A theoretical design comparison of a structural frame in prestressed and conventional reinforced concrete
by Mete Teoman

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University


Abstract:
The purpose of this thesis is to compare a structure designed in prestressed, and in conventional reinforced concrete. Both designs employ the same theoretical frame, and to obtain an accurate comparison the same live load is used. Allowable stresses in materials are as near the Same as codes and differing theories of the two systems will permit.

The thesis is divided into three parts. The first part covers the design of a theoretical frame in prestressed concrete. The design follows theories and practices used today.

The second part covers the design of the same theoretical frame in conventional reinforced concrete. The methods used follow those employed in this country.

The third part offers a comparison of the resultant structures of the two different designs.

# A THEORETICAL DESIGN COMPARISON OF A STRUCTURAL FRAME IN PRESTRESSED AND CONVENTIONAL REINFORCED CONCRETE

by

METE TEOMAN

A THESIS

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in

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## ABSTRACT

The purpose of this thesis is to compare a structure designed in prestressed, and in conventional reinforced concrete. Both designs employ the same theoretical frame, and to obtain an accurate comparison the same live load is used. Allowable stresses in materials are as near the same as codes and differing theories of the two systems will permit.

The thesis is divided into three parts. The first part covers the design of a theoretical frame in prestressed concrete. The design follows theories and practices used today.

The second part covers the design of the same theoretical frame in conventional reinforced concrete. The methods used follow those employed in this country.

The third part offers a comparison of the resultant structures of the two different designs.

# INTRODUCTION

## History

Continual improvement in concrete has made the present-day product much superior to the concrete used only thirty years ago. A high degree of uniformity in quality has been attained, and important advances in methods for analysis of indeterminate structures have resulted in more accurate determination of stresses than ever before.

The history of conventional reinforced concrete is well known to people associated with the field, but the development of prestressed concrete is relatively new.

The basic principle of prestressing has been used in wooden barrel construction for many centuries. Metal bands placed around wooden staves to form a barrel are tightened, subjecting them to tensile prestress which creates compressive prestress between the staves, enabling them to resist hoop tension caused by internal liquid pressure.

This principle was not used for concrete until about 1886, when P. H. Jackson[1] of San Francisco was granted patents for tightening steel tie rods in concrete arches to serve as floor slabs.

Soon after, C. E. W. Doehring[2] of Germany secured a patent for concrete reinforced with metal that had tensile stress applied to it before the slab was loaded.

Both of these early methods were unsucessful because the low prestress in the steel was lost as a result of the shrinkage and creep of concrete.

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1. "Prestressed Concrete Structures T. Y. Lin John Wiley & Sons N.Y.1955 pl
2. Ibid p. 1

In 1908, C. R. Steiner[3] of the United States suggested retightening the reinforcing rods after some shrinkage had taken place in the concrete.

Successful development of prestressed concrete is attributed to E. Freyssinet[4] of France, who began using high-strength steel wire for prestressing in 1928. These wires have an ultimate strength as high as 250,000 psi and a yield point strength of around 180,000 psi and are prestressed to about 150,000 psi before any shrinkage is taken place in the concrete.

E. Hoyer[5] of Germany first successfully used the method of pretensioning in which the steel is bonded to the concrete without end anchorage. His system consists of stretching wires between two buttresses several hundred feet apart, erecting form work between the units, placing the concrete, and cutting the wires after the concrete has hardened.

In 1939, Freyssinet[6] developed conical wedges for end anchorages and designed double-acting jacks which tensioned the wires and then thrust the male cones into the female cones for anchoring them.

Professor G. Magnel[7] of Belgium developed the Magnel system in 1940, in which two wires were stretched at a time and anchored with a simple metal wedge at each end.

Prestressed concrete first became important about 1940, perhaps partly as a result of a steel shortage in Europe. Much less steel is needed for prestressed than for reinforced concrete, since the steel used in

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3. Ibid p. 2
4. Ibid p. 3
5. Ibid p. 3
6. Ibid p. 4
7. Ibid p. 4

prestressed concrete is of higher strength. France and Belgium[8] led all other countries in the use of prestressed concrete.

The development of prestress concrete in the United States[9] followed a different course. Instead of prestressed concrete beams and slabs, or linear prestressing, circular prestressing was developed, especially in storage tanks.

Basic References Used

Gustave Magnel's book "Prestressed Concrete" was used as the main reference for the first part of this thesis, because the book embodies a good combination of theory and practical design problems in prestressed concrete. M. Guyon's "Béton Précontraint" and T. Y. Lin's "Prestressed Concrete Structures" were also followed.

In the second part of this thesis, "Reinforced Concrete Design" by Sutherland and Reese was the main reference together with the current A.C.I. Building Codes and Design Handbooks.

In designing girders and beams, uniformity was maintained by using I shapes and same size of wires in prestressed design, and rectangular shapes and same size of main reinforcements in conventional concrete design.

This thesis does not contain basic theories of the two designs, since such information is already available, but does employ them.

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8. Ibid p. 4
9. Ibid p. 5

The Frame

The frame to be used in both designs is a single-story concrete rigid frame structure as shown in fig. 1.

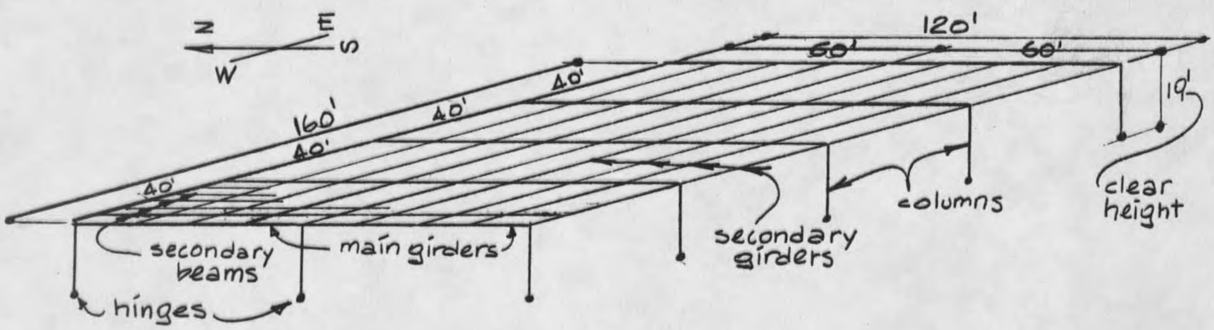


**fig. 1**
The Theoretical Frame

Center to center spacing between the columns in the E-W direction is 40', and center to center spacing between the columns in the N-S direction is 60'.

The slabs, secondary beams and secondary girders are the simply-supported type, and the main girder frames consist of two main girders and three columns of rigid-frame design. The spacing of secondary beams and secondary girders is given in fig. 2.

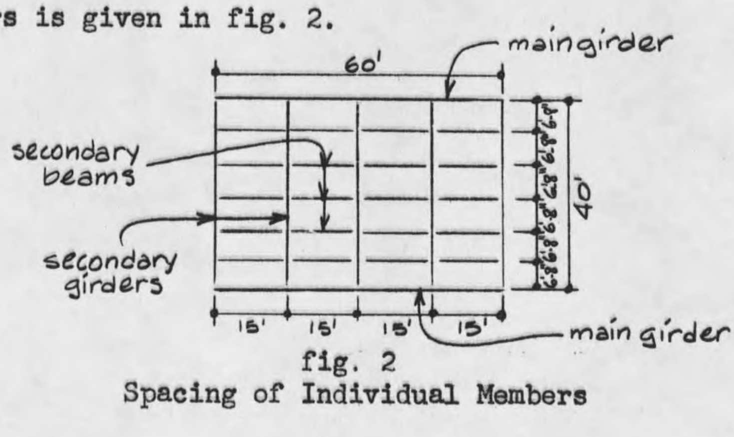


fig. 2
Spacing of Individual Members

Horizontal loads, such as earthquake loads and wind pressures, are omitted in the designs because they have the same effect on both designs of the structure. Reinforced concrete footings are assumed to be used in both designs and are therefore omitted.

The wide spacing of columns makes possible a good comparison of the two designs of beams and girders. Main girders and columns designed as a rigid frame result in more economical sections.

Loads on the Frame

Roof Live Loads:

Snow load = 25# per sq. ft. (flat roof)

Superimposed load = 30# per sq. ft.

Roof Dead Loads:

Lean concrete = 20# per sq. ft.

Built-up roofing = 5# per sq. ft.

Roof live loads and roof dead loads are the same in both designs.

Allowable Stresses

In Prestressed Design:

If the concrete is made carefully and with good aggregate a crushing resistance of 8,800 psi can be obtained without difficulty and consequently a working stress of 2,200 psi could then be adopted. For the tensile stresses due to shearing forces, the same stresses as in conventional reinforced concrete are acceptable. Tensile stresses due to bending moments do not, in theory, exist in fully-prestressed concrete, but there is no reason why some tensile stress in the top fibre during prestressing, and some tensile stress in the lower fibre under the greatest probable load, should not be permitted.

The working stress in wires is assumed to be 155,000 psi. Proportion of the initial stretching force that remains permanently is generally assumed to be 0.85, and this value is accepted in the design.

$f'_c$ = 8,800 psi

$f_c$ = 2,200 psi

$f_s$ = 155,000 psi

Tensile stress due to shearing forces (no web reinforcement).

$$0.03\ f'_c = 0.03\ (8,800) = 264 \text{ psi}$$

Tensile stress due to shearing forces (with properly designed web reinforcement)

$$0.12\ f'_c = 0.12\ (8,800) = 1,056 \text{ psi}$$

Tensile stress due to bending moments = 230 psi

$\eta = 0.85$

7mm round wires used in the design have an equivalent diameter in inches of 0.276, and an area of 0.0596 sq. in.

Because concrete can carry compressive load better without being precompressed by steel, and horizontal forces do not exist in the design, conventional methods are used in column designs.

In Conventional Design:

Although the working compressive stress in concrete in prestressed design is assumed to be 0.25 $f'_c$, in conventional design, in order to follow A.C.I. building code requirements of 0.45 $f'_c$, we have to use $f'_c$ = 4,890 psi to get $f_c$ = 2,200 psi.

$f'_c = 4{,}890$ psi

$f_c = 2{,}200$ psi

$f_s = 20{,}000$ psi

$n = \frac{30{,}000}{f'_c} = 6.13$

With the value of $\frac{f_s}{nf_c} = 1.48$, a very close estimate can be made for j and k, where j is the ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth and k is the ratio of distance (kd) between extreme fiber and neutral axis to effective depth.

$j = 0.866$

$k = 0.403$

Allowable unit shearing stress:

beams with no web reinforcement $= 0.03\, f'_c = 0.03\ (4890) = 146.7$psi

beams with web reinforcement $= 0.12\, f'_c = 0.12\ (4890) = 586.8$ psi

bond unit stress for deformed bars $= 350$ psi

Symbols and Notations

In Prestressed Concrete Design:

A, cross-sectional area of beam and girder.

$A_g$, gross area of concrete section.

$A_s$, cross-sectional area of the reinforcement or stretched wires.

a, length of the end-blocks of beam and girder.

add, additional.

b, width of rectangular beam or width of web in I-beams or girders; also the effective width (the width of the beam or girder at the end minus the width of the openings for the cables) in end-blocks of beam and girder; also the least lateral dimension of column.

C, constant relating to the ratio of stresses; also ratio of allowable concrete stress, $f_a$, in axially loaded column to allowable fibre stress for concrete in flexure.

C.O., carry-over factor.

$c_n$, compressive stress at the neutral axis.

$c_x$, compressive stress in the end-blocks of beam and girder.

$c_z$, bending stress in the end-blocks of beam and girder.

$c_{ab}$, $c_{at}$, calculated stresses in the bottom and top fibres respectively due to $w_a$.

$c_{db}$, $c_{dt}$, calculated stresses in the bottom and top fibres, due to loads acting at the time of prestressing.

comp., compressive, compression.

D, total depth of slab; also $D = \frac{t^2}{2R^2}$ = a factor, usually varying from 3 to 9 (the term R as used here is the radius of gyration of the entire column section.)

D.F., distribution factor

D.L., dead load

e, eccentricity of the stretched steel from the centroid; also eccentricity of the resultant load on a column, measured from the gravity axis.

$e_A$, eccentricity applicable to point A.

$e_{A_1}$, $e_{B_1}$, etc., actual eccentricity of the cable at sections $A_1$, $B_1$, etc.

F.E.C., far-end condition

F.E.M., fixed-end moment.

$f_a$, average allowable stress in the concrete of an axially loaded reinforced concrete column.

$f_c$, permissible compressive stress in the concrete.

$f'_c$, ultimate compressive strength of concrete.

$f_s$, permissible tensile stress in the steel.

$f_t$, permissible tensile stress in the concrete.

I, moment of inertia about the horizontal centroidal axis.

$I_b$, moment of inertia of main girder.

$I_c$, moment of inertia of column.

K, a non-dimensional coefficient used in calculating $c_z$, $K = 5\left(-1 + \frac{12z^2}{a^2} + \frac{16z^3}{a^3}\right)$, in end-block designs.

$K_1$ a non-dimensional coefficient used in calculating v in end-block designs. $K_1 = 5\left(\frac{1}{4} + \frac{z}{a} - \frac{4z^3}{a^3} - \frac{4z^4}{a^4}\right)$.

L, length of span; also length of column.

L.L., live load.

M, bending moment.

$M_a$, bending moment due to $w_a$.

$M_d$, bending moment due to $w_d$.

$M_{d+a}$, bending moment due to $w_d$ and $w_a$ acting together.

max., maximum.

mom., moment

$\Sigma M_q$, summation of moments around q.

N, axial load applied to conventional reinforced column.

n, ratio of modulus of elasticity of steel to that of concrete: assumed as equal to $\frac{30,000}{f'_c}$

N.A., neutral axis.

no., number, used in bar designation (no. 2 bars, no. 3 bars, etc.).

P, total allowable axial load on a column

$P_i$, initial stretching force.

$p_g$, ratio of the effective cross-sectional area of vertical reinforcement to the gross area $A_g$.

$p_t$, principal tensile stress.

$Q_1$, factor for moment of resistance; for a rectangular beam

$$Q_1 = \frac{0.775 f_c + f_t}{6}$$

$Q_m$, moment about the neutral axis of the area of a section on one side of the neutral axis.

R, vertical component of force in an inclined cable.

r, radius of gyration of the concrete section.

req'd, required.

sec., section; also secondary.

t, overall dimension of column.

u, unit str., unit stress.

V, shearing force ($V_{AE}$, shearing force at AE.).

v, shearing stress

$\Sigma V$, summation of shearing forces.

w, uniformly-distributed load (dead or live).

$w_a$ additional load per unit length applied after the prestress has been established.

$w_d$, load per unit length acting when the prestress is being established.

$w_{d+a}$, addition of loads $w_d$ and $w_a$.

x, distance from one support of any point in a beam or girder; also used in coordinate system of the end-block designs; also the distance from extreme fibre in compression to axis of zero stress in cracked-section design of column.

$x_L$, $\frac{x}{L}$ used in the equation for the shape of cables of continuous girder.

$y_1$, $y_2$, distances from the centroid to the top and bottom fibres respectively.

y, distance from the centroid to any fibre.

z, used in coordinate system of the end-block designs.

$\eta$, proportion of $P_i$ that remains permanently; generally $= 0.85$.

$\phi$, diameter of wires.

In Conventional Reinforced Concrete Design:

In addition to some of the symbols and notations used in the prestress concrete design:

$A_t$, area of temperature steel.

$A_v$, total area of web reinforcement in tension within a distance of s (measured in a direction parallel to that of the main reinforcement).

d, effective depth of flexural members.

$f_c$, actual stress in concrete.

$f_s$, actual stress in steel.

$f_v$, tensile unit stress in web reinforcement.

$I_T$, moment of inertia of transformed beam or girder areas.

j, ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth.

k, ratio of distance (kd) between extreme fibre and neutral axis to effective depth.

neg., negative.

$\sum_o$, sum of perimeters of bars.

poz., positive.

R, $\frac{M}{bd^2}$ coefficient of resistance.

s, spacing of stirrups in a direction parallel to that of the main reinforcement.

t, total depth of slab.

temp., temperature.

u, bond stress.

$V_c$, shear carried by concrete.

$V_s$, excess of the total shear over that permitted on the concrete.

$w_a$, all loads per unit length on the beam or girder, except their dead loads per unit length.

$w_d$, dead load per unit length of beam or girder.

x, distance (kd) between extreme fibre and neutral axis.

$Z_c$, $\frac{I_T}{x}$ used in moment of inertia method of finding actual extreme compressive fibre stress of concrete.

$Z_s$, $\frac{I_T}{d-x}$ used in moment of inertia method of finding actual tensile stress in steel.

# PART I

## DESIGN OF THE THEORETICAL FRAME IN PRESTRESSED CONCRETE

### A. Design of the Roof Slab

Precast slabs are designed as simply-supported beams of 12" width, supported by the top flanges of secondary beams.

$L = 6.66'$

$w_a = 20+5+55 = 80\ \#/\square' = 80\ \#/1'$ for each ft. of width.

Total depth (D)

$$M_a = \frac{80 \times 6.66^2 \times 12}{8} = 5{,}330 \text{ IN-LB}$$

$$Q_1 = \frac{0.775 f_c + f_t}{6} = \frac{0.775(2200)+230}{6} = \frac{1935}{6} = 323$$

$$\text{total depth } D = \sqrt{\frac{M_a}{Q_1 b}} = \sqrt{\frac{5330}{323 \times 12}} = 1.170''$$

Use $D = 2''$ (with min. fireproofing cover)

Initial stretching force & wires

$$w_d = 150 \times \frac{2}{12} = 25\ \#/\square' \quad , \qquad M_d = \frac{25 \times 6.66^2 \times 12}{8} = 1665 \text{ in-lb}$$

$$c_{dt} = c_{db} = \frac{1665 \times 6}{12 \times 2^2} = 208 \text{ psi}$$

$$c_{at} = c_{ab} = \frac{5330 \times 6}{12 \times 2^2} = 666 \text{ psi} \qquad \overset{f_c}{2200} > \overset{C_{dt} + C_{at}}{208 + 666}$$

$$C = \eta \frac{f_c - c_{dt} - c_{at}}{c_{db} + c_{ab} - f_t} = \frac{0.85(2200-208-666)}{208+666-230} = 1.75$$

$$r^2 = \frac{I}{A} = \frac{D^2}{12} = \frac{4}{12} = 0.333$$

$$e_A = \frac{(1-C)r^2}{y_1 + C y_2} = \frac{(1-1.75)\,0.333}{1 + 1.75 \times 1} = -0.091''$$

Ordinates for $e=0$ (fig 3)

Line 1. $-\frac{1}{(c_{dt}+f_t)A} = -\frac{1}{(208+230)A} = -\frac{228}{1000A}$

Line 2. $+\frac{1}{(f_c-c_{dt}-c_{at})A} = +\frac{1}{(2200-208-666)A} = +\frac{0.755}{1000A}$

Line 3. $+\frac{1}{(f_c+c_{db})A} = +\frac{1}{(2200+208)A} = +\frac{0.416}{1000A}$

Line 4. $+\frac{n}{(c_{db}+c_{ab}-f_t)A} = +\frac{0.85}{(208+666-230)A} = +\frac{1.320}{1000A}$

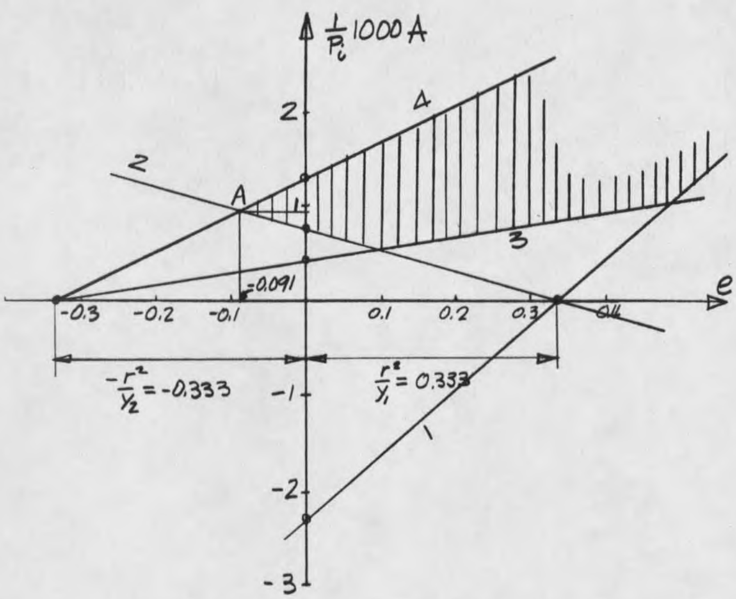


fig. 3.
The acceptable values
for $P_i$ & $e$.

$\frac{r^2}{y_1}=\frac{r^2}{y_2}=0.333$

According to fig. 3

$\frac{1000A}{P_i}=0.96$ $\quad P_i = \frac{1000\times2\times12}{0.96} = 25{,}000^{\#}$ $\quad \frac{P_i}{f_s}=\frac{25{,}000}{155{,}000}=0.161^{\square''}$

Use 3 $\phi=0.276''$ (7mm) wires

Shearing stresses.

ordinates of fig. 4 (due to $w_a$) $OA = \frac{80\times6.66}{2} = 267.0^{\#}$

" $DB = \frac{267}{4} = 66.6^{\#}$

(due to $w_d$) $OC = \frac{25\times6.66}{2} = 83.4^{\#}$

(due to bending up wires) $CE = 0$

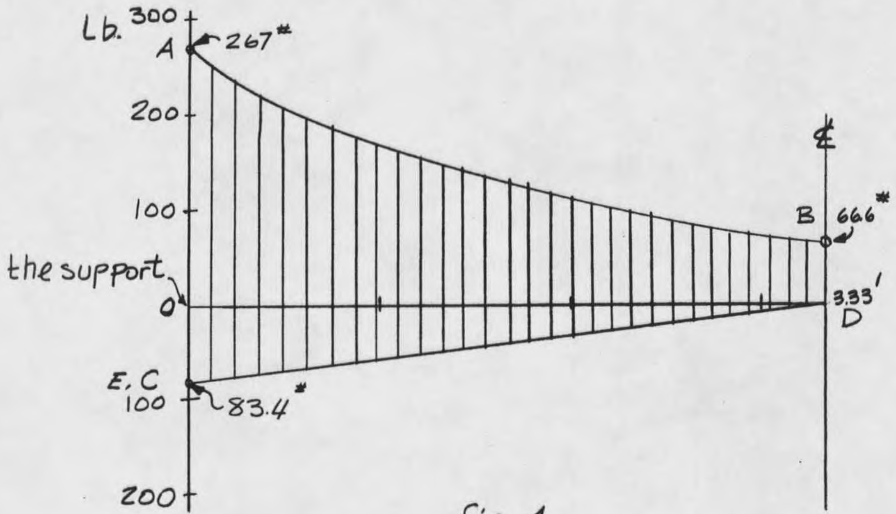


fig. 4.
Shearing stresses

Net shear at the support (fig. 4.) $= 267.0 + 83.4 = 350.4^{\#}$

Max. intensity of shearing stresses at the support

$$v = \frac{V_{AE}\, Q_m}{b\, I} = \frac{350.4 \times 6}{12 \times 8} = 21.9 \text{ psi}$$

$V_{AE} = 350.4^{\#}$

$Q_m = \frac{2}{2} \times 12 \times \frac{2}{4} = 6 \text{ in}^3$

$I = \frac{12 \times 2^3}{12} = 8 \text{ in}^4$

Horizontal compressive stress $= c_n = \frac{P_i}{A} = \frac{25{,}000}{2 \times 12} = 1040$ psi

Principal tensile stress $= P_t = \sqrt{v^2 + \frac{c_n^2}{4}} - \frac{c_n}{2}$

$$= \sqrt{21.9^2 + \frac{1040^2}{4}} - \frac{1040}{2} = \underline{2 < 230 \text{ O.K.}}$$

No further investigation is required since the greatest shearing stress is only <u>21.9 psi</u>.