Optimal Economic Control Strategies in Forest Resource Management
by AFFENDI ANWAR

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Applied Economics
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Abstract:
The investment decision is the most complex and difficult problem in forestry management because it involves large lapses of time and uncertainty of the future outcome resulting from the decision. This study attempts to analyze the forest production decision problem in general and management aspects related to investment, particularly those related to optimal thinning and rotation decisions. The decision model for this study is based on the application of modern control theory to the forest management problem. An appropriate line of approach to solve the control problem in forest production systems is stochastic dynamic programming which is capable of capturing the stochastic behavior of the system and incorporate risk factors of the decision explicitly. Another advantage of the method is thatit provides a powerful systematic search for obtaining numerical solutions to the problem, provided the required dynamic interrelationships of the system and transition probabilities can be estimated empirically. Based on this method a criterion function is formulated as maximization of the sum of expected discounted net returns to soil site as a modification of the Faustmann criterion in a statistical sense.

To approach a specific problem in forest management, however, this study pursues another method to solve the control problem of a disease-infected lodgepole pine stand due to lack of available basic data. The control model for this specific problem is based on a modified simulation model which was previously developed by Myers et al. [1971]. By a modification of this model, the redirected computer simulation program resulted in an economic model which has a specific objective function consistent with the problem of managing a diseased timber stand. The criterion function of the model is generalized present value as an extension of the Faustmann criterion.

By virtue of the certainty equivalence principle, the simulation model which is deterministic can be considered as a good approximation for solving stochastic problems in managing a forest production system due to the fact that the decision processes based on this model are carried out sequentially. The computer simulation model can serve as a management aid to help the forest manager in making decisions related to investment problems for any forest stand condition and economic factors which influence the system. Since the simulation results, in general, show a consistency with a priori logical reasoning, it is concluded that the approach can be expanded for application to other species and areas larger than the Rocky Mountain Region where this study was carried out.

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ABSTRACT

The investment decision is the most complex and difficult problem in forestry management because it involves large lapses of time and uncertainty of the future outcome resulting from the decision. This study attempts to analyze the forest production decision problem in general and management aspects related to investment, particularly those related to optimal thinning and rotation decisions. The decision model for this study is based on the application of modern control theory to the forest management problem. An appropriate line of approach to solve the control problem in forest production systems is stochastic dynamic programming which is capable of capturing the stochastic behavior of the system and incorporate risk factors of the decision explicitly. Another advantage of the method is that it provides a powerful systematic search for obtaining numerical solutions to the problem, provided the required dynamic interrelationships of the system and transition probabilities can be estimated empirically. Based on this method a criterion function is formulated as maximization of the sum of expected discounted net returns to soil site as a modification of the Faustmann criterion in a statistical sense.

To approach a specific problem in forest management, however, this study pursues another method to solve the control problem of a disease-infected lodgepole pine stand due to lack of available basic data. The control model for this specific problem is based on a modified simulation model which was previously developed by Myers _et al_. [1971]. By a modification of this model, the redirected computer simulation program resulted in an economic model which has a specific objective function consistent with the problem of managing a diseased timber stand. The criterion function of the model is generalized present value as an extension of the Faustmann criterion.

By virtue of the certainty equivalence principle, the simulation model which is deterministic can be considered as a good approximation for solving stochastic problems in managing a forest production system due to the fact that the decision processes based on this model are carried out sequentially. The computer simulation model can serve as a management aid to help the forest manager in making decisions related to investment problems for any forest stand condition and economic factors which influence the system. Since the simulation results, in general, show a consistency with _a priori_ logical reasoning, it is concluded that the approach can be expanded for application to other species and areas larger than the Rocky Mountain Region where this study was carried out.

CHAPTER I

INTRODUCTION

1.1. Motivation of Study

Among the most complex and difficult problems confronting a forest management authority are those involving investment decisions. Growing timber on forest land is an investment asset which has a changing value over time. Growing a timber stand can be considered as a long term investment decision where a long gestation period is needed between initial investment outlay and realization of all expected earnings. On the other hand, cutting timber at final harvest is an act of disinvestment in which capital represented by the value of timber is released and the forest land is made available for a subsequent timber crop or other uses. Hence, there is opportunity cost attached to the land presently occupied by standing timber.

Between the date of planting trees and final harvest, the forest manager may undertake a series of thinning actions and other stand improvement activities such as clearing, fertilizing, pruning, disease control, etc. which involve additional investment decisions, in order to improve the future stand condition and hence to obtain a higher value of final harvest. The land committed to timber production, growing stock of timber, and cumulated stand improvement outlays constitute the investment in a forest at any point in time. The long gestation period between investment outlays and receipt of revenues, combined with the complex biological interrelationships of all management activities over the economic life of the forest, is what makes forest investment decisions a difficult problem.

Throughout the development of the principles of traditional forest resource management, various theoretical and empirical studies have been made to serve as guides for sound investment decisions in forestry. In an era of rapidly changing technology, economic decisions grow in frequency and complexity, and wrong decisions increase in costs. With this additional impetus and advances in applied mathematics in recent years, quantitative methods for handling complex economic problems have been introduced to the forestry field. Among others are those which are related to determination of forest management, including simultaneous optimization of rotation and thinning decisions (see Näslund, 1969 and Schreuder, 1971). However, both authors have mainly concentrated their studies on the discussion of a general analytical framework for determining optimal rotations and thinnings without any empirical application of the results.

Determination of optimal rotation and thinning decisions in forestry, by and large, is influenced by physical-biological aspects of the forest growth which constitute an "engineering" side of forest production systems. Hence the optimal decision will be specific to a given forest environmental condition. Therefore, if there are any extraneous factors which influence this "engineering" aspect, the optimal decision will be affected.

Another factor which may influence the optimal decision is economic elements, including supply and demand of the commodity produced, capital, and input factors within the production system. Conceptually, optimal investment decisions in forest production systems can be approached from a more general equilibrium theory, but in practice, a model giving the general equilibrium solution would often be extremely unwieldy. Hence, in order to formulate an operationally manageable model, we must often be willing to settle the problem for something less. Thus the investment problem in forestry for this study will be limited to partial equilibrium analysis where the price and cost structure of the commodity, capital, and other input factors are determined outside the system of forest production. One specific aspect of the study will deal with the formulation of conceptual and empirical solutions to the forest investment problem so that it will help to give a better understanding of the overall problem.

Since the major portion of forest investment decisions is dominated by determination of the rotation and thinning policy, for clarity of further discussion, throughout the course of this study the term investment will refer to both rotation and thinning decisions interchangeably. Moreover, even though there are many intangible benefits that could be derived from the forest stand, this study will be limited to the measurable monetary benefits from wood production.

1.2. Objectives and Plan of Analysis

There are two general objectives to be pursued in this study. The first and more theoretical objective is to formulate and to evaluate the optimal control strategy in an operational model suitable for analysis of decision making involved in the choice of a sound forest management policy in cases where complete basic data can be obtained--for example, the basic data which could be obtained from a continuous forest inventory (C.F.I.). To obtain an optimal control strategy in forest production systems requires a multistage decision process; that is, a sequence of decisions that will optimize some objective function or criterion function. The appropriate criterion function for forest management policy is maximum discounted net returns with respect to soil site, but some method must be devised to account for the random occurrance of some measurable exogenous factors to the production system, such as disease, insects, and other factors that may affect the forest growth during evolution of the system. Therefore, the decision criterion of this study will be maximization of expected soil value over the entire planning horizon in a statistical sense--a modification of the well-known Faustmann criterion (Faustmann, 1849).

The strategy or conditional decision rule is applied sequentially as new information unfolds through time and the rule is chosen to maximize the above stated criterion. Mathematical analysis of this type of decision processes led to the development of dynamic programming, a la Bellman [1957]. 1/ This type of model provides more than just a computational procedure for obtaining numerical results. It is also a logical framework and in some cases, analytical results can be derived for specific problems, such as in optimal control of natural resources (see Burt, 1964, 1966 and Burt and Cummings, 1970). In fact, analytically, dynamic programming is one of the modern approaches for solving general control problems (see Intriligrator, 1971 and Robert and Schulze, 1973).

The realm of modern control theory also includes the maximum principle of Pontryagin et al. [1962] as an extension of the classical calculus of variations. This method will also be briefly reviewed in formulating the concept of optimal forest resource management.

The second objective, which is more applied, is to analyze forest management decisions regarding an optimal control strategy for dwarf mistletoe (Arceuthobium spicies). This disease has adverse effects on forest growth and constitutes one of the exogenous factors which affects the production system, and hence will affect the optimal investment decision. However, due to lack of available basic data,

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1/ Sometimes dated versions of linear programming are also called "dynamic programming" which is intrinsically not dynamic because the optimization is made once and for all.

a somewhat different approach for controlling diseased stands will be applied, as an approximate solution to more general control problems. The criterion function is still based on a stochastic version of the Faustmann criterion, but with some modification.

The physical-biological aspects of the production system for this study are obtained from a computer simulation program which has been developed by Myers et al. [1971]. This simulation model includes the dynamic physical-biological interrelationships between the level of disease infestation and growth of the forest which can be altered by thinning decisions at various stages of maturity of the forest. This simulation study provides the necessary information which enables us to construct a dynamic decision model for controlling forest production.

In order to build a decision model based on economic criteria, the simulation model required some modifications by inserting the computation of some economic elements into the model. By varying some of the parameters of the system, results could then be obtained by calculating a performance measure of the system. The performance measure for this study is present value of the stream of net returns from wood production over an infinite planning horizon. A search procedure on the parameters of the modified simulation model provides an approximation to the optimal decision rule. Information derived from this analysis could be of assistance to the forest management authority which is responsible for the execution of current forest management policy in the Rocky Mountain Region where the study was carried out.

Chapter II of the study discusses the basic structure of the forestry investment decision, and a simple investment decision model is also described. In Chapter III, the general optimal control problem in relation to forest investment decisions is dealt with. Then a more operational model of the investment problem with a dynamic programming solution is presented. In Chapter IV the empirical model of the study using the simulation model as an approximate solution to the general control problem is presented. Finally, Chapter V contains an interpretation of the results of the empirical study and some conclusions.

CHAPTER II

INVESTMENT MODEL FOR FORESTRY

2.1. The Concept of Forest Production

Forestry, the business of growing timber, has characteristics that distinctly give it special status among agricultural production activities. First, the long gestation period between initial input and the first harvested output is quite unique. Second, there is an extended period of autonomous growth in value associated with the initial investment decision. Then eventually, there is a gradual decline in the productive capacity of trees to grow over an extended time due to the biological aging process.

To produce a wood crop, one has to plant the trees and care for them through several years before there is any opportunity to receive revenues. There is a long interval between the date when growing timber reaches a minimum size of marketability and the date when trees finish their growth. At any time between these two dates the timber may be judged ready for cutting on the basis of an economic criterion. The land has many alternative uses, but the trees on the land have virtually no use other than production of standing timber. Thus, an understanding of resources committed on a long-run basis is important to any consideration of timber production.

Let us assume there exists an expected intertemporal production function with many inputs and many outputs. The production function associated with initial time t is assumed to be twice differentiable:

$$\Phi_t(\underline{Q}) = \Phi_t(q_1, q_2, \ldots, q_n) \tag{1}$$

where q_j $(j = 1, 2, \ldots, n)$ are discrete dated inputs and outputs and n is the total number of dated inputs and outputs over some arbitrarily long but finite planning horizon. The condition $q_j < 0$ signifies an input and $q_j > 0$ indicates an output. A given set of dated inputs will result in a unique set of dated outputs which the firm values at v_j $(j = 1, 2, \ldots, n)$. The v_j are prices adjusted by the time discount structure.

Assuming profit maximizing behavior of the firm and perfect competition in the factor and product markets, an optimal production plan can be derived by means of the Lagrangian function:

$$L(\underline{Q},\lambda) = \sum_{j=1}^{n} v_j q_j - \lambda \Phi_t(\underline{Q}). \tag{2}$$

The first order necessary conditions for a maximum of this Lagrangian will give a solution for output supply and input derived demand functions:

$$q_j^* = q_j^*(v_1, v_2, \ldots, v_n) \quad , \quad j = 1, 2, \ldots, n. \tag{3}$$

Given initial expectations, the demand function for inputs shows how the firm would desire to allocate inputs over time to maximize profit. It also shows the stream of outputs which the firm expects as a result.

The t-subscript on Φ implies that a different production function exists at each time period t when a set of decisions are made. This production function is determined by all previous decisions and reflects the number of acres of standing timber of various ages at the point in time t, as well as any other fixed conditions resulting from all previous decisions of the firm.

Moreover, because of some changes in production conditions, it is assumed that the firm makes periodic revisions of its plans based on new expectations. Thus each period the firm maximizes an appropriate Lagrangian of the above form. This maximization may in some periods call for the firm to invest in planting new trees expressed as derived demand for land.

When prices were sufficiently favorable in some previous period trees were planted. This decision process occurs recursively where in each period the forest manager uses updated expectations as timber becomes one period older. As long as reasonable economic stability is maintained, we may assume that there exists a wide span of periods where it is more profitable to keep the trees, rather than to cut them for timber, until it comes the time where additional net returns from presently standing timber are equal to opportunity cost of the land for other uses. One use of the land which may determine its opportunity cost is growing a new stand of trees.

Each optimization problem faced by the firm at a particular time period t involves variables associated with time t and all future periods in the firm's planning horizon. The future variables must be considered jointly with the current variables in the optimization problem to determine optimum levels of the current variables, but these future variables are then ignored until the perpetually recurring optimization problem is faced in the next time period, say t+1. At that time the entire process is repeated to get optimal levels of the variables associated with period t+1. In a sense, the variables in the optimization which are associated with future time periods are merely instruments or artificial variables used to derive optimal levels of current variables.

The underlying justification for this sequential dynamic optimization approach is certainty equivalence decision theory of Simon [1955] which was later expanded by Theil [1957] and Madansky [1960]. Although the theory only applies to quadratic criterion functions subject to linear constraints, it would appear to be a reasonable approximation to the general case of decision under uncertainty.

The above description of the concept of forest production is approached mainly from neo-classical theory of production related to investment decisions. However, in order for the concept to be meaningful in practice it must be operationally feasible. The following section will discuss a more realistic model of investment decisions consistent with the above concept.

2.2. Multiperiod Investment Decisions

Before we can go further to a formal investment decision model it is necessary to describe the procedure which must be employed to compare present and future receipts and outlays. Hence, a discussion of the discounting or present value concept is important to any investment decision model.

In real world situations, most decisions are periodic; hence, in facing economic problems which deal with time, the time span or planning horizon can be divided into several periods usually with equal intervals. Multiperiod investment decisions are characterized by a situation where factors of production invested during one time period will affect the level of output during subsequent periods. Therefore, there exists a functional relationship between input and output which has different dating. Further, we assume that there exists a market of capital where capital can be borrowed or lent at some given rate of interest. By using this interest rate, outlays or revenues incurred during different time periods can be made comparable by discounting (or compounding) them to one common period.

Suppose P dollars were invested or borrowed for one time period at the rate r. The value of r expresses the proportion of the amount of capital borrowed or lent which constitutes a cost of capital to the borrower or income to the lender (investor) per period. Hence at the end of one period the investor would expect rP dollars in interest, and with return of the principal P, this would give the sum $S_1 = P + rP = P(1+r)$ dollars. If the investment rate does not change over time, the investment would yield $S_2 = S_1(1+r) = P(1+r)^2$ dollars at the end of the subsequent period after the first, or in general, it would yield $S_t = S_{t-1}(1+r) = P(1+r)^2$ after t time periods. Therefore, an amount of P dollars expected to be received after t time periods can always be exchanged, or equals $P(1+r)^{-t}$ at the present, because the firm or individual who expects to receive the amount of P dollars after t time periods can borrow the amount of $P(1+r)^{-t}$ dollars at the present and he can repay with the amount $P(1+r)^{-t}(1+r)^t = P$ dollars at the later period.

The expected future receipts $S_t = P(1+r)^t$ from investment are called the future or compounded value of P, while the amount $P = S_t(1+r)^{-t}$ is called present value or discounted value of S_t after t periods of investment. Hence, the factor $(1+r)^{-t}$ by which the future value should be multiplied to get the present value P is called the discount factor.

If we assume that within one time period we have more than one time compounding, say m times, then the future value S_t becomes:

$$S_t = P\left(1 + \frac{r}{m}\right)^{mt}. \tag{4}$$

Let us designate $\frac{m}{r} = k$, then

$$S_t = P(1 + \frac{1}{k})^{rkt} = P[(1 + \frac{1}{k})^k]^{rt}. \tag{5}$$

As compounding within the time period becomes more and more frequent, in the limit as $m \to \infty$, $k \to \infty$, hence:

$$\lim_{m\to\infty} S_t = \lim_{k\to\infty} P[(1 + \frac{1}{k})^k]^{rt} = P[\lim_{k\to\infty} (1 + \frac{1}{k})^k]^{rt}. \tag{6}$$

Since by definition $\lim_{k\to\infty} (1 + \frac{1}{k})^k = e$, where e is the base of natural logarithm, then

$$\lim_{m\to\infty} S_t = Pe^{rt} \tag{7}$$

or

$$P = (\lim_{m\to\infty} S_t)e^{-rt}. \tag{8}$$

Discounting provides a method of transforming future incomes and outlays so that they are commensurate with the present. Through discounting, a future stream of net returns can be expressed as a single number which is called present value of the entire set of future returns. For example, an entrepreneur expects to receive returns from an investment equal to R_0, R_1, ..., R_T during time periods 0, 1, 2, ..., T, respectively, and the corresponding discount factors are $\beta^t = 1/(1+r)^t$, t = 0, 1, 2, ..., T. Present value of this revenue stream is:

$$R = \sum_{t=0}^{T} R_t \beta^t \quad , \quad t = 0, 1, 2, \ldots, T \qquad (9)$$

or for instantaneous discounting,

$$R = \int_0^T R(t) e^{-rt} dt \quad , \quad 0 \leq t \leq T.$$

While static theory assumes that the entrepreneur will maximize his profit in a single time period, in dynamic multiperiod cases, it is assumed that the entrepreneur will maximize present value of the difference between future incomes and outlays; or in short, he will maximize present value of the streams of future net returns from the investment. 2/

Having established the concept of multiperiod investment decisions, now we are in a position to apply this concept to forestry enterprises. The following discussion will deal with a very simple case of investment decisions concerning a growing stand of timber.

2.3. An Investment Decision for Timber Stands

In the forestry enterprise, the investment decision is characterized by a relatively large amount of initial outlays for timber

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2/ If the decision maker is a public agency who is concerned with public investment projects, the same criteria still can be applied. A primary difference between private and public investment criteria lies in the rate of discount. The private rate reflects private opportunity cost of capital, while the social rate reflects the social opportunity cost of public projects as the value to society of the next best alternative use to which the funds employed on the project could have been put.

stand establishment, and receipts are dominated by the income from terminal harvest as contrasted to the flow of annual costs of upkeep and returns from thinnings. In order to formulate a criterion function for the investment decision, let us use the following notation:

T = length of the rotation cycle;

$B(T)$ = present value of all net returns;

$R(t)$ = net returns associated with management of the timber stand at the moment t, except for net sale value of final harvest;

$H(T)$ = net sale value of final harvest;

r = interest rate; and

$\beta^t = 1/(1+r)^t$ is the discount factor for any period t.

Present value of the stream of net returns from the timber stand for a single rotation cycle can be expressed as

$$B(T) = \sum_{t=0}^{T} R(t)\beta^t + H(T)\beta^T \quad , \tag{10}$$

or for the continuous time discounting case: 3/

$$B(T) = \int_0^T R(t)e^{-rt}dt + H(T)e^{-rT} \tag{11}$$

However, formulation of the present value criterion in (10) and (11) considers only one rotation cycle, which is not applicable to the

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3/ In some cases, a continuous version of the present value criterion is preferable, especially for analytical treatment of the model.

forestry enterprise based on a sustained yield principle. Based on this principle, the enterprise would evaluate the income stream not only from one rotation cycle, but from all future cycles within the planning horizon. Hence, the appropriate present value criterion for investment decisions in forestry should be based on an expected income stream over an infinite planning horizon. Therefore, present value of net returns from growing stock of timber over an infinite chain of rotations can be expressed as follows:

$$B(T) = [\int_0^T R(t)e^{-rt}dt + H(T)e^{-rT}][1 + e^{-rT} + e^{-2rT} + \ldots + e^{-irT} + \ldots] = \frac{1}{1 - e^{-rT}}[\int_0^T R(t)e^{-rt}dt + H(T)e^{-rT}]. \quad (12)$$

The expression in (12) is essentially the Faustmann criterion. 4/

Having established the criterion function, we can analytically deduce a decision rule for determining optimal length of the rotation cycle. For simplicity, we assume that other decisions such as the thinning level are given; the forest entrepreneur is only concerned with determination of the optimal rotation. The object is to maximize

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4/ Faustmann's solution to the problem, was later confirmed by G. A. D. Preinreich, who was apparently unaware of Faustmann's work in forestry nearly a century earlier, in "The Economic Life of Industrial Equipment," Econometrica, VIII (January, 1940), pp. 12-44.

present value of the infinite series of discounted future returns with respect to rotation length T. Therefore, the problem of investment becomes:

$$\underset{T}{\text{Max}}B(T) = \underset{T}{\text{Max}}\left[\frac{1}{1 - e^{-rT}}\left\{\int_0^T R(t)e^{-rt}dt + H(T)e^{-rT}\right\}\right]. \tag{13}$$

The necessary condition for the above problem can be obtained by differentiating (16) with respect to T and equating the results to zero, which gives the following:

$$\frac{dB}{dT} = -\frac{re^{-rT}}{(1 - e^{-rT})^2}\left\{\int_0^T R(t)e^{-rt} + H(T)e^{-rT}\right\} + \frac{1}{(1 - e^{-rT})}$$

$$\left\{R(T)e^{-rT} + \frac{dH}{dt}e^{-rT} - rH(T)e^{-rT}\right\} = 0. \tag{14}$$

Multiplying (14) by $e^{rT}(1 - e^{-rT})$ yields

$$-rB(T) + R(T) + \frac{dH}{dT} - rH(T) = 0 \tag{15}$$

or

$$R(T) + \frac{dH}{dT} = r[H(T) + B(T)]. \tag{16}$$

A literal economic interpretation of (16) is that under an optimal rotation policy, we shall cut the timber when the current return R(T) for having the stand one more year plus the increase in the harvest value $\frac{dH}{dT}$ equals the interest on the sum of harvest value and soil rent. Soil rent is equal to B(T) which is present value of net returns starting with bare timber land, following an optimal rotation policy over an infinite planning horizon.

The above analytical method, using elementary calculus for determining the optimal rotation in forestry, gives some insight as to the optimal decision rule to be followed by the forest entrepreneur. However as the problem becomes more and more realistic, and consequently more complicated, more sophisticated mathematical methods are required. The next chapter will examine some more comprehensive models for forest investment decisions. Before going into the analysis of more realistic models for investment, we will discuss the problem of uncertainty which is an important difficulty faced by any decision maker in a real world situation.

2.4. Consideration of Risk and Uncertainty

No discussion of economic decisions--such as determination of optimal investment in growing timber--can be complete without recognizing and taking into account the effect of uncertainty of future events. The analytical model of investment in the last section was simplified by assuming perfect knowledge of input-output relations and prices in the production system within the firm's planning horizon. However a good economic decision model should be capable of providing managers with guides to take action when they face choices between alternative courses of action where the results are uncertain. Introduction of risk and uncertainty into the analytical model will complicate the problem, but from a practical decision viewpoint, it is justified by the additional realism obtained in approaching a real world problem.

Although no one knows the exact outcome of future events, a real world problem of investment could be treated as if the probability distribution of the future outcomes were known rather than pretending to know the exact future outcomes. Knight [1921] distinguished risk and uncertainty as two different phenomena. Following his ideas, risk refers to the situation when parameters of probability distributions of future outcomes can be estimated so as to be actuarially insurable; or in other words, the variability of future outcomes can be empirically measured. Uncertainty, on the other hand, refers to a situation in which the parameters of the probability distribution of the outcomes cannot be empirically or quantitatively estimated.

The importance of risk factors affecting forest management decisions has long been recognized by forest economists, and most forest land managers are aware of the advantage in maintaining a flexible forest management program to reduce the difficulties of changing output of the forest in response to changes in the market. Markowitz made an important contribution to the theory of investment under risk when he developed his methods for determining an optimally diversified portfolio [1959]. He developed a logical framework to guide investment decisions by explicitly recognizing the risk factor. His model provides a formal method for evaluating the advantage of diversifying investments as a means of reducing risk associated with the expected returns from a number of investments. Dowdle [1962] has applied the EV investment guide of Markowitz to diversify cost expenditures in forest investment activities. 5/ However this method only applies to static investment decisions in forestry.

A promising method applicable to dynamic investment in forestry which recognizes risk factors has been developed by Burt [1965]. He analyzes the problem of investment in the context of the general replacement model under risk. His discussion is related to a special case of Howard's model (Howard, 1960), for which an analytical solution is derived. The replacement model is concerned primarily with the economic decision problem of asset life under conditions of chance failure or loss, i.e., possible loss of the asset due to exogenous influences outside the system. Though his model pertains to dynamic decisions of investment in forestry, the exogenous factors described

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5/ E and V refer to expected income and variance of income, respectively. The efficiency criterion of investment decisions can be considered as a combination of individual investments such that the variance for the entire set of investments cannot be reduced without also reducing the mean. All possible combinations of such investments traces out what is commonly called the EV efficiency frontier.

in the analysis refer to a sudden extraneous shock, such as insect attack or fire which may cause a sudden destruction of the asset in a short period of time. This model is not applicable to our problem where the exogenous influence of dwarf mistletoe on the standing timber system is chronic in nature.

In the next chapter we present a stochastic dynamic programming solution to the control problem which can incorporate the risk factor into the investment decision model.

CHAPTER III

OPTIMAL CONTROL IN FOREST MANAGEMENT

3.1. Management Problem in Forestry

Sound management practices in forestry require knowledge about physical-biological aspects of the forest resource. If the primary objective in forest management is to produce timber, the basic physical resource with which a forest manager is concerned is land and the timber growing on it. The productivity of timber as a growing "machine" where the wood product is part of that "machine" depends on many factors. First, growth of timber depends on the site quality which reflects the capacity of a given area of land to grow a certain timber species. Another important factor to be considered in growing timber is stocking (tree density) which affects the rate of growth of timber at a given age and site quality. Genetic make-up of a tree species determines the capacity of timber to grow and withstand any exogenous influences which might affect its development. Those exogenous factors to be accounted for in growing timber are disease, insects, adverse climate, etc. For a certain tree species, there exists an optimum physical-biological combination among the elements comprising the forest environment which give an optimum growth of timber. Hence, a good knowledge of how to use those resources effectively is essential in forest management.

In order to obtain maximum benefit from the forest resource, full advantage of knowledge, skill, and art of silviculture and forest measurement should be taken. Those technical aspects of forestry which are important should be considered in making any decision to solve management problems. However the concern of this study is investment decisions related to the economic problem of how to grow a better timber stand which will result in maximum benefits over a given time horizon. In order to achieve this objective, the study will focus on economic decisions, particularly the ones that are associated with the intensity of thinning and length of rotation cycle.

Thinning the standing stock of timber can be beneficial for the following reasons: (1) to eliminate undesirable trees and thereby concentrate growth on the desirable trees which produce a high value of timber, (2) to prevent excessive competition on site resources among trees which would result in wasting energies needed for total growth, (3) to obtain revenues before final harvest. The fourth reason often advanced is to increase the net growth of timber. However, evidence on this aspect of thinning is quite inconclusive, at least for most timber species.

Timber production involves a large lapse of time between planting and harvesting. Hence time is an important variable in management decisions of forestry. In a sense, decisions must be made in the forestry enterprise at each moment in time over the entire planning horizon of the firm. A mathematical model adapted to this kind of problem is commonly called "control theory."

3.2. Formulation of General Control Problem

The essence of an optimal control problem is to choose a feasible time path for the control (decision) variables which will maximize the objective function. The choice of time paths for the control variables implies a set of differential equations called equations of motion, and also determines time paths for certain additional variables called state variables which describe the state of the system. Employing this concept in the forest management problem, let us consider a forest production system which can be described by a set of state variables $x_1, x_2, \ldots, x_r$. Hence the state of the system at any given time t can be represented by the state vector $\underline{X}(t) = [x_1(t), x_2(t), \ldots, x_r(t)]$.

Suppose the objective of forest management is to maximize total discounted net benefits over a given planning horizon. In order to achieve this objective, a sequence of decisions is required that will maximize the objective function. Let us designate the set of managerial decisions by decision variables $u_1, u_2, \ldots, u_s$, so that at any time t, the related decision is represented by the vector of decision $\underline{U}(t) = [u_1(t), u_2(t), \ldots, u_s(t)]$.

From the definition of state and decision variables, there should exist a relation such that the rate of change in a state variable at any time t is a function of the present state of the system, the date, and the decision taken which is implied by the decision vector. This relation can be expressed in the form of a system of differential equations:

$$\dot{\underline{X}}(t) = \frac{dX}{dt} = \omega(\underline{X}(t),\underline{U}(t),t). \tag{17}$$

Expression (17) constitutes the equations of motion which can trace values of the state variables in a dynamic procress.

Decisions influence the system in two different ways: (1) the rate at which net returns are earned at that moment in time, and (2) the rate at which the growing stock of timber is changing, or in more general terms, the future path of the state vector.

Let us designate $R(\underline{X}(t),\underline{U}(t),t)$ as the marginal return with respect to time generated from a stand of timber at time t, where the state is described by state vector $\underline{X}(t)$, and $\underline{U}(t)$ implies the decision taken at that time. Further we define $H(\underline{X}(T),\underline{U}(T),T)$ as net harvest value at the end of rotation cycle T resulting from the decision implied by $\underline{U}(T)$, and when the state of the system is described by $\underline{X}(T)$. The objective of timber management can be translated into the following problem:

$$\max_{\underline{d}} J(\underline{X}_o,\underline{d}) = \max_{T,\underline{U}(t)} \sum_{i=0}^{\infty} e^{-irT}\Big[\int_0^T R(\underline{X}(t),\underline{U}(t),t)e^{-rt}dt$$

$$+ H(\underline{X}(T),\underline{U}(T),T)e^{-rT}\Big]$$

$$= \max_{T,\underline{U}(t)} \Big[\int_0^T R(\underline{X}(t),\underline{U}(t),t)e^{-rt}dt$$

$$+ H(\underline{X}(T),\underline{U}(T),T)e^{-rT}\Big]/(1 - e^{-rt}). \qquad (18)$$

Both T and the vector function of time $\underline{U}(t)$ are variables in the maximization subject to the following constraints:

$$\dot{X}(t) = \omega(\underline{X}(t),\underline{U}(t),t)$$
$$\underline{U}(t) \in \Psi(\underline{X}(t)). \qquad (19)$$

If the initial state of the control problem is included in the system and the state vector is confined to a given region, then

$$\underline{X}(0) = \underline{X}_o$$
$$\underline{X}(t) \in S \qquad (20)$$

where $\underline{X}_o$ is the initial state of the standing timber system and S is the set of possible states of the system.

The problem of (18) subject to (19) and (20) is a model in optimal control theory which can be applied to timber management decisions in the continuous time case $0 \leq t \leq \infty$. Theoretically this problem can be solved by either the classical calculus of variations or the maximum principle of Pontryagin et al (see Intriligrator, 1971). An application of the maximum principle to solve the optimal thinning and rotation problem in timber management has been presented by Näslund [1969], and an excellent interpretation of the principle in an economic context has been given by Dorfman [1969]. Mathematically, this principle is a generalization of the Lagrange multiplier method to solve optimization problems over time. Derivation of the necessary conditions for an optimal solution according to this principle boils down to finding the Hamiltonian function, so to speak, which underlies the basic theory of the principle. 6/

However, for the discrete time situation, any optimal control problem can be formulated in a dynamic programming framework, because in that situation, both the maximum principle and dynamic programming give equivalent solutions. Hence for practical purposes in which the decision is taken periodically like the one in timber management decisions, dynamic programming often has some advantages, especially in getting numerical solutions. The following section will discuss a more concrete and detailed formulation of optimal control to approach the investment problem in timber management. The numerical procedure for the problem will be described in a dynamic programming framework.

6/ The name of the function is after the astronomer-physicist W. R. Hamilton who first used such a function to unify mechanics and optics. See D. J. Wilde and C. S. Brightler, Foundations of Optimization (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1967), p. 431.

3.3. Dynamic Programming Solution

In order to formulate optimal control in a dynamic programming context, first we have to specify a stage of the decision process which in timber management may be taken as a year or some longer interval of time. A stage of one year gives the most sensitive decision rule, but little precision is lost in timber management applications by using a somewhat longer period.

Next we have to determine the state variables which are capable of describing the future state of the system completely. Since the decision process of our problem is growing a stand of timber, age of the stand and a density index can be considered as state variables, because once we know the present age of the stand and the stand density, the future condition (state) of the stand can be predicted by the present state of growing timber. However, any exogenous factors such as the disease of dwarf mistletoe which influences growth conditions of the stand will change the future state of the forest system also. Hence, the level of disease infestation will constitute another state variable.

Then we have to determine the appropriate decision variables that are going to be used to control future states of the standing timber system. Since we know that there is a relationship between thinning action and the future level of mistletoe infestation, and of course the stand density (see Hawksworth and Hinds, 1964; Baranyay and Safranyik, 1970), the level of thinning constitutes a decision or control variable.

Another decision variable which will change the future state of the forest system is the harvest variable which is a dichotomous variable which specifies to cut or let the stand grow. In fact, both of these decision variables can be considered as one decision variable because the harvest variable is an extreme case of the thinning decision.

One important aspect of the dynamic programming method requires estimation of biological relationships which express each state variable in period t+1 as a function of state and control variables in the previous period t. These relationships constitute first-order difference equations in the state variables and are discrete counterparts of the differential equations in (17).

In order to be more specific, let us introduce the following notation:

x_1 = age of the stand;
x_2 = stand density index;
x_3 = dwarf mistletoe rating; and
u = thinning level, including harvest as an extreme.

The set of dynamic relationships will be three in number:

$$\begin{aligned} x_{1,t+1} &= f(u_t, x_{1t}, x_{2t}, x_{3t}) = x_{1t}+1 \\ x_{2,t+1} &= g(u_t, x_{1t}, x_{2t}, x_{3t}) \\ x_{3,t+1} &= h(u_t, x_{1t}, x_{2t}, x_{3t}) \end{aligned} \tag{21}$$

The equation of x_1 happens to be extremely simple, so its specific algebraic form was written out in the above set of equations. The relationships in the last two equations of (21) can be estimated by some kind of regression analysis, if adequate data could be found. These three equations in (21) also constitute a discrete version of the equations of motion which trace the dynamic process over time, starting from some initial state.

Now we define a strategy or conditional decision rule (not necessarily optimal) which states how to control the standing stock system at any given stage for each set of values of the state variables. Mathematically, this rule expresses each decision variable as a function of all state variables, at any given stage. Following the above notation and applying it to our problem, this rule can be expressed as follows:

$$u_t = d_t(x_{1t}, x_{2t}, x_{3t}). \tag{22}$$

If we could estimate the relationship of (21) and set up the rule of (22), we would be able to trace the value of each state variable through time starting from an initial state of the process $\underline{X}_o$ by iterating the equations in (21). Therefore, the strategy expressed in (22) can be evaluated at any stage and state of the process. Complete enumeration of all possible decision rules given by (22) is not feasible except in the simplest of problems; dynamic programming provides a powerful method to systematically determine the optimal decision rule.

Since some of the state variables are subjected to random fluctuation, the decision rule of (22) which is a function of random variables is also a random variable. The fact that the decision rule is random, makes it difficult to evaluate a strategy by the above iterative method. In cases where a decision process involves some random state variable, a Markov decision model solved by dynamic programming can overcome the difficulty. In addition, stochastic dynamic programming provides a systematic method to obtain the optimal policy, i.e., the one that maximizes the criterion function.

Since the decision to be made involves some random variables, the appropriate model for optimal control in the discrete case will be a stochastic decision model solved by dynamic programming to obtain the optimal policy. The following discussion will first present the deterministic version of dynamic programming and then modify it into a stochastic framework by incorporating a Markov chain into the model.

A characteristic of the dynamic programming method is that decisions are made sequentially in a multi-stage process. One important aspect of this multi-stage decision process is the effect of a decision at a given stage upon the state variables in the immediately following stage. A particular decision within the set of admissible alternatives will change the state of the process in the passage from one stage to the next. If the optimal decision to be made at a particular stage of the process depends only on the state of the process at that stage, then the decision process satisfies the Markovian requirement. 7/ In a deterministic case, the Markovian requirement is equivalent to the difference equation of (21) being first order instead of second or higher order difference equations, and also assuming that all relevant state variables have been included in the set.

The implied change of the state of the process from one stage to the next resulting from a decision is referred to as state transition. Transition from one state to another can be made deterministically (with certainty) or stochastically--according to a probability distribution of the future state of the process. The latter is called a stochastic decision process.

Now let us introduce dynamic programming with a deterministic model. In order to be able to formulate the dynamic decision model, the first order difference equation of (21) must be shifted into dynamic programming notation in terms of stages as follows:

$$
\begin{aligned}
x_{1,n-1} &= f(u_n, x_{1n}, x_{2n}, x_{3n}) = x_{1n} - 1 \\
x_{2,n-1} &= g(u_n, x_{1n}, x_{2n}, x_{3n}) \qquad\qquad (21') \\
x_{3,n-1} &= h(u_n, x_{1n}, x_{2n}, x_{3n})
\end{aligned}
$$

7/ A formal definition of the Markovian requirement can be found in Richard E. Bellman, Adaptive Control Process (New Jersey: Princeton University Press, 1961), p. 54.

where n refers to the number of stages remaining in the planning horizon. The relation between stages and chronological time can be illustrated by the following diagram:

time	0	t	t+1	T	end of the planning horizon
stage	N	n	n-1	0	

Figure 1. The Relation Between Stages and Chronological Time.

Bellman's principle of optimality states "an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (Bellman,1957). 8/ By application of this principle, the entire problem can be summarized by the following recurrence relation:

$$F_n(x_1,x_2,x_3) = \max_u [R_n(u,x_1,x_2,x_3) + \beta F_{n-1}\{(x_1-1),g(\ ),h(\ )\}] \qquad (23)$$

where: $f_n(x_1,x_2,x_3)$ is present value of total net returns from soil site in the next n stages under an optimal policy, when the initial state is that implied by the variables x_1, x_2, x_3.

8/ A direct mathematical derivation of the recurrence relation as an alternative to the application of the principle of optimality can be found in R. E. Bellman and S. E. Dreyfus, Applied Dynamic Programming (New Jersey: Princeton University Press, 1962), p. 15.

$R_n(u,x_1,x_2,x_3)$ is the immediate net return resulting from decision u in stage n.

$F_{n-1}\{(x_1-1),g(\ ),h(\ )\}$ is the future net returns from the site in the next n-1 stages to go. The functions g( ) and h( ) are evaluated with their arguments set equal to values of the variables in stage n, i.e., the arguments are as given in (21').

$\beta = 1/(1+r)$ is a discount factor with interest rate r.

The above procedure describes how to formulate the multi-stage decision model to control the production system in growing timber in a deterministic framework. If we want to incorporate stochastic elements into the model which would more closely represent real world phenomena, the dynamic equation (21') and the decision model (23) need slight modifications as follows:

$$\begin{aligned} x_{2,n-1} &= g(u_n,x_{1n},x_{2n},x_{3n},e_n) \\ x_{3,n-1} &= h(u_n,x_{1n},x_{2n},x_{3n},e'_n) \end{aligned} \qquad (24)$$

where e and e' are random variables. Also (23) becomes an expected discounted net returns function:

$$\begin{aligned} F_n(x_1,x_2,x_3) &= \max_u E[R_n(u,x_1,x_2,x_3) + \beta F_{n-1}\{(x_1-1),g(\ ),h(\ )\}] \\ & \max_u [\bar{R}_n(u,x_1,x_2,x_3) + \beta E F_{n-1}\{(x_1-1),g(\ ),h(\ )\}] \end{aligned} \qquad (25)$$