Metamorphism of dry snow as a result of temperature gradient and excess vapor density
by Edward Eagan Adams

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Engineering Mechanics
Montana State University

Abstract:
A heat conduction equation to determine the temperature profile in a snowpack is developed. The magnitude of temperature gradient tends to increase as the snow surface is approached, with local minimums through high snow density layers and local maximums above and below these layers. Calculations are made which determine the excess vapor density over the ice grain surfaces which border the pore space. In the presence of a temperature gradient faceted crystals will develop near the top of the pore, as ice is sublimated off of the surfaces in the lower region. Necks will deteriorate most readily, causing an overall weakening of the snowpack. There will be a reduction in the percentage of rounded grains as the faceted form develops. The process is enhanced at warmer temperature and larger temperature gradients. Temperature and excess vapor density are known to determine the habit of ice crystals grown in air. The model predicts excess vapor densities in the snowpack which are similar to those which exist in the atmosphere. Comparison of crystal habits predicted by the model are in good agreement with experimental evidence, when the pore geometry and temperature conditions are specified.

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ABSTRACT

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Chapter I

INTRODUCTION

The metamorphism of snow is an ongoing process from the time of its inception in the atmosphere as individual ice crystals, until it eventually returns to the liquid state, as spring runoff or glacial melt water. Ice crystals are formed in the atmosphere when water vapor in clouds is supercooled in the presence of a condensation nuclei, such as dust. A great variety of crystal shapes may result, depending on supersaturation with respect to ice and the temperature under which they form (Nakaya, 1954; Kobayashi, 1961; Mason, Bryant and Van den Heuvel, 1963). All of the crystal types, however, display the same crystal structure. The structure is that of a basal plane with hexagonal symmetry, which is oriented perpendicularly to the principal crystallographic axis.

It is the accumulation of these ice crystals on the ground which forms the basis of a snowpack. Snow is a granular material consisting of ice particles and interstitial pore spaces filled with water, air, and water vapor. It may be classified as dry snow—ice, air, and water vapor, saturated snow—ice and water, and wet snow—where all four constituents are present.

Naturally occurring snowpacks generally exhibit a complex stratigraphy, composed of individual layers of snow which have different properties. These differing snow layers result from the environment to which the snow is subjected when it is on the ground, such as thermal effects, wind, rain or subsequent snowfall, as well as the atmospheric conditions which exist during formation and deposit.

Ice crystals formed in the atmosphere as described above are in a state of unstable thermodynamic equilibrium. In a snowpack, there is a tendency for these intricate crystal shapes to metamorphose into a more spherical configuration. In the presence of isothermal or near isothermal conditions, this is accompanied by the sintering or bonding together of the ice grains. Sintering leads to an overall strengthening of the snowpack. The entire process is known as destructive or equi-temperature metamorphism. The term equi-temperature may be misleading, since isothermal conditions rarely, if ever, exist naturally in a dry snowpack.

Temperatures at the base of a seasonal snowpack usually remain just below 0° C throughout the winter. Snow is being warmed from below, as the ground gives up heat accumulated during the warmer months. If this geothermal heating condition coincides with cooler ambient air temperatures, characteristic of the winter months, a temperature gradient is established across the snowpack. A similar situation will also exist on temperate glaciers or polar ice which has been warmed during the summer. Temperature gradients, measured from the ground toward the snow surface, between -1.0 to -10.0 deg m^{-1} are reported to be typical in alpine snowpacks (Yosida, 1963), while gradients of -30.0 deg m^{-1} occur annually on the Greenland ice sheet (Benson, 1962). Trabaut and Benson (1972) report normal gradients of -100.0 deg m^{-1} and as high as -200.0 deg m^{-1} in central Alaska.

A variation in vapor pressure, resulting from the temperature differential, will cause a mass flux of vapor from the zone of high pressure, the deeper warmer region, to the zone of lower vapor pressure. Since dry snow, in which this mechanism is most effective, consists of a solid and a vapor phase of the same material, the migration of mass need not take place exclusively through the tortuous labyrinth of pore spaces. The transfer can take place through the solid matrix itself, as a "hand to hand" transfer of mass (Yosida and

Kojima, 1950, from Akitaya, 1974). Vapor will evaporate from the top of an ice grain diffuse across the pore and deposit as a solid on the bottom of the grain above. The ice molecules on the top of this grain will likewise sublimate and redeposit in a similar fashion.

When snow of low to medium density is subjected to the correct combination of temperature and temperature gradient, angular flat surfaced crystals, known as depth hoar, will develop at the expense of those grains with a more rounded configuration. This effect is most prevalent in low density snow at relatively warm temperatures subjected to large temperature gradients. Consequently, depth hoar crystals develop most readily early in the snow season, when the pack is shallow and therefore large temperature gradients exist, in association with a relatively warm average snow temperature.

Two general types of depth hoar are recognized (Akitaya, 1974); a solid type, which develops under smaller temperature gradients, and a skeleton type which predominates under large temperature gradients. Skeleton type depth hoar is composed of large faceted crystals, which are poorly bonded together and form a weak snow layer. The solid type consists of small faceted crystals which do not show the marked reduction in strength characterized by the skeleton type.

Snowpack development in which faceted crystal growth predominates is known as constructive or temperature gradient metamorphism. Crystal development will take place in three stages. Early crystal development is known as anhedral, partially developed crystals are called subhedral, and the advanced form are termed euhedral crystals. Snow which has undergone this temperature gradient process is characterized by an overall reduction in strength, particularly during the intermediate subhedral crystal development (Bradley, Brown, and Williams, 1977; Adams and Brown, 1982). Weak zones in the pack resulting from temperature gradient metamorphism are a major cause of snow avalanches. An understanding of the process is therefore of practical concern.

Akitaya (1974), in what is perhaps the most complete experimental work carried out on the topic of temperature gradient metamorphism, has shown that pore size, initial ice grain size and geometry effects depth hoar development. A large pore will enhance the rate of growth and the size of the crystal which will develop. This is in agreement with Marbouty's (1980) observation that fine grained snow with a density in excess of approximately 350 $KG\text{-}M^{-3}$ subjected to even a large temperature gradient will develop solid type depth hoar. Lower density snow, which has a larger pore space tends toward the skeleton type in the presence of a sufficient temperature gradient.

Faceted crystals which develop in the snow have a preferential direction of growth (Akitaya, 1974). Crystals grow toward the direction of higher temperature. This was demonstrated by imposing positive and negative temperature gradients on the snow samples. The direction of crystal growth was shown to be dependent on the thermal gradient and not on gravity.

Several types of crystals will develop simultaneously in a particular layer with one form being predominant. The predominant crystal type was observed to be a function of temperature and temperature gradient (Figure 1) (Akitaya, 1974; Marbouty, 1980). Earlier studies by Nakaya (1954), Hallet and Mason (1958), Kobayashi (1961), Mason, Bryant, and Van den Heuvel (1963) and others have investigated the growth of ice crystals in air. Their results show the correlation between crystal habit, temperature and excess vapor density (or supersaturation) in air relative to the equilibrium vapor pressure over ice. Results observed by Kobayashi are displayed in Figure 2.

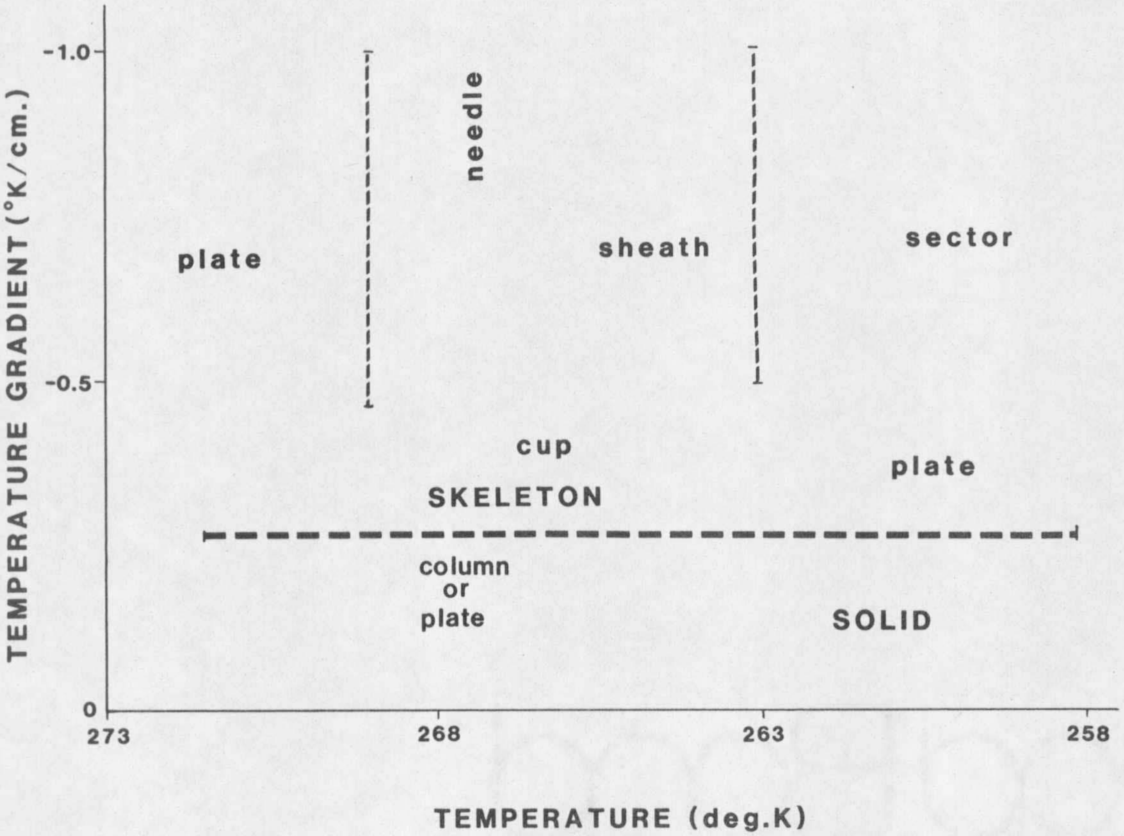

Figure 1. Dependence of Crystal Habit Development in a Snowpack on Temperature and Temperature Gradient (Akitaya, 1974).

It seems as if there must be some correlation between crystals grown from vapor in the air and those which grow in snow subjected to a temperature gradient. This becomes apparent when the observances of Kobayashi (1961) (Figure 2) for crystals grown in air are compared to those of Akitaya (1974) (Figure 1), for the same temperature range.

Examining these diagrams it becomes evident that the temperature gradient in the snow must in some way affect the excess vapor density. It is the temperature of formation which governs the crystal habit and the excess vapor density (or supersaturation) which determines the secondary growth features (Mason, Bryant, and Van den Heuvel, 1963). The basic crystal habit may take the form of either a plate or a prism. Whether a crystal is considered a plate or a column is determined by the ratio of the principal (c) axis to the basal (a) axis. For a plate structure c/a is less than 1, whereas if c/a is greater than 1, the crystal is considered a prism. Secondary growth features alter the appearance of the crystal habit such as dendritic extensions on plate crystals or prisms taking on a cup shaped appearance.

It is the purpose of this paper to further the understanding of depth hoar development. An analytic development is presented which represents an attempt to describe the thermodynamic processes involved in temperature gradient metamorphism.

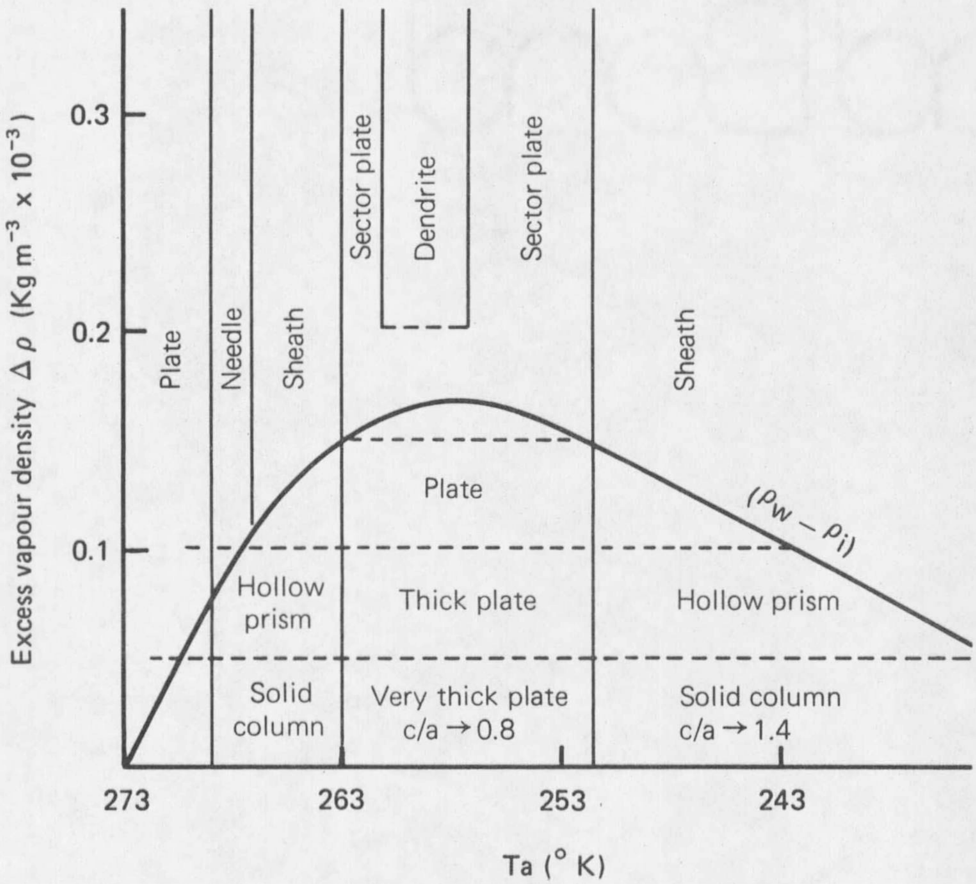

Figure 2. Crystal Habit of Ice Grown in the Atmosphere Based on Temperature and Excess Vapor Density (Kobayashi, 1961).

Chapter II

HEAT FLOW IN A SNOWPACK

When all mechanical effects are neglected the general isotropic form of the Fourier heat conduction equation may be written as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + Q = \rho_s C\frac{\partial T}{\partial t} \qquad (1)$$

Where k is the thermal conductivity, T is temperature, Q is the internal heat generation per unit time per unit volume, $\rho_s C\ \partial T/\partial t$ is the rate of internal energy change per unit volume, t is time, ρ_s is the snow density, C is the specific heat of ice (Bozey and Wienke, 1960).

In the presence of a temperature gradient such as that discussed in the Introduction, heat flow in a natural snowpack occurs in the direction normal to the slope of the ground. This is taken to be the z coordinate direction. There will be no flow of heat in the x and y component directions, which are taken as tangent to the slope, since the snow cover may be considered as extending infinitely in this plane.

Using the ground snow interface as the origin for the z axis, a negative temperature gradient $\partial T/\partial z$ is established in the snowpack, since the ground is in general warmer than the upper regions of the snow. The flow of heat will be from the warmer ground toward the cooler snow surface. Positive gradients may occur near the snow surface due to diurnal fluctuations or periods of warmer weather. A positive gradient throughout the pack will not generally persist naturally, since this indicates a prolonged surface temperature above 0°C, in order to be significantly warmer than the ground. This will cause melting in the upper part of the pack. Meltwater then percolates downward through the snow, finally achieving a constant temperature near 0°C throughout. This type of wet or saturated isothermal snowpack is characteristic of late spring snow conditions in alpine regions.

Internal heating within the snowpack will be generated during metamorphism as mass is sublimated off of, and deposited onto the ice surfaces. The net quantity of heat produced or lost may be calculated from the energy released due to the latent heat of sublimation, as the mass of water undergoes this phase transformation. Internal heat generation for Equation 1 may be expressed as

$$Q = -L \frac{\partial J_s}{\partial z} \tag{2}$$

Where L is the latent heat of sublimation and J_s is the mass flux of the snow.

Equation 1 may now be written for snow as

$$\frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - L \frac{\partial J_s}{\partial z} = \rho_s C \frac{\partial T}{\partial t} \tag{3}$$

Once again neglecting mechanical effects, a change in the snow density is the result of water vapor being driven upward through the snow. The conservation of mass in the snow may then be expressed by

$$\frac{\partial J_s}{\partial z} = -\dot{\rho}_s \tag{4}$$

$\dot{\rho}_s$ is the time rate of change of snow density.

Thermal conductivity of snow is a function of a number of parameters such as density, temperature, intergranular bonding and to a lesser degree on grain size and grain shape, as well as any other parameters which would facilitate or impede the transfer of heat. The thermal conductivity of ice is much greater than that of water vapor at the same temperature. Consequently the major portion of heat transfer by pure conduction will take place through the solid ice network, rather than the pore. This is the reason thermal conductivity is strongly dependent on snow density. However, part of the heat is carried upward by diffusion through the pore space as well. Because of this, an effective thermal conductivity coefficient instead of the true conductivity is used to determine the snowpack temperature profile.

Palm and Tveiteroid (1979) have concluded theoretically, that in deep layers of highly permeable snow subjected to very large temperature gradients, thermal convection may be an effective means of vapor flux and therefore heat flux. In conditions occurring naturally in central Alaska, U.S.A., it has been shown that convection can indeed be significant to the flow of mass and heat. The significance of convection in this region is the result of the extreme conditions prevalent there. A shallow snowpack 0.5 to 0.8 m thick is subjected to approximately 200 consecutive days of excessive thermal gradients. Gradients of -200 deg m^{-1} have been observed and -100 deg m^{-1} are considered common.

Investigation by Akitaya (1974), on natural snow samples, indicate that convection did not occur even when gradients of -200 deg m^{-1} were artificially induced, although the duration of the experiment was much shorter than those observed in central Alaska. He was able to achieve convection only when an extremely permeable artificial snow was constructed. Individual ice grains for the sample were represented by cubes of fine grained compacted snow 0.15 m on a side. Using this "snow," temperature gradients greater than 100 deg m^{-1} were necessary before convection was assumed to occur.

The model presented in the paper will ignore the effects of thermal convection. Porosity, temperature gradient magnitude and the duration necessary to initiate significant thermal convection are much larger than those encountered in most alpine regions of the world or even on the Greenland ice sheet.

Empirical expressions which are generally used to determine the thermal conductivity coefficient are based solely on snow density. For a partial list of examples, see Mellor (1964). Since density is certainly not the sole parameter by which conductivity should be measured, the data accumulated by Voitkovsky, Golubey, Lapteva, Troshkina, Ushakova, and Pavlov (1976) has been utilized. Voitkovsky et al. obtained what they considered to be values near the true conductivity as a function of density. These values were obtained by taking measurements at very low temperatures, where diffusion is not as effective a means of heat transfer. Statistical methods were used to arrive at a true thermal conductivity coefficient.

$$k_t = 0.030 + 0.303\,\rho_s - 0.177\,\rho_s^{\,2} + 2.250\,\rho_s^{\,3} \tag{5}$$

where the snow density ρ_s is in c.g.s. units.

Data collected in the same region (Yakutsk, USSR) for conduction as a function of temperature is represented in Figure 3, by the solid line. The relationship of the conductivity as a function of temperatures is approximately exponential. The measurements were taken for snow at a density of 0.25 gm cm^{-3}.

Using the data collected in Yakutsk, an expression to determine the effective thermal conductivity coefficient as a function of two parameters, density and temperature, is

$$k = \gamma \exp[\beta T]\, k_t \tag{6}$$

The constants γ and β were determined by evaluating Equation (6) for ρ_s equal to 0.25 gm cm^{-3} and temperatures of 253°K and 268°K. Values for conductivity, as a function of temperature, are based on the data collected in Yakutsk shown in Figure 3. Values