Frequency of peak flows predicted from rainfall frequencies

by Lee Robinson

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Civil Engineering
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Abstract:
A theoretical equation is derived which may be used to predict floods on small Montana watersheds. The equation derived uses the mean annual peak discharge rate of the stream (which may be estimated from short streamflow records) together with three theoretically based factors. These three factors were evaluated for Montana east of the continental divide.

The rainfall intensity ratio, defined as the ratio of the 50-year rainfall intensity of given duration to the mean annual rainfall intensity, of the same duration, was evaluated using published rainfall data. An isoplethal map was plotted showing values of the rainfall intensity ratio for Montana east of the continental divide.

The rainfall-discharge recurrence factor, a factor expressing the relationship between the rainfall intensity recurrence relation and the recurrence relation of peak annual discharges produced by rainfall, was evaluated using published rainfall data, as well as water-stage and discharge data for twelve Montana watersheds.

Special techniques were required to separate rainfall-induced flow from the remaining portions of streamflow hydrographs so that peak annual discharges from rainfall could be determined. Values of the rainfall discharge-recurrence factor determined for the twelve watersheds were used to plot an isoplethal map applicable to Montana east of the continental divide.

The rain-snow-base flow interaction ratio, a factor reflecting the relative dependence or independence of the frequency curves of rainfall-induced flows and snow-melt-induced flows, was evaluated for the twelve Montana watersheds. An average value is recommended for use in Montana east of the continental divide.

Estimates of the 50-year flood obtained by extreme value (Gumbel) and log-normal recurrence analyses were compared with estimates from the method derived in this study for 46 watersheds. From this comparison, the derived method is considered to be more reliable than the extreme value and log-normal methods for discharge records 50 years or shorter in length.

The derived method is applicable for watersheds less than 500 square miles in area for which large events occur most often during the snow melt season.

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CHAPTER I

INTRODUCTION

As a result of the intensive highway construction program which is currently in progress in Montana, larger and more expensive drainage structures are being built on small Montana watersheds than have been in the past. As the size and cost of these structures increases, the need for reliable estimates of the magnitude of floods which may occur during the lifetime of the structures also increases.

Predicting floods on small watersheds in Montana is difficult. Only since the inception of the U.S. Geological Survey Small-area Peak-flow Highway Program in 1955 have peak-flow measurements been made on more than a few such watersheds. These records are generally too short to be used without additional information in predicting floods of more than very short recurrence intervals. No satisfactory flood-prediction methods using data other than streamflow records have been developed for use in Montana.

Precipitation measurements, on the other hand, have been made for many years at stations located throughout Montana. Since peak flow rates are related to rainfall, at least in part, a logical approach to predicting floods is to utilize the long rainfall records which are available. This approach is taken as one phase of tho Drainage Correlation Research Project.

The Drainage Correlation Research Project, sponsored by the Montana State Highway Commission and the Bureau of Public Roads, was instituted at Montana State University in 1963 to develop and evaluate procedures for predicting floods from Montana watersheds 1 to 100 square miles in area. The investigation is divided into two concurrent phases:

1) Runoff data from a number of watersheds and precipitation data from weather stations throughout Montana are studied to develop a correlation between peak flows and precipitation data.

2) Comprehensive hydrologic studies are being made of four watersheds included in the U.S. Geological Survey Peak-flow Program. Data collected is used to develop and evaluate flood prediction procedures.

The research reported herein was conducted under the first phase of the project investigation.

In this study a theoretical equation is derived which may be used to predict floods on small Montana watersheds. The equation derived uses the mean annual peak discharge rate of the stream (which may be estimated from short streamflow records) together with three theoretically based factors. These three factors were evaluated for Montana east of the continental divide.

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The rainfall-discharge recurrence factor, a factor expressing the relationship between the rainfall intensity recurrence relation (for specified durations) and the recurrence relation of peak annual discharges produced by rainfall, was evaluated using published rainfall data and water-stage and discharge data for twelve Montana watersheds. Special techniques were required to separate rainfall-induced flow from the remaining portions of streamflow hydrographs so that peak annual discharges from rainfall could be determined. Values of the rainfall discharge-recurrence factor determined for the twelve watersheds were used to plot an isoplethal map applicable to Montana east of the continental divide.

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Data for developing the method was obtained from twelve water-stage-gaged watersheds in Montana east of the continental divide. The method was tested on a number of water-stage-gaged and crest-stage-gaged watersheds located east of the continental divide.

CHAPTER II

REVIEW OF LITERATURE

Since 1940 wide scale development of hydrologic techniques applicable to small watersheds has taken place. Some of the most useful developments have been in the application of frequency analyses. Below, traditional and modern techniques of flood prediction which incorporate rainfall data are outlined; the current methods of frequency analysis are discussed; and the methods available for predicting floods on small Montana watersheds are presented.

RAINFALL-RELATED FLOOD FORMULAS

The rational runoff formula, proposed and used by Mulvany as early as 1851, (Dooge, 1957) is an empirical method using watershed area and rainfall intensity as variables. Since that time numerous formulas have been proposed to predict peak discharge rates from rainfall intensities, for example, Gregory and Arnold (1932), Kringold (1938), and Roe and Snyder (1943). These formulas take the general form

$$Q = K I^a A^b \qquad (1)$$

where Q is the peak discharge rate, I is the rainfall intensity, A is the area of the watershed, and K, a, and b are empirical constants. The formulas are not considered valid unless the duration of the storm is at least equal to the travel time of the watershed.

MULTIPLE CORRELATION METHODS

During the past several decades hydrologists have shown that the total depth of rainfall (excess rainfall) which appears as the surface runoff part of the hydrograph may be estimated for a given watershed from a multiple correlation using such variables as antecedent precipitation index, season of year, and storm duration and intensity. The time distribution of discharge resulting from this excess rain has then been determined by unit hydrograph methods, or by a graphical correlation of the amount of excess rainfall during a short time increment vs runoff rate during the increment (Kohler and Linsley, 1951; Kresge and Nordenson, 1955; Miller and Paulhus, 1957; and Paulhus and Miller, 1957).

Unit Hydrograph: Since the physical characteristics of a basin--shape, size, slope, etc.--are constant, the hydrographs from storms of similar characteristics might be expected to have considerable similarity in shape. This concept is the basis of the unit hydrograph. The unit hydrograph method, proposed by Sherman (1932), yields a "typical" hydrograph for the watershed.

The unit hydrograph is defined as the hydrograph of one inch direct runoff (total runoff minus base flow) from a storm of specified duration. It is obtained by adjusting the ordinates of the hydrograph resulting from a storm of the specified duration so that the volume of direct runoff under the hydrograph is one inch. This is usually done for several storms of similar duration, and the hydrographs are superimposed to give a typical (average) unit hydrograph. Theoretically, a separate unit hydrograph is required for each duration of rainfall; however, unit single duration (Linsley, Kohler, and Paulhus, 1958).

Synthetic unit hydrograph: Often on small watersheds insufficient streamflow records are available from which to construct unit hydrographs. In this case the synthetic unit hydrograph method introduced by Snyder (1938) may be used. In this method the time lag, usually defined as the time interval between the centroid of rainfall and the hydrograph peak, is determined as an empirical function of the length and slope of the stream channel. The unit hydrograph peak for a given rainfall duration is then expressed as a function of drainage area and time lag. The hydrograph is assigned a shape from another empirical relation. The hydrograph thus synthesized can be analyzed by the same techniques as ordinary unit hydrographs to predict the peak discharge from a given rainfall.

Synthetic unit hydrographs have been applied specifically to small watersheds by Hickock, Keppel, and Rafferty (1959). They point out that hydrograph synthesis can only be applied on watersheds where major floods result from single storms producing runoff from the entire watershed. In the Southwest, this limits the use of synthetic hydrographs to watersheds of less than 1000 acres. The limit for eastern Montana is expected to be of the same order of magnitude. Renard and Keppel (1966) have developed a special technique for using synthetic unit hydrographs on larger ephemeral streams in the Southwest. In this technique, unit hydrographs are synthesized for portions of the watershed and routed to the mouth accounting for transmission losses.

For watersheds larger than a few square miles in area, the labor required to apply this method is very great. The labor involved may be reduced somewhat by the use of the electronic computer.

Multivariate methods: Lewis (1968) (in progress) has applied multivariate analysis to the four watersheds gaged by the Drainage Correlation Research Project in order to obtain a multiple correlation applicable to eastern Montana watersheds. Lewis's method relates peak discharge rates to watershed-related variables and meteorological variables in a single correlation. His results are not available at this writing.

Multiple correlation methods can be used to estimate peak floods for long recurrence intervals for watersheds with short streamflow records. They are, however, comparatively laborious to use.

FREQUENCY ANALYSIS

Frequency analysis is a technique well adapted to use on small watersheds. The major difficulty in the use of frequency analysis to determine the magnitude of floods from streamflow data is the shortage of long records. Harrold (1945) used streamflow records of less than ten years' length to predict floods of various recurrence intervals. He adjusted the data for non-representativeness of sample by use of rainfall frequency data. More recently Chow (1958) has suggested that synthetic peak discharges with corresponding recurrence intervals obtained by multiple correlation methods (as described in the preceding section) may be used to extend short records on small watersheds in frequency analysis.

The method of frequency analysis treats hydrologic data as a statistical variable. The frequency distribution of the data is examined by an analytical approach and the magnitudes of the variable for various recurrence intervals are determined. The recurrence interval is defined as the average interval of time within which a given magnitude of the variable is equaled or exceeded (Andrews, 1957).

Figure (1-a) shows a set of recorded hydrologic data. Only the portion of the data of large magnitude is significant in predicting extreme events. Two methods are used to exclude the smaller portion of the data from the analysis. One method, the annual maximum series, uses the series of single largest values recorded in each year. The other, the partial-duration or exceedence series, uses the series of values above a selected base value irrespective of time of occurrence. The series of exceedence values is more difficult to study statistically, and Chow (1951) has shown that results from its use do not differ significantly from those obtained from the annual series for recurrence intervals longer than ten years. For these reasons, the annual series will be used for the frequency analyses in this study.

The annual maximum series for the data of Figure (1-a) is shown in Figure (1-b). The recurrence interval for the data is computed from a plotting position formula. The generally recommended formula is

$$T = \frac{N + 1}{m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

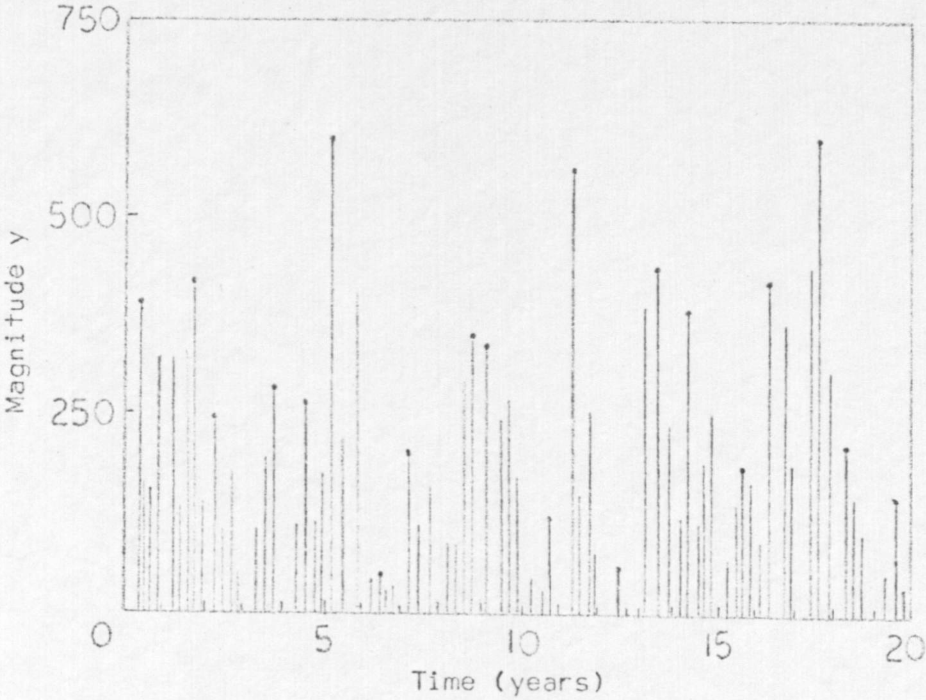

Figure (I-a): Original hydrologic data used to illustrate frequency analysis of hydrologic data.

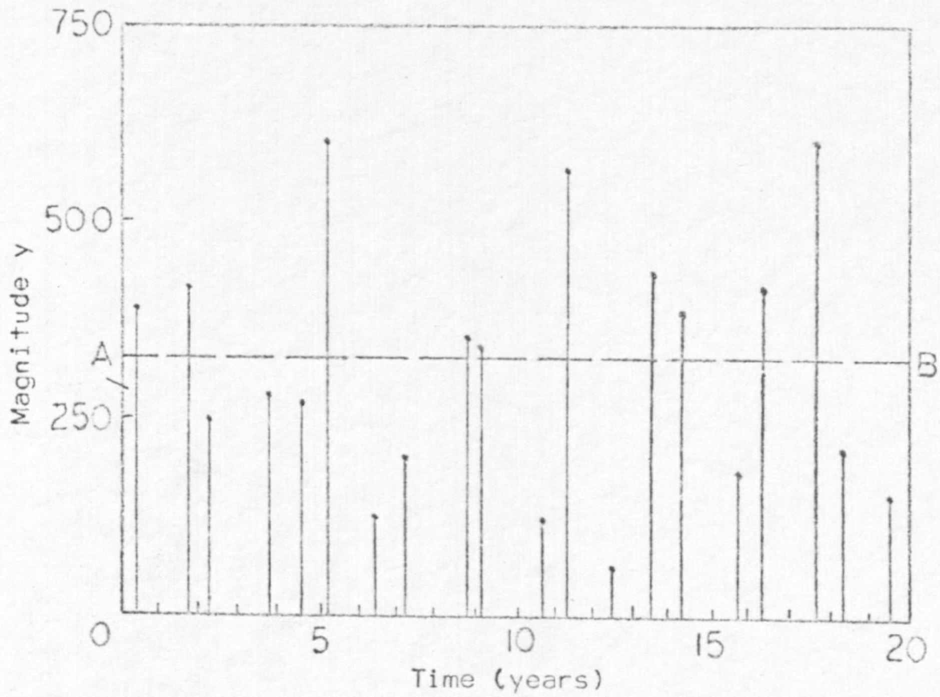

Figure (I-b): Annual maxima for the original data of Figure (I-a).

in which T is the recurrence interval, m is the rank of the values arranged in an order of decreasing magnitude, and N is the number of data values. Line AB in Figure (1-b) indicates the mean, $\bar{y}$, of the observed data whose magnitudes are represented by a variable, y. Any particular value of y can then be expressed by

$$y_i = \bar{y} + \Delta y_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which Δy_i is the departure from the mean and i is a subscript denoting the particular value of y. By statistics it has been shown that Δy_i can be expressed by the product of the standard deviation, σ, defined as,

$$\sigma = \sum_{j=1}^{n} [(y_j - \bar{y})^2] / n \quad \ldots\ldots\ldots\ldots\ldots (4)$$

and a frequency factor, K (Chow, 1958). Values of K depend on the frequency distribution used, and increase as the recurrence interval increases. Thus,

$$y_i = \bar{y} + \sigma K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The selection of a frequency distribution is arbitrary and any distribution which fits the data can be used.

The extreme value (Gumbel) method is easy to apply and gives comparable answers to several commonly used distributions (Huff and Neill, 1959; Majumdar and Sawhney, 1965).

The extreme value distribution is obtained from the following assumptions (Gumbel, 1958):

1) The data is divided into N samples each of size n.

2) The distribution becomes defined as N and n become infinite.

3) A finite set of data can be fitted to the distribution by the method of least squares.

To apply the distribution to hydrologic analysis the following additional assumptions must be made:

1) The number of recorded values during one year is large enough to be considered infinite.

2) The distribution of the original data is of the exponential type.

3) The sample size is constant.

It is clear that these last assumptions are never exactly met. Experience has shown, however, that the fit is usually better than might be expected from these considerations.

Chow (1958) has derived the value of K for the extreme value (Gumbel) distribution as

$$K = \frac{1}{\sigma_n} (\ln T - \bar{y}_n) \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

in which σ_n and $\bar{y}_n$ are the reduced standard deviation and reduced mean and depend only on sample size (years of record) (tabulated in Gumbel, 1958), and T, as before, is the recurrence interval.

(ln T is the logarithm of T taken to the Naperian base e).

The log-normal distribution (log-probability law) is another frequency distribution commonly used in hydrology. It is based on the assumption that the logarithm of the variable, y, follows a normal distribution. Use of the log-normal distribution is equivalent to applying a logarithmic transformation to the data and fitting the transformed data to a normal distribution.

Chow (1958) has shown that the frequency factor, K, for the log-normal distribution is given by

$$K = \frac{e^{(\sigma K_y - \sigma/2)} - 1}{(e^{\sigma^2} - 1)^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

where $K_y = (y_i - \bar{y})/\sigma$. The other quantities are defined as above. For computational purposes, K has been tabulated as a function of the coefficient of variation, C_v, for given recurrence intervals (see Appendix A). The coefficient of variation is defined as

$$C_v = \sigma/\bar{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Application of the log-normal distribution to hydrologic analysis is based on the following assumptions (Chow, 1958):

1) The hydrologic event is the result of the joint action of many causative factors.

2) The variable y is equal to the product of a large number of r