



Shock wave propagation in two-phase flow
by Michael James Weaver

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in
Mechanical Engineering
Montana State University
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Abstract:

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MONTANA STATE UNIVERSITY
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Michael James Weaver

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Date

May 30, 1986

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>
A	Cross-sectional area
a	Thermal Diffusivity
a _d	Droplet projected area
B	Gas constant
C	Speed of sound
C _{VM}	Virtual mass coefficient
D	Thermal damping coefficient
D _c	Drag constant
F	Oscillation frequency
f	Frequency
g	Accélération due to gravity
g _c	Gravitational constant
g _o	Newton's constant
K	Polytropic gas constant
k	Wave number
L	Latent heat of evaporation
N	Number of points
N _{GuSM}	Dimensionless parameter
N _L	Dimensionless parameter
n	Droplet density
P	Pressure
R	Universal gas constant
R _b	Bubble radius
R _o	Reynold's number
r	Droplet radius
T _x	Temperature, where x refers to any phase
t	Time
U _x *	Velocity, where x refers to any phase
V _{GS}	Modified Froude number
v	Velocity
W	Mass flowrate
w	Speed of propagation
X	Volumetric fraction
Y	Expansion factor
Z	Position
α	Void fraction
γ	Change in parameter $\gamma = \Delta\xi_j / \Delta\xi_j^\circ$
δ	Ratio of specific heats, $\delta = C_p / C_v$
μ	Absolute viscosity
ν	Kinematic viscosity
ρ _x	Density, where x refers to any phase
ρ̄	Density of equilibrium state
π̄	Partial pressure of equilibrium state

NOMENCLATURE (continued)

σ	Surface tension
ω	Angular frequency
τ	Time delay
ϕ_x	Phase, where x refers to which hydrophone

ABSTRACT

The frequency dependence of acoustic velocity in upward, vertical, two-component, two-phase flows was experimentally investigated. The two fluids used were air and water in annular and annular mist flow regimes.

Weak shock waves were introduced into the two phase flow by pressurizing the downstream (top) side of a diaphragm until the diaphragm ruptured. As the pressure pulse propagated through the flow, hydrophones, tangentially wall mounted, were used to record the wave form at two separate locations. A spectral analysis was then performed on the waveforms.

In comparison to the single phase (still air) spectral (phase) sonic velocity, the lower frequency components of the pressure pulse were slowed more than the higher frequencies as the air quality decreased.

CHAPTER I

INTRODUCTION

Classical fluid mechanics can be used to describe the behavior of motion for either phase in two-phase flow, but unfortunately trying to apply fundamentals, such as the Navier-Stokes equations, proves to be a hopeless task except for the most trivial two-phase models.

There are several factors that influence the speed of propagation of a shock wave in two-phase flow, such as the speeds of sound of both phases, the densities of the phases, and the volume fraction of the dispersed phase. A homogeneously mixed gas-liquid (air-water) two-phase flow has a sound velocity that is smaller than the velocity of sound of either the gas or liquid component of the mixture because of the high compressibility of the gas (air) and the large density of the liquid (water)¹.

The past twenty to thirty years has seen a considerable increase in the amount of research and study in the area of two-phase flow because of its many diverse applications. Acoustic propagation effects are important considerations to safely design the vessel, piping, and suppression pool of a boiling water reactor power system. The prediction of exit choking, critical flow rates, and certain oscillation phenomena in pipe flows rely on the velocity of propagation of pressure waves in two-phase

(liquid-gas) flows. The detection of thermal-hydraulic instability in two-phase media and the exploration and prediction of earth movements are all related to the speed of sound in two-phase mixtures. In theory, void fractions in boiling water systems and the free-gas concentration in liquids can be determined by measuring the sonic velocity in the two-phase mixture. This leads to an interesting application in the early detection of gas bubbles associated with decompression sickness (the "bends") in divers, and to aid in the treatment through monitoring the concentration of gas bubbles in the diver's body.

The purpose of this study was to experimentally investigate the frequency dependence of acoustic velocity in upward, vertical, two component, two-phase flows. The two components used in the two-phase flow were water and air. The mixture flowed vertically upward in a 1 1/4 inch inside diameter pipe. Annular and annular mist flow regimes were used in this experiment. The shocks, which contained a large range of frequencies, were induced to travel downward through the flow. The shocks were produced by rupturing a layer of aluminum foil by pressurizing the downstream (top) side until the diaphragm burst. The shocks were recorded by two separate hydrophones located one foot apart along the inside wall of the flow pipe. A spectral analysis was performed on the waveforms to analyze the data.

CHAPTER II

LITERATURE REVIEW

The study of acoustic characteristics in two-phase media is a very diverse and complicated field because of the large number of factors that must be taken into account. Some of the factors that influence two-phase studies are: 1) The large variety of flow regimes and interfacial geometries, such as slug, bubbly, annular flows, etc. 2) Nonequilibrium events, such as nucleation during phase change. 3) The large influences of small impurities, such as foaming agents. 4) The geometrical configuration of a test apparatus. Due to the large number of variables influencing two-phase media studies, the study of two-phase acoustic characteristics has been labeled an "insecure" science by Kenneth Boulding². An "insecure" science is defined as one which studies a very large universe with a very small and biased sample, and the available data only cover a small part of the total field in which the structures and relationships are extremely complex. The danger here lies in the temptation of claiming an analytical precision that is greater than the degree to which the problem can be defined.

Wallis³ describes six different methods of analysis, the most common of which is descriptive-experimental. This method of investigation involves observing and trying to explain what

happens. Since the investigation of this paper is of this method, the following discussion will be mainly limited to previous investigations of the descriptive-experimental nature.

The existence of an interface in two-phase flow causes a wide variety of flow patterns or flow regimes, depending on the flow rates and the physical properties of the phases. McQuillan and Whalley⁴ defined four main flow patterns for upwards flow in vertical tubes. They are bubble flow, plug flow, churn flow, and annular flow. Their paper deals with a method for predicting the likely flow pattern in vertical upflow of a gas-liquid two-phase mixture. For an annular flow regime to exist:

$$v_{Gs}^* \geq 1$$

Where v_{Gs}^* is a modified Froude number, representing a comparison between inertia and gravity forces. The critical value of unity was empirically observed by Hewitt and Wallis⁵ for an air-water system.

A more comprehensive study on flow patterns was done by Mukherjee and Brill⁶. Their investigation derived empirical equations for predicting flow regime transitions as a function of the inclination angle in the pipe for both upflow and downflow in two-phase gas-liquid systems. The inclination angles ranged from 0 degrees (horizontal) to 90 degrees (vertical).

The transition from slug to annular mist flow was found to be identical for all horizontal and all upflow and downflow angles. The liquid viscosity, μ , has a significant influence on this transition. As the liquid viscosity increases, the

transition from slug to annular mist flow accelerates. The transition is defined by:

$$N_{GvSM} = 10^{**}(1.401 - 2.694N_{Lv} + 0.521N_{Lv}^{.329})$$

where the dimensionless parameters;

$$N_{GvSM} = v_{SG}(\rho_L/g\sigma)^{1/4}$$

$$N_{Lv} = \mu_L(g/\rho_L\sigma^3)^{1/4}$$

and v_{SG} = superficial gas velocity

ρ_L = liquid density

σ = surface tension

g = acceleration due to gravity

Hijikata et al.¹ experimentally and theoretically studied the hydrodynamical behavior of a large bubble (4 to 8 mm diameter), subjected to a shock wave in a homogeneous two-phase flow consisting of small bubbles (.5 mm in diameter). They determined the large bubbles were adiabatically compressed and move with a velocity different from the liquid velocity behind the shock due to the bubble's inertia force of the virtual mass.

Martindale and Smith⁷ found that both sonic velocity and pressure drop data were good indicators of the flow regime transition from annular to churn-froth flow. Because the sonic velocity data showed little or no change, they also concluded that interface transport processes such as heat and mass transfer were negligible in the separated flow region from 100 percent quality down to the quality of the transition between annular and churn-froth flow patterns and at this rate of wave propagation or

pressure change.

The effects of a gas-liquid interface and tube geometry on pressure wave propagation were experimentally studied by Sutradher et al.⁸ by using "tee" sections in the piping. They concluded that momentum transfer occurred across the gas-liquid interface because of the distortion in the pressure distribution across the duct. The geometry effects caused by flow through the "tee" section significantly enhanced these pressure wave propagation phenomena.

Evans, Gouse, and Bergles⁹ reported that wall mounted pressure transducers do not measure the characteristics of the shock wave itself, rather the liquid boundary layer's response to the shock wave. They came to this conclusion after failing to explain why the shock picture from the transducers had considerably different characteristics depending on whether it was flowing up or downstream.

Evans et al. found that little or no acoustic energy is capable of being transmitted in the liquid film at the pipe wall due to thermal conduction and viscous drag. The flow is usually turbulent with extremely high shear forces which cause all but very high frequencies to be completely damped out. The result of this is that the pressure signal propagates down the core of the flow. Therefore, the core characteristics govern the propagation phenomena. Evans et al. were able to measure the pressure signal traveling down the core of flow at the pipe wall by treating the interaction between pressure disturbances in the core and the

wall film as the interaction of bulk flow over a thin boundary layer rather than acoustic phenomena. This situation is analogous to single-phase boundary layer flow, with the exception of extremely high boundary layer density. By subtracting the liquid flowing along the pipe wall from the total liquid flow rate before calculating the void fraction, the mean acoustic velocity data collected by Evans et al. compared well with previous tests done by Hinkle¹⁰.

Radovskii¹¹ derived an equation for the speed of propagation for a disturbance of an arbitrary nature in a slightly nonequilibrium adiabatic two-phase flow:

$$W^2 = c_3^2 - \gamma_1(c_1^2 - c_0^2) - \gamma_2(c_2^2 - c_1^2) - \gamma_3(c_3^2 - c_2^2)$$

where

W = speed of propagation

$\gamma = \frac{\Delta \xi_j}{\Delta \xi_j^0}$, change in parameter ξ_j , which characterize the independent processes

c = speed of sound

and the subscripts denote the number of processes that are considered as "frozen", the other processes are equilibrium ones.

An expression to predict the speed of propagation of longitudinal acceleration waves in bubbly two-phase flows was derived by Dobran¹².

$$W^2 = (A_1 \pm A_2) / A_3$$

where

W = speed of propagation

$$A_1 = \bar{\rho}_1 c_1 (\bar{\rho}_2 + \Delta_{11}) + \bar{\rho}_2 c_2^2 (\bar{\rho}_1 + \Delta_{11})$$

$$A_2 = (\bar{\rho}_1 C_1^2 (\bar{\rho}_2 + \Delta_{11}) - \bar{\rho}_2 C_2^2 (\bar{\rho}_1 + \Delta_{11}))^2 + 4\bar{\rho}_1 \bar{\rho}_2 C_1^2 C_2^2 \Delta_{11}^2$$

$$A_3 = (\bar{\rho}_1^2 \bar{\rho}_2^2 + \Delta_{11} (\bar{\rho}_1 + \bar{\rho}_2))$$

$$C_1^2 = (\delta \bar{\pi}_1 / \delta \bar{\rho}_1) s_1$$

$$C_2^2 = (\delta \bar{\pi}_1 / \delta \bar{\rho}_2) s_2$$

$$\Delta_{11} = (\bar{\rho}_1 \bar{\rho}_2) / (\bar{\rho}_1 + \bar{\rho}_2) < 0$$

$\bar{\pi}$ = partial pressure of equilibrium state of the phase
(1 or 2)

$\bar{\rho}$ = density of equilibrium state of the phase (1 or 2)

To see if this relation could be used to model the speed of propagation of shock waves, Dobran assumed that C_1 and C_2 could be approximated by a_1 and a_2 , the speeds of sound in phases 1 and 2 respectively. The expression for the speed of propagation of shock waves, W , became:

$$(W/a_{g1,2})^2 = (B_1 \pm B_2^{1/2}) / B_3$$

where $B_1 = (\rho_g / \rho_l) (1 + (x/(1-x)) C_{VM}) + (a_g^2 / a_l^2) (\rho_g / \rho_l) + C_{VM}$

$$B_2 = ((\rho_g / \rho_l) (1 + C_{VM} x / (1-x)) - (a_l^2 / a_g^2) (\rho_g / \rho_l + C_{VM}))^2 + 4x / (1-x) (\rho_g / \rho_l) (a_l^2 / a_g^2) C_{VM}^2$$

$$B_3 = 2(\rho_g / \rho_l) + C_{VM} ((x/(1-x)) (\rho_g / \rho_l) + 1)$$

$$C_{VM} = .3 \tanh(4\alpha) \text{ Virtual mass coefficient}$$

X = volumetric fraction of gas bubbles

ρ = density

The subscripts ℓ and g refer to the liquid and gas phases respectively.

There are two physical solutions for W . One solution, W_1 , is independent of α and C_{VM} and $W_1 = A_1$. The second solution, W_2 , strongly depends on C_{VM} and is much less than either a_1 or a_g . The values obtained from the expression agreed well with the experimental data of Akagawa et al.¹³ and Miyazaki et al.¹⁴

Cheng et al.¹⁵ presents an expression for the "frozen" velocity of sound in the bubbly flow regime assuming a homogeneous fluid and considering compressibility effects. The homogeneous "frozen" velocity of sound, C_H , refers to the state of the fluid in which the speed of sound is being measured. As a steep pressure pulse passes through a mixture, the fluid does not have time to adjust to a new equilibrium state and is referred to as "frozen". The relation is:

$$1/C^2 = ((1-\alpha)\rho_\ell + \alpha\rho_g) \left((1-\alpha)/(\rho_\ell C_\ell^2) + \alpha/(kP_g) \right)$$

where α = void fraction

P = pressure

ρ = density

k = polytropic gas constant

C = speed of sound

Subscripts ℓ and g refer to liquid and gas phase respectively.

The thermodynamic process (adiabatic, isothermal, etc.) the fluid undergoes as the pressure pulse passes through it determines the method used to calculate the speed of sound in each phase.

This model was extended to include relative motion (virtual

mass) and viscosity effects by Crespo¹⁶. The relations derived by Crespo predict the velocity of sound for liquid in the frozen state, isothermal state, or isentropic state depending on the radius of the gas bubbles, as compared to viscous length and the thermal diffusion length.

$$\frac{\nu \ell}{\omega R_b} = \text{viscous length} \quad \frac{a_g}{\omega R_b} = \text{thermal diffusion length}$$

a_g = thermal diffusivity R_b = bubble Radius

ν = Kinematic viscosity ω = angular frequency

The bubbles behave isothermally (i.e. $k = 1$) when the bubble radius is less than the viscous length which is less than the thermal diffusion length. Crespo's relation agreed with the relation derived by Cheng et al.¹⁵ When the bubble radius is large when compared to the viscous length, but small when compared to the thermal length, the bubbles will still behave isothermally but do not move at the same velocity as the liquid. The sonic velocity is given by:

$$c^2 = [(1 + \alpha[1 - \alpha]/C_{VM})P_\ell] / [\rho_\ell \alpha(1 - \alpha)]$$

where C_{VM} = Virtual volume coefficient. The rest of the symbols and subscripts are the same as in the previous equation.

Crespo found that when the bubble radius is greater than both the viscous and thermal diffusion lengths, there is slip between the two phases and the bubbles behave isentropically. The velocity of sound is given by:

$$c^2 = ((1 + \alpha(1 - \alpha)/C_{VM})\delta P_\ell)/(\rho\alpha(1 - \alpha))$$

where δ is the ratio of the specific heats in the gas (C_p/C_v).

For Crespo's equation derived for the case when the bubble radius is greater than the viscous length but less than the thermal diffusion length, the frequency must be below the bubble resonance frequency. Henry¹⁷ included a simplified virtual mass term in his derivation and assumed $P_1 = P_g$ to take into account the frequency restriction.

$$c_{2\phi}^2 = ((1 + \alpha(1 - \alpha)/C_{VM})kP_\ell)/(\alpha(1 - \alpha)\rho_\ell)$$

For $k = 1$, the isothermal case, this equation reduces to Crespo's relation for isothermal behavior. Similarly, when $k = \delta$, the isentropic case, the above relation reduces to Crespo's isentropic relationship. For the case of homogeneous flow, $C_{VM} \rightarrow \infty$, Henry's relation reduces to an approximation of the relation derived Cheng et al¹⁵.

By assuming that the gas compressibility term was a function of the void fraction ($k = k(\alpha)$), Henry derived a correction factor that was linear in void fraction. He derived an expression for homogeneous, isothermal two-phase velocity of sound, C_{HT} :

$$C_{HT} = [([1 - \alpha]\rho_\ell + \alpha\rho_g)([1 - \alpha]/[\rho_\ell C_\ell^2] + \alpha/P_\ell)]^{-1}$$

This relation agrees well with experimental data for void fractions up to 0.5 and appears to accurately describe both one- and two-component bubbly flow momentum transfer processes.

By treating the interface of one phase as the elastic boundary of the other, Nguyen et al.¹⁸ derived a relation for the velocity of sound given by:

$$C_H = [([1 - \alpha]\rho_\ell^{1/2} + \alpha\rho_g^{1/2})([1 - \alpha]/[\rho_\ell C_\ell^2] + \alpha/[\rho_g C_g^2])]^{1/2}^{-1}$$

The results of this equation also agree well with one- and two-component sound velocity data.

Van Wijngaarden¹⁹ presents an expression in two-phase flows where liquid forms the continuous phase, for the speed of sound, C_0 , that is similar to Crespo's. The following relation needs correction when the void fraction, α , is either close to zero or unity.

$$C^2 = (\delta P)/(\rho_\ell \alpha [1 - \alpha])$$

where P = pressure

ρ_ℓ = density of liquid phase.

Levich's²⁰ model for calculating the frictional force, W , experienced by a bubble in two-phase flow was experimentally verified by van Wijngaarden. The relation is:

$$W = 12\pi\mu R(v-u)$$

where μ = viscosity

u = liquid velocity

R = bubble radius

v = bubble velocity

Nakoryakov et al.²¹ experimentally verified the Landau relation for the speed of sound, C_1 , in a vapor-liquid mixture on the saturation line. Assuming slow processes and sound propagation due to phase transition, the relation is:

$$C_1^2 = Lp^2/\rho_1^2 C_v B^3 T^3$$

where ρ_1 = liquid density P = pressure

C_v = speed of sound in vapor

B = gas constant

T = temperature

L = latent heat of evaporation

A relation between shock strength and velocity of propagation (U_{shock}) for isothermal, homogeneous two-component two-phase flow was derived and experimentally verified by Campbell and Pitcher¹⁹.

$$U_{\text{shock}}^2 = \frac{P_2}{P_1} C_1^2 = \frac{P_2}{\rho_1 \alpha_1 (1-\alpha)}$$

where the subscripts 1 and 2 stand for conditions in front and behind the wave front, and the isothermal two-phase velocity C_1 is calculated from the expression of Cheng et al.¹⁵ for determining the "frozen" velocity.

Akagawa et al.²³ used a one-component, two-phase horizontal bubbly flow to predict the relationship between the magnitude of the potential surge, ΔP_{ps} , and the propagation velocity, C_{TP} .

$$\Delta P_{ps} = C_{TP} W_{10} \rho_1$$

where W_{10} = superficial velocity of the liquid

ρ_1 = density of the liquid

Experimental results show that this relation holds for two-component two-phase flow as well as one-component two-phase flow.

For a liquid with gas bubbles, Malykh and Ogorodnikov²⁴ found that as the incident wave pressure of the pulse was increased, the damping of the pulse increased. They also experimentally determined that as the void fraction was decreased, the speed of wave propagation increased.

Moody²⁵ studied acoustical damping in liquid-gas systems considering only thermal and mechanical interface irreversibilities without phase changes. Thermal damping in two-phase mixtures occur when gas bubbles undergo pressure changes and temperature variations due to heat transfer between the liquid and gas in a bubbly mixture. Mechanical drag dissipation results from the relative motion between liquid droplets and the surrounding gas in a droplet mixture.

Moody found the speed of sound in a bubbly mixture, C_b , to be:

$$C_b = [x(\rho/\rho_g)^2 / C_g^2 + (1-x)(\rho/\rho_L)^2 / C_L^2]^{-1/2}$$

where:

ρ = density

x = mixture quality

C = speed of sound

subscripts L and g refer to liquid and gas phases respectively.

Moody derived expressions for the acoustic penetration of sound waves in bubbly mixtures and droplet mixtures. The propagation into the fluid, $P(z,t)$, is a function of position, z ,

and time, t .

$$P(z,t) = P_0 e^{-\sigma z} \sin \omega(t - z/c)$$

For a bubbly mixture:

$$\sigma = (D/C_b) \left[\left(\left[\frac{\omega}{D} \right] \left(\left[\frac{\omega}{D} \right]^2 + 1 \right)^{1/2} - \left(\frac{\omega}{D} \right)^2 \right) / 2 \right]^{1/2}$$

$$C = C_b / \left(\left(\left(1 + \left[\frac{D}{\omega} \right]^2 \right)^{1/2} + 1 \right) / 2 \right)^{1/2}$$

For a droplet mixture:

$$\sigma = (\omega/C_g) \left[(C_1 - C_2) C_3^2 / C_4 \right]^{1/2}$$

$$C = C_g / \left[(C_1 + C_2) C_3 / C_4 \right]^{1/2}$$

with

$$C_1 = \left(\left[1 + \left(A_3 / [C_g^2 \omega] \right)^2 \right] \left[1 + \left(A_3 / [C_g^2 \omega] \right)^2 (\rho / \rho_g)^2 \right] \right)^{1/2}$$

$$C_2 = 1 + \left(A_3 / [C_g^2 \omega] \right)^2 \rho / \rho_g$$

$$C_3 = 1 + \rho_g C_g^2 (1 - \alpha_g) / (\rho_l C_l^2 \alpha_g)$$

$$C_4 = 2 \left(1 + \left[A_3 / (C_g^2 \omega) \right]^2 \right)$$

$$A_3 = [g_0 D_c n C_g^2] / [\alpha_g (1 - \alpha_g) \rho (1 + \rho_g C_g^2 (1 - \alpha_g) / (\rho C_l^2 \alpha_g))]]$$

where

D = thermal damping coefficient

α = volume fraction

D_c = drag constant

ω = circular frequency

g_0 = Newton's Constant

C = speed of sound in
mixture

n = droplet density

P = pressure

C_g = speed of sound in gas phase

It can be determined from these relations that higher frequencies penetrate shorter distances, while lower frequencies tend to penetrate further without attenuation in both bubbly or liquid drop mixtures. This explains why fog horns are used to warn ships instead of whistles.

An expression for acoustic decay was formulated by Moody to be:

$$P(z,t) = P_0 e^{-Rt} (\cos Tt) \sin(kz)$$

where

$$R = (C_g^2 K^2) / (((3D_c g_o) / (4\alpha_g r \rho_l a_d)) (\rho / \rho_g))^2$$

F = oscillation frequency

K = wave number

a_d = droplet projected area

r = spherical droplet radius

By increasing the wave number (shorter wave length), the sound wave decays faster. Also large values of the density ratio cause faster decay.

Chug ringout occurs when steam is discharged into cool water and the sudden condensation creates a void that causes an acoustic disturbance. Moody discovered that thermal dissipation in a fine bubbly mixture was capable of providing the strong damping associated with chug ringout, and that further study is needed to determine the relationship between the two.

The majority of the studies performed on predicting the

propagation speed of waves have generally been limited to predicting the leading edge velocity of large amplitude waves. In effect, this corresponds to predicting the sonic velocity for the highest frequency contained in the wave.

High frequency waves propagate at a faster rate than low frequency waves because there is less time for the dispersed phase (water) and the continuous phase (air) to reach equilibrium with respect to mass, momentum, and energy transport processes.²⁶ This means that the number of active degrees of freedom is reduced which results in an increase in the speed of wave propagation through the media.

Of the three transport processes (mass, momentum and energy), mass transfer (condensation and evaporation), may be shown to be the slowest mode of energy distribution. Theoretically, high frequency propagation in one-component media should not differ from that in two-component media, which does not experience mass transfer, because there is no time for mass transfer to occur. Experimental results support this view.²⁶

Zink and Delsasso²⁷ studied how sound velocity as a function of frequency was affected in a gas with solid particles suspended in it. The experimental procedure involved comparing the original signal to the signal after it passed through the two-phase media. An oscilloscope was used to compare the signals and measure the magnitude and phase changes. They found that the change in velocity for low frequencies was greater than the velocity change in the higher frequencies. Figures 1 and 2 show their

