



Demodulation of narrow-band FM by a phase-locked loop
by Michael D Berkoff

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
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Abstract:

The subject of the thesis is the comparison of performances of the first- and second-order loops at low carrier-to-noise ratios.

It was determined in this thesis that the first-order phase-locked loop is capable of suppressing more "clicks", or FM anomalies, than a second-order loop when demodulating a narrow-band FM signal and while subject to a constraint on the size of the phase error. The phase error constraint was chosen so that the loop would tend to operate in its linear region and the FM modulation was limited to very small modulation indexes (.2 and less). The ability of the loop to suppress "clicks" was computed in conjunction with Hess' "Click" model [8]. The phase error variance of the loop was estimated by an approximate quasi-stationary linear model. An extensive computer search over the fields of all first- and second-order loops led to the conclusion of the thesis about the superiority of the first-order loop under the above-mentioned operating conditions.

DEMODULATION OF NARROW-BAND FM.
BY A PHASE-LOCKED LOOP

by

MICHAEL D. BERKOFF

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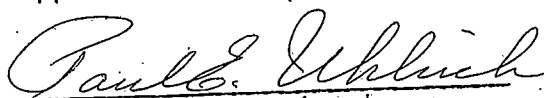
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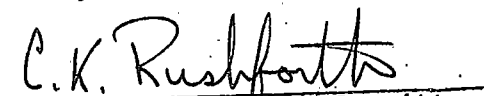
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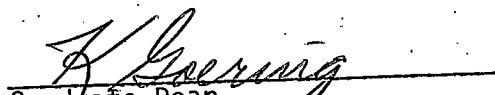
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VITA

Michael D. Berkoff, son of Mr. and Mrs. Dmitry N. Berkoff, was born in Harbin, China, on October 4, 1937. He attended the Soviet Middle School there and was graduated in July 1955. He received the B. S. degree in Electrical Engineering in June 1962 and the M. S. degree in Electrical Engineering in June 1964, both from Montana State University.

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$E\left[\left[\phi(t) - \mu_0\right]^2 \mid a(t) > A_0\right]$ versus A_0 [a threshold of the normalized carrier amplitude $a(t)$].
 Errors are calculated with reference to the results of the Monte Carlo computer simulations. 117

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