. His overall TPACK scores were 3 and 4 for the first and second individual lessons respectively. His TPACK survey responses and his teaching
philosophy statement’s description of the role of technology did not match up with the level of understanding he displayed in his practicum classroom. His students were the main users of technology in his lessons.
CHAPTER 5

CONCLUSIONS

Introduction

The purpose of this chapter is to present conclusions based on the analysis of the data in Chapter 4, to hypothesize implications for the development of TPACK in mathematics pre-service teachers, to discuss limitations and delimitations of this study, and to provide recommendations for further study. Four overall research conclusions emerged from the results and these will be presented first to help illuminate conclusions on the research questions.

Overall Research Conclusions

A detailed analysis of the results presented in Chapter 4 demonstrates that the development of pre-service teachers’ mathematics TPACK is complex. This is because there are a number of factors that are at play such as the pre-service teachers’ prior experiences with technology, their mathematical backgrounds, and their beliefs about the use of technology in mathematics instruction. Assessing the development of TPACK in pre-service teachers is complicated by the fact that the available model for mathematics teachers’ TPACK was developed using observations of in-service mathematics teachers. Obviously, pre-service teachers are different from in-service teachers. Thus, using a lesson plan rubric that was based on the in-service teacher model required modifications as it was applied to pre-service teachers.
Despite the challenge of applying the TPACK lesson plan rubric to pre-service teachers, this study was able to extract four significant conclusions about the development of TPACK in pre-service teachers. The following are the four overall research conclusions from the results in presented in Chapter 4.

Development of TPACK in Pre-service Teachers vs In-service Teachers

The development of TPACK in pre-service teachers is not the same as the development of TPACK in in-service teachers. This conclusion emerged from attempts to apply the rubric developed by Lyublinskaya and Tournaki (2011) to analyze the pre-service teachers’ lesson design and implementation. There are sections of the rubric, such as the section on Knowledge of Curriculum and Curricular Materials that Integrate Technology in Learning and Teaching Subject Matter Topics, where certain statements only apply to in-service teachers. There are two possible explanations for this. First, the lesson plan rubric was designed to assess the development of TPACK in four in-service teachers who were teaching integrated algebra (Lyublinskaya & Tournaki, 2011). Second, the model of TPACK on which the TPACK lesson rubric is based was developed from observing in-service teachers. In the original model, Pedagogical Content Knowledge (PCK) is already in existence and is anticipated to intersect the knowledge of with technology (see Figure 15). In other words, the available model of TPACK development assumes that the teachers already have PCK, which is not the case for pre-service teachers.
The rubric had to be modified in order to assess pre-service teachers’ TPACK. However, a question arises from this. “Is it enough to simply modify indicators on a TPACK rubric so that they apply to pre-service teachers?” Perhaps instead, we should be thinking about a different model in terms of TPACK development for pre-service mathematics teachers. I view the development of TPACK in pre-service teachers as involving three knowledge components coming together to form TPACK as in Figure 16.
If Figure 16 depicts the nature of TPACK development in pre-service teachers, then maybe we need not describe it the same way that we describe the development of TPACK in in-service teachers. At the beginning of the semester, the five pre-service teachers who participated in the study exhibited different strengths in three knowledge domains in Figure 16. Ethan, Eddy and Heidi had a strong content knowledge. Only Ethan had a strong technological knowledge. Eddy and Heidi had a strong pedagogical knowledge. Ally did not show strength in any of the three knowledge domains. The five pre-service teachers’ TPACK developed differently and as a result it was conjectured that the differences in strengths in the three knowledge domains impacted how their TPACK developed. Maybe we should not have a one-size-fits-all model of TPACK development for pre-service mathematics teachers. I suggest having different models for different
knowledge strength combinations that is, unique models of TPACK development for pre-service teachers who are strong in TK, PK, CK, TPK, TCK, PCK, and TPCK.

**Enacted TPACK vs Self-Reported TPACK**

There is a mismatch between the enacted TPACK and the self-reported TPACK of the participants. For each participant, the self-reported TPACK scores were higher than enacted and observed TPACK behaviors. For instance, even halfway into the semester, Ally could not integrate technology in her lesson on “the slope of a line” her TPACK score for the Likert-type part of the TPACK survey was relatively high, a 3.8 at the beginning of the semester. This could be a result of the fact that the participants had little or no understanding of what it meant to teach mathematics with technology. Doug’s self-reported scores at the beginning and at the end of the semester were 4.4 and 4.2 respectively. This did not match up with his performance in two ways. First, in the only instance where Doug used technology on his own, he fit a linear regression line to a simple data set, a technique which is not even part of the 8th curriculum at the school he was in. This performance did not match Doug’s 4.4 TPACK score for the Likert-type part of the TPACK survey at the beginning of the semester. Second, Doug’s self-reported TPACK score declined from 4.4 to 4.2, yet there was evidence of improvement in terms of his enacted TPACK. These disconnections between pre-service teachers’ conceptions of their practice and the reality of that practice are not new (Fung and Chow 2002).

One explanation for the mismatch is to consider potential flaws and differences in the development of the assessment tools. The Likert part of the TPACK survey and the TPACK rubric were developed based on different views of the concept of TPACK. The
Likert part of the survey was developed by Schmidt et al. (2009) who simply view TPACK as the intersection of technology, pedagogy, and content as illustrated by the Venn diagram in Figure 17.

Figure 17: TPACK as an intersection of Technological, Pedagogical and Content Knowledge

This view of TPACK is not content specific, thus the Likert statements developed from it are very general. The statements in Figure 18 are typical of the Likert statements on the survey that the participants completed to self-report their TPACK.
Figure 18: Likert Statements on the TPACK Survey used by the Participants to Rate Themselves

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<thead>
<tr>
<th></th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I can teach lessons that appropriately combine mathematics, technologies and teaching approaches.</td>
</tr>
<tr>
<td>2</td>
<td>I can select technologies to use in my classroom that enhance what I teach, how I teach and what students learn.</td>
</tr>
<tr>
<td>3</td>
<td>I can use strategies that combine content, technologies and teaching approaches that I learned about in my coursework in my classroom.</td>
</tr>
<tr>
<td>4</td>
<td>I can provide leadership in helping others to coordinate the use of content, technologies and teaching approaches at my school and/or district.</td>
</tr>
<tr>
<td>5</td>
<td>I can choose technologies that enhance the content for a lesson.</td>
</tr>
</tbody>
</table>

Such general statements are subject to different interpretations. For instance, pre-service teachers may differently interpret the statement “I can teach lessons that appropriately combine mathematics, technologies and teaching approaches.” One may think of demonstrating how to multiply fractions using a fraction applet and then have students practice, as in the case of Heidi, or having students follow demonstrate demonstration of how to fit a linear regression, in the case of Doug, or presenting material with a PowerPoint, in the case of Ally. At least in the cases of Heidi and Doug, technology is used to do some mathematics; Ally’s use of technology only improved the efficiency and presentation of the work. Each of these three participants rated Statement 1 as “agree” (3), yet their classroom performances were very different. Could it be that the pre-service teachers needed clarifications about the meaning of each of these statements?

In contrast, the TPACK lesson rubric was based on the description of TPACK specific to mathematics suggested by Neiss (2005) and refined by Neiss, Sadri and Lee (2007). As a result the rubric is a matrix with levels of TPACK (Recognizing, Accepting, Adapting, Exploring and Advancing) matched against the four TPACK domains.
suggested by Neiss (2005). This rubric is specific to mathematics, whereas the Likert statements in the survey instrument can apply to any subject in addition to mathematics.

Also, self-reported TPACK scores may not be a good measure of one’s TPACK development since they may be subject to misinterpretation by the respondents, in this case pre-service teachers. What, then, is an appropriate use or interpretation of the self-reported TPACK scores? The results of this study suggest that self-reported TPACK scores, such as those from the survey, are useful for exposing the beliefs of a pre-service teacher but not adequate for analyzing a pre-service teacher’s true application of TPACK. You can use a self-reported survey to assess a teacher’s self-perception of their TPACK, but not to measure or compare how TPACK plays out in the classroom.

Potential Influences on the Demonstration of TPACK

Results of this study suggest that TPACK can be influenced by the grade level or type of curriculum (i.e., whether middle school or high school) and whether the lessons were developed by a group of teachers or by individual teachers.

Grade Level Two of the participants, Eddy and Ethan, were placed in high schools for their practicum field experience, whereas the other three (Ally, Doug and Heidi), were placed in middle schools. A comparison of their performance shows that Eddy and Ethan demonstrated higher levels of enacted TPACK overall than the other three participants. Ethan had the highest TPACK scores on both individual lessons whereas Eddy started off weak but made tremendous progress on his second individual lesson. Eddy’s performance could be attributed to his openness to learning. He
consistently asked for feedback on his lessons throughout the semester. Could it also be as a result of the fact that he was placed in a high school classroom?

The group lesson design and implementation also provide another lens to look at the influence of the grade level. The groups did not do well on their first group lesson design and implementation for an 8\textsuperscript{th} grade algebra class. It is possible that the low TPACK scores were a result of the fact that this was their first group lesson design. However, the performance improved tremendously on the second group lesson that was for a high school AP statistics class. It is possible that the improvement was a result of the lessons learned from the first group lesson design and implementation. If so, wouldn’t it have carried over to the individual lessons design and implementation?

Another angle to look at the influence of the grade level is to consider the case of Eddy and Ethan. Although Eddy and Ethan were both placed in high school, they were in different types of classes. Eddy was in a normal high school class, while Ethan was in an AP class. Ethan had higher TPACK scores than Eddy. It is possible that the classroom environment and the content being taught were factors in coming up with good lessons that integrate technology.

**Type of the Curriculum.** The above observations on the influence of the grade level warrant a closer look at the level and quality of the curriculum in a given classroom. The middle school curriculum in the schools in which the pre-service teachers were placed does not make large use of calculator or computer technology, whereas the high school curriculum does. This observation could have resulted in the differences in
TPACK scores between lessons designed for the middle school and those designed for high school classes.

*Group Lessons vs Individual Lessons.* The results of data analysis also suggest that assessing group lesson designs does not give an accurate assessment of the individual’s TPACK. With the exception of Ethan, the other four participants displayed higher levels of TPACK in groups than they did individually. The pre-service teachers teach their own classes, for the most part, when they graduate. It is therefore important that they become proficient users of technology as individuals than as groups.

Considering the participants’ performance on group lessons, is it possible that certain group members were carrying the groups? If so, who could have been carrying the groups? In Team A (Ally, Doug and Ethan) I suspect that Ethan was carrying the group, since he outperformed all the other participants on the individual lesson design and implementations. In Doug and Heidi’s group there were two other pre-service teachers who did not participate in the study. These were a graduate student and another senior student who had been outstanding in her work as a pre-service teacher. Although I cannot say for sure without assessing their lessons for TPACK, it is possible that these two carried their group. It is also possible that group interactions in general produce better lesson designs just because of the opportunities for more ideas to be expressed. The results of this study seems to suggest that TPACK develops best in contexts where there is such collaboration, just like mathematical knowledge.

Whether or not group lessons an accurate assessment of the pre-service teachers’ ability to teach with technology, they can be a good pedagogical tool. In the methods
class used for this study, group lessons were used to either develop or enhance the pre-service teachers’ lesson plan writing skills. Ally, Doug and Ethan had not taken a methods course before this one and as a result they did not have any experience in lesson plan writing. Writing two group lessons was meant to ensure that they develop lesson plan writing skills at least to the same levels as more experienced group members. By writing lessons to be implemented by an in-service teacher rather than themselves, the pre-service teachers were forced to write well-detailed lesson plans that could easily be followed by an outsider. This helped the pre-service teachers to learn to write detailed lesson plans even for themselves. Another subtle reason for having pre-service teachers engage in group lesson design and implementation was to introduce them to how they can collaborate with other teachers on designing and teaching a lesson.

**Conclusions on the Research Questions**

The preceding are overall conclusions that emanated from the results presented in Chapter 4. The following are conclusions on each research question.

**Conclusions on Research Question 1**

**Research Question 1:** How do pre-service teachers’ conceptions about the purposes for incorporating technology in teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?
The participants’ first group lesson design and implementation suggests that the participants’ initial conceptions were that technology is used for drill-and-practice. In both group lessons, the teacher solved problems by hand first and then introduced technology to either solve the same problems in the case of Team B or different problems in the case of Team A. This drill-and-practice approach could be a result of how pre-service teachers have seen technology being used by their own teachers, because all of them at least had been in a classroom where technology was used. It is also possible that they did not have good experience learning elementary mathematics of the first lesson plans with the help of technology. Furthermore, they only had a limited notion of how well the middle school students were able to use the TI-83 in contrast to the mathematics they presumably know based on the section of the book they were given to use in their lesson plans. This might have made them reluctant to use technology for anything essential.

In the second group lesson design and implementation the participants seemed to have progressed in their TPACK. Team A showed evidence of being at the adapting level. Team B’s lesson showed indicators of being between the adapting and exploring levels. It is possible that the in-class reflections which came after watching a video of the implementation of Team B’s 8th grade Algebra lesson impacted their second group lesson design. On the other hand, the second group lesson design was for a high school AP statistics class in which the students are more mature than the 8th graders. Could it be that the participants were more comfortable with having their AP students use technology than the 8th graders? Team B displayed a higher level of TPACK than Team A on the
second lesson design and implementation. This could be a result of the fact that their first
lesson plan was the one that was chosen for implementation by the 8th grade teacher.
They then revised the lesson at the request of the teacher so that she could teach it again.
In a way they had an extra lesson design experience that the first group did not have.

The participants’ performance declined on their first individual lessons from the
second group lesson performance, possibly pointing to individual weaknesses in terms of
technology integration. I can find evidence that that is the case for Ally, Doug and Eddy,
whose individual TPACK levels were judged low by both the lesson plan and the
supplementary data provided from the survey. However, for Heidi, it was possibly a
result of the conflict she had within herself about the use of technology in the classroom.
She was at the recognizing level for her first individual lesson, when I expected her to be
either at the adapting or exploring level of TPACK level judging from her survey
response. Also, instead of using technology to explore mathematics, Heidi’s use of the
fraction multiplication applet simply added efficiency to what she had demonstrated on
the board. At the end of the semester, only Ethan was able to fully explore with
technology.

Conclusions on Research Question 2

Research Question 2: How does pre-service teachers’ knowledge of students’
understandings, thinking, and learning of mathematics with technology emerge as they
experience a coordinated methods-mathematics-practicum curriculum designed to elicit
the specific components of TPACK?
The participants initially valued knowing how to use the technology, as evidenced by their initial survey responses. The responses supported the notion that the participants thought knowing how to use the technology was important for students to learn mathematics with technology. It was also supported by the way they approached technology use with their students in the first group lesson design and implementation. The teacher was to demonstrate how to use the technology first and then have students practice. The activity worksheets that Team B created had step-by-step instructions on how to use the TI-83 to accomplish the day’s activity. This could be because they were in an 8th grade classroom where they felt the need to give the students detailed instructions. This could also be because they felt less confident about what they should assume students already knew in the area of technology, as opposed to the mathematics of the lesson.

In the second group lesson design, there was a distinct shift in the way the pre-service teachers approached technology use by students. They did not give students step-by-step instructions on how to use the TI-83s. They assumed the students could use the calculators to accomplish the tasks. The fact that the second group lesson was taught to an AP statistics class raises the question of whether this shift was due to designing, teaching, reflecting on the first lesson, or due to the more advanced skills and knowledge of the AP student audience. This observation agrees with Zbiek (2005, p. 301) who reported pre-service teachers’ movement from seeing technology as “another things to teach” or “another thing to learn” to a tool to help students develop a deep understanding of concepts.
For the first individual lessons, three of the participants (Ally, Doug and Eddy) did not have their students use technology. As a result, it was not possible to assess how they have their students use technology. Heidi simply had her students use the calculators for computation rather than focusing on student understandings, thinking and learning mathematics with technology. Only Ethan gave his students an opportunity to work on a technology related activity. As with the second group lesson design, he did not have to give his students step-by-step instructions on how to use their TI-83s. Again, this was in an AP Calculus class; the situation may have played out differently in a middle school or lower level class.

For the second individual lesson, there was not much improvement in terms of thinking about students’ understandings, thinking and learning mathematics with technology. Ally totally missed the spirit of this aspect when she considered using a PowerPoint presentation as a way of teaching with technology. Doug gave his students step-by-step instructions as they followed along while he fit a simple linear regression. Heidi first did a demonstration making sure she gave the students detailed instructions, and then had the students take turns to practice. Eddy did not have his students use any technology but did all the demonstrations himself. However, Ethan had his students explore on their own without prior teacher modeling of the exploration.

Similarly, the post-survey responses showed little or no improvement as compared to the pre-survey responses in terms of thinking about the students’ understandings and thinking when learning mathematics with technology. The participants still valued knowing the operations as important when learning mathematics
with technology. Heidi seemed to contradict her initial response to the question on the pre-survey where she expressed the need for students to “understand why they are doing something with technology and also the fundamentals,” by noting on the post-survey that students “need to know the basics of using technology.” Doug’s post survey response, on the other hand, showed a better understanding of what students should know when learning mathematics with technology. However, this was in contrast to his second individual lesson in which he used technology where he simply had his students follow along as he demonstrated how to fit a simple linear regression. This is another example of a mismatch between the enacted TPACK and the self-reported TPACK as I reported earlier. One question arises. “Was the contrast between Doug’s belief and his use of technology possibly due to being in a middle school classroom?”

The cases of Heidi and Doug’s especially support the notion that pre-service teachers struggle with what they encounter in the classrooms during their field experiences. Heidi initially thought that students need to understand why they are doing something with technology but after spending more than 60 hours in a 6th grade classroom she backed down on that. This could be because of the few times she used technology the students had to learn how to use the technology. It could also be because that was how the cooperating teacher used technology.

It is likely that the participants’ limited experiences learning mathematics with technology and also teaching mathematics with technology contributed to their inability to think beyond using technology for technology’s sake.
Conclusions on Research Question 3

**Research Question 3**: How does pre-service teachers’ knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics emerge as they experience a coordinated methods-mathematics-practicum curriculum designed to elicit the specific components of TPACK?

The participants were initially not familiar with any specific curriculum but they knew about curricular materials that integrate technology. At the end of the semester, the participants still did not indicate awareness of any specific curriculum that integrate technology despite having completed a mini-project from the CME curriculum on difference tables as part of their modeling course assignment. The Center for Mathematics Education (CME) project is a National Science Foundation (NSF) funded high school program that is organized around Algebra 1, Algebra 2 and Precalculus. The main goal of the CME project is “to help students acquire a deeper understanding of mathematics” (EDC 2009, p. T3). It is guided by six principles among which is “using technology to sharpen, not to replace, thinking” (http://www2.edc.org/cme/principles.html). However, their teaching performance and their teaching philosophy statements revealed that they had a good sense of the mathematics they are going to teach as teachers.

Conclusions on Research Question 4

**Research Question 4**: How does pre-service teachers’ knowledge of instructional strategies and representations for teaching and learning mathematics with technologies
emerge as they experience a coordinated methods-mathematics-practicum curriculum
designed to elicit the specific components of TPACK?

The participants initially focused on how to use the technology rather than how to
explore mathematics with technology in their first group lesson design and
implementation. This showed a lack of awareness of the strategies and representations for
teaching mathematics with technology such as trivialization, experimentation,
visualization and concentration suggested by Kutzler (2003). Their second group lesson
design and implementation showed progress in terms of focusing more on the
mathematics than on the technology and also allowing for student reflection. For the first
individual lesson had the participants’ four out of five of the participants failed to
complete the assignment by not using technology to explore the mathematics. There
were improvements on the second individual lesson but only Ethan was able to get to
exploring with technology.

I hypothesize that the participants are still learning how to teach and are not very
confident integrating technology before they are comfortable with other aspects of
teaching. Both Heidi’s and Eddy’s statements during the exit interviews suggests that
because pre-service teachers are struggling to integrate technology into their teaching
they display the tendency to describe their teaching strategies and the role of technology
separately. Ethan demonstrated confidence in his teaching and this could be because he
was a teaching a college course at the time as a graduate teaching assistant and that could
have given him more experience and helped him gain confidence. If this is the case,
maybe researchers should think about ways of extending the TPACK development model
suggested by Neiss, Sadri and Lee (2007) so that it accommodates a strong foundation in PCK prior to expectations for growth in TPACK.

Implications for the Development of TPACK in Pre-Service Teachers

The overall development of TPACK is influenced by the four components that make up the mathematics teacher’s TPACK according to Neiss (2005):

1. Overarching conceptions about the purposes for incorporating technology in teaching mathematics;
2. Knowledge of students’ understandings, thinking, and learning of mathematics with technology;
3. Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics; and

Data from the case study suggest that it is very difficult to evaluate growth in TPACK between pre-service teachers because of several factors that were non-uniform before the study such as mathematical background and prior experience with technology; during the study such as grade level being taught and within the data such as the contrast between self-reported and enacted TPACK.

The pre-service teachers demonstrate differences in terms of their TPACK scores on each of these components. The pre-service teachers’ TPACK scores were not uniform for any lesson, except for Ethan who had uniform scores on his second individual lesson.
When assessing a teacher’s TPACK there is the need to consider the individual contribution of each of these TPACK components. This is especially important because the TPACK lesson rubric assigns a TPACK level based on the least score on all the TPACK components. For instance, for the second individual lesson design and implementation, Eddy’s scores for each TPACK components were 3, 1.5, 2.5, and 2 (scores for the TPACK components in the order in which the components are listed above). Although Eddy had a three on the Overarching Conceptions about the Purposes for Incorporating Technology in Teaching Mathematics TPACK domain his overall aggregate score was 1.5. Simply looking at this score might mask the fact that Eddy had a better idea of purposes for incorporating technology in teaching mathematics than he has for the “knowledge of students’ understandings, thinking, and learning of mathematics with technology” because his score was a 1.5. These differences reveal the need to any TPACK development program to focus on the different TPACK domains and not just the aggregate TPACK. This may help reveal any inconsistencies or TPACK domains that need to be focused on.

Moreover, these differences in performance also point toward the fact that the pre-service teachers are comfortable with the mathematics they teach but not with the teaching strategies that allow for effective use of technology. As Heidi and Eddy pointed out on their exit interviews, they need to be comfortable with the teaching itself, that is, teaching without technology before integrating technology. It may be beneficial to have the pre-service teachers teach in classrooms without technology first until they are comfortable with the setting and then have them introduce technology into their teaching.
Alternatively, pre-service teachers can have more extensive opportunities to teach, whatever those are. This suggestion is supported by the fact that the two participants who did not perform as well as others, Ally and Doug, had not taken the middle school methods course at this university which provides pre-service teachers with lots of teaching experience. As a result this was their first time learning about mathematics teaching. Although Ethan did not have another methods course either, he was teaching a college algebra course as part of his assistantship that likely helped him develop teaching technique.

Content Knowledge Influence on TPACK

Results of this study suggest that stronger content knowledge leads to stronger or faster development of TPACK. Ally struggled with the mathematics content in the modeling course more than the other study participants. She was not able to think creatively about integrating technology in teaching mathematics. She used a PowerPoint presentation to present her lesson on solving systems of equation using the substitution method. On the other hand, Ethan, who was more comfortable with the mathematics content, was able to explore with technology in his practicum classroom. Eddy was also comfortable with his mathematics although once in a while he would present some misconceptions. His performance at the end showed a lot of improvement. Heidi was a very good mathematics student and she had the right ideas about technology integration in the classroom. However, she had a conflict with her beliefs about technology use. Were it not for those beliefs, I believe she would have displayed a higher level of TPACK than she did.
Context’s Influence on TPACK

The differences in demonstration of TPACK between the participants who taught their individual lessons in the middle schools and those who were in the high schools suggest that there may be differences in terms of teaching with technology in a middle school and teaching with technology in a high school. There is need to approach TPACK development issues differently depending on the grade level and sophistication of both students and the mathematics content.

Technology Skills’ Influence on TPACK

The participant with the highest level of TPACK, Ethan, had a lot of prior experience with technology outside of the classroom, which may have made him more comfortable with trying out new things with technology. On the other hand Ally, whose only experience was using the Geogebra and GSP in a geometry course she took during the spring semester, did not successfully integrate technology into her classroom lesson. This suggests that pre-service teachers need more personal experience with technology in order for them to gain confidence in using technology in their classrooms. This experience might come from outside classroom experience as with Ethan, but more likely should be made available through integrating technology into more coursework during teacher preparation.

Use Appropriate Technology

The primary form of technology used to help the participants learn about teaching mathematics with technology (i.e. in the modeling class) was the TI-Nspire CAS CX.
This calculator and the associated software were not available for use in any middle school classrooms where the participants were placed and were not widely available in the two high school classrooms where the participants were placed. In other words, they learned mathematics with the TI-Nspire CAS CX but had to teach with a different type of technology. This required a more mature level in terms of integrating technology in mathematics instruction, which could affect the participants’ ability to teach with technology. Thus for helping pre-service teachers develop TPACK, a good starting point might be having the pre-service teachers learn mathematics with the technology that is already in the classroom.

**Measure TPACK in Multiple Ways**

When assessing the development of TPACK in pre-service teachers there is need to use multiple data sources to confirm observations. Any good measure of TPACK should not solely rely on any one instrument to assess TPACK. In particular, this study revealed a mismatch between the participants’ self-reported TPACK and enacted TPACK with a tendency among participants to rate themselves highly.

For this research study, although not the original intention, the lessons that the pre-service teachers taught became the main data sources. It was in the lesson designs and implementation that the pre-service teachers actively combined their knowledge of technology, pedagogy and mathematics to form a coherent body of knowledge that could be assessed by an external observer. Thus, any meaningful endeavor towards helping pre-service teachers develop TPACK should include opportunities for them to actively combine their knowledge of the TPACK domains. The methodology of this study
required pre-service teachers to design and teach lessons that integrate technology. Practically, not all lessons are most effectively taught with technology. Teachers need to be able to decide when technology is appropriate and when it isn’t. Thus, instead of specifying when the pre-service teachers should implement technology and alternative would be giving the pre-service teachers responsibility for determining whether the lessons they design are more effectively taught with technology.

Limitations of the Study

The intent of this study was to understand the development of TPACK in five pre-service teachers who were enrolled in three courses (methods, mathematics and practicum) that were offered in collaboration. This study not only adds to research literature on TPACK development in pre-service teachers, but also is valuable to teacher educators as they prepare learning opportunities for pre-service mathematics teachers to effectively use technology in mathematics instruction. There are several potential limitations of this study that should be noted.

First, my sample for this study consisted of five pre-service secondary mathematics teachers. Although, the teacher preparation program at this university is fairly representative of other teacher preparation programs in the region, the results of this study may not be generalizable to other teacher preparation programs or larger populations.

Second, the modeling instructor has extensive experience with using technology in mathematics and modeling, particularly regarding Texas Instruments (TI) calculators.
He is a strong advocate for technology integration in mathematics instruction. As a result there is a possibility of instructor bias in terms of what the participants were exposed to and were assigned to do.

Third, the researcher had no control over the practicum field placements. As a result the pre-service teachers were placed in classes regardless of whether the cooperating teachers used technology or not. The classroom experiences, including opportunities to teach and use technology, depended heavily on the beliefs, practices and the support of the cooperating teacher.

**Delimitations of the Study**

In this multiple case study, although the focus was on understanding the development of TPACK in pre-service teacher, data for this study involved snapshots of teaching examples. This therefore limited the depth of the case study. In addition, the data on individual pre-service teachers was largely limited to the beginning and end of the semester. Since the researcher was a course assistant in two of the courses (methods and modeling) and a practicum field supervisor, it is possible that the participants may have responded to TPACK survey and exit interview questions in a way that reflects what they perceived was expected by the researcher.

The rich environment of this study where three courses (methods, mathematics and practicum) are offered concurrently is likely not replicable in other settings without program restructuring.
Recommendations for Further Research

One of the conclusions from this research study is that the development of TPACK in pre-service teachers is not the same as the development of TPACK in in-service teachers. Research is needed to establish and characterize the differences between pre-service teachers and in-service teachers’ in terms of TPACK development. Is it simply a matter of the difference between adding Technology to Pedagogical Content Knowledge and having Technology, Pedagogy, and Content coming together? What assumptions should we make about the development of TPACK in pre-service teachers? This can be done through grounded theory studies on the development of TPACK in pre-service teachers.

Further research is needed to investigate the nature of the mismatch between enacted and self-reported TPACK uncovered in this study. Is this a consistent finding or merely a result of using two different instruments as in this study? Research might focus on developing a survey that aligns better with the TPACK lesson rubric used in this study, or modifying the lesson rubric to better match the existing survey.

Assigning groups to design a lesson is an effective pedagogical tool to use with pre-service teachers. However, group lessons may not give an accurate assessment of an individual’s TPACK. Is it possible that group lesson design tasks can be assigned that can enhance the ability of group lesson designs to provide an accurate measure of pre-service teachers’ TPACK? This is a question for further research. Possible studies would involve pre-service teachers who are involved in an alternating cycle of designing group and individual lessons. The level of TPACK would then be assessed at the end of each cycle.
using the TPACK survey, lesson plan rubric, teaching philosophy statements and interviews. This is meant to establish if any particular type of a group lesson design gives an accurate assessment of the individual TPACK development or level.

Another recommendation for further research is exploring whether a pre-service teacher’s placement in a particular grade band or type of course affects his or her TPACK development. This can be done in a number of ways. One way would be to randomly place a number of pre-service teachers in middle and high schools for a certain period varying from 6 weeks to a semester, assess their TPACK during that time and then switch them for an equal amount of time and then assess their TPACK again. Another way would be to have pre-service teachers randomly placed in either middle or high schools for the entire period without any switching to either middle or high school, and then assess their TPACK development.

**Summary of Conclusions**

TPACK development in pre-service teachers should not be viewed the same way as TPACK development in in-service teachers. For pre-service teachers, the three main components that make up TPACK (technology, pedagogy, content) exist as separate entities whereas in in-service teachers, pedagogy and content are assumed to be intertwined and only technology has to be added for them to have TPACK. Thus a new model of TPACK that applies to the pre-service teachers is needed. The ability to demonstrate TPACK by pre-service teachers can be influenced by the grade level, type of the curriculum and whether the lessons were designed in groups or by individual
teachers. The lessons that the pre-service teachers taught proved to be very valuable in terms of assessing their TPACK because they demonstrated what the pre-service teachers could not demonstrated what they said they can do. I believe that research focused on assessing the development of TPACK in teachers should allow participants teaching opportunities.
REFERENCES CITED


Hardy, M. D. (2003). "It should have been stressed in all education classes": Preparing pre-service teachers to teach with technology. Report: Searcy, AR. (Eric Document Reproduction Service No. ED 478379).


APPENDIX A

TPACK SURVEY
Introduction
Thank you for taking time to complete this survey. Please answer each question to the best of your knowledge. Your thoughtfulness and candid responses will be greatly appreciated. Your responses will be kept completely confidential. Please note that participation is voluntary, and you can choose to not answer any question that you do not want to answer, and you can stop at anytime. Your participation or non-participation will not affect your grade or class standing.

Name:

Technology is a broad concept that can mean a lot of different things. For the purpose of this survey, technology refers to digital tools we use such as computers, laptops, iPods, handhelds (calculators), interactive whiteboards, clickers, digital cameras, software programs, etc.

6. What does the phrase “teaching mathematics with technology” mean to you? Describe this in the context of a high school class.

7. (a) Describe and compare a good use of technology in teaching mathematic to a trivial or unproductive use of technology in teaching mathematics.

(b) Describe what it means to learn mathematics with technology.

8. If technology is used, what should students know in order to learn mathematics with technology?

9. Do you know of any curriculum and/or curricular materials that integrate technology in learning and teaching mathematics? If so, which ones?

10. In terms of technology integration in mathematics teaching and learning:
    a) What online resources are you aware of?
b) What textbook resources are you aware of?

11. Do you know of any instructional strategies and representations for teaching and learning mathematics with technology? If so which ones?

12. (a) What is the role of exploration and inquiry in the learning of mathematics?

(b) How can exploration and inquiry fit into the teaching of mathematics?

Please answer all of the questions and if you are uncertain of or neutral about your response you may always select "Neither Agree or Disagree"

<table>
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<th>Technological Knowledge</th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree or Disagree</th>
<th>Agree</th>
<th>Agree Strongly</th>
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<tr>
<td>13. I know how to solve my own technical problems.</td>
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<td>14. I can learn technology easily.</td>
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<td>15. I keep up with important new technologies</td>
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<td>16. I frequently play around the technology.</td>
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<td>17. I know about a lot of different technologies.</td>
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<td>18. I have the technical skills I need to use technology.</td>
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<td>19. I have had sufficient opportunities to work with different technologies.</td>
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Content Knowledge

| 20. I have sufficient knowledge about mathematics. | | | | | |
| 21. I can use a mathematical way of thinking. | | | | | |
| 22. I have various ways and strategies of developing my understanding of mathematics | | | | | |

Pedagogical Knowledge

<p>| 23. I know how to assess student performance in a classroom. | | | | | |
| 24. I can adapt my teaching based-upon what students currently understand or do not understand. | | | | | |
| 25. I can adapt my teaching style to different learners. | | | | | |
| 26. I can assess student learning in multiple ways. | | | | | |
| 27. I can use a wide range of teaching approaches in a classroom setting (collaborative learning, direct instruction, inquiry learning, problem/project based learning etc.). | | | | | |
| 28. I am familiar with common student understandings and misconceptions. | | | | | |
| 29. I know how to organize and maintain classroom | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th>25% or less</th>
<th>26%-50%</th>
<th>51%-75%</th>
<th>76%-100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.</td>
<td>I can assess student learning in multiple ways.</td>
<td></td>
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</tr>
<tr>
<td><strong>Pedagogical Content Knowledge</strong></td>
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<tr>
<td>31.</td>
<td>I know how to select effective teaching approaches to guide student thinking and learning in mathematics.</td>
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<tr>
<td><strong>Technological Content Knowledge</strong></td>
<td></td>
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</tr>
<tr>
<td>32.</td>
<td>I know about technologies that I can use for understanding and doing mathematics.</td>
<td></td>
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<td></td>
</tr>
<tr>
<td><strong>Technological Pedagogical Knowledge</strong></td>
<td></td>
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</tr>
<tr>
<td>33.</td>
<td>I can choose technologies that enhance the teaching approaches for a lesson.</td>
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</tr>
<tr>
<td>34.</td>
<td>I can choose technologies that enhance students' learning for a lesson.</td>
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<tr>
<td>35.</td>
<td>My teacher education program has caused me to think more deeply about how technology could influence the teaching approaches I use in my classroom.</td>
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<tr>
<td>36.</td>
<td>I am thinking critically about how to use technology in my classroom.</td>
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<tr>
<td>37.</td>
<td>I can adapt the use of the technologies that I am learning about to different teaching activities.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Technology Pedagogy and Content Knowledge</strong></td>
<td></td>
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</tr>
<tr>
<td>38.</td>
<td>I can teach lessons that appropriately combine mathematics, technologies and teaching approaches.</td>
<td></td>
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</tr>
<tr>
<td>39.</td>
<td>I can select technologies to use in my classroom that enhance what I teach, how I teach and what students learn.</td>
<td></td>
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<tr>
<td>40.</td>
<td>I can use strategies that combine content, technologies and teaching approaches that I learned about in my coursework in my classroom.</td>
<td></td>
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</tr>
<tr>
<td>41.</td>
<td>I can provide leadership in helping others to coordinate the use of content, technologies and teaching approaches at my school and/or district.</td>
<td></td>
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</tr>
<tr>
<td>42.</td>
<td>I can choose technologies that enhance the content for a lesson.</td>
<td></td>
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</tr>
<tr>
<td><strong>Models of TPACK (Faculty)</strong></td>
<td></td>
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</tr>
<tr>
<td>43.</td>
<td>My mathematics education professors appropriately model combining content, technologies and teaching approaches in their teaching.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44.</td>
<td>My educational foundation professors appropriately model combining content, technologies and teaching approaches in their teaching.</td>
<td></td>
<td></td>
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<tr>
<td>45.</td>
<td>In general, approximately what percentage of your teacher education professors have provided an effective model of combining content, technologies and teaching approaches in their teaching?</td>
<td></td>
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<tr>
<td>46.</td>
<td>In general, approximately what percentage of your professors outside of teacher education have provided an effective model of combining content, technologies and teaching approaches in their teaching?</td>
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</tbody>
</table>
Describe a specific episode where an MSU professor or instructor effectively demonstrated or modeled combining content, technologies and teaching approaches in a classroom lesson. Please include in your description what content was being taught, what technology was used, and what teaching approach(es) was implemented.

Describe a specific episode where you effectively demonstrated or modeled combining content, technologies and teaching approaches in a classroom lesson. Please include in your description what content you taught, what technology you used, and what teaching approach(es) you implemented. If you have not had the opportunity to teach a lesson, please indicate that you have not.
APPENDIX B

EXIT INTERVIEW
I have planned this interview to last no longer than 30 minutes. During this time, I am going to ask you some questions about your experience in the methods, modeling and practicum courses this semester. This is not intended to be a test of any kind, but to help me understand your experiences in these courses and how they prepared you to teach with technology. Nothing you say here will be revealed to your instructors in a way that you can be identified, and will have no affect on your grade. To facilitate my note-taking, I will audio tape our conversation today. For your information, only I will be privy to the tape and I will destroy it after I have transcribed it. Please speak clearly and verbalize any thoughts you have.

I have a calculator here to use for demonstration and/or reference as we talk about how it was used in the modeling course. Please answer the specific questions as completely as possible. At the end of the interview you will be given the opportunity to share any thoughts or opinions that you feel were not addressed by any of the questions. With your assistance, we are working to find ways to enhance pre-service teachers’ technology-related training. Thank you for your agreeing to participate.

Do you have any questions before we begin?

Background and prior experience with technology
Your modeling course introduced you to online applets and the TI Nspire CX CAS and curriculum that integrate technology.

1. What other types of technology have you used for previous math classes?
   
   o **Probe:** Were they required, optional, or did you get one on your own?
   o **Probe:** How were the technologies used by the teacher?
   o **Probe:** How did you use the technologies in class and out of class?

2. How was the TI Nspire CX CAS that you used this semester different from previous calculators you have used?
Probe: Are there things that you did with this calculator that you weren’t able to (or never tried) with other calculators?

3. Did you have any prior experience with online applets? If so which ones?

TPACK
Our main goal for this semester was to train you to teach mathematics with technology through offering three courses (methods, practicum, and modeling) in collaboration. The modeling class was supposed to offer you an opportunity to learn mathematics with technology, the methods class an opportunity to design technology-related lessons and the practicum an opportunity to practice teaching your lessons in a real classroom.

4. As a mathematics student, how did technology empower you or broaden your vision of what mathematics could be?
   • Probe: Can you give specific examples?

5. How was your experience in learning how to teach with technology?
   - Probe: Why was it hard/easy for you to learn how to teach with technology?
   - Probe: Describe, if any, the difficulties you had learning how to use teach with technology.
   - Probe: What could be done in future to best train pre-service teachers to teach with technology?

6. How did you use the TI Nspire CX CAS calculator or any other type of technology during class?

7. How did you use the TI Nspire CX CAS calculator or any other type of technology during your practicum?
   • Probe: What informed your classroom decision?

8. What is your view of the role of technology in mathematics teaching? Has this view changed over the course of the semester?

9. Of the three courses (methods, modeling, practicum) which one impacted you most on how to teach with technology? Which one impacted you the least?

10. What are your weaknesses in terms of teaching mathematics with technology?

11. What are your strengths?
12. Do you see yourself teaching mathematics with technology when you graduate from MSU? Why/Why not?

13. Do you feel adequately prepared to teach with technology?  
If not, what could we have done to help you?

Opinions and attitudes
14. What do you feel we should know from a pre-service teachers’ perspective about learning to teach with technology?

15. Do you have any thoughts to add?
APPENDIX C

CONSENT FORM
Montana State University
Informed Consent Form
Examing the Nature of TPACK Development in Pre-Service Secondary Mathematics Teachers

Rejoice Mudzimirii
2-232 Wilson Hall
(406) 580-3042

The study in which you will be participating seeks to understand the development of TPACK in pre-service secondary mathematics teachers who are enrolled concurrently in three courses (methods, modeling and practicum) that offer opportunities to explore technology, pedagogy, and content knowledge (TPACK).

If you agree to participate in this study you will complete a TPACK survey online and you will also be asked to participate in a 20-30 minute interview regarding your knowledge of the interaction between mathematics, technology and pedagogy. I will ask you questions about your use of technology in teaching mathematics. The interview will be audio taped. Rejoice Mudzimirii will transcribe the tapes verbatim. Only Rejoice Mudzimirii will have access to the tapes from the interviews. These tapes will be erased by May 7, 2012. I may also contact you at a future date to clarify questions or to provide insight into my interpretation of the data.

Your participation in this research is voluntary. You are free to stop participating in the research at any time, or to decline to answer any specific questions. You may ask me about the research procedures and I will answer your questions to the best of my ability. Your participation in this research study is confidential. Following our initial conversations, I will identify information about you using a code number. I am the only person with access to the key linking your name with this code number. Results from this study will be reported using pseudonyms. If I believe that information from this interview could result in you being uniquely identifiable, I will decline to disclose this information.

If you have any questions regarding this research project you can contact Rejoice Mudzimirii (406-580-3042) at Montana State University. Any additional questions about the rights of human subjects can be answered by the Chair of the MSU Human Subjects Committee, Mark Quinn, (406) 994-5721.

I agree to participate in an investigation the development of TPACK in pre-service secondary mathematics teachers. I understand the information given to me, and I have received answers to any questions I may have had about the research procedures. I understand and agree to the conditions of this study as described. I understand that my participation in this research is voluntary and that I may withdraw from this study at any time by notifying Rejoice Mudzimirii.

Participant’s Signature ___________________________ Date ___________________________

Researcher’s Signature ___________________________ Date ___________________________

APPROVED
MSU IRB
08/10/2011
Data approved: N/A
Expiration date

Page 1 of 1
APPENDIX D

TPACK LESSON PLAN AND TEACHING LEVELS RUBRIC
<table>
<thead>
<tr>
<th>Recognizing (1)</th>
<th>Accepting (2)</th>
<th>Adapting (5)</th>
<th>Exploring (4)</th>
<th>Advancing (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>An overarching conception about the purposes for incorporating technology in teaching subject matter topics.</strong></td>
<td></td>
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</tr>
<tr>
<td>☐ Technology is used for practice only, and all learning of new ideas done through the teacher mostly without technology</td>
<td>☐ Larger part of Technology use is for demonstrating, which include presenting new knowledge</td>
<td>☐ Teacher is using Technology in a way that is new and different from teaching without this technology (dynamic nature, linked representations) and used for learning new knowledge by students</td>
<td>☐ Larger part of Technology use is by students who explore and experiment with it for new knowledge and for practice</td>
<td>☐ Technology tasks provide students with deeper conceptual understanding of mathematics and its processes.</td>
</tr>
<tr>
<td>☐ Technology activities do not include inquiry tasks. Technology procedures concentrate on drills and practice only.</td>
<td>☐ Technology activities do not include inquiry tasks. Technology procedures concentrate on teacher demonstration and practice.</td>
<td>☐ Technology activities include inquiry tasks. Technology procedures concentrate on mathematical tasks with connections and on inquiry activities that use or develop connections.</td>
<td>☐ Technology activities include inquiry tasks. Technology procedures concentrate on mathematical tasks with connections and doing mathematics – and on inquiry activities that use or develop mathematical knowledge of high value: connections (especially between multiple representations) and strategic knowledge.</td>
<td></td>
</tr>
</tbody>
</table>

| **Knowledge of students’ understandings, thinking, and learning in subject matter topics with technology** | | | | |
| ☐ Technology is used primarily for student practice. | ☐ Technology is mostly used for teacher demonstration or teacher-led student-follow work with technology; it is rarely used for students’ independent explorations. | ☐ Teacher focuses on students’ thinking of mathematics while students are using technology on their own – both for learning new knowledge and review of prior knowledge | ☐ Technology teacher focuses on students’ mathematics conceptual understanding and serves as a guide of student learning with technology, not a director. | ☐ Teacher facilitates students’ high level thinking with technology (linked representations, reasoning and proofs) |
| ☐ A Technology document does not present any new material, and only provides space for applications and drills. | ☐ A Technology document mirrors the structure of the textbook presentation of mathematics without active explorations. | ☐ A Technology document provides an environment for students to do mathematics with teacher guidance. | ☐ A Technology document provides an environment for students to deliberately take mathematically meaningful actions on objects. Teacher guidance is necessary in order for students to see the mathematically meaningful consequences of those actions. | ☐ A Technology document provides an environment for students to deliberately take mathematically meaningful actions on objects and to immediately see the mathematically meaningful consequences of those actions. |

**Knowledge of**

☐ Teacher

☐ Teacher uses

☐ The

☐ Students are

☐ Teacher uses
<table>
<thead>
<tr>
<th>curriculum and curricular materials that integrate technology in learning and teaching subject matter topics</th>
<th>Technical condition</th>
<th>Knowledge of instructional strategies and representations for teaching and learning subject matter topics with technologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>does not use technology for learning mathematics.</td>
<td>□ Technology if used is not aligned with one or more curriculum goals.</td>
<td>□ Teacher focuses on how to use technology rather than how to explore math ideas, using teacher-directed lectures followed by student practice.</td>
</tr>
<tr>
<td>standard approach to the curriculum topics with Technology being used as add-on.</td>
<td>□ Technology is partially aligned with one or more curriculum goals. Teacher has difficulty in identifying topics in mathematics curriculum for including technology as tool.</td>
<td>□ The instructions are teacher-led. Teacher structures lesson plan with limited student explorations with technology.</td>
</tr>
<tr>
<td>Technology is used as a replacement for non-technology based tasks in a traditional curriculum approach</td>
<td>□ Technology is aligned with one or more curriculum goals. Teacher chooses topics from school mathematics curricula; however, technology use is not always appropriate for the chosen curriculum topics.</td>
<td>□ Technology document is not built around learning objects and does not promote student reflection.</td>
</tr>
<tr>
<td>given problem solving tasks with technology and are asked to expand math ideas on the basis of technology explorations</td>
<td>□ Technology is partially aligned with one or more curriculum goals.</td>
<td>□ Technology document is built around learning objects but does not promote student reflection – especially the posing of questions for sense-making.</td>
</tr>
<tr>
<td>technology in a fully constructive way, including tasks for development of higher level thinking and deepening understanding of mathematics concepts.</td>
<td>□ Technology is strongly aligned with curriculum goals. Teacher chooses essential topics of school mathematics curricula. Technology use is effective for the chosen curriculum topics.</td>
<td>□ Technology document is built around learning objects and must explicitly promote student reflection – especially the posing of questions for sense-making.</td>
</tr>
</tbody>
</table>

- Teacher uses deductive (teacher-directed) approach with more focus on mathematics discoveries with technology rather than inductive, traditional curriculum.

- Teacher uses various instructional strategies (deductive and inductive) and focuses on students thinking about mathematics. Teacher’s use of technology is beyond traditional approaches to curricular topics.

- Teacher focuses on students’ hands-on and experimentation of new mathematics ideas with technology, and focuses on conceptual development.

- Technology document is built around learning objects and must explicitly promote student reflection – especially the posing of questions for sense-making and reasoning, including explanation and justification.
APPENDIX E

METHODS COURSE SYLABUS
Materials:
Required:
Developing Essential Understanding of Functions: Grades 9-12 NCTM: 2010
Teach Like a Champion, Doug Lemov, Jossey Bass Teacher: 2010.

Recommended:
NCTM membership (www.nctm.org) (includes a discount on PSSM, online subscription to Mathematics Teacher, and limited free downloads of past journal articles.)
MCTM membership (www.montanamath.org)

Course description:
This course will examine curricular issues, learning theories, teaching strategies, instructional materials, and assessment procedures for teaching secondary school mathematics. The course is designed to enable you to become an effective professional decision maker in the context of standards-based teaching. In particular, given a set of standards, you will learn and practice strategies to:
- Understand what students are to learn and the contextual factors that influence their learning
- Create an assessment plan
- Select teaching strategies
- Implement the strategies
- Implement the assessment plan
- Assess the effectiveness of your teaching.

The course will look at various aspects of a professional mathematics educator’s work, including the study of:
- Teaching strategies. You will observe, discuss, analyze, and practice various teaching skills and strategies which are consistent with the goals for 9–12 mathematics education, as defined by state and national organizations.

- Mathematical access. You will read selections from mathematics and mathematics education sources and formulate a plan for ensuring mathematical access among all students. This includes examination of issues related to social justice, gender and multiculturalism and the teaching of mathematics.

- Tools for teaching and learning. You will read about and practice the use of various resources for enhancing the teaching of mathematics, including the appropriate use of manipulatives, calculators, and computers. This course has a particular focus on the integration of technology, pedagogy, and mathematical content knowledge.
Research skills. You will conduct searches for sources of classroom activities and lessons that make use of the tools mentioned above, and for articles and research results that have direct impact on your effectiveness in the mathematics classroom.

The profession of teaching. You will read selections relating to professionalism, including the nature and role of professional organizations.

Course Components:
Written reports (and discussions) & lesson plans (and presentations) (60%). Each class period will involve discussions of issues in mathematics education. The catalysts for these discussions will be articles you have read, ideas you have researched, or observations you have made in your field experience. Each pre-service teacher will prepare complete lesson plans and will present small lesson excerpts to the class. Some discussions will take place on the D2L web site.

Field experience/reading response journal (10%). You will make regular entries in a field experience journal, recording observations and reflections about the classrooms you observe, the schools you work in, and your own teaching. These journal entries may be made through the D2L course site. These journal entries will overlap with your practicum journals (EDSD 301/EDSD 461 section 2-8). They should be well thought out reflections, displaying analysis and insight.

Teacher candidate work sample (30%). You will create a teacher candidate work sample that demonstrates your capacity to fulfill the six goals of the course listed above. Your final exam will include creating and participating in a peer-interview regarding the contents of your work sample and the written work sample itself. The work sample must show evidence of your understanding of important mathematics for school students and your understanding of appropriate pedagogy and technological pedagogy to create environments conducive to students' learning of important mathematics. The sample will also include a statement of your teaching philosophy and other documents providing evidence of your development as a teaching professional. This portfolio will become part of the evidence you use to demonstrate proficiency with the teacher program assessment framework (http://www.montana.edu/ehhd/educ/curriculum/tepp/docs/ConceptualFramework.pdf)

D2L Expectations: The use of the EDSD 461 D2L course page will be a regular part of this course. The website is accessible from my home page, http://www.math.montana.edu/~burrough, or from the D2L page, https://ecat.montana.edu. You will access articles from the course D2L page. It is your responsibility to ensure that you obtain these articles and read them by the posted dates.
Additionally, during the month of October you will post your Field Experience Journal (FEJ) entries using the D2L course page. You are responsible for posting two entries each week during the month of October.

Then, you must read your classmates’ postings and respond to at least two per week with further analysis, insight, or questions. Then, you must respond to at least two of these postings (either posted to you or posted to someone else).

Your total D2L requirement for the month of October is
1. Eight FEJ postings, posted regularly at a rate of 2 per week (counts towards your 10% FEJ course component)
2. Eight further analyses, insights, or questions to the FEJ postings, posted regularly at a rate of 2 per week (counts towards your 60% written reports and discussions component)
3. Eight responses to analyses, insights, or questions (counts towards your 60% written reports and discussions component)

Grading. I expect that you will approach each assignment seriously, investing the necessary time and energy to prepare your responses. Most questions will be scored using the following four-point scale. Each assignment will receive a score that reflects the average points-per-question, or in the case of written essays, points-per-category.

<table>
<thead>
<tr>
<th>Score</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Excellent. Outstanding work that exhibits comprehensive and thoughtful understanding of the content of the assignment. Grammatical, mathematical, pedagogical, or technological errors do not occur in the assignment.</td>
</tr>
<tr>
<td>3</td>
<td>Good. Work reflects a solid understanding of the content of the assignment. Work may not exhibit original perspectives. Several errors may occur in the assignment.</td>
</tr>
<tr>
<td>R</td>
<td>Redo. Work satisfies the requirements of the assignment while missing the spirit of the assignment. May be an incomplete rendering of the assignment. The work may contain inconsistencies or demonstrate a lack of thought. May be incoherent or poorly written. Assignment must be redone and turned in <strong>within one week. You must include a critical analysis/reflection of what was missing in the first attempt.</strong> Work successfully redone will receive a grade of 3.</td>
</tr>
<tr>
<td>1</td>
<td>Poor. The work shows a lack of understanding of the assignment.</td>
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<tr>
<td>0</td>
<td>No credit. The problem was not seriously attempted.</td>
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</tbody>
</table>

Letter grades will be assigned according to the following minimums: 3.75 (A), 3.0 (B), 2.5 (C), below 2.5 (F). Problems marked with an "R” **must** be redone and turned in within one week. Staple the revised solution to the top of the original assignment. Label
the revision with your name, the assignment number, and if applicable, the page and problem number. **You must include a reflection/analysis of why the original solution was lacking and how you know the revised solution is correct.** Problems marked "R" that are not redone in the expected timeframe will be scored "0."

Late work is not accepted, except in the case of illness. Late work will be scored a "0."

If you have more than three "0" assignments due to turning assignments in late, unrevised "Rs," class absences resulting in "0" participation grade, or missed assignments, the highest grade you will earn for the class is a C.

**In the event of illness,** please do not come to class, do not go to campus to turn in work, and do not go to your field experience practicum. You should email me if you will miss class due to illness as soon as practical. If necessary, we will use D2L to conduct class business at a distance.

**Daily Timetable - subject to change.** You must check your MSU email account daily and the D2L course page at least twice a week to stay informed of syllabus changes.

<table>
<thead>
<tr>
<th>Monday</th>
<th>Topics</th>
<th>Wednesday</th>
<th>Topics</th>
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</thead>
<tbody>
<tr>
<td>August 29</td>
<td><strong>Mathematics as a Process</strong></td>
<td>August 31</td>
<td><strong>Finding Balance in Mathematics Education</strong></td>
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<tr>
<td></td>
<td>PSSM Chapters 1 &amp; 2 (nctm.or)</td>
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<td>The Algebra Solution to Mathematics Reform, Chapter 1 -3</td>
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<td></td>
<td>CCSS Mathematical Practices</td>
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<td></td>
<td>Hands-on Equations</td>
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<tr>
<td>Labor Day</td>
<td>No Class</td>
<td>September 7</td>
<td><strong>Technology, Pedagogy, and Mathematical Content Knowledge</strong></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>The Algebra Solution to Mathematics Reform, Chapter 4 - 7</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>CCSS High School Standards</td>
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<tr>
<td>September 12</td>
<td><strong>Big Ideas in the Courses We Teach</strong></td>
<td>September 14</td>
<td>Independent work on group lesson</td>
</tr>
<tr>
<td></td>
<td>Small &amp; Lin Chapter 1, page 210</td>
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<tr>
<td></td>
<td>CCSS Group Lesson Design in Algebra</td>
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</tbody>
</table>

**Unit 2: Create Assessment**

<p>| September  | Essential Understanding Chapter 1          | September  | Essential Understanding Chap |</p>
<table>
<thead>
<tr>
<th>Date</th>
<th>Chapter/Section</th>
<th>Date</th>
<th>Chapter/Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 26</td>
<td>Planning for Instruction</td>
<td>September 28</td>
<td>The Role of Assessment</td>
</tr>
<tr>
<td></td>
<td>Essential Understanding Chapter 3</td>
<td></td>
<td>Small &amp; Lin Chapter 2</td>
</tr>
<tr>
<td></td>
<td>Group Lesson Debrief</td>
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<td></td>
</tr>
<tr>
<td>October 3</td>
<td>Appropriate Teaching Strategies in Lesson Planning</td>
<td>October 5</td>
<td>Appropriate Teaching Strategies in Lesson Planning</td>
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<tr>
<td>October 10</td>
<td>Unit Planning</td>
<td>October 12</td>
<td>Addressing Student Misconceptions</td>
</tr>
<tr>
<td>October 17</td>
<td>Differentiated Instruction</td>
<td>October 19</td>
<td>Differentiated Instruction</td>
</tr>
<tr>
<td></td>
<td>Small &amp; Lin Chapters 3-4</td>
<td></td>
<td>Small &amp; Lin Chapters 5-6.</td>
</tr>
<tr>
<td></td>
<td>Group Lesson Debrief</td>
<td></td>
<td>Lesson plan for practicum</td>
</tr>
<tr>
<td>October 24</td>
<td>Differentiated Instruction</td>
<td>October 26</td>
<td>Differentiated Instruction</td>
</tr>
<tr>
<td></td>
<td>Small &amp; Lin Chapters 3-4</td>
<td></td>
<td>Small &amp; Lin Chapters 5-6.</td>
</tr>
<tr>
<td>October 31</td>
<td>Microteaching</td>
<td>November 2</td>
<td>Microteaching</td>
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<tr>
<td>November 14</td>
<td>Teaching for Social Justice</td>
<td>November 16</td>
<td>Principles of Assessment Practices</td>
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<td></td>
<td>Required: Schedule Field Experience observation between October 31 and November 11.</td>
<td></td>
<td>Lemov Chapter 1, 3</td>
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<tr>
<td>November 21</td>
<td>Lemov Chapter 4</td>
<td>Thanksgiving break</td>
<td>No Class</td>
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<tr>
<td>November 15</td>
<td>Micro Teaching</td>
<td>November 17</td>
<td>Micro Teaching</td>
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<tr>
<td>Date</td>
<td>Activity</td>
<td>Date</td>
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<td>November 28</td>
<td><strong>Revisit NCTM Principles</strong></td>
<td>November 30</td>
<td><strong>Teacher Candidate Work Sample</strong></td>
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<tr>
<td></td>
<td>Required: Schedule Field Experience observation between November 28 and December 7.</td>
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<tr>
<td>December 5</td>
<td><strong>Putting it All Together</strong></td>
<td>December 7</td>
<td><strong>Teacher Candidate Work Sample</strong></td>
</tr>
</tbody>
</table>

Thursday, December 15: Final work sample due by 5pm
APPENDIX F

MODELING COURSE SYLLABUS
Math 428 Mathematical Modeling for Teachers Fall, 2011

Objectives:
(1) To familiarize you with technological tools for mathematical modeling in the classroom; (2) to help you develop a better understanding of some standard mathematical models; (3) to help you develop your own mathematical modeling habits of mind, (4) to provide you with modeling activities, technologies, and materials that you can use in the secondary classroom; and (5) to connect your mathematical, technological, and didactical perspectives as you enter the teaching profession. This is a senior “Capstone” course. Therefore, expect to review and to use much of the mathematics you ever learned. Expect to reflect, philosophically speaking, on the enterprises of doing mathematics, teaching mathematics, and learning mathematics. The course will involve you in problem solving, communicating convincing mathematical arguments to support models you develop, connecting mathematics to numerous real world domains, representing mathematical and physical phenomena in various ways, and reasoning (algebraic, probabilistic, geometric, analytic, and statistical).

Schedule:
Unit 2: Sequences, Functions, Difference Equations and Curve Fitting Oct. 4 – Nov. 10.
Unit 3: Measurement, Data Gathering, Motion Simulation, Dynamic Systems Nov. 15 – Dec. 15.

Exams:
There will be three short end-of-unit exams. There is no “Cumulative Final Exam” but class will meet at the scheduled final time, if necessary, to complete the short test over Unit 3. That time is 2:10-3:30 pm, Monday, December 12. The third test is brief and only counts for 20% of the test totals. It will include a performance component on the TI Nspire CX with Vernier Data Collection Devices.

Projects:
For each unit you will complete a mathematical modeling project which includes developing an extended lesson involving mathematical modeling in the context of Grades 9-12 mathematics. The third project will focus on textbook analysis and teaching trajectories. Projects will be discussed in online “Project Conferences.”

Papers:
You will write two reflection papers based on readings and Daily Journal Entries. These will be discussed in online “Paper Conferences.”
ERMO Summaries:
There will be three papers that you will read, carefully summarize, and then present an opinion on, in writing. These will be discussed in groups.

Other Readings:
There will be five readings that will be discussed but not glorified by ERMO summaries.

Journal:
You will make a journal entry for each lesson in the course. These are short, “what did I learn today” reflections posted to your personal folder on D2L. These form the backbone of Paper 2 in the course.

D2L Participation:
There will be regular online discussions and paper/project conferences that are required. All assignments and your homework will be posted in the online platform. I will post my daily reflections as news bulletins after each class. Syllabus and readings will be posted there.

Homework:
All homework, unless otherwise stated or permitted, is to be submitted via the dropbox in D2L.

In-Class Participation:
You are expected to attend and participate in all the class sessions. To prepare for a session you need to read the posting for that day in the discussion section of the D2L forum for the course. All reflection pieces will be posted on D2L. Presentations are routine and expected.

Grading:
Unit Exams 45%, Papers and Projects 25% Ermo Summaries 10% Homework Problems 20%

Please note: University policy states that, unless otherwise specified, students may not collaborate on graded material. Any exceptions to this policy will be stated explicitly for individual assignments. If you have any questions about the limits of collaboration, you are expected to ask for clarification. You may
### Daily Schedule

<table>
<thead>
<tr>
<th>Aug. 30</th>
<th>Sept. 1</th>
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<tbody>
<tr>
<td>Probability – Pesky Bets Brenda</td>
<td>Probability – Square Root Lotto</td>
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<tr>
<td><strong>S6</strong></td>
<td><strong>S8</strong></td>
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<tr>
<td>TI Nspire CAS CX Algebra</td>
<td>Word Problems in Algebra</td>
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<tr>
<td><strong>S13</strong></td>
<td><strong>S15</strong></td>
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<td>Stat Simulations: Cell Phones</td>
<td>Confidence and Error</td>
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<tr>
<td><strong>S20</strong></td>
<td><strong>S22</strong></td>
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<tr>
<td>Creating Random Populations</td>
<td>Hypothesis Testing of Means</td>
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<tr>
<td><strong>S27</strong></td>
<td><strong>S29</strong></td>
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<tr>
<td>Expected Value Simulations</td>
<td><strong>Exam 1 (36%)</strong></td>
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- **Oct. 4 STAT PROJECT DUE**
- Sequences as Models

<table>
<thead>
<tr>
<th>O6</th>
<th>O13</th>
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<td>Data and Infinity of models</td>
<td>Sums and Fundamental Theorem of Diff Sequences</td>
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- **O11**
- Difference Sequences
- **O18**
- Newton’s law of Differences and Pascal

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<tr>
<th>O20</th>
<th>O25</th>
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| Write a draft of Paper 1 In class. | Difference Equations

- **O27** PAPER 1 DUE
- Applications of Difference Eqs.

- **N3**
- Unit Analysis and Curve Fitting

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<th>N8</th>
<th>N10</th>
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<tr>
<td>Curve Fitting with Difference Eq. and Temperature probe</td>
<td><strong>Exam 2 (44%)</strong></td>
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- **N15 FUNCTION PROJECT DUE**
- Measurement and Data

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<tr>
<th>N17</th>
<th>D6</th>
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| Cord of Wood | Visual Data: Picture Analysis

- **D8**
- Write a draft of Paper 2 In class.

- **D15 PAPER 2 DUE**

### Notes:
- **D6** Visual Data: Picture Analysis
- **D8** Write a draft of Paper 2 In class.
- **D15** PAPER 2 DUE
- **D12 NOTE: THIS IS A MONDAY! Exam 3(20%)**
- **FINALS WEEK**
APPENDIX G

PRACTICUM COURSE SYLLABUS
EDU 395 - Teaching Practicum

Course Description:
This course has two components: the on-campus weekly seminar and the field experience. Each component will complement the other. The on-campus seminar meets every week at the scheduled time. Practicum students will complete their placements in a K-8 classroom, 5-12 classroom, or after school program in which they will be observed teaching. All practicum students will complete a mini-Teacher Work Sample (TWS) in addition to other assignments. While methods courses treat issues relevant to the teaching and learning context of specific curricular domains at different developmental levels, this course will treat issues and topics relevant to all prospective professional educators.

This course is designed to provide an introduction to the professional practice environment of public school and to prepare practicum students for the student teaching experience. Topics will include classroom management, differentiation, engaging parents and community members, co-teaching, and applying data-driven decision-making to inform the instructional process.

Professional Expectations:
As a practicum student in the MSU Teacher Education Program, you are expected to hold yourself to the high behavioral and ethical standards of the teaching profession. Please review the Professional Expectations for Prospective Teachers on the Department of Education’s website (linked on the D2L site). If you present unprofessional behavior to the instructor, your Field Supervisor, or your Cooperating Teacher, the instructor will complete a Student Concern Form to document your behavior. This form will be reviewed by the faculty and placed in your permanent file. Egregious unprofessionalism will result in removal from the Teacher Education Program.

The expectations to behave professionally are especially critical in your field experience. Remember that you are a guest in the classroom or after school program and are there to learn from a veteran teacher. Act professionally and respectfully at all times.

The seminar aspect of the course stresses learning about yourself in new ways. You are expected to take risks with your learning, to stretch your understanding, and to experience some discomfort regarding your experiences and understanding. Throughout the seminar sessions, you are expected to exhibit professional behavior. Professionalism demands open-mindedness; personal accountability; intellectual curiosity; engagement in the learning process; and respect for the beliefs, voices, experiences, and feelings of others. You will be asked to assess your own professionalism, growth, and learning at various times throughout the course.

Expectations of Course Instructor and Field Supervisors
The course instructors design the course, teach the on-campus seminar, assess student work, and supervise the Field Supervisors (FSs). We are committed to creating a valuable
and transformative experience at this critical stage in professional development. If you have a concern about any component of this course or questions about your transition from college student to professional educator, please feel welcome to contact me.

FSs will be the first point of contact for you and for Cooperating Teachers/After School Program Coordinator. FSs will observe you in the field and provide structured feedback on observed lessons. In addition, FSs may facilitate small-group discussions during the on-campus seminar and assist with assessment review.

Attendance & Communication
This course requires active participation—physical and mental attendance—from all class members. Each person has a valuable perspective and set of experiences related to education in K-12 classrooms throughout the United States. Furthermore, your experiences as connected to your field placement, contribute to our collective learning. Please treat this course as if it were a professional teaching job.

- Come to every class on time, prepared and ready to participate. **You are expected to attend all seminars.** These sessions provide a foundation for successful completion of course requirements.
- Deadlines for assignments must be honored, unless arrangements are made in advance and in writing via email.
- In order to create a classroom environment conducive to learning, we need to keep our classroom free of distractions. **Do not use your cell phones or other communication devices during class.** In the event of an emergency that requires you to use your phone, please leave the classroom.
- It is your responsibility to **check your email account(s) and the D2L announcement page regularly** for course updates, questions about your work, etc. You are expected to respond to messages from classmates and/or instructors in a timely manner (no more than 2 days).
- Any changes to your field experience schedule, including absences, must be approved by your FS. You will not receive a grade for this course unless you complete the requirements of your field experience.

Use of Desire2learn (D2L)
D2L is our online learning environment. It is also an essential link in our communication system for this course. You will be expected to log into your D2L account regularly and manage your engagement with all aspects of this course accordingly. It is your responsibility to ensure that you can access D2L and navigate the site. You are reminded that communication in this online environment is still expected to be professional in nature.

Readings & Resources
Readings & other resources will be posted on D2L. Please make sure you have familiarized yourself with the week’s resources prior to the start of each seminar session.

**Guidelines for Written Assignments:**
Teachers are considered to be the academic leaders of their communities. Colleagues, administrators, community members, and students closely scrutinize every document teachers produce—from reports that are sent home to flyers hanging on the school walls. As such, the same standard is applied in this course. Unless otherwise noted, written assignments must:

- be typewritten, double spaced, and submitted on or before the due date.
- include your name, course #, assignment, and date in the upper left hand corner of the first page (no cover sheet should be attached).
- be spell-checked, edited, & proofread for errors in punctuation, grammar, etc.
- present ideas coherently & with evidence from resources, examples, &/or experiences (not primarily your preferences or opinions).
- exhibit your understanding of the assigned resources, course topics, and discussions.

**Guidelines for Electronic Submissions:**
You will submit several of the assignments for this course using the D2L online dropbox. Please let me know if you are unsure how to use the online platform or if you have trouble accessing the site, reviewing assignments, or uploading documents. Unless otherwise noted, electronically submitted assignments must:

- adhere to the above “guidelines for written assignments.”
- be submitted using the D2L dropbox, unless other arrangements have been made.
- use Microsoft Word (make sure your files have either a .doc or .docx extension).
- be submitted by 11:59 p.m. on the assignment due date.

**Grading Information**
Seminar Activities & Attendance: 80 (5 points per session)
Reflection & Field Notes (D2L Discussion): 100 (10 points per week/6 points for original response & 4 points for two responses)
Peer Observation of Lesson: 20 (5 points for each section)
Field Experience: 100 (50 points each)
Lesson Plans & Enrichment Activities: 100 (10 points per week)
Mini-TWS elements: 100 Total
   - Section 1 – 20 points
   - Section 2 – 20 points

**Signature Assignments**
   - Section 3 – 30 points
   - Section 4 – 30 points
Final: 100 (graphic organizer of articles 10% of final)

**Total Points Possible: 600**
Seminar Activities & Attendance (80 points/5 points per session): The on-campus seminar depends on your preparation and engagement. Each week, you will complete various readings, which can be found on D2L, prior to class. You must also participate in whole group and small group discussions during each seminar.

Reflection & Field Notes (100 points/10 points per week starting Week 3): Each day you spend in the practicum setting, you should take notes about your observations and professionally reflect on successful or unsuccessful strategies used throughout the day and week. This reflection will be posted by Thursday at 11:59 pm in your D2L discussion group once a week during your placement (6 points), and you will be responsible or responding to two other posts by Sunday at 11:59 pm in that group (Total 4 points/2 points for each response).

Original post must include:
- A paragraph (at least 7 sentences) describing what you observed during the week and/or your experience interacting with students.
- Include one thoughtful question related to your observations and/or experience to facilitate conversation within the discussion thread.

Responses to posts must include:
- Include a direct quote from the post to which you are responding
- At least a 3 sentence response to your peer in addition to the direct quote

Peer Observation of Lesson (20 points/5 points per section): You will be required to arrange a time with a site-peer to be observed and evaluated based on the Performance Assessment. Your site-peer must observe your lesson for at least 15 minutes and provide feedback through the Performance Assessment tool and follow-up conference. Using the information and feedback from the discussion and evaluation by your site-peer, compose a one-page review. The review must include:
- Description of your lesson and assessment (5 pts)
- How you feel the lesson went (5 pts)
- Overview of feedback from your site-peer (5 pts)
- How you would improve the lesson and instruction (5 pts)

**Field Experience (100 points/50 points each observation):** You will be evaluated by your Field Supervisor and Cooperating Teacher according to each of the following areas: 1. Content, 2. Diversity, 3. Pedagogy, 4. Assessment, and 5. Professionalism. (Please see the TEP’s Conceptual Framework on the Department’s website).

**Lesson Plans (100 points/10 points each):** You will develop one lesson plan per week that is relevant to your placement. These lessons should reflect the content observed in your placement and can comprise an instructional sequence (i.e. two lessons in the same subject intended to be taught on consecutive days). Lessons should be 1) clearly aligned with state/national standards, 2) student learning outcomes, 3) pre-, formative, and summative assessments, 4) step-by-step lesson procedures & list of materials, and 5) differentiation based on interests modality, and/or readiness. You will share your lesson plan with your peers, Cooperating Teacher or Program Coordinator, and Field Supervisor as part of the development and revision process.

**Mini-TWS (100 points)**
The Teacher Work Sample (TWS) is a key component of your student teaching semester and the capstone project for the Teacher Education Program. For this course, you will complete a Mini-TWS to prepare you for the work of the TWS. The Mini-TWS consists of scaled-down versions of the four sections of the TWS (info. available at: [http://www.montana.edu/fieldplacement](http://www.montana.edu/fieldplacement)).

**Section 1: Contextual Background & Learner Characteristics (20 points)**
Look at the community, school, and learning environment of your placement.
- Describe the community, school, and learning environment and how they affect learner achievement and your instruction.
- Create a “Student Characteristics Table.” This table will include a list of learners using a coding system, gender, age, and one indicator that would assist in differentiating instruction.

**Section 2: Standards & Assessment (20 points)**
The instructional sequence must be aligned with:
- Content Standards (limit 2)
- Unit Learning Outcomes (limit 2)
- Unit Assessment (limit 1)

Focus on two sequential lessons:
- Lesson Objectives (*include Bloom’s Taxonomy verbs*) (limit 2)
- Lesson Assessments (1 pre, 1 formative, and 1 summative assessment)

**Section 3: Instructional Planning and Student Achievement**
(Signature Assignment – 30 points)
Provide a rationale of why you chose to deliver each lesson in a certain order.

Show learner growth in a particular lesson or sequence of lessons through a graphic representation or written description. What can you learn as a pre-service teacher by looking at student achievement?

Section 4: Self Assessment and Reflection
(Signature Assignment – 30 points)
Reflect on:
- Unit and/or lessons
- Changes to improve future instruction of the lessons
- Personal growth as an educator

Course Topics & Due Dates
Week 1 (August 31st): Introduction to Practicum:
To begin to apply theory to practice
21st Century Skills
In-school/After school options
Due Sunday, September 11, in the Dropbox by 11:59pm:
- Letter of Intro & Resume – 1 page each
- Read through the Teacher Work Sample

Week 2 (September 7th): Lesson Plans, Classroom Management, Formal and Informal Settings in Learning Environments
Collaboration

Week 3 (September 14th) Professional Assessment & Reflection
Marilyn King, Assistant Superintendent of Curriculum of Instruction
Megan Brenna, United Way
Due Sunday, September 18th, in the Dropbox by 11:59pm:
- Lesson Plan 1

***Week of September 19th - December 2nd: Out in schools ***

Week 4 (September 21st): Intro to The Teacher Work Sample
Designing a Unit/Instructional Sequence
(standard lesson & enhancement)
Section I: Contextual Factors
Section II: Standards
Due Sunday, September 25th, in the Dropbox by 11:59pm:
- Lesson Plan 2

Week 5 (September 28th): Co-Teaching Model
Video Procedures
Permission Form for Videotaping
Due Sunday, October 2nd, in the Dropbox by 11:59pm:
• Lesson Plan 3

Week 6 (October 5th): Working with Your Supervisor
Due Sunday, October 9, in the Dropbox by 11:59pm:
• Peer Observation of Lesson Plan (First Observation Reflection of Video Recording & Conference with Site Peer)
• Sections I & II
• Lesson Plan 4

Week 7 (October 12th): Differentiation
Integrating Technology
IEFA
Differentiating Instruction
Due Sunday, October 16th, in the Dropbox by 11:59pm:
• Lesson Plan 5

Week 8 (October 19th): Assessment/Formative & Summative
Section III of TWS Observation Techniques
Due Sunday, October 23rd, in the Dropbox by 11:59pm:
• Lesson Plan 6

Week 9 (October 26th): Best Practices (Instructional Strategies)
Project Based Learning
Motivating the Unmotivated
Cooperative Learning
Due Sunday, October 30th, in the Dropbox by 11:59pm:
• Lesson Plan 7

Week 10 (November 2nd): Legal & Ethical Issues & Professional Assessment
Due Sunday, November 6th, in the Dropbox by 11:59pm:
• Lesson Plan 8

Week 11 (November 9th): Integrated Curriculum/Curriculum Mapping
Graphic Organizer
Due Sunday, November 13th, in the Dropbox by 11:59pm:
• Lesson Plan 9

Week 12 (November 16th): Discipline/Behavior Management
Due Sunday, November 20th, in the Dropbox by 11:59pm:
Lesson Plan 10 (for Week of November 30th)

**Week 13** (November 23rd): No Class (Thanksgiving)

**Week 14** (November 30th): Home/School Connection

Due Sunday, December 4, in the Dropbox by 11:59pm:

- Final TWS (Sections III & IV)

**Week 15** (December 7th): Regroup and Review

Public, Private & Charter Schools

School Law

**Week 16** (December 14th): FINAL
APPENDIX H

AN EXAMPLE OF A GUIDED EXPLORATION PROBLEM
This problem is a variation of a problem in the Navigating through Algebra in Grades (9-12).

Problem

Devil: “Daniel, I need some money and I know of a fabulous investment opportunity.”

Daniel: “What’s that got to do with me?”

Devil: “If you put $1000 into the “Webster Investment Account” I have set up for you, I will double the amount of money in your account by the end of the first day. My commission for that day will be 10% of your initial investment, or $100. It will be deducted as the “Devil’s due” for that day, leaving you $1900 in your account at the end of the first day! On each successive day, I will double the amount in your account and double the commission to be placed in the Devil’s due for that day. But you need to promise to stick with my schemes for at least 30 days so that I can build up some capital of my own. You could be a rich man, Daniel. What do ya say?”

Daniel: “Hand me a spreadsheet.”

Solution Process

A spreadsheet is used to solve this problem and to extend it through exploration as shown by the sequence of spreadsheets in figure 3. This exploration is done as a whole class, the instructor leads the exploration and he uses a TI-Nspire that is projected on a SmartBoard. The pre-service teachers are supposed to be actively involved by responding to and asking questions and also entering the information of their calculators.
Figure 19: A Sequence of Spreadsheets that Enhance Visualization of the Devil and Daniel’s Problem
Spreadsheet A represents the initial set up of the problem in which three columns are labeled as follows:

- day - day,
- dd - the Devil’s Due for that day, and
- wia - the balance of the Webster Investment Account (wia) at the end of that day.

The day column has numbers 0 – 30 that represents the 30 days. Zero is entered in B1 for the Devil’s Due on day 0 and a 1000 in C1 represents the initial amount placed in the Webster account. To create the rest of the spreadsheet, the formula \((= c1 * .1)\) is entered in cell B2 yielding the Devil’s commission of $100 for day 1. The formula \((= 2 * c1 - b2)\) is entered in cell C2 indicating the amount in the Webster account at the end of day 1. Since the Devil’s commission on day 2 is determined by a different process than that used in day 1, the formula \((= 2 * b2)\) is entered into B3, yielding $200. The formulas in C2 and B3 are then copied down to day 30 in order to calculate both the Devil’s due and the balance of the Webster Investment Account on each of the 30 days. An examination of the spreadsheet reveals that, on day 20 the Webster account drops to 0 (Spreadsheet B) and by day 30, Daniel is in debt to the tune of $536,870,912,000.

When Daniel’s initial investment of $1000 is changed to $5000 in cell C1, the spreadsheet automatically updates the cells, but once again Daniel has $0 in his account on day 20 (Spreadsheet C). Varying the amount in C1 leads to similar results, with Daniel always going broke on day 20. Is this true in general? To help answer this a variable \(x\) is entered in cell C1 to represent his initial balance and it appears Daniel always goes broke on day 20 regardless of how much he invests(Spreadsheet D). To
fully understand why Daniel goes broke on day 20 parameters are introduced as follows; the formula in cell B2 is changed to ($p * c1$), and the multiplicative factor 2 in cells C2, C3, and B3 is replaced with a variable $m$. A clear pattern emerges (Spreadsheet E) and the pre-service teachers are asked to identify and describe the pattern. First, the pattern in the Devil’s Due sequence is geometric with explicit function

$$dd(n) = p \times x \times m^{n-1}.$$  The pattern in the Webster Investment Account is

$$wia(n) = m^{n-1} (m - np) \times x.$$  This makes it clear why Daniel goes broke on day 20 when $p = .1$ and $m=2$, regardless of how much he invests. Putting these values in for $p$ and $m$ and simplifying, we get $dd(n) = 100 \times 2^{n-1}$, and $wia(n) = 2^n(1000 - 50n)$. This kind of conclusion would not be reached by the pre-service teachers without assistance from the instructor at the beginning.
APPENDIX I

IRB APPROVAL
INSTITUTIONAL REVIEW BOARD
For the Protection of Human Subjects
FWA 00000165

MEMORANDUM

TO: Rejoice Mudzimiri
FROM: Mark Quinn, Ph.D. Chair
Institutional Review Board for the Protection of Human Subjects
DATE: August 10, 2011
SUBJECT: Examining the Nature of TPACK Development in Pre-service Secondary Mathematics Teachers [RM081011-EX]

The above research, described in your submission of August 10, 2011, is exempt from the requirement of review by the Institutional Review Board in accordance with the Code of Federal Regulations, Part 46, section 101. The specific paragraph which applies to your research is:

X (b)(2) Research involving the use of educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures or observation of public behavior, unless: (i) information obtained is recorded in such a manner that human subjects can be identified, directly or through identifiers linked to the subjects; and (ii) any disclosure of the human subjects’ responses outside the research could reasonably place the subjects at risk of criminal or civil liability, or be damaging to the subjects’ financial standing, employability, or reputation.

Although review by the Institutional Review Board is not required for the above research, the Committee will be glad to review it. If you wish a review and committee approval, please submit 3 copies of the usual application form and it will be processed by expedited review.