The effects of taxes and inflation on the composition of inputs to agriculture
by Douglas Roger Hart

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Applied Economics
Montana State University

Abstract:
The substitution effect between horsepower (farm machinery) and labor is analyzed when wage rates, tax rates, discount rates, depreciation methods, investment credits and inflation rates are varied. A simulated wheat farm is developed and the effects of the above mentioned variables are analyzed on the horsepower (machinery)/ labor ratio. When either the price of labor or horsepower (machinery) is altered directly or indirectly, there is a change in the ratio of horsepower to labor. This thesis explores the causes and extent of these changes.

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Signature Douglas Roger Hard

Date September 1, 1981

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DOUGLAS ROGER HART

A thesis submitted in partial fulfillment
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ABSTRACT

The substitution effect between horsepower (farm machinery) and labor is analyzed when wage rates, tax rates, discount rates, depreciation methods, investment credits and inflation rates are varied. A simulated wheat farm is developed and the effects of the above mentioned variables are analyzed on the horsepower (machinery)/labor ratio. When either the price of labor or horsepower (machinery) is altered directly or indirectly, there is a change in the ratio of horsepower to labor. This thesis explores the causes and extent of these changes.

Chapter 1

INTRODUCTION

The composition of the agricultural community in the United States has changed dramatically over the past fifty years. It has changed from small labor intensive units to large, highly specialized, capital intensive units. This result has been the product of a number of social-political events; an agricultural depression[1/] lasting two decades, two world wars and the rapid growth of United States industry and technology during the past two decades. But these events have all subsided in the early 1970's, industrial productivity has actually been declining, yet the trend continues. The purpose of this thesis is to explore some of the underlying, and perhaps, unsuspected reasons for the changing structure of U.S. agriculture.

Fifty years ago taxes played a minor role in the operating and investment practices of most farming enterprises. But in the last couple of decades the importance of tax laws has had increasing significance on farmers' financial planning. The main objective of this thesis is to explore the effects of income tax rates, and depreciation methods on the use of labor and machinery in the

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[1/] The agricultural depression lasted from 1919 to 1939, and is documented in a number of history books such as American Epoch, by Arthur Link and William Catton, Vol. II. 4th Ed.

structure of agriculture. That is, when the composition of inputs change the structure changes. For example, when machinery is substituted for labor for any reason, the structure of agriculture becomes more machinery intensive. This study will also explore the effects of inflation on this composition, and how inflation enhances or diminishes the effects of the tax rates and depreciation methods.

The maintained hypothesis of this thesis is that with changes in income tax rates, wage rates, social security tax rates and depreciation methods, the relative prices of the inputs change. When these prices are changed relative to each other, there is a reallocation of inputs.[2/]

It may be useful to analyze this effect by showing the substitution effect between two inputs. The substitution effect is the rate at which the producer substitutes one input for another when the price of an input changes and he moves along a given isoquant.[3/] Henderson and Quandt[4/] prove that this effect is always negative; i.e., when the price of an input decreases, the quantity used of that input

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[2/] This is proved mathematically in Chapter 3.

[3/] Microeconomic Theory; a Mathematical Approach, Henderson and Quandt; 3rd Ed., p. 27; the words inputs and isoquant were substituted by the author for the words, commodity and indifference curve, respectively.

[4/] Ibid, p. 47.

increases.

For purposes of illustration, the effects of an input price change will be observed utilizing an isoquant graph. Let the isoquant represent a measure of the number of acres worked by the farmer and define this isoquant to be a function of horsepower and hours, where hours represent the amount of labor time required to work a given number of acres.

The equation of the isoquant is:

$$\text{Acres} = K \cdot HP \cdot HRS$$

where K = a constant

HP = the amount of horsepower used

HRS = the amount of labor hours used.[5/]

If acres are held constant, the equation of the isoquant will be a rectangular hyperbola, as illustrated in Figure 1. Units of horsepower are represented on the y axis and units of labor hours are measured on the x axis.

With a change in the relative prices of the inputs (Price of horsepower decreases), the budget line shifts from BB to B'B'. The least cost combination of inputs shift from point A to point C. The amount of labor used decreases (HR° -- HR') and the amount of

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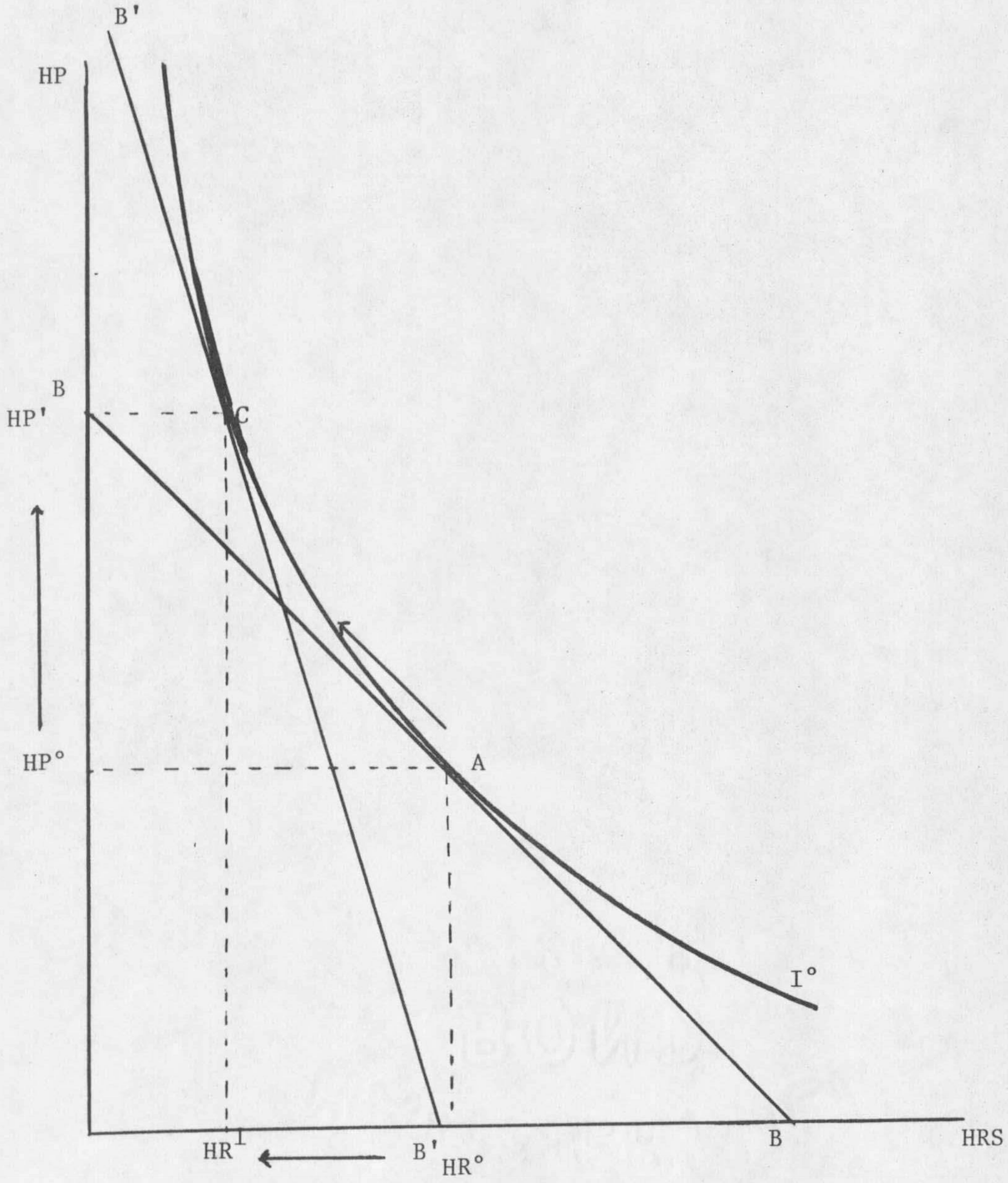
[5/] The actual derivation of this function is found in Chapter 4. K = a constant and its value can vary with given variables, this relationship is also found in Chapter 4.



BB = original budget line
B'B' = new budget line
I° = isoquant, holding acres constant

Figure 1. Demonstration of the Substitution Effect Between Two Inputs

capital (horsepower) used increases (HP° -- HP'). So, as is illustrated with a change in the relative price of an input there is a change in the least cost combinations of inputs to produce a specified level of output.

This is not the total effect. There is also a scale effect where the budget line shifts to a higher (or lower) isoquant. This effect is not explored in this thesis since the amount of acres farmed is assumed to be held constant, i.e., the isoquant is not allowed to shift.

Need for the Project

When tax reforms are initiated there should be specific objectives to be achieved. These objectives may be diverse and differ with the different environments for reform that were prevalent at the time. However, if all of the consequences of the new laws are not taken into consideration, there may be perverse effects on the system, totally opposite of the objectives that initiated the process. For example, when the amount of investment credit allowed is increased, the hoped for result would probably not include a reallocation of resources used in agriculture. But that is exactly what happens because the price of machinery, an input to agriculture, has effectively been decreased. One of the objectives of this thesis is to illustrate how this occurs and estimate the extent of the influence.

The other objectives of this thesis are specifically: 1) to measure the substitution effect between labor and machinery, for a simulated dryland wheat farm, when tax rates and depreciation methods change the relative prices of these two inputs; 2) to develop optimal replacement strategies for the farm machinery given discount rates, wage rates, and tax rates; and 3) to measure the impact of inflation on the first two objectives.

Chapter 2 is a review of the literature concerning optimal replacement strategies. A number of different methods are presented and the pertinent parts of each are discussed. Chapter 3 develops the theory and maintained hypothesis of this thesis. Chapter 4 presents the simulation model used and its assumptions. Chapter 5 contains the results of the model and the conclusions drawn from them.

Chapter 2

LITERATURE REVIEW

The thesis deals primarily with the effects of taxes, depreciation schemes and inflation on the optimal size and replacement ages of finite life, depreciable inputs in agriculture. This essentially constitutes a replacement problem. Therefore, the literature review deals with the development and theory of optimal replacement strategies.

The first section of the chapter deals with some of the pioneering work in the area of replacement theory, then traces its development to the present state of the science. The second section of the chapter deals with the application of replacement theory to agriculture and also discusses works which support some of the basic assumptions of this thesis.

A number of different replacement models will be presented in the literature review, so that the reader will have a basis by which to compare the model and replacement strategy presented in this study.

Replacement Theory

Martin Faustmann [translated 1968], a German forester, is credited with first applying the principal of discounted cash flow to a replacement problem. His article appeared in a forestry journal in

1849, and was written in response to an article which appeared just two months prior, dealing with the same problem.

Faustman addressed himself to the optimal cutting age of a stand of trees on a tract of land. The maturing age of the trees was twenty years so the tract was to be divided into twenty equal sections in order to afford an annual income.

To solve this problem, Faustmann developed and used the idea of net discounted cash flows. His symbols and equations, although not explicitly stated as such, can be reduced to:

$$\text{Net Discounted Revenue (NDR)} = \sum_{n=0}^{t} \frac{Sn - Cn}{(1+r)^n}$$

where:

Sn = sales in year n,

Cn = costs in year n,

r = the discount factor,

0-t = the planning period,

n = the year.

Thus, he maximized NDR and solved the problem of an optimal cutting age.

Faustmann's examination of problems in forestry is not as specialized and narrowly applicable as first thought. When he analyzed these problems, he really tackled the broader problem of how long capital assets should be kept before being replaced, i.e., the question of finding an optimal rate of turnover for capital stock.

As noted, this basic formulation was developed for use in forestry in 1849. It was not until years later that a similar theory was adapted for use in economics. Dr. Harold Hotelling [1925] was an early pioneer in this field, in a paper presented in 1925, he presented a model which formed the basis of many modern theories of replacement. This model was:

$$1)\quad \beta = \int_0^T [w(Q(t)) - E(t)]e^{-\int_0^T i(t)dt}\,dt + S(t)e^{-\int_0^T i(t)dt}$$

where:

β = original cost of a single machine

T = an unknown date at which it ought to be discarded.

w = unknown unit cost (plus interest) of the product

Q(t) = rate of production.

E(t) = combined rate of all expenses, except depreciation and interest

i(t) = rate of interest

S(t) = selling price or scrap value.

By differentiating with respect to T, the unit cost can be written:

$$2)\quad w = \frac{E(T) + i(T)S(T) - S'(T)}{Q(T)}\ .$$

This equation states that the cost of a unit of product is found by adding the operating cost [E(T)] of the machine, at the time when it is least efficient and about to be scrapped, to interest [i(T)S(T)] on scrap value and the rate of depreciation [-S'(T)] of the scrap value and divide this sum by the machine's rate of production. The result will be a minimum when T is determined by subtracting equation 2 from equation 1 and solving. This value of T will be the optimal time period for holding the machine.

In a 1940 publication, Gabriel A. D. Preinreich [1940] modified the Hotelling formula by defining the optimal replacement for a single machine as:

$$1) \quad V = \int_0^T [Z(Q(t)) - E(t)]e^{-it}dt + Se^{-it}$$

where:

V = capitalized value of the machine

Z = unit market price

Q(t) = rate of production

E(t) = combined rate of all expenses, except depreciation and interest

i(t) = interest rate

S(t) = selling price (scrap value)

This formulation is the rule in which a machine will not be replaced.

Preinreich then took the derivative of the valuation formula with respect to the time period T:

2) $\frac{dV}{dT} = ZQ(t) - E(t) - iS$

He then solved set (2) equal to zero and solved for the most lucrative life span (optimal replacement age) of the machine (T). T was then plugged back into equation 1) to find the capitalized value of the machine. Thus it was discovered that the economic life of the machine was independent of the price at which it was bought and sold.

Preinreich then looked at this replacement strategy for several different situations. These situations may be classified under three different headings.

A. Scope

1. A single machine;
2. A finite chain of replacements;
3. An infinite chain;
4. A number of parallel chains, whose replacement dates are evenly staggered;
5. A large plant continuously renewed in accordance with natural variations in the behavior of similar machines;

B. Limitations

1. Scarcity of new machines available for replacement;
2. Scarcity of various operating facilities or ingredients of production;

3. scarcity of demand for product;

4. scarcity of capital;

5. regulation of profit by law;

C. Economic Conditions

1. the static case where only variations due to the age of the machine are considered;

2. variations due to the number of co-operating machines;

3. change in ownership and outlook;

4. change in the type of machine used (obsolescence);

5. the general dynamic case, embracing extraneous influences as well.

After analyzing a variety of replacement problems, Preinreich concluded:

> "The general rule of replacement, which is simply the theory of maxima and minima, has a separate solution for every kind of rigid scarcity and for every volume of the supply so limited. When the volume required by a single machine becomes insignificant in comparison to the total, the problem is simplified into making the excess profit (goodwill) per unit of that ingredient a maximum. In the case of demand, that means making the cost per unit of demand (output) a minimum. In all other instances, the limitation operates at the other end of the productive process and therefore the first description applies. The excess profit per new machine, per square foot of space, per hour of labor, per ton of fuel, etc., must be made a maximum, depending on where the shortage is felt."

He observed that the reason many plants are in a rundown condition is because the resultant rise in the rate of profit hides the more significant decline in its amount. To correctly calculate replacement lives for these plants one should substitute the unknown rate of profit for the rate of interest, the original cost of a machine will always be equal to the net rental and the scrap value, discounted at the rate of profit.

Although Preinreich observed a number of variations in Hotelling's formula (properly called the Hotelling-Taylor formula), his general conclusions were that Hotelling's idea of minimizing unit costs was the most valuable single rule of thumb which can be laid down for the general guidance of entrepreneurs, at least when the number of machines is very large and no radical change in type is imminent.

Dr. Paul A. Samuelson [1937] in an article published in 1937 took a slightly different view of replacement theory. He describes N(t) as the income stream in time (t) where $0 \leq t \leq b$, 0 and b are the boundaries of the time period. The value of this stream is then defined as:

$$1) \quad V = V(t'r) = \int_t^b N(x)e^{r(t-x)}dx$$

where:

r = rate of interest

x = variable of integration.

He then defined the rate of depreciation (or appreciation) as:

$$2) \quad \frac{\partial V}{\partial t} = r[V(t)] - N(t)$$

This is to say, "The rate of depreciation at any instant of time is equal to the difference between net income and the returns on value of the investment account at that instant of time. This is equivalent to saying the net income includes the return on given value of investment plus the rate of depreciation."

If the value remains constant, and net income is considered perpetual, by performing the integration in equation 1 (if $N(X) = N$, or returns are constant):

$$3) \quad r \equiv \frac{N}{V}$$

That is, the rate of depreciation becomes equal to zero and the rate of interest may be expressed as the ratio between perpetual net income and value.

This is the same conclusion Faustmann would have reached a century earlier, if his equation had been simplified; although both theories were developed along different lines.

Samuelson observed that the value of an investment account will necessarily be given by the integration of the income stream, discounted at the market rate of interest. This follows because if the market price of the account is greater than its capitalized value, it would pay the owner to sell it and lend out the resulting sum of money at the current market rate of interest. But no one would be willing to pay any price for it above its capitalized value since they could always do better with their money elsewhere. Thus, the market price of an asset can never exceed its capitalized value.

He also proved that under a varying rate of interest, the value of an investment account is equal to:

$$V'(t) = r(t)V(t) - N(t).$$

This is the same relationship that held under a constant rate of interest. Even with a varying rate of interest, net income in his definition of the term contains a return on the value of an investment and also an amount equal to the rate of depreciation. "Thus the time shape of interest being given and income being known, the capital invested up to any time is always equal to the value of the account at that time, the value being a capitalization of subsequent income."

Samuelson's theory of replacement has been the basis of much of the work done in the area to date. It is the main theory upon which this thesis is based.

Samuelson [1962] also developed a "corallary" to his theory in which taxes and depreciation play a role. It is known as "The Fundamental Theorem of Tax Invariance" and states, "If, and only if, time loss of economic value is permitted as a tax deductible depreciation expense will the present discounted value of a cash-receipt stream be independent of the rate of tax."

This thesis will explore the effects of accelerated depreciation gimmicks on the costs of finite life inputs in agriculture. From this corollary it can be concluded that these gimmicks will affect the costs of these inputs in a negative direction. If the costs of these inputs are altered then a reallocation of these inputs relative to annual life inputs will occur. The causes and support for this hypothesis is presented in detail in the next chapter.

Replacement Theory in Agricultural Economics

There has been little actual work done on the effects taxes have on replacement theory but its occurrence has been mentioned. Kay and Rister [1976] briefly mention these effects in their article by stating,

> "The effects of these tax regulations is to lower the present value (cost) of any replacement policy. This result has, probably, in past, encouraged the trend towards larger equipment and the substitution of machinery for labor. The net result is a larger overall investment in farm machinery than would have existed without these incentives."

There has been a substantial amount of work done in the area of replacement theory in agricultural economics, although the effects of taxes and tax gimmicks have not been included in much of this work.

Dr. Mason M. Gaffney [1957] developed a procedure whereby he specified a machine should be kept another period if the marginal costs of retaining it another period were less than the "average" periodic costs of a replacement machine.

R. K. Perrin [1972] developed a model which specifies that an asset should be held to the age in which marginal revenues equal marginal opportunity costs with the latter being interpreted as the flows of earnings which would be realized from some given-year replacement policy. He also suggested calculating present values for each replacement year may be a better procedure than evaluating the marginal criteria.

Perrin also evaluated the effects of discount rates on optimal replacement ages. From this he obtained the conclusion that, "Some assets may be replaced earlier with rising discount rates while others may be replaced later; and in fact, a given asset may be replaced later up to a given rate but earlier thereafter." This depends upon the shape of the flows.

Perrin also addressed himself to the question of what the appropriate discount rate should be. The cost of capital and the return on alternative investment possibilities on the time of personal consumption may both be factors in determining what the appropriate discount rate should be. Neither of these choices are universally acceptable. The cost of capital may be appropriate if the entrepreneur faces a perfect capital market. But a particularly destitute entrepreneur may value present earnings high relative to future earnings so a high personal discount rate may be appropriate. If there were no capital markets, the internal rate of return may be the appropriate discount rate. The internal rate of return is determined by the market prices of the inputs; thus, if the internal rate is above the market rate for ventures of similar riskiness, the market price of the inputs will be driven up. Therefore, market rates of return for ventures of similar riskiness can be viewed as the appropriate discount rate if equilibrium prices of all inputs are expected to prevail by the first replacement date. The choice of appropriate discount rates for use in this thesis will be discussed in chapter four.

Anthony H. Chisholm [1974] developed a present value cost model incorporating income tax rates and investment credits. He estimated the effects of different investment credit rates and different tax rates, given certain discount rates, on the optimal replacement age of machinery. He found that the investment credit significantly influenced the optimal replacement age for machinery, and that different depreciation methods biased the optimal replacement time. He states:

> "There is no simple general rule for predicting the direction of bias on replacement age of a particular method of depreciation. Perhaps more important is the fact that in no instance was the magnitude of the bias stemming from a particular method of depreciation of sufficient size to change the optimal replacement age."

Kay and Rister [1976] also did some work on the effect of tax depreciation and investment credit on replacement ages and concluded that:

a) the after-tax discount rate had the greatest effect,

b) the tax rate causes only slight differences in replacement policy,

c) depreciation methods have little effect (they analyzed straight-line and double-declining balance).

They observed that additional first year depreciation and investment credits had the greatest affect on the optimal replacement ages. They also found that tax regulations which permit using double declining balance depreciation, additional first year depreciation and investment credit, affect the present value (cost of obtaining the constant annual stream of tractor services. This led to their comment about the effects of taxes on the allocation of capital and labor quoted at the beginning of this section.

Bates, Rayner and Custance [1979] viewed a farm tractor replacement model in a continuous-time framework. They used this model to observe the effects of inflation on replacement ages. The model they represented was:

$$PVn = \frac{1}{1-e^{-rn}} \{(Co - Cne^{-rn}) + (1-T)Rk - T(A(n)e^{-(r+f)} -$$

$$T(\int_{K=1}^{n} DKe^{-(r+f)K} dK)^{n} - Ine^{-r(r+f)}\}$$

where:

PVn = present value of the total cost in year n,

r = after-tax real discount rate,

Co = initial cost,

Cn = resale price at the end of year n,

T = marginal income tax rate,

Rk = repair cost in year K,

An = additional first year depreciation policy; with a policy in n years,

dK = regular depreciation allowance in year K,

In = investment credit

e^f = rate of inflation.

Their conclusions were that inflation affects the cost incurred by farmers in three ways:

1) Taxes are based typically on historic costs. The allowances that can be claimed for "depreciation" of equipment are a significant element in tax allowances. If inflation is significant, the model must properly allow for the loss in the real value of these allowances.

2) Receipts and benefits from tax allowances are lagged, typically by about one year. With inflation these receipts are in depreciated money.

3) When a farmer sells his equipment, the difference between the resale price and the unexpired depreciation allowance is subject to tax. In inflationary times, resale prices for any given age of equipment are likely to be increasing and may well exceed the unexpired depreciation allowances which are based on historic costs.

Watts and Helmers' [1980] research into this area explored the actual central theme of this thesis which is the substitution of capital for labor with the imposition of taxes. They incorporated a model in which an after-tax profit function was constrained by a strictly concave and continuous production function:

$Y = F(F,S)$,

where:

Y = quantity of output,

L = quantity of annual inputs,

S = quantity (size) of depreciable inputs.

The constrained present value profit function was defined as:

$$\pi = \int_0^n QYe^{-\beta ri}di - \int_0^n wLe^{-\beta ri}di - S[V(0) - V(n)e^{-\beta rn}] -$$

$$\int_0^n ST[QY - wL + V'(i)]e^{-\beta ri}di - \lambda[Y-F(L,S)].$$

where:

π = present value of after-tax profit,

Q = price of output assumed to be constant over time,

w = price of the annual input, assumed to be constant over time,

V(i) = price of a unit of depreciable input as price per unit of size of a depreciable input (which implicitly assumes price is a linear function of size at age i.

V'(i) = change in price of depreciable input as the input age (this value is negative under most circumstances),

T = the tax rate in decimal form which is assumed to be between zero and one and constant over the relative income range,

$\beta = 1-T$,

i = time or age,

n = length of planning period which is assumed to be consistent with the ownership life of the machine,

r = before-tax discount rate which is assumed to be positive.

The first order conditions for profit maximizing levels of Y, L, and S are:

$$\frac{\partial\pi}{\partial Y} = \beta Q\int_0^n e^{-\beta ri}di - \lambda = 0$$

$$\frac{\partial\pi}{\partial L} = -\beta w\int_0^n e^{-\beta ri}di + \lambda F_L = 0$$

$$\frac{\partial\pi}{\partial S} = -[V(0) - V(n) + T\int_0^n V'(i)e^{-\beta ri}di] + \lambda F_S = 0$$

$$\frac{\partial\pi}{\partial Y} = Y - F(L,S) \overset{set}{=} 0.$$

The first order conditions reduce to:

$$F_L = \frac{\beta w\int_0^n e^{-\beta ri}di}{\beta QP\int_0^n e^{-\beta ri}di} = \frac{w}{Q}$$

$$\frac{F_S}{F_L} = \frac{V(0) - V(n)e^{-\beta rn} + T\int_0^n V'(i)e^{-\beta ri}di}{\beta w\int_0^n e^{-\beta ri}di}$$

$$Y = F(L,S).$$

Since $V'(i)$ is not constant the integral cannot be factored out, therefore the tax rate is implicit in the maximizing levels of S and L. They arbitrarily set $\frac{F_S}{F_L} = R$ and explored the derivative $\frac{\partial R}{\partial T}$ to investigate the effects of the tax rate on the optimal (maximizing) amounts of inputs to use. By exploring this derivative they reasoned that $\frac{\partial R}{\partial T}$ was likely to be less than zero which meant that as T increases, S increases and Y increases, so as long as S and L are economic substitutes, L decreases. So as the tax rate increases the amount of machinery will increase and the amount of labor will decrease. This theme will be explored in some detail in the theory part of this thesis.

In another paper, Watts and Helmers [1980] developed budgeting techniques and concepts which will be used in this thesis. For instance, assume an after-tax basis, then the original outlay for a depreciable asset is considered to be placed on an after-tax basis by the inclusion of depreciation credits. Depreciation credits are found by multiplying depreciation by the marginal tax rate. Depreciation and investment credit are on a nominal or real basis and depreciation and investment credit recapture are estimated in nominal terms. They state that:

> "The net present after-tax cost is achieved by discounting either nominal or real after-tax flows by the appropriate nominal or real after-tax discount rate. This net present cost is then placed on an annual after-tax basis by amortizing the net present cost by the real after-tax discount

> rate. The resultant real annual after tax cost can then (if desired) be converted to a before-tax basis by dividing the amortized tax cost by the complement of the marginal tax rate."

They also present a proof showing the relation between the traditional and capital budgeting fixed cost estimates of opportunity cost and depreciation. The proof is as follows:[1/]

Discrete Time Cast

Prove that: $$V(0) - \frac{V(n)}{(1+r)^n} = \sum_{i=1}^{n} \frac{D(i) + OC(i)r}{(1+r)^i}$$

where:

$D(i)$ = depreciation in year t,

$= V(i) - V(i-1)$ = market depreciation,

$OC(i)$ = opportunity cost in year $i - V(i-1)r$

$V(n)$ = value of the machine in time n.

$$\sum_{i=1}^{n} \frac{D(i) + V(i-1)r}{(1+r)^i} = \frac{V(0) - V(1) + V(0)r}{1+r} + \ldots + V(n-1) - \frac{V(n) + V(n-1)r}{(1+r)^n}$$

$$= V(0) - \frac{V(1)}{1+r} + \frac{V(1)}{1+r} - \frac{V(2)}{1+r} + \ldots + \frac{V(n-1)}{(1+r)^{n-1}} - \frac{V(n)}{(1+r)^n}$$

1/ Myles J. Watts and Glenn Helmers, "Machinery Costs and Inflation," unpublished research, Appendix A.

$$= V(0) - \frac{V(n)}{(1+r)^n}$$

For the continuous time case show that:

$$V(0) - \frac{V(n)}{e^{rn}} = \int_0^n \frac{D(i) + OC(i)r}{e^{ri}} di$$

note that $d\frac{V(i)}{e^{ri}}/di = \frac{V'(i)}{e^{ri}} - \frac{V(i)r}{e^{ri}}$

$$= \frac{V'(t) - V(t)r}{e^{ri}}$$

Furthermore, since $-V'(i) = D(i)$ and $V(i)r = OC(i)$ then:

$$\int_0^n - \frac{D(i) - OC(i)}{e^{ri}} di = - [\frac{V(n)}{e^{rn}} - \frac{V(0)}{e^{r0}}]$$

$$= V(0) - \frac{V(n)}{e^{rn}}$$

The relevance of the proofs by Watts and Helmers and the models used by the others will become clear as the next chapter is read. Assumptions and implications of most of the models discussed in this section of the literature review are incorporated into the model used in this thesis.

Chapter 3

THEORETICAL DEVELOPMENT AND MAINTAINED HYPOTHESIS

The goal of any profit maximizing firm is to minimize costs for a given level of output and thus maximize profits for that level of output. Income tax rates, depreciation schedules and investment credit allowances are all implicit variables in the cost functions facing agricultural entrepreneurs. The effects these variables have on the cost functions and thus on the profit maximizing combination of inputs are the main subject of this thesis. This chapter will explore how these variables affect the cost functions and the profit maximizing conditions facing the farming firm. Different effects will be illustrated for different types of inputs. Annual inputs, infinite life inputs and finite life inputs, which depreciate in value over their productive lives, will all be analyzed.

Annual Inputs

Assume the farmer faces the production function $Y = F(x, w)$, where: Y = the quantity of output or product and x and w are the annual inputs. Also assume x and w are totally diminished during the production period and the total amount of Y, output, is sold at the end of the production period. The planning period is from 0 to n production periods, where n can vary.

The profit function can now be defined as the Lagrangean:

1) $\pi = \int_0^n PiYe^{-ri}di - \int_0^n V_w we^{-ri}di - \int_0^n V_x xe^{-ri}di -$

$\lambda\,[Y-F(w,x)].$

Which can be reduced to:

2) $\pi = Y \cdot \int_0^n Pie^{-ri}di - w \cdot \int_0^n Vwe^{-ri}di - x \cdot \int_0^n V_x e^{-ri}di -$

$\lambda\,[Y-F(w,x)].$

The variables are defined as:

π = Present value of profit,

r = discount rate,

0-n = planning period,

P_i = price of one unit of Y at time i,

V_w = price of one unit of w at time i,

V_x = price of one unit of x at time i.

The first-order conditions for profit maximization are:

3) $\frac{\partial\pi}{\partial Y} = \int_0^n Pie^{-ri}di - \lambda = 0$

4) $\frac{\partial\pi}{\partial w} = - \int_0^n V_w e^{-ri}di + \lambda\,[\frac{\partial F(x,w)}{\partial w}] = 0$