Natural convection heat transfer from a trombe wall geometry to a rectangular enclosure
by Peng-Cheng Lin

A thesis submitted in partial fulfillment of the requirements for the degree of MASTERS OF SCIENCE in Mechanical Engineering
Montana State University
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Abstract:
A 1/18th scale experimental model was developed to characterize the natural convection heat transfer in passive solar heating with a Trombe wall geometry. Steady natural convection heat transfer was measured from an isothermal heated Trombe wall (inner body) with various gap sizes, 2.54 cm gap, 0.635 cm gap, and zero gap, to a rectangular enclosure (outer body). Temperature profiles within the test space were obtained and fluid flow patterns were observed. Dow Corning 20 cs fluid was utilized as the working fluid with Prandtl numbers which ranged from 125 to 277. The inner body with 2.54 cm gap appears to give a highest heat transfer from the Trombe wall, especially in the low temperature range, due to the exchange of mass between the Trombe wall space and living space. However, when the gap size was increased the temperature stratification in the living space was also increased. A circulation cell around the upper portion of the living space was observed. The recommended empirical equation describing the heat transfer using two independent parameters was \( N_u = 1.6106 \cdot Ra^{0.1760} \cdot (H/H_i)^{-0.2159} \) for \( 6.2 \times 10^8 < RaH < 1.5 \times 10^9 \) with an average percent deviation of 3.01.
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NATURAL CONVECTION HEAT TRANSFER FROM A TROMBE WALL GEOMETRY TO A RECTANGULAR ENCLOSURE

by

PENG-CHENG LIN

A thesis submitted in partial fulfillment of the requirements for the degree of

MASTERS OF SCIENCE

in

Mechanical Engineering

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Bozeman, Montana
September, 1982
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ACKNOWLEDGEMENT

The author wishes to express his thanks and appreciation to the following for their contribution to this investigation:

Dr. R. L. Mussulman and Dr. R. O. Warrington, for their guidance, advice and instruction.

Dr. A. Demetriades, for serving as a committee member and reviewing this thesis.

Pat Vowell and Gordon Williamson for their helpful assistance in the construction and maintenance of the heat transfer apparatus.

The Mechanical Engineering Department of Montana State University, for funding of this investigation.

The Theater Arts Department of Montana State University for providing lighting equipment for this investigation.

Lastly, yet foremosly, Sue-May, for her constant encouragement, and for typing this thesis.
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ABSTRACT

A 1/18th scale experimental model was developed to characterize the natural convection heat transfer in passive solar heating with a Trombe wall geometry. Steady natural convection heat transfer was measured from an isothermal heated Trombe wall (inner body) with various gap sizes, 2.54 cm gap, 0.635 cm gap, and zero gap, to a rectangular enclosure (outer body). Temperature profiles within the test space were obtained and fluid flow patterns were observed. Dow Corning 20 Cs fluid was utilized as the working fluid with Prandtl numbers which ranged from 125 to 277. The inner body with 2.54 cm gap appears to give a highest heat transfer from the Trombe wall, especially in the low temperature range, due to the exchange of mass between the Trombe wall space and living space. However, when the gap size was increased the temperature stratification in the living space was also increased. A circulation cell around the upper portion of the living space was observed. The recommended empirical equation describing the heat transfer using two independent parameters was

$$\text{Nu}_H = 1.6106 \text{ Ra}_H^{0.1760} (H/H_i)^{-0.2159}$$

for $6.2 \times 10^8 < \text{Ra}_H < 1.5 \times 10^9$ with an average percent deviation of 3.01.
CHAPTER I
INTRODUCTION

The study of natural convection heat transfer within enclosures has been receiving increasing attention in recent years. Despite this, there is still a dearth of knowledge in this field. The study of natural convection within enclosures has not advanced sufficiently to the point where an analytical or a numerical investigation can be developed without simplifying the governing equations, idealizing the boundary conditions, or applying the boundary layer assumptions. As a result, experimental studies are needed to tackle the problem of natural convection heat transfer within enclosures. This is particularly true for predicting fluid flow behavior from a body to an enclosure. Interest has been concerned with solar heating design, nuclear reactor technology, cooling electronic devices, aircraft cabin design, and numerous other applications. Most recently applications in passive solar heating have required a better understand of natural convection within enclosures.

Since 1967, when the prototype "Trombe Wall" house was built and monitored in Odeillo, France, several experimental and analytical investigations have been presented pertinent to this geometry in passive solar heating. The passive solar heating with a Trombe wall geometry is illustrated in
Two disadvantages of full-scale building studies were observed: (1) the thick concrete Trombe wall has a long conductive time constant resulting in a time dependent heat transfer problem, (2) solar irradiation changes too rapidly making full-scale data difficult to evaluate. Nevertheless, the convective time constant is only a few seconds, which is calculated by dividing Trombe wall height by airflow velocity [30]. Thus, steady convective data are appropriate for application to the time dependent heat transfer behavior of a Trombe wall system. Since passive solar heating designs, including the Trombe wall itself, are becoming economically competitive with conventional heating systems, there is a need to obtain steady convective data for application to a Trombe wall system. These considerations led Mussulman and Warrington [1] to propose to build a 1/18 scaled experimental model to eliminate the above two disadvantages. The criteria for designing this experimental model will be described in chapter III.

The objective of the present study was to develop a 1/18th scale experimental model to characterize the natural convection heat transfer in passive solar heating with a Trombe wall geometry. In order to accomplish this the heat
Figure 1.1 Solar Passive Heating with a Trombe Wall Geometry
transfer from an isothermal heated Trombe wall geometry with three different gap sizes to a rectangular enclosure was measured. Temperature profiles within the Trombe wall space and living space were obtained and the fluid flow patterns in the living space were observed. Dow Corning 200 (silicone) fluid with a kinematic viscosity of 20 centistokes at 32°C (90°F) was utilized as the test fluid. The Rayleigh number based on the enclosure height varied from $6.2 \times 10^8$ to $1.5 \times 10^{10}$. Moreover, empirical equations to determine the heat transfer within the living space were determined for several different combinations of parameters.
Chapter II
LITERATURE REVIEW

The phenomenon of natural convection is a buoyancy-driven effect, that is, the fluid motion results from density differences caused by temperature gradients in the fluid. A considerable number of analytic and experimental studies of natural convection heat transfer have been made over the last 70 years. The most thoroughly studied case is that of external natural convection from a body to an infinite fluid medium [2-8]. Some of the more recent studies [9-20] have been focused on natural convection within simple rectangular vertical enclosures or rectangular enclosures inclined from the vertical. Cylindrical and spherical enclosure geometries have also been investigated [21-23]. The study of natural convection heat transfer from several different geometric bodies such as spheres, cubes, and cylinders to both cubical and spherical enclosures has also been presented [24]. Moreover, several recent studies of natural convection within rectangular enclosures have investigated passive solar heating geometries [19-20, 24-32].

The focus of the following discussion is directed toward published articles on natural convection. The discussion is organized into two sections: vertical flat plates and rectangular enclosures. In addition, studies in
passive solar heating with a Trombe wall geometry are presented.

**VERTICAL FLAT PLATES**

Numerous early works dealt with natural convection heat transfer from vertical flat plates [2-8]. Researchers have developed theoretical solutions to heat transfer problems by simplifying the governing equations, idealizing the boundary conditions, and applying the boundary layer assumption. These solutions were in good agreement with experimental results.

Eckert and Jackson [4] analyzed turbulent natural convection on an isothermal vertical plate by applying momentum integral equations to boundary layer theory. A heat transfer equation was derived,

\[ Nu = 0.0246(Gr)^{2/5}(Pr)^{7/15}[1+0.494(Pr)^{2/3}]^{-2/5} \]  

for Grashof numbers greater than $10^{10}$. Nu, Gr, and Ra without a subscript are based on the height of the vertical flat plate. A comparison with the experimental heat transfer results of Jakob [2] and McAdams [3] showed good agreement. They also pointed out that the transition region was between Rayleigh numbers of $10^8$ to $10^{10}$ with air as the working fluid.

Five years after Eckert and Jackson's work, Bayley [5]
proposed a theoretical analysis of turbulent natural convection from an isothermal vertical plate. Two equations were presented

\[ \text{Nu}_x = 0.10 (\text{Gr}_x \text{Pr})^{0.33}, \text{ for } 2 \times 10^9 < (\text{Gr}_x \text{Pr}) < 10^{12} \]  \hspace{1cm} (2-2) 

and

\[ \text{Nu}_x = 0.183 (\text{Gr}_x \text{Pr})^{0.31}, \text{ for } 10^{12} < (\text{Gr}_x \text{Pr}) < 10^{15} \]  \hspace{1cm} (2-3) 

In addition, an approximation of the heat transfer for mercury (Pr = 0.01) was

\[ \text{Nu}_x = 0.06 (\text{Gr}_x)^{0.25}, \text{ for } 10^{10} < \text{Gr}_x < 10^{15} \]  \hspace{1cm} (2-4) 

With the same geometry, Warner and Arpaci [6] performed an experimental investigation of turbulent natural convection along a vertical isothermal heated flat plate with air as the test medium. Several plots of temperature profile data near the hot wall were obtained. The heat transfer results of their study have shown good agreement with the analytical correlation of Bayley [5] for Rayleigh numbers up to $10^{12}$. The one third power of this correlation showed that when turbulent free convection is encountered, the local heat transfer coefficient is essentially constant with \( x \).

Vliet and Liu [7] experimentally studied turbulent natural convection boundary layers for a constant heat flux surface condition using water as the working fluid. The
isothermal surface condition data of Eckert and Jackson [4] and Bayley [5] have revealed quite good agreement with their heat transfer data, but exhibited a much earlier transition point due to the different heating modes.

A general correlation of laminar and turbulent free natural convection from an isothermal vertical surface was developed by Churchill and Chu [8]. This correlation is

$$\text{Nu}^{1/2} = 0.825 + 0.387 \frac{Ra^{1/6}}{[1+(0.437/\text{Pr})^{9/16}]^{8/27}} \quad (2-5)$$

which may be applied to the entire range of Rayleigh numbers. Although this equation is suitable for most engineering calculations, slightly better accuracy can be obtained for laminar flow by using

$$\text{Nu} = 0.68 + 0.67 \frac{Ra^{1/4}}{[1+(0.492/\text{Pr})^{9/16}]^{4/9}} \quad (2-6)$$

for $Ra < 10^9$. The two resulting equations may be applied for constant heat flux conditions as well as for constant surface temperature conditions.

RECTANGULAR ENCLOSURES

Natural convection within enclosures has been investigated analytically from the middle of this century. However, the interactions of the boundary layers of the enclosure with its core region cause complex flow patterns making analytical solutions difficult to obtain. Theoretically, at large Rayleigh numbers the heat transfer
across each of the turbulent boundary layers might approach the heat transfer from a vertical plate in an infinite medium; this was not found experimentally. The existing analytical solutions for natural convection within rectangular enclosures dealt only with the steady two-dimensional case. Such solutions have compared poorly with the experimental results for high Rayleigh numbers with aspect ratios less than one because three-dimensional effects are important. The aspect ratio is defined as the ratio of the enclosure height to length normal to the hot wall. In general, rectangular enclosures with two opposite vertical walls at different temperatures and insulated horizontal surfaces have been extensively examined. The depth (the distance between the remaining walls) of this rectangular enclosure was made sufficiently large to assure two-dimensional flow in the central region of the cavity.

Natural convection in a rectangular cavity with adiabatic top and bottom surfaces and two opposing isothermal vertical walls at different temperatures was first investigated analytically by Batchelor [9]. He expanded temperature and stream functions in power series of the Rayleigh number and defined various flow regimes. The resulting equation was
\[ \text{Nu}_L = 0.48 \text{Ra}_L^{1/4}(A)^{3/4} \text{ for } (\text{Ra}_L/500) > A. \]  

(2-7)

Eckert and Carlson [9] made an experimental study of natural convection in a rectangular enclosure with three different aspect ratios (2.5, 10, and 20) using water as the working fluid. Temperature distributions within the enclosed space were obtained using an interferometer to describe conduction, transition, and boundary regimes. Grashof numbers based on the enclosure height were on the order of \(10^6\) in this investigation. Flow fluctuation and wave motions were observed in some cases.

Elder [11-12] performed an extensive experimental investigation in a rectangular cavity with aspect ratios ranging from 1 to 60 for laminar and turbulent flow regimes. Medicinal paraffine, silicone oil (Pr~1000) and water were utilized as test fluids. The flow was made visible by using aluminum powder suspended in the fluid. The experiments were conducted especially to gain further insight into the fluid flow behavior. Travelling wavelike motions growing up the hot wall of the slot and down the cold wall were observed for Rayleigh numbers above approximately \(8 \times 10^8(\text{Pr}^{1/2}/A^3)\). These waves developed most readily midway between the two endwalls. Near \(\text{Ra} = 10^{10}/A^3\) an intense entrainment and mixing process between the region near the
wall and the interior was initiated. As the Rayleigh number increased, the turbulent middle portion of the flow extended further toward the endwalls.

Numerical and experimental studies of natural convection between vertical planes with moderate and high Prandtl number fluids were carried out by MacGregor and Emery [13]. The vorticity and stream function were substituted into the governing equation and a numerical solution was obtained by using a finite difference technique for isothermal and constant heat flux boundary conditions. They concluded that the net heat transfer was a strong function of the aspect ratio when correlated with $Ra_L$. The correlations for both the isothermal and constant heat flux conditions obtained from the experiments were:

$$Nu_L = 0.42(A)^{-0.30}Pr^{0.012}Ra_L^{0.25} \text{ for } 10^4 < Ra_L < 10^7 \quad (2-8)$$

with aspect ratios from 1 to 40. However, the following one parameter correlation could be used:

$$Nu_L = 0.046 \ Ra_L^{1/3} \text{ for } Ra_L > 10^6 \quad (2-9)$$

Ostrach and Raghaven [14] conducted an experimental study which described the effect of stabilizing thermal gradients on natural convection in rectangular enclosures with aspect ratios of 1 and 3. In order to track the streamlines and measure the approximate velocity of the
fluid, Pliolite plastic particles were mixed into silicone oils with kinematic viscosities of 10,000 and 2,000 centistokes at 25°C. Thermal boundary conditions were established by heating, cooling, or insulating on opposite walls of the enclosure so that simultaneous horizontal and vertical heat flows were achieved. Different streamline patterns were observed by varying the ratio of vertical to horizontal Grashof numbers and the aspect ratio. The results showed that a stabilizing effect on the flow was established by heating the upper surface and cooling the lower surface.

An approximate analysis of natural convection within the rectangular enclosures was proposed by Raithby et al. [15]. The resulting solutions were

\[ \text{Nu}_L = 0.75C_\zeta (\text{Ra}_L/A)^{1/4} \quad (2-10) \]

and for the laminar regime

\[ \text{Nu}_L = 0.29C_t (\text{Ra}_L)^{1/3} \quad (2-11) \]

for the turbulent regime where \( C_\zeta = 0.50/(1+(0.49/\text{Pr})^{9/16})^{4/9} \) and \( C_t = 0.14\text{Pr}^{0.084} \). Good agreement was found in comparing these equations to the data available up to 1975. There is a lack of data at high Rayleigh numbers \( (\text{Ra} > 10^{10}) \) and low aspect ratios \( (A < 5) \). The validation of predictions covering this range of parameters has to rely on further
experiments.

Berkovsky and Polevikov [16] developed a numerical solution for natural convection heat transfer within vertical slots. The heat transfer solution was

$$\text{Nu}_L = 0.22(A)^{-0.25}(RaLPr/(0.2+Pr))^{0.28} \quad (2-12)$$

for $2 < A < 10$, $Pr < 10^5$ and $Ra_L < 10^{10}$.

Catton, Ayyaswamy, and Clever [17] studied convection in a rectangular cavity at various angles of tilt. Of interest in this analytical investigation was the comparison of their results for adiabatic and perfectly conducting horizontal surfaces. The results for a vertical slot showed that the overall heat transfer across the gap is lower for perfectly conducting horizontal surfaces than for adiabatic horizontal surfaces. Examination of the local heat transfer indicates that at high aspect ratios, there is very little difference except very near the horizontal surface. A significant difference can be seen at the lower aspect ratios ($A = 1$ and $A = 0.2$). This is due to the more pronounced thermal interaction at the perfectly conducting boundaries. With a similar geometry, Elsherbiny, Raithby, and Holland [18] obtained an empirical correlation for a vertical air slot with perfectly conducting horizontal surfaces:
$$\begin{align*}
\text{Nu}_1 &= 0.0605 \frac{Ra_L^{1/3}}{} \\
\text{Nu}_2 &= \left[1+\{0.104Ra_L^{0.293}/(1+(6310/Ra)^{1.36})\}^{1/3}\right]^{1/3} \\
\text{Nu}_3 &= 0.242\left(\frac{Ra_L}{A}\right)^{0.272}
\end{align*}$$

and the maximum of $\text{Nu}_1$, $\text{Nu}_2$, and $\text{Nu}_3$ was recommended for $5 < A < 110$ and $10^2 < Ra_L < 2 \times 10^7$.

More recently the works of Bauman et al. [19] and Nansteel et al. [20] were motivated by studies of natural convection heat transfer within buildings. Bauman et al. [19] conducted an experimental and numerical study of the natural convection heat transfer in a rectangular enclosure which simulated a full scale room with an aspect ratio of 0.5. The enclosure consisted of two vertical copper endwalls at different temperatures and adiabatic plexiglas horizontal surfaces. The Rayleigh numbers achieved, based on the enclosure length, were about $10^{10}$ using water as the working fluid. The experimental heat transfer result in the enclosure appeared somewhat higher than the approximation of Raithby et al. [15]. This may have occurred, because the vertical boundary conditions were not perfectly isothermal and the horizontal surfaces were not well insulated. With a similar geometry and the same working fluid, Nansteel and Greif [20] focused on an experimental study of natural convection in undivided and partially divided rectangular
enclosures. Rayleigh numbers based on the enclosure length in the range of $2.3 \times 10^{10}$ to $1.1 \times 10^{11}$ were obtained. It seemed that no fully developed turbulent flow was observed within the enclosure, even for $Ra_L$ as high as $10^{11}$. The recommended heat transfer correlations were

$$Nu_L = 0.748 \, Ap^{0.256} \, Ra_L^{0.226} \quad (2-16)$$

for a conducting partition and

$$Nu_L = 0.726 \, Ap^{0.473} \, Ra_L^{0.226} \quad (2-17)$$

for an adiabatic partition, where $Ap$ is the ratio of the central opening to the height of the enclosure, and is called the aperture ratio.

From this review of the existing work on heat transfer by natural convection within a rectangular enclosure, the important dimensionless parameters are

$$Nu_S = \frac{hs}{k} \quad (Nusselt \ Number),$$

$$Gr_S = \frac{(g \beta \rho^2 s^3 \Delta T)/\mu^2}{Pr} \quad (Grashof \ Number),$$

$$Pr = \frac{(\mu C_p)/k}{(Prandtl \ Number)},$$

and

$$A = \frac{H}{L} \quad (Aspect \ Ratio),$$

where $s$ is some characteristic length. The dimensionless grouping $(Gr_S \, Pr)$ is widely used as the Rayleigh number, which is

$$Ra_S = Gr_S \, Pr = \frac{(C_p \beta \rho^2 s^3 \Delta T)/\mu k}{.}$$

In general, the heat transfer by natural convection can be
determined in functional form as

$$\text{Nu}_s = f(A, \text{Ra}_s, \text{Pr})$$

Since 1967, when the prototype Trombe wall house was built and monitored in Odeillo, France, several analytical and experimental investigations have been presented pertinent to this geometry in passive solar heating.

Balcomb et al. [27-29] conducted several full scale experimental studies in passive solar heating with either unvented or vented Trombe-type thermal storage walls. Temperature variations of the building walls and the ambient room temperature on a daily or monthly basis were obtained. The effect of storage capacity on annual energy delivery for a Trombe-type passive system was also examined. However, no convective heat transfer data were obtained, nor could it be calculated from their experimental data. It is probably difficult to reach the steady state condition for the full scale building, because the solar irradiation changes too rapidly and the Trombe wall is rather thick providing a long conductive time constant (10 to 15 hours) to transfer heat from one side to another.

Free convective laminar flow within the Trombe wall space was assumed similar to the flow between two parallel, infinitely wide vertical plates with an adiabatic bottom
surface by Akbari and Borgers [30]. By this assumption, they solved nondimensional boundary layer equations by using a forward-marching, line by line implicit finite difference technique. Several velocity and temperature profiles were obtained for different wall temperatures.

An experimental investigation of the Trombe wall passive solar energy system was carried out by Casperson and Hocevar [31]. The wood-frame test room was constructed with outside dimensions of 3.66 m x 4.27 m x 3.05 m (12 ft x 14 ft x 10 ft). The Trombe wall consisted of a 30.48 cm (12 in.) thick solid concrete block wall with a movable double-glazed Kalwall cover unit, which allowed the wall gap (Trombe wall space) to be varied from approximately 2.54 cm (1 in.) to 25.4 cm (10 in.). Velocity and temperature profiles obtained in the Trombe wall space with room temperature at 14.8°C (55°F) and 15.6°C (60°F) indicated that the flow is likely to be turbulent. No heat transfer data were obtained in this study.

Stotts, Warrington, and Mussulman [25] developed a computer model to simulate direct gain, indirect gain, isolated gain, and Trombe wall passive solar heating systems. The Trombe wall model performance was verified using data from passive test cells at NCAT (National Center
for Appropriate Technology) [32], located in Butte, Montana. Radiative interactions between room's interior surfaces were considered. Convection from the walls to the air was also considered. The simulated globe temperature showed good agreement with the measured values.

The study of natural convection within enclosures has increased rapidly in the last decade. Results describing the heat transfer within rectangular enclosures are not applicable for the full-scale building in passive solar heating due to aspect ratio mismatches. Although several studies have been made pertinent to the passive solar heating with a Trombe wall geometry, no natural convection heat transfer data have been published to date. There is a definite need to study the phenomena of natural convection in passive solar heating with a Trombe wall geometry with either scaled models or full-scale structures. These studies will help the development of computer simulations and the design of Trombe wall systems. The intent of this investigation was to develop a scale model for convective heat transfer which simulated a full-scale structure with Trombe wall geometries.
CHAPTER III
EXPERIMENTAL APPARATUS AND PROCEDURE

EXPERIMENTAL APPARATUS

A 1/18th scale parametric study of passive solar heating with a Trombe wall geometry was carried out as part of the present experimental investigation. The present experiment was to simulate a two story high rectangular room with inside dimensions of 5.5 m in height, 8.2 m in length, and 5.5 m in depth. Typical values of the dimensionless parameters characterizing the convection process for this room, filled with air at 21 °C, and with a maximum of 9 °C temperature difference [26] between a Trombe-type thermal storage wall and the inside building are:

- \( A = \frac{H}{L} = 0.67 \)
- \( Pr = 0.71 \)
- \( Ra_H = Gr_H Pr < 1.4 \times 10^{11} \)

The range of \( Ra_H \), based on the height of room, suggests that natural convective flow within this building can be either laminar or turbulent, governed by the specific temperature distributions on the boundary surfaces and the configuration of the enclosure. In order to characterize buoyancy-driven convection in this two story room, the design of a heat transfer apparatus has to meet the above requirements. The experiment covered the following range of parameters.
A = 0.67
124.7 < Pr < 276.9
6.2 \times 10^8 < Ra_H < 1.5 \times 10^{10}

using 20 cs fluid as a working fluid was appropriate because it permitted typical \( Ra_H \) to be approached in a 1/18th scale apparatus; Raithby et al. [15] have pointed out that natural convection processes are insensitive to \( Pr > 5 \). Although the range of present Prandtl numbers is higher than the Prandtl number of air, the Nusselt numbers attained in this experiment, the general flow pattern, and heat transfer can be expected to be similar to the full-scale building.

A heat transfer apparatus was then designed to provide the capability to study the heat transfer from an isothermal heated Trombe wall geometry to a rectangular enclosure. Moreover, the temperature profiles and the natural convective flow around this Trombe wall geometry were investigated. A photograph of the assembled apparatus and peripheral components is shown in Figure 3.1. A schematic of the entire experimental system, consisting of a rectangular test enclosure, a water jacket, a vertical heated wall, a water cooling system, and temperature controlling and monitoring instruments, is shown in Figure 3.2. These will be described in detail below.
Figure 3.1 Heat Transfer Apparatus
Figure 3.2 Schematic of the Heat Transfer Apparatus
The rectangular test enclosure with inside dimensions of 30.48 cm (12 in.) in height, 45.52 cm (17.92 in.) in length, and 30.48 cm (12 in.) in depth, was fabricated from 1.27 cm (0.50 in.) thick clear sheet plexiglas. However, to provide a higher heat transfer between the Trombe wall space and the outside of the enclosure, one endwall was constructed from 1.27 cm (0.50 in.), type 6066 sheet aluminum. This was done to simulate a double glazing on south facing wall. The water jacket enclosure, with inside dimensions 41.40 cm x 41.91 cm x 57.15 cm, surrounding the rectangular test enclosure was also made of plexiglas. This water jacket consisted of a separate 3.81 cm wide rectangular channel for each face of the rectangular test enclosure except the bottom face which was insulated by R-11 fiberglas. The two enclosures were fastened together with machine screws and sealed with silicone sealant. Access to the test chamber was gained by removing a rectangular lid on the top water jacket and an inner lid which formed the ceiling of the test space. A groove, 1.27 cm wide and 0.64 cm deep on the inside surface of each lid, was milled to fit a 0.24 cm (0.094 in.) thick rubber gasket. The top was then secured in place with wing nuts and machine screws. This arrangement facilitated access to the test space.
A vertical electrically heated wall with the same duct gap size at the top as the bottom, was employed inside the rectangular test enclosure as an inner test body to simulate the Trombe wall. Three different duct gap sizes, 2.54 cm, 0.635 cm, and zero cm (no duct gap), were studied in this investigation by stopping down the 2.54 cm gap to the desired size with aluminum blocks. A front view and an end view of the inner body are presented in Figure 3.2. The inner body was fabricated from two 0.47 cm thick and 30.23 cm square type 6061 aluminum sheets. Aluminum was selected for the inner body to minimize any temperature gradients on the inner body surface. To construct the duct gaps, the top and bottom edges of each aluminum sheet were milled out providing two notches, 2.54 cm in height and 25.4 cm in length. A 0.79 cm (0.31 in.) thick aluminum spacer was tightly adhered around the margins of the inner surface of each aluminum sheet using a high temperature silicone rubber sealant. This aluminum spacer was 2.54 cm wide along the sides and 1.27 cm wide along the top and bottom of the wall margin. The spacers and aluminum sheets were assembled together with machine screws and the high temperature sealant to form the inner body. The edges of the assembled wall, which contacted the water jacket, were covered with a
0.038 cm (0.015 in.) thick rubber gasket to minimize heat losses and to eliminate fluid flow.

Electrical resistance heat tape supplied the heating for the inner body. The heat tape, 0.064 cm thick and 0.32 cm wide, consisted of an electrical resistance wire sandwiched between an adhesive backed fabric and a metallic insulative foil. The electrical resistance wire was rated at 28.87 ohms/m (8.8 ohms/ft) and a maximum power of 75.46 Watts/m (78.53 Btu/hr/ft). Five sets of these heater tapes were attached to each inner surface of test body. Each set consisted of 10 rows of heater tapes, connected in series. High temperature silicone cement was spread over the tapes after installation to insure adhesion to the surface. Figure 3.3 shows the heater tape arrangement, thermocouple location, and the interior of the test body. The inside of the test body was filled with a styrofoam sheet and fiberglas insulation to minimize internal heat transfer. Input power to the tapes was individually controlled by means of Ohmite Variable Power resistors (0-35 ohms or 0-50 ohms, 150 watts, 2.07 amperes or 1.68 amperes maximum) connected in series with each set of the heater tapes. This combination allowed the two sides of the test body to be maintained at essentially equal and constant
Figure 3.3 Interior of the Inner Body
The cooling system consisting of a chiller, pump, filter, and insulated storage tank provided water to cool the test enclosure. The cooling water was collected from the water jacket through a drain manifold system and pumped through the chiller, into the storage tank, through the filter and a supply manifold system, and back into the water jacket. A uniform flow rate through each channel of the water jacket was accomplished by valves which fed several inlet ports for each channel. This arrangement allowed the temperature of the bottom face of the test chamber to vary while providing a uniform temperature on the remaining faces of the test space. Since the apparatus cannot withstand cooling water pressure above 41 KPa (6 psi), a regulator valve and a 122 cm tall stand pipe were placed in the main outlet from the water jackets to maintain the pressure around 13.8 KPa (2.0 psi) under normal operation and emergency shutdown conditions. Three aircocks were placed on the water jacket lid to release air bubbles from the jackets.

The inside surface temperature of the rectangular test space enclosure was monitored by using 27 copper-constantan thermocouples epoxied 0.08 cm from the inner surface. There
were five thermocouples per face of the test enclosure except the bottom face which had only two. All thermocouple leads exited through holes drilled in the cooling water jackets. The holes were sealed tightly with epoxy.

The temperature of the heated vertical test wall was monitored using 10 copper-constantan thermocouples placed in each face of the heated wall. The thermocouple leads and heat tape leads passed through holes in the sides of the spacers, one lead per hole, and were brought downward inside slots in the sides of the vertical heated wall. The holes and the slots were sealed with silicone rubber caulk. The leads were directed toward the plexiglas endwall where they exited through two 1.9 cm inside diameter, 10.2 cm long stainless steel tubes, which passed through a hole in the bottom of the apparatus and then threaded into the test chamber at the corners of plexiglas endwall. At both ends of the tubes, the leads were sealed off from the test enclosure with silicone rubber sealant.

Local fluid temperatures in the test space were measured using seven thermocouple probes to traverse a plane in the middle of the enclosure and perpendicular to the inner body. These probes consisted of a copper-constantan thermocouple coated with magnesium oxide insulation and
sheathed with a 0.32 cm diameter, type 304 stainless steel tube. The region between each thermocouple bead and tubing end was streamlined with an epoxy cone. This arrangement was intended to limit heat conduction along the probe shaft from the measuring point. Three 0.38 cm diameter holes were drilled in the plexiglas endwall and four holes were drilled in the aluminum endwall along the vertical center. These holes were counter-bored to a diameter of 0.51 cm to a depth of one half the wall thickness to allow placement of O-rings and aluminum or plexiglas tubes in the holes. This design allowed the probes to be passed through the tubes and into the test space without seepage of the working fluid or cooling water. Four of the probes were positioned in the aluminum endwall of the enclosure at 0.18 cm, 10.16 cm, 20.32 cm and 30.30 cm from the enclosure top. The other probes were positioned in the plexiglas endwall at 2.54 cm, 10.16 cm, and 22.86 cm from the enclosure top.

The rectangular test space enclosure and the inner body were painted flat black except for a light source slit the length of the chamber located in the center of the ceiling and one clear side face allowing flow observations and photographs to be taken. A lighting system equipped with two 650 Watt, high intensity, tungsten halogen lamps
provided a thin collimated beam of light necessary for flow visualization.

**EXPERIMENTAL PROCEDURE**

The inner body was placed perpendicular to the floor at 2.54 cm (1.0 in.) from the aluminum endwall in the rectangular test space. Hence, the rectangular test space was separated into two parts by the inner body as shown in Figure 3.2. One was the left hand side test space (or Trombe wall space) with aspect ratio 12. The other was the right hand side test space (or living space) with aspect ratio 0.75. The lead wires from the inner body along the floor were sealed off from the test space with silicone rubber caulk and connected as shown in Figure 3.2. Final assembly involved securing the inner lid and the rectangular lid on the top water jacket and connecting the plumbing associated with the chilling system.

Distilled water was added to the system through the stand pipe until all of the air was forced out of the water jacket. The cooling system was activated. The test fluid, 20 cs silicone oil, was then introduced into the rectangular test space by gravity feed. The gravity fed fluid flowed through a filler stem located on the bottom of the apparatus and exited out of a 0.79 cm i.d. stainless tube which
threaded into the ceiling of the test chamber. The tube was flush with the inside surface of the ceiling. Power was applied to the heater tapes and adjusted until the desired isothermal temperature condition was achieved on the surface of the inner body. At this condition, no conduction could occur between these faces. Simultaneously, cooling water flow rates were adjusted to attain an isothermal condition on all faces but the bottom face of the test enclosure. The establishment of the isothermal conditions and thermal equilibrium within the test chamber were of primary importance. Once thermal equilibrium was reached (approximately six hours) the following data were recorded for each run: (1) The inner body and outer body thermocouples millivolt readings, (2) The input amperage to each of the inner body heater tapes, (3) The input voltage to each of the inner body heater tapes. The program used to reduce the data is presented in Appendix I.

A minimum of 12 heat transfer data points were obtained with each duct gap arrangement. A list of the partially reduced data is provided in Appendix II. In addition, twelve temperature profile sets were collected in conjunction with selected data runs. Each of seven thermocouple probes was inserted into the test space until
it contacted the inner body. It was then withdrawn in small increments until the probe tip was flush with the endwall of the test space. The thermocouple readings and position along the traverse were recorded at each step of the withdrawal. Positions were measured within 0.05 cm.

In order to minimize errors in the data, all instruments were calibrated before the test. Comparison of some of the thermocouples with an OMEGA type 2809 Digital Thermocouple meter revealed that the standard calibration was accurate, with a maximum error of about plus 1.6°C (2.8°F). Measurements of power input at the inner body were accurate to within plus or minus one percent. All the data were collected at the condition of room temperature in the range of 21.1°C (70°F) to 26.7°C (80°F) and pressure around one atmosphere.

The percent temperature variation for either the inner body or the outer body was defined as

\[
\text{Temperature Variation \%} = \frac{T_{\text{local, max}} - T_{\text{local, min}}}{T_h - T_c} \times 100
\]

(3-1)

where \(T_{\text{local, max}}\) (\(T_{\text{local, min}}\)) represents the maximum (minimum) inner or outer body temperature. The average temperature variation for the left face of the inner body
was 9.6%, with a maximum variation of 15.9%. For the right face of the inner body, these quantities were 11.4% and 18.5%, respectively. For the outer body, they were 13.65% and 24.37%, respectively.

The heat transferred by natural convection was calculated by

\[ Q_{\text{conv}} = Q_{\text{tot}} - Q_{\text{cond}} - Q_{\text{rad}} \]  

Since the 20 cs fluid was opaque to radiation only the heat loss due to conduction had to be determined.

To determine the conduction heat loss, the inner body was moved to the central position of the test space. The test space and the two end water jackets were then filled with styrofoam sheets and fiberglas. This arrangement permitted neither convection nor radiation to occur within the test space and limited conduction in the space. With the complete system assembled as described, the cooling flows to the end water jackets were cut off and a minimum of six heat transfer data points were taken over the range of temperature difference for the zero cm gap (no duct gap) and 2.54 cm duct gap arrangements. The temperature difference was calculated as the average temperature on each face of the inner body minus the average temperature of the top and sides of the outer body. The data are shown in Figure 3.4.
Figure 3.4 Heat Loss by Edge Conduction

- □ - 2.54 cm (1 in.) gap
- ○ - Zero gap

Solid symbols - left face
Open symbols - right face
These data were used in a least squares curve fit to develop an equation for each face of the zero gap and 2.54 cm gap. For the inner body with zero gap, the resulting equation for the left face was (Q_{cond} in watts and ΔT in degrees Kelvin)

\[ Q_{\text{cond}} = 0.2574 (\Delta T)^{1.0495} \]  

(3-3)

with an average percent deviation of 5.39 and maximum percent deviation of 13.42, and for the right face

\[ Q_{\text{cond}} = 0.2483 (\Delta T)^{1.0115} \]  

(3-4)

with a 5.82% average deviation and 15.17% maximum deviation.

For the inner body with a 2.54 cm gap, the resulting equation for the left face was

\[ Q_{\text{cond}} = 0.188 (\Delta T)^{1.0709} \]  

(3-5)

with a 6.13% average deviation and 10.32% maximum deviation, and for the right face

\[ Q_{\text{cond}} = 0.1825 (\Delta T)^{1.0304} \]  

(3-6)

with a 7.1% average deviation and 14.43% maximum deviation.

No heat conduction heat loss data were taken for the inner body with 0.635 cm gap, the heat conduction losses were calculated from the resulting equations of the case of the 2.54 cm (1 in.) gap because the same edge area contacted the test enclosure. These equations were used to correct the heat transfer data in subsequent measurement. The average percentage of heat loss by conduction to the heat
transferred by convection was 7.99 for the no gap, 6.1 for
the 0.635 cm gap, and 5.8 for the 2.54 cm gap.

Once the heat transferred by natural convection was
known, the average heat transfer coefficient was obtained
from

\[ h = \frac{Q_{\text{conv}}}{A_i (T_h - T_C)} \]  \hspace{1cm} (3-7)

where \( A_i \), 929 cm\(^2\) (1.0 ft\(^2\)), is the surface heat transfer
area of the inner body with zero gap. This area is the same
as the surface area on the south facing aluminum wall. For
the ease of the engineering calculations, this area was
chosen for all the inner body gap arrangements. The fluid
properties were evaluated by an existing function subroutine
[23] based on the fluid film temperature which is defined by

\[ T_f = \frac{(T_h + T_C)}{2}. \]  \hspace{1cm} (3-8)

To observe the flow phenomena within the test space,
tracer particles were made by spraying and mixing
fluorescent paint particles into the 20 cs test fluid.
These paint particles were visible when the midplane was
illuminated by high intensity lights. At the steady state
condition, the flow pattern was photographed with a Calumet
4x5 professional camera using Polaroid, type 52 positive
film.
CHAPTER IV
DISCUSSION OF RESULTS

The heat transfer data are correlated in terms of
dimensionless parameters which are derived from the
nondimensional form of the governing equations of mass,
momentum, and energy. The independent dimensionless
parameters are the Rayleigh number, the Prandtl number, and
the geometric aspect ratio. The dependent dimensionless
parameter, the Nusselt number, was calculated in the
following functional forms to fit the heat transfer data by
using a least squares method of curve fitting:

\[ \text{Nu}_S = C_1 \text{Ra}_S^{C_2} \quad (4.1) \]
\[ \text{Nu}_S = C_1 \text{Ra}_S^{C_2} \text{Pr}^{C_3} \quad (4.2) \]
\[ \text{Nu}_S = C_1 \text{Ra}_S^{C_2} \text{Ac}^{C_3} \quad (4.3) \]
\[ \text{Nu}_S = C_1 \text{Ra}_S^{C_2} \text{Pr}^{C_3} \text{Ac}^{C_4} \quad (4.4) \]

where \( s \) is some characteristic length (\( H, L_1 \) or \( L_2 \)) and \( A \) is
an aspect ratio (\( H/L \)). For all the data combined, the
aspect ratio \( A \) was replaced by the height ratio \( H_i/H \) which
is defined as the ratio of height of the inner body to the
outer body. All the notations of the inner and outer bodies
are given in to Figure 1.1. Three characteristic lengths,
the height \( H \), Trombe wall space length \( L_1 \), and living space
length \( L_2 \) of the rectangular enclosure were adopted to
calculate the Rayleigh and Nusselt numbers. The range of the
correlating parameters is provided in Table 4.1.

The following sections will discuss the effect of geometry on heat transfer. Temperature profiles, flow visualizations, and comparisons will be included in each section as they are applicable.

**GEOMETRIC EFFECTS**

When the gap of the inner body was decreased from the 2.54 cm (1 in.) gap to the 0.635 cm (0.25 in.) gap or the zero cm gap, there was a decrease in the heat transferred. This decrease was exhibited for both the Trombe wall space and the living space. This decrease is most pronounced in the low Rayleigh number range and becomes less in the high Rayleigh number range as shown in Figure 4.1. For the overall heat transfer, the 0.635 cm gap has an average Nusselt number 4.14% less than the 2.54 cm gap's average Nusselt number and the zero gap has an average Nusselt number 3.88% less than the 2.54 cm gap's average Nusselt number.

Since each face of the inner body was separately heated to maintain isothermal conditions, a similar analysis was possible for the left hand test space (Trombe wall space) and right hand test space (living space) as shown in Figure 4.2 and Figure 4.3. These two separate test spaces are
TABLE 4.1
RANGE OF DIMENSIONLESS PARAMETERS

<table>
<thead>
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<th>DIMENSIONLESS PARAMETER</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
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</thead>
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<tr>
<td>$N_u_H$</td>
<td>55</td>
<td>109</td>
</tr>
<tr>
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<td>73</td>
<td>130</td>
</tr>
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<td>9.3</td>
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<td>$8.2 \times 10^6$</td>
</tr>
<tr>
<td>$Pr$</td>
<td>125</td>
<td>277</td>
</tr>
</tbody>
</table>
OVERALL HEAT TRANSFER

□ - 2.54 cm (1 in.) gap
△ - 0.635 cm (0.25 in.) gap
○ - Zero gap

\[ \text{Nu}_H = 1.5079 \text{Ra}_H^{0.1823} \]

Figure 4.1 Geometric Effect for All Gaps and A Heat Transfer Correlation with All Data Combined
HEAT TRANSFER TO LEFT HAND SIDE SPACE

- □ - 2.54 cm (1 in.) gap
- △ - 0.635 cm (0.25 in.) gap
- ○ - Zero gap

Figure 4.2 Geometric Effect for All Gaps and A Heat Transfer Correlation with All Data Combined

\[ \text{Nu}_H = 1.3657 \text{Ra}_H^{0.1894} \]
Figure 4.3 Geometric Effect for All Gaps and A Heat Transfer Correlation with All Data Combined
denoted LHS and RHS, respectively, in the following discussions. For the heat transfer in LHS, the 0.635 cm gap and the zero gap have average Nusselt numbers 3.84% and 4.26% less than the 2.54 cm gap's average Nusselt number. For the heat transfer in RHS, the values are 5.57% and 4.3%. For the 2.54 cm gap geometry the warmer fluid rose from the left face of the inner body and passed through the upper gap into the RHS test space causing an exchange of mass between these two spaces. Flow visualization data also support this fact as seen in the flow patterns of Figure 4.4 and Figure 4.5 for the zero gap and the 2.54 cm gap, respectively. Photographs of the flow patterns at the upper portion of the living space are shown in Figures 4.6 and 4.7. This exchange of mass may have resulted in the higher heat transfer for lower Rayleigh numbers for the 2.54 cm vented inner body than for the unvented inner body within the test space since no exchange of mass could happen for the unvented inner body. Since the 0.635 cm gap is rather small, it only shows a slight difference (below 1%) in heat transfer coefficients compared to the zero gap.

Figure 4.4, which shows the flow pattern in the living space for the inner body with no gap, is described by the following:
Figure 4.4  A Sketch of Flow Pattern for the Inner Body with Zero Gap
\[ \Delta T = 30.80^\circ K (55.44^\circ R), \quad \text{Ra}_H = 4.79 \times 10^9 \]
Figure 4.5 A Sketch of Flow Pattern for the Inner Body with 2.54 cm Gap

$\Delta T = 31.33^\circ K (56.39^\circ R), \text{Ra}_H = 4.89 \times 10^9$
Figure 4.6 (a) A Photograph of Flow Pattern in the Living Space for Zero Gap (Upper Left)
Figure 4.6 (b) A Photograph of Flow Pattern in the Living Space for Zero Gap (Upper Right)
Figure 4.7 (a) A Photograph of Flow Pattern in the Living Space for 2.54 cm Gap (Upper Left)
Figure 4.7 (b) A Photograph of Flow Pattern in the Living Space for 2.54 cm Gap (Upper Right)
1) The boundary regions comprise a relatively high velocity clockwise circulation cell A in which the streamlines closely followed the shape of the living space.

2) A clockwise circulation cell B at the upper portion of the living space resulted from the lower density fluid rising from the hot wall and interacting with the conducting cold ceiling.

3) A streamline C was observed below the cell B. The streamline C started near the cold vertical boundary and ended near the hot wall.

4) Weak motion was not detected in the core region until the shutter speed of the camera was reduced to 24 seconds, and outside this region near the walls again is a horizontal entrainment flow.

5) A clockwise eddy D was observed in the upper corner adjacent to the heated wall.

The flow pattern in the living space for the inner body with a 2.54 cm gap is different from the inner body with no gap as shown in Figures 4.4 and 4.5. The distinguishing difference was observed inside the gap region. The clockwise circulation of cell A was deformed and interacted with the counter-clockwise circulation of main cell in the
Trombe wall space due to the existence of the gap. A streamline which passed through the upper gap into the living space was separated from the counterclockwise circulation of the main cell. A clockwise eddy (perhaps more than one) was observed at the upper gap region. For the flow pattern with the 0.635 cm gap, no distinguishing difference was observed in comparison to the no gap except in the gap regions where there was an exchange of mass.

A comparison of temperature profile data with all of the inner body gaps at approximately the same temperature difference is shown in Figure 4.8. The plot uses a dimensionless temperature and dimensionless length which are defined as

\[ T^* = \frac{T - T_c}{T_h - T_c} \]

where \( T \) is the local temperature measured at any vertical position, \( Y^* = y/H \), along midplane within the test space and \( X^* = x/L_1 \), where \( x \) is the local distance from the aluminum endwall to the test point in the test space and \( L_1 \) is the distance between the inner body and the aluminum endwall. The temperature profile was plotted in two different scales. For \( X^* \) from 2.5 to 17 the smaller scale plot is inset in the
Open Symbols - $\Delta T = 30.9^\circ K (55.5^\circ R)$, $Ra_H = 4.4 \times 10^9$, 2.54 cm gap
Solid Lines - $\Delta T = 30.8^\circ K (55.4^\circ R)$, $Ra_H = 4.5 \times 10^9$, 0.635 cm gap
Solid Symbols - $\Delta T = 30.3^\circ K (54.6^\circ R)$, $Ra_H = 4.4 \times 10^9$, no gap

Figure 4.8 Comparison of Temperature Profiles for All Inner Body Gaps
right upper portion of the figure. In order to display the temperature distribution adjacent to the walls, a larger scale plot is presented in the main portion of the figure. When the gap size of the inner body was increased, the thermal stratification within the living space also increased as shown in Figure 4.8. This suggests that the vented inner body tends to provide a larger thermal stratification than the unvented inner body in the living space due to the exchanges of mass, momentum, and energy between the Trombe wall space and the living space.

The temperature profile along $Y^* = 0.08$ and $Y^* = 0.33$ within the living space is increased with increasing gap sizes. This implies that the larger gap size tends to increase convective heat transfer to the upper portion of the living space. Along $Y^* = 0.75$, the 0.635 cm gap has a higher temperature profile in the RHS than the 2.54 cm gap. Throughout the LHS space, the no gap inner body gives the highest temperature profiles. This is not surprising, since no lower temperature mass could pass through the lower gap from the RHS space.

Figures 4.9 through 4.11 present the dimensionless temperature profile plots at different $\Delta T$ for all inner body gaps. The magnitude of mid-range dimensionless temperature
Figure 4.9 Variation of Temperature Profile with $\Delta T$, Inner Body with Zero Gap
Figure 4.10 Variation of Temperature Profile with ΔT, Inner Body with 0.635 cm Gap
Figure 4.11 Variation of Temperature Profile with $\Delta T$, Inner Body with 2.54 cm Gap

- Open Symbols - $\Delta T = 65.6^\circ K (118.1^\circ R)$, Pr = 125.5
- Solid Symbols - $\Delta T = 46.1^\circ K (83.1^\circ R)$, Pr = 162.5

- $\triangle - Y^* = 0.01$
- $\bigcirc - Y^* = 0.08$
- $\bigdiamond - Y^* = 0.33$
- $\blacktriangledown - Y^* = 0.33$
- $\square - Y^* = 0.67$
- $\blacksquare - Y^* = 0.75$
- $\bigtriangledown - Y^* = 0.99$
profiles generally increased with increasing $\Delta T$. Several features common to each of the temperature profiles were observed. These are (1) the temperature profiles are almost antisymmetric about the center line either of the Trombe wall space or of the living space. The major deviations from antisymmetry are near the hot and cold walls where fluid viscosities were significantly affected by temperature. (2) The appearance of thermal boundary layers can be observed near the surfaces and the profiles are horizontal in the central region in the living space. (3) The profiles throughout the Trombe wall space have no horizontal parts. In a sense this situation can be considered as one in which the boundaries are not thin relative to the Trombe wall space and, therefore, they interact. (4) Near $Y^* = 0$ the temperature distribution is dominated by the heat lost through the cold ceiling, and nearly linear profiles were observed in the Trombe wall space. Near $Y^* = 1$ below the bottom vent in and the bottom portion of the living space, the temperature distribution was nearly constant because the bottom surface was insulated. Similarities of the temperature profiles to Warrington's [23] were a steep drop at the inner body, an inner transition region, a level zone, an outer
transition zone, and a fairly steep drop at the cold endwall. The five zones are convincingly observed only in the RHS test space. In the profiles for the LHS test space, no level zone was observed due to the narrow space between the hot and cold walls (i.e., high aspect ratio 12).

**HEAT TRANSFER RESULTS FOR EACH INNER BODY GAP**

The data obtained from the inner body with each gap size were correlated separately for both the left face and right face of the inner body. This is also referred to the Trombe wall space and the living space. Moreover, a separate correlation for the overall heat transfer (the sum of heat transfer for the left and right faces) data is presented. The notation throughout the remainder of this discussion will consist of presenting a two-part designation in which the gap size will be given first, followed by the inner body's face (i.e., zero-left, zero-right). When combined data are presented the last term will be all.

The present data from the zero-left and zero-right experiments are plotted in Figures 4.2 and 4.3, respectively. The recommended equation which correlates the heat transfer with a single parameter for zero-left data was

\[ \text{Nu}_H = 0.904 \text{Ra}_H^{0.208} \] (4.5)

with an average percent deviation of 1.09 and maximum
percent deviation of 3.28. The percent deviation at a point is defined as the absolute difference between the data value and the equation value divided by the data value. The average percent deviation is the sum of the individual deviations divided by the number of data points. The recommended best correlation for the zero-right data with one parameter was

$$\text{Nu}_H = 0.862 \text{Ra}_H^{0.205}$$ (4.6)

with 1.38% average deviation and 5.54% maximum deviation.

The best equation using a single correlation parameter for all the zero gap data was

$$\text{Nu}_H = 0.887 \text{Ra}_H^{0.206}$$ (4.7)

with 5.27% average deviation and 8.27% maximum deviation. A correlation including aspect ratio resulted in an average deviation of 1.22% with a maximum deviation of 5.49%. This correlation was

$$\text{Nu}_H = 0.847 \text{Ra}_H^{0.206}A^{0.038}$$ (4.8)

The Prandtl number of the 20 cs fluid varied from 124.7 to 276.9 in this investigation. The best correlation including the Prandtl number effect was

$$\text{Nu}_H = 0.1184 \text{Ra}_H^{0.252}\text{Pr}^{0.191}$$ (4.9)

with 5.25% average deviation and 8.01% maximum deviation.

The effective heat transfer area was used to calculate
the heat transfer data for the zero gap only when making a comparison with Raithby's predication. The method for defining this effective heat transfer is described in Appendix III. Comparison to the results of Raithby et al. [15] was made with a plot of Nu versus Ra based on enclosure length, L1, for the zero-left data as shown in Figure 4.12. It appears to give higher average Nusselt numbers than Raithby's analytical approach in the laminar regime, especially in the lower Rayleigh number range. This is because the heat transfer in the lower Rayleigh number range was significantly affected by the upper horizontal conducting boundary. The flow in the Trombe wall space was dominated by the entrainment flow between vertical boundaries in the higher Rayleigh number range. As the Rayleigh number increased, the direct flow from the vertical hot wall to cold wall probably was increased thereby lessening the role played by the upper horizontal surface in the heat transfer. Figure 4.13 shows that when Rayleigh number increases, the flow pattern is modified. Probably the direct flow from the hot wall to the cold wall yielded higher velocities at high Rayleigh numbers than the flow at lower Rayleigh numbers. Hence, for the higher aspect ratios and higher Rayleigh numbers, the heat transfer in the cavity
Zero-Left (A = 12)

- Present Data
- Raithby et al.

Turbulent Regime
Laminar Regime

Figure 4.12 Comparison of Present Data with Raithby et al.
Figure 4.13 Flow Patterns for Zero-Left

(a) $\text{Ra}_H = 4.8 \times 10^9$  (b) $\text{Ra}_H = 9.0 \times 10^9$
with the conducting upper horizontal surface and insulated lower horizontal surface tends to be similar to the case of the adiabatic horizontal surfaces.

For the zero-right, a comparison with Raithby's general approximation was also made as shown in Figure 4.14. The present heat transfer data give slightly lower results with Raithby's analytical prediction. This is not surprising, since Raithby's correlation is not claimed to be valid for \( A < 5 \).

Figure 4.15 shows that the heat transfer in the LHS test space is higher than the heat transfer in the RHS test space. The average Nusselt number of the LHS space was 9.84% greater than the average Nusselt number of the RHS space. This is due to the different flow patterns in the spaces as mentioned previously.

Table 4.2 and Table 4.3 present the calculated constants, average deviation, and maximum deviation for the correlations for each inner body gap size in the LHS test space and RHS test space, respectively. The equation using a single correlation parameter for all three inner body gap sizes in the LHS test space is

\[
\text{Nu}_H = 1.3657 \text{ Ra}_H^{0.1894}
\]  

(4.10)

with 2.37% average deviation and a 8.26% maximum deviation.
Figure 4.14 Comparison of Present Data with Raithby et al.

Zero-Right (A = 0.75)

- Present Data
- Line - Raithby et al.

Laminar Regime
Figure 4.15 Comparison of Heat Transfer Results between LHS and RHS for Zero Gap
### TABLE 4.2
CORRELATIONS FOR LHS TEST SPACE

<table>
<thead>
<tr>
<th>EQUATION FORM</th>
<th>EMPIRICAL CONSTANTS</th>
<th>AVERAGE % DEVIATION</th>
<th>MAXIMUM % DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C₁</td>
<td>C₂</td>
<td>C₃</td>
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<tr>
<td>Zero cm gap</td>
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<tr>
<td>4.1</td>
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<td>4.2</td>
<td>.1518</td>
<td>.2483</td>
<td>.1689</td>
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<td></td>
<td></td>
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<td>.2848</td>
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<td>4.2</td>
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<td></td>
<td></td>
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<tr>
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<td>.1647</td>
<td>.2542</td>
</tr>
<tr>
<td>4.2</td>
<td>.0330</td>
<td>.2354</td>
<td>.2542</td>
</tr>
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<td>All</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>.1894</td>
<td>.3704</td>
</tr>
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<td>4.3</td>
<td>1.395</td>
<td>.1877</td>
<td>-.2078</td>
</tr>
<tr>
<td>4.4</td>
<td>.0419</td>
<td>.2709</td>
<td>.3183</td>
</tr>
<tr>
<td>EQUATION FORM</td>
<td>EMPIRICAL CONSTANTS</td>
<td>AVERAGE % DEVIATION</td>
<td>MAXIMUM % DEVIATION</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------</td>
<td>-------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td></td>
<td>( c_1 )</td>
<td>( c_2 )</td>
<td>( c_3 )</td>
</tr>
<tr>
<td>Zero cm gap</td>
<td>( 0.1088 )</td>
<td>( 0.2049 )</td>
<td>( 1.37 )</td>
</tr>
<tr>
<td>4.1</td>
<td>0.635 cm (0.25 in.) gap</td>
<td>4.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( 0.3651 )</td>
<td>( 0.3832 )</td>
<td>( -0.2159 )</td>
</tr>
<tr>
<td>4.2</td>
<td>2.54 cm (1 in.) gap</td>
<td>4.20</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>( 1.5734 )</td>
<td>( 0.1778 )</td>
<td>( 3.09 )</td>
</tr>
<tr>
<td>4.1</td>
<td>( 0.00793 )</td>
<td>( 0.3032 )</td>
<td>( 4.809 )</td>
</tr>
<tr>
<td>4.3</td>
<td>( 1.6107 )</td>
<td>( -0.2159 )</td>
<td>( -0.1726 )</td>
</tr>
<tr>
<td>4.4</td>
<td>( 0.01340 )</td>
<td>( 0.2897 )</td>
<td>( 0.4350 )</td>
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</table>
The correlation which provides the best fit of all combined data is

$$\text{Nu}_H = 0.0419 \text{Ra}_H^{0.2709} (\text{Pr})^{0.3183} (H_i/H)^{-0.172} \quad (4.11)$$

with an average percent deviation of 1.77 and maximum percent deviation of 4.82. The equation with one parameter using all of the data combined in the RHS test space is

$$\text{Nu}_H = 1.5734 \text{Ra}_H^{0.1778} \quad (4.12)$$

with a 3.09% average deviation and 8.65% maximum deviation. The best correlation for the RHS test space of all three inner body gap sizes combined is

$$\text{Nu}_H = 0.0134 \text{Ra}_H^{0.2897} (\text{Pr})^{0.435} (H_i/H)^{-0.1726} \quad (4.13)$$

with 2.37% average deviation and 6.45% maximum deviation.

Correlations for all three inner body gaps only dealt with the overall heat transfer data, (as shown in Figure 4.1), from the inner body to the outer body. These are presented in Table 4.4. The correlation which provides the best fit of all combined data with three parameters is

$$\text{Nu}_H = 0.0329 \text{Ra}_H^{0.2719} (\text{Pr})^{0.3496} (H_i/H)^{-0.1543} \quad (4.14)$$

with a 1.98% average deviation and 5.07% maximum deviation. Table 4.4 also presents the calculated constants for the data of all gap sizes using one parameter. The equation is

$$\text{Nu}_H = 1.5079 \text{Ra}_H^{0.1823} \quad (4.15)$$

with 2.65% average deviation and 8.11% maximum deviation.
### TABLE 4.4
OVERALL HEAT TRANSFER CORRELATIONS

<table>
<thead>
<tr>
<th>EQUATION FORM</th>
<th>EMPIRICAL CONSTANTS</th>
<th>AVERAGE % DEVIATION</th>
<th>MAXIMUM % DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>C₁</td>
<td>C₂</td>
</tr>
<tr>
<td>Zero cm gap</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>.9131</td>
<td>.2047</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>.1616</td>
<td>.2442</td>
<td>.1641</td>
</tr>
<tr>
<td>0.635 cm (0.25 in.) gap</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>1.2962</td>
<td>.1886</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>.0571</td>
<td>.2632</td>
<td>.2812</td>
</tr>
<tr>
<td>2.54 cm (1 in.) gap</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>3.0013</td>
<td>.1522</td>
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</tr>
<tr>
<td>4.2</td>
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<td>.2321</td>
<td>.2878</td>
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<tr>
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<td></td>
</tr>
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</tr>
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<td>.3958</td>
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<td>4.3</td>
<td>1.5395</td>
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<td>-.1940</td>
</tr>
<tr>
<td>4.4</td>
<td>.0329</td>
<td>.2719</td>
<td>.3496</td>
</tr>
</tbody>
</table>
The best correlation for overall heat transfer data combined including geometric effects is

\[ \text{Nu}_H = 1.5395 \text{Ra}_H^{0.1807} \left( \frac{H_1}{H} \right)^{-0.194} \]  \hspace{1cm} (4.16)

with an average percent deviation of 2.45 and maximum percent deviation of 6.17.

All of the above correlating parameters are only based on the height of the enclosure, H. In order to obtain the empirical equation based on the length of the enclosure, a transformation was derived for the living space

\[ \text{Nu}_{L_2} = C_1 \left( \frac{\text{Ra}_{L_2}}{C_2} \right)^{3C_2^{-1}} \frac{\text{Pr}^{C_3}}{A^C} \]  \hspace{1cm} (4.17)

and for the Trombe wall space

\[ \text{Nu}_{L_1} = C_1 \left( \frac{\text{Ra}_{L_1}}{C_2} \right)^{3C_2^{-1}} \frac{\text{Pr}^{C_3}}{A^C} \]  \hspace{1cm} (4.18)

where \(H/L_1 = 12\) and \(H/L_2 = 0.75\). These transformations will not change the average and maximum deviations of the correlations.

The flow regime in the living space may be similar to a vertical plate in the infinite medium since the aspect ratio of 0.75 could be considered small. Eckert and Jackson [4] found the transition (to turbulent) regime was in the Rayleigh number range between \(10^8\) to \(10^{10}\) for their isothermal vertical plate study. This may imply that the flow in the living space seen in the present study is in the
transition regime, since $Ra_H$ is in that range. The flow in the Trombe wall space may be expected to resemble flow in a rectangular enclosure with adiabatic horizontal surfaces. As Elder [12] mentioned, for $Ra_L A^3/Pr^{1/2}$ greater than $10^9$ a turbulent behavior may appear. In the present study, $(Ra_L A^3)/Pr^{1/2}$ is in the range of $4.1 \times 10^7$ to $9.8 \times 10^8$. This suggests that the flow in the Trombe wall space is also in the transition regime.
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

This investigation has presented a study of natural convection heat transfer from an isothermal vertical heated wall with the same size gap at the top and the bottom to a rectangular enclosure. Three different gap sizes, 2.54 cm gap, 0.635 cm gap, and zero gap, were investigated. Several conclusions can be drawn:

1) The 2.54 cm vented inner body, shows a higher heat transfer behavior than the 0.635 cm vented and unvented inner body within the rectangular test space, especially in the lower Rayleigh number ranges.

2) As the gap size of the inner body increased, thermal stratification within the living space also increased.

3) The dimensionless temperature profile increased with increasing vertical distance from the enclosure bottom. An exception occurs very near the top where the dimensionless temperature approaches zero.

4) The temperature profile is nearly linear near the top of the Trombe wall space. Temperature is nearly constant in the bottom of the test space. Similarities of the temperature profiles to Warrington's are a steep drop at the inner body, an inner transition region, a nearly constant temperature zone (not in Trombe wall
space), an outer transition zone, and a fairly steep drop at the cold wall.

5) The flow pattern at the upper portion of the living space for all of the inner body gap sizes shows a clockwise circulation cell instead of a single circulation cell enclosing the entire region of the living space as seen in other studies. This is due to an asymmetry of horizontal surface boundary conditions.

6) For the higher aspect ratios and higher Rayleigh numbers, heat transfer in the cavity is independent of the horizontal boundary conditions.

Since the results of this investigation are the first reported concerning the natural convection heat transfer from a vertical heated wall with various vent sizes to a rectangular enclosure, no direct comparison with previous results can be made. The present experiment is a 1/18th scale model study of the natural convection process in a two story high room in passive solar heating with a Trombe wall geometry. However, steady convection heat transfer data for the full-scale structure has not been published to date, nor could it be calculated from the past investigations of similar Trombe wall houses. Direct comparison of the present results with full-scale structures is truly difficult at the
present time. This study, however, has provided not only a method to tackle the problem of natural convection within a rectangular enclosure but also a method to obtain data relevant to passive solar heating with a Trombe wall geometry.

The present temperature profile results show a large thermal stratification in the living space because of the existence of a circulation cell at the upper portion of the living space. This stratification could possibly be reduced by two methods. One method is the top gap of the inner body could be moved downward to somewhere above the central position of the inner body. The other is that the upper horizontal surface could be insulated as well as the bottom surface to create a single circulation cell enclosing the entire region of the living space according to the previous rectangular enclosure studies with adiabatic horizontal surface boundary conditions.

Several correlations, in terms of one, two, or three correlating parameters, were developed to predict the heat transfer in the living space. The recommended empirical equations are

\[
\text{Nu}_H = 1.5734 \text{ Ra}_H^{0.1778} \quad (5.1)
\]

\[
\text{Nu}_H = 1.6107 \text{ Ra}_H^{0.1760} (H_1/H)^{-0.2159} \quad (5.2)
\]
and \( \text{Nu}_H = 0.0134 \text{Ra}_H^{0.2897} \text{Pr}^{0.4235} (\text{H}_i/\text{H})^{-0.1726} \) (5.3) for \( 6.2 \times 10^8 < \text{Ra}_H < 1.5 \times 10^{10}; 124.7 < \text{Pr} < 276.9; 0.833 < (\text{H}_i/\text{H}) < 1.0 \) which have average percent deviation of 3.50, 3.01 and 2.37, respectively.

This study could be extended to determine the extent of the three-dimensional effects by making additional planar observations and flow velocity measurements. Reflections from the tracer particles that have settled onto the upper vent surface of the inner body and the bottom face of the outer body caused visual and photographic problems that need to be eliminated. Future work with the apparatus is planned. Other test fluids are to be used to extend the Prandtl and Rayleigh number ranges to detect the transition from laminar to turbulent flow.
APPENDICES
APPENDIX I
HEAT TRANSFER DATA REDUCTION PROGRAM

The following is a data reduction program which computes and correlates all of the dimensionless groups. All of the variables, subroutines and function subprograms are defined within the program.
HEAT TRANSFER DATA REDUCTION AND CORRELATION PROGRAM

**ENCLOSURE LENGTH**

GL = 18.0

102 FORMAT(1X,F4.2)
66 FORMAT(' TROMBE WALL POSITION AT ',F3.1,' IN')
READ(105,88) AL(1)
OUTPUT '  '
READ(105,102) DG
88 FORMAT( '  TROMBE WALL DUCT GAP EQUALS ',1X.F4.2,' IN')
AL(1) = GL - AL(1) * 2.0
CALL TAV(TEMP, TB, TM)
TM(S) = 0.0; TM(IO) = 0.0
GTOT = 0.0

**CALCULATE HEAT FLUX**

OUTPUT '  HEAT TAPE VOLTAGE CURRENT(A) TEMP(F) TEMP(K) '  
DO 43 NI = 1, 10
TK(NI) = (TM(NI) + 459.63) / 1.8
TK(S) = 0.0; TK(IO) = 0.0
READ(105,333) EMF, VOL
333 FORMAT(1X,F5.1,1X,F6.4)
OHM(NI) = 1.0
CURNT = VOL / OHM(NI)
111 FORMAT(7X,I2,7X,F5.1,4X,F8.4,4X,' * ',F7.3,F9.0)
POWER = EMF * CURNT
WRITE(108,111) NI, EMF, CURNT, TM(NI), TK(NI)
GTOT = GTOT + POWER
IF(NI.EQ.5) THEN
43 CONTINUE

**LHS: LEFT HAND SIDE HEAT FLUX INPUT**

GTOT(1) = GTOT + 3.413
GTOT = 0.0
ELSE
END IF

**RHS: RIGHT HAND SIDE HEAT FLUX INPUT**

GTOT(2) = GTOT + 3.413
QTOTAL(3)=QTOTAL(1)+QTOTAL(2)

C GCONV(1)=QTOTAL(1):GCONV(2)=QTOTAL(2):GCONV(3)=QTOTAL(3)
OUTPUT ' ':
OUTPUT ' ** AVERAGE TROMBE WALL TEMPERATURE (R) **'
OUTPUT '   LEFT-HAND SIDE   RIGHT-HAND SIDE   OVERALL'
TAVGI(1)=TB(1):TAVG(1)=TB(5)
TAVGI(2)=TB(2):TAVG(2)=(TB(3)+TB(4)+TB(6)+TB(7))/4.0
TAVGI(3)=(TAVGI(1)+TAVGI(2))/2.0
TAVG(3)=(TAVG(1)+4.0*TAVG(2))/5.0
DO 888 MM=1,3
TH(MM)=TAVGI(MM)/1.8
TC(MM)=TAVG(MM)/1.8
888 CONTINUE
WRITE(108,222) (TH(I),I=1,3)
OUTPUT ' ** ITS AVERAGE SURROUNDING COOLED WALL TEMPERATURE (K) **'
WRITE(108,222) (TC(I),I=1,3)
OUTPUT ' ':
222 FORMAT(5X,3(F7.3,11X))
BL(1)=12.0
J=2
OUTPUT ' I  BL  DT (K) NUSSELT NO. PRANDTL NO. GROSHOF NO. * RALEIGH NO. '
DO 7 M=1,2
IF(M.EQ.2) THEN
OUTPUT ' LHS CAVITY TEMP BASED ON AVERGING COLD WALLS TEMP'
TAVG(1)=(TB(3)+TB(4)+TB(5)+TB(7))/4.0
ELSE
END IF
DO 5 J=1,2
C HA: HEAT TRANSFER SURFACE AREA
HA=1.0
DO 6 I=1,3
DT=TAVGI(I)-TAVG(I)
DO 38 II=1,3
DTT=TAVGI(II)-TAVG(II)
DTK(II)=DT/1.8
38 CONTINUE
IF(DG.EQ.0.00) THEN
HLOSS(1)=(EXP(ALOG(.257443)+1.048533*LOG(DTK(1))))*3.413
HLOSS(2)=(EXP(ALOG(.248315)+1.011473*LOG(DTK(2))))*3.413
ELSE
HLOSS(1)=.187955*(DTK(1)*1.070927)*3.413
HLOSS(2)=.182527*(DTK(2)*1.030395)*3.413
END IF
HLOSS(3)=HLOSS(1)+HLOSS(2)
HLOSS(I) = HLOSS(1) + HLOSS(2)
GCONV(I) = GTOTAL(I) + HLOSS(I)
IF(I.EQ.1) BL(2) = AL(I)
IF(I.EQ.2) BL(2) = AL(2)
IF(I.EQ.3) BL(2) = AL(1), HA = 2.0*HA
H = GCONV(I)/(DT*HA)
TAVG = (TAVG(I) + TAVG0(I))/2.0
VIS = U(TAVG,JJ)
SH = CP(TAVG,JJ)
DEN = RHO(TAVG,JJ)
BET = BETA(JJ)
PR(I) = VIS*SH/COND
GR(I) = 32.174*BET*(BL(J)/12.0)**3*DEN*DEN*300*300/(VIS*VIS)
ZZNUSt(I) = H*BL(J)/12.0
RA(I) = GR(I)*PR(I)
WRITE(106,233) I, BL(J), DTK(I), ZZNUS(I), PR(I), OR(I), RA(I)
233 FORMAT(X,11,2X,F4.1,2X,F7.3,2X,F10.3,4X,2(E10.3,4X))
6 CONTINUE
5 CONTINUE
7 CONTINUE
TC(I) = TAVG0(I)/1.8
WRITE(106,999) TC(I)
999 FORMAT(X,'**LHS AVERAGING COLD WALLS TEMP: ',F7.3,'K')
OUTPUT ' I GTOTAL GCONV HLOSS (WATT)* PER CENT LOSS '
DO 8 I = 1,3
GTOTAL(I) = GTOTAL(I)/3.413; GCONV(I) = GCONV(I)/3.413
HLOSS(I) = HLOSS(I)/3.413
HPC = HLOSS(I)/GTOTAL(I)*100
WRITE(106,232) I, GTOTAL(I), GCONV(I), HLOSS(I), HPC
8 CONTINUE
232 FORMAT(2X,I11,3(2X,F8.3),10X,F5.2)
END
C
C CSUBROUTINE TAV(TEMP,TB,TM)
C**** TEMP(R) IS A TEMPERATURE CONV. FROM VOLTAGE READ(MV)
C**** TB IS AVERAGE TEMPERATURE
C**** IL-II+1 IS NUMBER OF DATA POINTS OF THE TEST FLUID
DIMENSION TEMP(300), TB(300), TM(300)
READ(105,88) LW
88 FORMAT(1X,13)
WRITE(108,100)
100 FORMAT(28X,'DATA POINT',' READING(MV)',' TEMP(R)',' TEMP(F)')
NC=0
99 FORMAT(1X,F5.3)
17 READ(105,99) VOLT
NC=NC+1
IF(NC.EQ.1) OUTPUT ' LEFT HAND SIDE TROMBE WALL ' IF(NC.EQ.11)OUTPUT ' RIGHT HAND SIDE TROMBE WALL' IF(NC.EQ.21) THEN OUTPUT ' ** ENCLOSURE SURFACE **' OUTPUT ' FRONT' ELSE END IF
IF(NC.EQ.26) OUTPUT ' REAR' IF(NC.EQ.31) OUTPUT ' LEFT' IF(NC.EQ.36) OUTPUT ' RIGHT' IF(NC.EQ.41) OUTPUT ' TOP' IF(NC.EQ.46) OUTPUT ' BOTTOM'
TEMP==(VOLT)
TEMP(NC)=TEMP(NC)-459.69
11 WRITE(108,101) NC,VOLT,TEMP(NC),TT(NC)
101 FORMAT(32X,13.7X,F8.3,6X,F8.2,F8.2)
DO 13 N=1,8
GO TO (1,2,3,4,5,6,7,8) N
1 I=1:L=10
GO TO 15
2 I=11:L=20
GO TO 15
3 I=21:L=25
GO TO 15
4 I=26:L=30
GO TO 15
5 I=31:L=35
GO TO 15
6 I=36:L=40
GO TO 15
7 I=41:L=45
GO TO 15
8 I=46:L=47
15 TA=0.0
DO 12 J=I,L
12 TA=TA+TEMP(J)
TB(N)=TA/(L-I+1)
TM(N)=TB(N)-459.69
13 CONTINUE
IF(NC.NE.LW) GO TO 17
RETURN
FUNCTION T(E)
DIMENSION C(8)
C(1)=-491.96562
C(2)=46.381884
C(3)=-1.3918864
C(4)=-.15260112
C(5)=-.020201612
C(6)=.0016456956
C(7)=-6.628709/(10.**5)
C(8)=1.0241343/(10.**6)
T0T=0.
DO 10 I=1,8
10 T0T=T0T+C(I)*(E**(I-1))
T=T0T
RETURN
END

C**** CORRELATE THE DIMENSIONLESS RESULTS
DIMENSION X(6,400),C(8),S(10,10),ANP(10),RNRL(400),RNRLS(400)
DIMENSION RA(100),SH(100),XNU(100),PR(100)
READ(105,102) DG
102 FORMAT(1X,F4.2)
H=12.0
WRITE(108,103) DG
103 FORMAT( ' ** TROMBE WALL DUCT GAP EQUALS ',1X,F4.2,' IN')
C**** DETERMINE THE CONSTANTS USING THE LEAST SQUARE AND
C##** ERROR CURVEFITTING SUBROUTINES
READ(105,100) NV,N0B
100 FORMAT(1X,1I,1X,1I)
DO 10 N=1,N0B
READ(105,99) RA(N),XNU(N),PR(N),SH(N)
99 FORMAT(1X,E10.5,2(1X,F7.3),1X,F4.I)
10 CONTINUE
DO 3 I=1,3
DO 1 NV=2,4
DO 11 N=1,N0B
X(I,N)=1.0
IF(NV.EQ.2) GO TO 55
IF(NV.EQ.3) GO TO 66
IF(NV.EQ.4) GO TO 77
55 X(2,N)=LOG(RA(N))
IF(I.EQ.2) X(2,N)=LOG(PR(N)/(.2+PR(N))*RA(N))
IF(I.EQ.3) X(2,N)=LOG(RA(N)/(H/SH(N)))
GO TO 2
66 X(2,N)=LOG(RA(N))
X(3,N)=LOG(PR(N))
IF(I.EQ.2) X(2,N)=LOG(SH(N)/H); X(3,N)=LOG(PR(N)/(.2+PR(N))*RA(N))
IF(I.EQ.3) X(2,N)=LOG(RA(N)); X(3,N)=LOG(H/SH(N))
GO TO 2
77 X(2,N)=LOG(RA(N))
X(3,N)=LOG(SH(N)/H)
X(4,N)=LOG(PR(N))
IF(I.GT.1) GO TO 3
2 M=NV+1
X(M,N)=LOG(XNU(N))
11 CONTINUE
CALL CURFT(NV,NOB,X,C)
CALL ERROR(NV,NOB,X,C,SSD)
C(I)=EXP(C(I))
DO 5 J=1,NV
WRITE(10B,105) J,C(J)
105 CONTINUE
5 CONTINUE
1 CONTINUE
3 CONTINUE
END
SUBROUTINE CURFT(NV,NOB,X,C)
DIMENSION X(6,400),C(8),S(10,10)
DOUBLE PRECISION S.D
LEAST SQUARES - NV INDEPENDENT VARIABLES
Y=X(NV+1,I) - NOB- NO. OF OBSERVATIONS
C(I)=COEFFICIENT OF X(I)
M=NV+1
MP=M+1
DO 1 I=1,M
DO 1 J=1,MP
1 S(I,J)=0.D0
DO 2 I=1,NOB
DO 2 J=1,M
DO 2 K=1,M
2 S(J,K)=S(J,K)+X(J,I)*X(K,I)
S(1,MP)=1.D0
IF (NV-I) 997,997,998
997 S(1,1)=S(1,2)/S(1,1)
GO TO 999
998 DO 14 J=1,M
14 S(M,J)=S(J,M+1)/S(1,1)
DO 15 1=2,NV
D = S(1,1)
DO 15 J=1,M
15 S(1,J)=S(1,J+1)-D*S(M,J)
GO TO 998
999 DO 16 LK=1,NV
11 IF (S(1,1)) 13,12,13
12 WRITE (10B,20)
20 FORMAT (' EQUATIONS IN SUBROUTINE CURFT ARE DEPENDENT - IGNORE FOL
LOWING ERROR ANALYSIS')
DO 30 III=1,NV
30 C(III)=0.
GO TO 31
13 DO 14 J=1,M
14 S(M,J)=S(1,J+1)/S(1,1)
DO 15 I=2,NV
D=S(1,1)
DO 15 J=1,M
15 S(I-1,J)=S(I,J+1)-D*S(M,J)
GO TO 31
999 DO 3 I=1,NV
       CL(I)=S(I,1)
  3 WRITE (108,4) I,C(I)
  4 FORMAT (' ',10X, 'COEFFICIENT OF X', 12, ' = ', E13.9)
  31 RETURN
END

SUBROUTINE ERROR(NV,NOB,X,C,SSQ)
DIMENSION X(8,400),C(6),S(10,10),ANP(10),RNRL(400),RNRLS(400)
DOUBLE PRECISION YC,TS,TE

ERROR ANALYSIS - NOB OBSERVATIONS

Y=X(NV+1,I) - SSQ=STANDARD DEVIATION
C(I) - CONSTANTS

M=NV+1
TS=O.DO
TE=O.DO
EMX=O.
DO 5 I=1,5
   ANP(I)=O.
  5 WRITE (108,1)
  1 FORMAT (' ',2X, ' Y EXPERIMENTAL', 4X, ' Y CALCULATED', 2X,
     1 ' NUMERICAL ERROR', 3X, ' PER CENT ERROR')
  DO 2 I=I,NOB
     YE=X(M,I)
WORD EQUATION FOR YC AT THIS POINT
     YC=O.DO
  20 YC=YC+C(IJ)*X(IJ,I)
     YE=EXP(YE)
     E=YC-YE
     EP=100.*E/YE
     EPA=ABS(EP)
     IF (EPA-EMX) 6,6,7
  7 EMX=EPA
  6 DO 8 J=1,5
     ACD=J*S
     IF (EPA-ACD) 9,9,8
  9 ANP(J)=ANP(J)+1.
  8 CONTINUE
     TS=TS+E*E
     TE=TE+ABS(EP)
  2 WRITE (108,3) YE,YC,E,EP,I
  3 FORMAT (4E17.8,15)
     A=NOB
     SSQ=(TS/A)**.5
     TE=TE/A
  4 WRITE (108,4) SSQ,TE
  4 FORMAT (' ',5X, 'STANDARD DEVIATION=', E15.8, 3X, 'AVERAGE PER CENT D')
1EVATION='E15.8')
WRITE (108,10) EMX
10 FORMAT ('0',20X,'MAXIMUM PER CENT DEVIATION='E15.8)
DO 11 I=1,5
XNP=ANP(I)
XNP=100.*XNP/A
ACD=1*5
WRITE (108,12) XNP,ACD
11 CONTINUE
12 FORMAT (1X,E15.8,2X,'PER CENT OF DATA WITHIN',2X,E15.8,2X,
1 'PER CENT OF EQUATION')
RETURN
END
FUNCTION U(T,IDF)
CO TO (1,2) IDF
C**** ABSOLUTE VISCOSITY OF AIR
1 C0=134.375
C1=6.0133834
C2=1.8432299
C3=1.3347050
U=T**C2/(EXP(C1)*(T+C0)**C3)
GO TO 10
C**** ABSOLUTE VISCOSITY OF 20CS DOW 200 SILICONE
2 V=.03875*(4.6*10**5)/(T-359.89)**1.912
C1=52.754684
C2=.04537533
C3=5.183233/(10.**5)
RHO=C1+(C2-C3-T)*T
U=RHO*.949*V
10 RETURN
END
FUNCTION CP(T,IDF)
GO TO (1,2) IDF
C**** SPECIFIC HEAT OF AIR
1 C0=2.236775/(10.**5)
C1=.22797749
CP=C1+C0*T
GO TO 10
C**** SPECIFIC HEAT OF 20CS DOW 200 SILICONE
2 TP=5..*(T-491.69)/9.
C1=.3448334
C2=7.7499/(10.**5)
C3=4.167/(10.**8)
CP=C1+(C2+C3*TP)*TP
10 RETURN
END
FUNCTION CON(T,IDF)
GO TO (1,2) IDF
C**** THERMAL CONDUCTIVITY OF AIR
1 XP=.1
C0=-0.5964965
C1=34490.89
C2=868.23837
C3=8056583.8
7 X=XP
F=C0+C1*X+C2*X*X+C3*X*X*X-T
FP=C1+2.*C2*X+3.*X*X*X
XP=X-F/FP
IF (ABS((XP-X)/X)-.0001) 6,6,7
8 CON=XP
GO TO 10
C**** THERMAL CONDUCTIVITY OF 20CS DOW 200 SILICONE
2 CON=.00034/0.004134
10 RETURN
END
FUNCTION RHO(T,IDF)
GO TO (1,2) IDF
C**** DENSITY OF AIR AT ATMOSPHERIC PRESSURE (LBM/FT**3)
1 A=12.5
RHO=A*144./(53.36*T)
GO TO 10
C**** DENSITY OF 20CS DOW 200 SILICONE
2 C1=.045437533
C2=51.832336/(10.**5)
RHO=.949*(C1+(C2-C3*T)*T)
10 RETURN
END
FUNCTION BETA(T,IDF)
GO TO (1,2) IDF
C**** THERMAL EXPANSION COEFFICIENT OF AIR
1 BETA=1./T
GO TO 10
C**** THERMAL EXPANSION COEFFICIENT OF 20CS DOW 200 SILICONE
2 BETA=.00107/1.8
10 RETURN
END
### APPENDIX II

#### PARTIALLY REDUCED DATA

<table>
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<tr>
<th>GAP (cm)</th>
<th>FACE</th>
<th>( T_h (^\circ K) )</th>
<th>( T_c (^\circ K) )</th>
<th>( Q_{\text{conv}} (W) )</th>
<th>( Q_{\text{cond}} (W) )</th>
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APPENDIX III
EFFECTIVE HEAT TRANSFER AREA

The inner body was not perfectly isothermal in the present experiments because the heater tapes did not cover the entire inner surface. Thus, the effective heat transfer area on each face of the inner body had to be obtained to calculate the heat transfer coefficient.

UNVENTED INNER BODY

The following steady-state heat conduction problem was solved for the unvented inner body (refer to Figure A3.1). The problem was initially set forth in cartesian coordinates as

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{A3.1}
\]

with boundary conditions

\[
\begin{align*}
\frac{\partial T(0,Y,Z)}{\partial x} &= 0, & \frac{\partial T(L,Y,Z)}{\partial x} &= 0, \\
\frac{\partial T(X,0,Z)}{\partial y} &= 0, & \frac{\partial T(X,L,Z)}{\partial y} &= 0, \\
\frac{\partial T(X,Y,0)}{\partial z} &= \begin{cases} -q''/k & \text{for } b < X < L-b; \ a < Y < L-a \\ 0 & \text{otherwise} \end{cases} \\
\frac{\partial T(X,Y,Z)}{\partial z} &= (h/k) \left[ T_\infty - T(X,Y,Z) \right] 
\end{align*}
\]

By applying the method of separation of variables, a Fourier
Figure A3.1 Unvented Inner Body
series solution was obtained:

\[ T(X,Y,Z) = \sum_{n,m=0}^{\infty} C_{nm} \cos(\lambda_n X) \cos(\mu_m Y) \left\{ \sinh\left[ (\lambda_n^2 + \mu_m^2)^{1/2} Z \right] \right. \]

\[ - D_{nm} \cosh\left[ (\lambda_n^2 + \mu_m^2)^{1/2} Z \right] \left. \right\} + q''(L-2a)(L-2b)(1+h/k(\ell - Z))/(kL^2) \]

where \( \lambda_n = n\pi/L \) \((n = 0, 1, 2, \ldots)\) and \( \mu_m = m\pi/L \)

\((m = 0, 1, 2, \ldots)\) are eigenvalues, and

\[ D_{nm} = \frac{(\lambda_n^2 + \mu_m^2)^{1/2} + (h/k) \tanh[(\lambda_n^2 + \mu_m^2)^{1/2} L]}{(\lambda_n^2 + \mu_m^2)^{1/2} \tanh[(\lambda_n^2 + \mu_m^2)^{1/2} L] + (h/k)} \]

The Fourier series constants were obtained by orthogonality

\[ C_{0m} = \frac{-2q'' (L-2b) \{ \sin[\mu_m (L-a)] - \sin(\mu_m a) \}}{k \pi^2 L^2} \]

for \( n = 0 \) and \( m = 1, 2, \ldots \),

\[ C_{n0} = \frac{-2q'' (L-2a) \{ \sin[\lambda_n (L-b)] - \sin(\lambda_n b) \}}{k \pi^2 L^2} \]

for \( n = 1, 2, \ldots \), and \( m = 0 \),

\[ C_{nm} = \frac{-4q'' \{ \sin[\lambda_n (L-b)] - \sin(\lambda_n b) \} \{ \sin[\mu_m (L-a)] - \sin(\mu_m a) \}}{k \pi^2 L^2} \]

for \( n = 1, 2, \ldots \), and \( m = 1, 2, \ldots \).

The effective heat transfer area was obtained:

\[ A_{\text{eff}} = A \overline{T}_{Z=\ell} / T(L/2, L/2, \ell) \]

where \( A = L^2 \) and \( \overline{T}_{Z=\ell} = Q/h \). Figures A3.2 and A3.3 show plots of \( Q \) versus \( A_{\text{eff}} \) for the left and right faces of the inner body, respectively. These figures were used to
Figure A3.2 The Effect Heat Transfer Area for the Left Face of the Unvented Inner Body
Figure A3.3 The Effect Heat Transfer Area for the Right Face of the Unvented Inner Body
correct the heat transfer data only when making a comparison with Raithby's predictions.
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