An investigation of load entry into the edge beam of a concrete hyperbolic paraboloid
by Allen G Thurman

A THESIS Submitted to the graduate faculty in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University

Abstract:
This thesis is a report of load tests made on a hyperbolic paraboloid model. The model, made of concrete, was 10 ft 0 in. x 10 ft 0 in. x 0 ft 2 in. thick with an edge beam rise of 4 ft 0 in. The encompassing edge beams, 3 in. x 4 in. deep, extended into rigid buttresses, 1 ft 10 in. from the low corners of the shell. Uniform, unsymmetrical and concentrated incremental loads were applied to the shell with sand bags. Throughout the model, strains were measured by SR=4 strain gages arid a Baldwin-Lima-Hamilton strain indicator at each increment of load. Hook-up and test procedures are thoroughly described. Tests were made to determine the modulus of elasticity of the model. After E was determined, mean strains were . calculated and converted to stress by an IBM 650 computer. Stresses in the edge beams and shell have been plotted in several manners for each loading condition.

Theoretical action for uniform load was derived by employing several simplifications, particularly in regard to boundary conditions. This analysis declares that the applied load is distributed equally to tension and compression parabolas where it is transferred into edge beams without causing bending stresses; further, that the load in the edge beams is axial and continues into the buttresses with uniform compressive stress. Conflicting with this theory, the model studied; had large bending stresses in the edge beam, in the shell, and between the shell and the edge beam. Although compression and tension parabolas were created, their stresses were not equal. Primary reasons for the discrepancies between theory and tests appeared to be the existence of a strong rigid arch between buttresses , distortions of the shell and edge beams, and eccentric entry of diaphram shear into the edge beams.

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ALLEN G. THURMAN

ABSTRACT

This thesis is a report of load tests made on a hyperbolic paraboloid model. The model, made of concrete, was 10 ft 0 in. x 10 ft 0 in. x 0 ft 2 in. thick with an edge beam rise of 4 ft 0 in. The encompassing edge beams, 3 in. x 4 in. deep, extended into rigid buttresses, 1 ft 10 in. from the low corners of the shell. Uniform, unsymmetrical and concentrated incremental loads were applied to the shell with sand bags. Throughout the model, strains were measured by SR-4 strain gages and a Baldwin-Lima-Hamilton strain indicator at each increment of load. Hook-up and test procedures are thoroughly described. Tests were made to determine the modulus of elasticity of the model. After E was determined, mean strains were calculated and converted to stress by an IBM 650 computer. Stresses in the edge beams and shell have been plotted in several manners for each loading condition.

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INTRODUCTION

Problem Defined

This thesis is an attempt to evaluate the entry of load into the edge beam of a hyperbolic paraboloid shell by measuring accumulative shears and bending moments in the edge beams and the diaphram.

An endless hyperbolic paraboloid structure of infinitesimal thickness would have pure compressive stress in the convex paraboloid and pure tensile stress in the concave paraboloid,[1] but a real structure must be given a location of final support. This usually is done by encompassing the shell surface with edge beams, an enlargement at the periphery of the structure, and extending these beams into two buttresses located at the two lowest points of the curved surface.[2] The forces caused by the dead and live loads are transferred from the diaphram into the beams and then into these buttresses.

A real structure must have a specific thickness, the amount depending upon the material used and its strength. For ease of placing concrete and for fire protection of the reinforcing steel, three inches is often considered a minimum thickness for concrete shell construction. Such a thickness is capable of a sizable bending moment and one will be created

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[1] See General Analysis, p. 18. [2] See Fig. 4, p. 19.

at any shell edge.[1] An edge beam, which is quite stiff compared to a free shell edge, will greatly increase this bending moment. Therefore, only interior portions of large hyperbolic paraboloids approach the condition of pure compressive and tensile stress, that is, freedom from bending stresses.

Concrete Hyperbolic Paraboloid Use

The hyperbolic paraboloid shape is beautiful in itself, uttering simplicity and rejecting the ornamentation found on many buildings. Its natural strength, created by the double curvature, allows it to economically span large areas entirely uninterrupted by interior supports — an awesome spectacle to the engineer as well as the layman. With support required at only two points, the walls become mere barriers of the elements, allowing windows of any size to be placed at any point including the corners under the high peaks.

A building which fully utilizes these advantages is a Roman Catholic Church recently completed in Spokane, Washington.[2] The simplicity of its smooth, curving lines, its humbling spaciousness, its strong, powerfully uplifting altar give it a functional and esthetic value seldom achieved in a building. To see it after looking at other buildings is like seeing a figure by Michelangelo after walking through a park viewing

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[1] H. H. Bleich and M. G. Salvadori, "Bending Moments on Shell Boundaries," Journal of Structural Division, Proceedings of the American Society of Civil Engineers, LXXXVIII, No. ST8, (October 1959), pp. 91-100.

[2] St. Charles Catholic Church, Spokane, Washington; Funk, Murray & Johnson, Architect; T. Y. Lin, Consulting Engineer; Johnson, Busboom & Rauh, General Contractor.

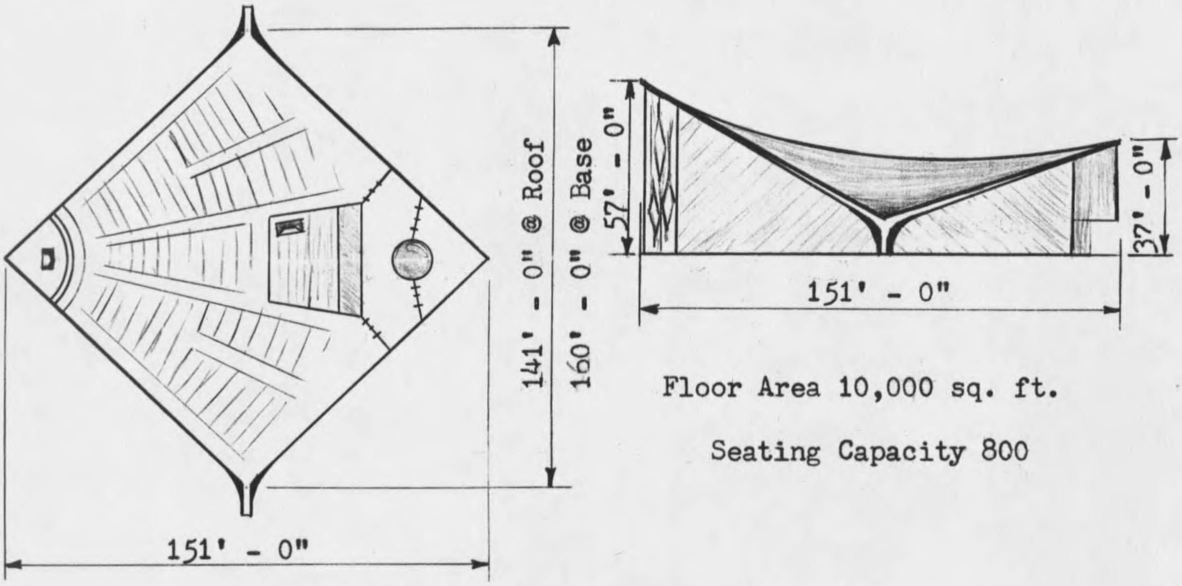
the cold, dead statues of forgotten generals. Similar opinions are held by many non-Catholics whose judgments have a minimum of biased distortion.

The building's structure is a hyperbolic paraboloid, diamond shaped in plan (Fig. 1, p. 11). The buttresses, at the two corners closest together, carry the gentle curve of the shell into the ground with undoubted strength. By rotating the entire shape about the buttresses, one corner has been raised, greatly emphasizing its majestic cantilever. Edge beams exist, but their usual heavy appearance has been entirely eliminated by putting a reversed curve on the outside face leaving a distinct edge only two or three inches thick, and by graduating the total thickness from a minimum at the peak to a maximum at the buttresses. Smooth, white, architectural concrete complements the building shape.

The entrance curves inward under the lower cantilevered peak. The vertical freedom beneath this peak is emphasized by the simplicity of the entry hall and ante-rooms which fill this portion of the plan. Thus, the lower peak is well utilized without detracting from the grandeur of the main peak.

Lines which are formed by the sanctuary floor sloping downward toward a curved altar rail and the parabolic roof thrusting to a peak high above, create a natural altar having an atmosphere of aspiration and reverent mystery. Intensifying this impression is a wide, modern, stained glass corner window extending from the floor to the very roof peak and providing a major portion of the light for the building.

The organ and choir are located on a balcony centered under the roof area and open to all sides including the entry hall. Thus, with



Schematic Plan and Elevation - No Scale

ST. CHARLES CATHOLIC CHURCH

FIGURE 1

minor exceptions in the rear of the church, the entire area enclosed by the exterior walls is open to the roof, transmitting sound and light with unusual freedom.

This use of the hyperbolic paraboloid, employing the shape as a complete structure in itself, is perhaps the classic use of this shape as it combines its natural strength with its inherent beauty. Two or more hyperbolic paraboloids often are connected to form one large roof area. While these arrangements maintain their strength allowing large areas to be economically covered with few or no interior obstructions, more than four hyperbolic paraboloids joined together lose much intrinsic beauty as the identity of the shape is lost. The hyperbolic paraboloid has also been used for foundation work where the shape, which is buried in the earth, becomes economically valuable only for its natural strength.[1]

Model Studies of Hyperbolic Paraboloids

So that the functional beauty of the hyperbolic paraboloid may be properly utilized, it is important to attain a complete understanding of the structural properties of this shape. Being very complicated, exact mathematical analyses must be substantiated by thorough test model studies. A review of literature on this subject at the Montana State College Library revealed a conspicuous lack of previous formal testing of concrete hyperbolic paraboloids. American Doctoral dissertation abstracts from 1952 through 1960 show that there has been no dissertation devoted to this subject,[2] although one discusses stress theory and a few mention hyperbolic paraboloids along with discussion of other shells. The past two and one-half years of government translations of foreign material[3] include no treatment of hyperbolic paraboloids. Dr. T. Y. Lin, while speaking before the

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[1]Arthur R. Anderson, "Precast Prestressed Stadium Floats on Hyperbolic Paraboloids," Engineering News-Record, CLXIV, No. 7 (February 18, 1960), 62.

[2]Dissertation Abstracts, (Ann Arbor, Michigan: University Microfilms, Inc., Jan. 1956-Oct. 1960), Vol. 16-22. Doctoral Dissertations Accepted by American Universities, The Association of Research Libraries, New York: H. W. Wilson Co., 1952-1955), No. 20-22. The idea of using hyperbolic paraboloids for structures dates back to at least 1930, but almost all use of the shape has been since World War II, originating primarily in Latin America. Therefore, these volumes appear to include all the work done in the U.S.A.

[3]Technical Translations, U.S. Dept. of Commerce, Office of Technical Services, (Washington: Government Printing Office, Jan. 2, 1959-March 28, 1961), Vol. 1, No. 1-Vol. 5, No. 6.

Architect-Engineers' Convention in Great Falls, Montana, January 1958, mentioned that tests utilizing prestressing of concrete hyperbolic paraboloids were being conducted at the University of California. No report of this work was found. Although the amount of library material is limited, this survey confirms the lack of research on concrete hyperbolic paraboloids.

The Portland Cement Association has completed some experiments and has made public some important aspects of these tests.[1] Some discussions of theory have recently appeared in the Proceedings of the American Society of Civil Engineers and other well circulated engineering publications. A British journal, Concrete and Construction Engineering,[2] notes several uses of hyperbolic paraboloids and also some tests of scale models of structures which were built following these tests. Unfortunately almost no design data is given nor are sources of further information listed.

Test reports of wooden hyperbolic paraboloids are of some value, however such structures do not have the inherent stiffness of component parts, such as slab to edge beams, found in concrete structures. In addition, there is an enormous difference in the ability of the two materials to yield in bending without fracture, thus redistributing the loads. Therefore, extreme caution must be used when considering concrete design based on results of tests of wooden structures.

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[1] Elementary Analysis of Hyperbolic Paraboloid Shells, Reinforced Concrete Bulletin 35, (Chicago, Illinois: Portland Cement Assoc., 1960).

[2] Concrete and Construction Engineering, London: Concrete Publications Ltd., Monthly.

Hyperbolic paraboloid model study at Montana State College was initiated during 1959-60. Dennis Nottingham, the Ideal Cement Company Fellow for that school year, constructed the model and started testing it. His test report[1] gave indications of definite disagreement with some theoretical ideas.

The tests described in this report are in many ways a duplication as well as an extension of the work done by Nottingham. The duplication was warranted because:

1. More strain gages were required to better describe the action of the model under load.
2. Total strains were small, indicating greater accuracy could be obtained safely by applying larger loads.
3. Some strains were erratic, indicating a more accurate strain measurement was desirable.
4. Because stress (pounds per square inch) is a unit more easily compared with theoretical analyses than strain (inches per inch), determination of the modulus of elasticity of the model and conversion from strain to stress made the tests more valuable.

Thus, by extending and increasing the precision of Nottingham's model studies, the results became more convincing by their greater reliability and utility.

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[1]Dennis Nottingham, "Experimental Testing of a Saddle Type Hyperbolic Paraboloid Using Three Different Load Conditions" (Unpublished M.S. thesis, Dept. of Civil Engineering, Montana State College, 1960).

REVIEW OF THEORY

A complete structural analysis, discussing the effects of various boundary and loading conditions is not within the scope of this report. Only a limited, elementary discussion using a uniformly applied load will be presented. However, for those interested in more thorough treatment of theoretical analyses, a list of suggested references is included at the end of this section.

General Analysis

This material is from the Portland Cement Association bulletin, Elementary Analysis of Hyperbolic Paraboloid Shells.[1] As presented here and in the bulletin, the analysis employs several simplifications that are not entirely correct. Some of these points will be discussed briefly following this analysis.

The hyperbolic paraboloid is a doubly curved surface which may be generated in either of two ways. One method which may be described by using Fig. 2a, p. 16, is: A saddle-shaped surface called a hyperbolic paraboloid will be generated if a convex parabola AOE is draped over a fixed concave parabola GOC and moved keeping the parabolas at right angles and keeping the vertex O of parabola AOE on the parabola GOC. The other method may best be described by using Fig. 2b, p. 16: A hyperbolic paraboloid is formed by

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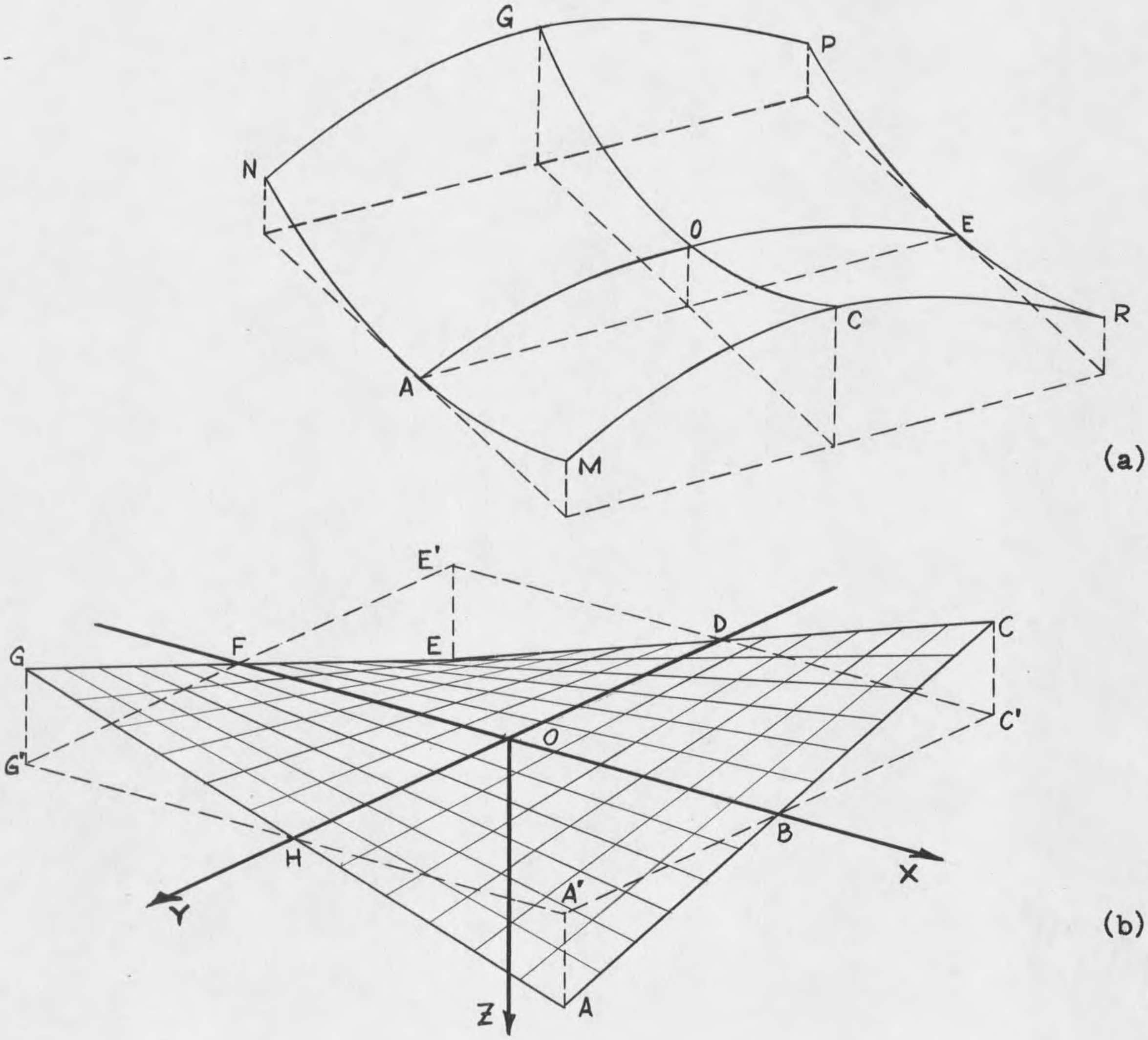
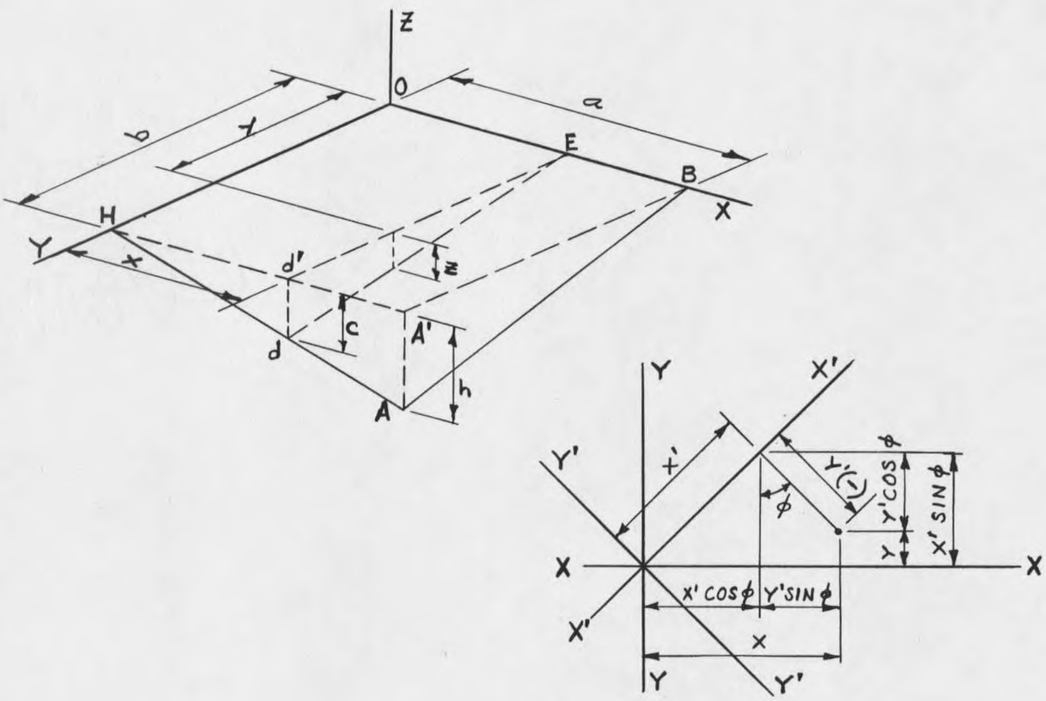
[1] Elementary Analysis of Hyperbolic Paraboloid Shells, Reinforced Concrete Bulletin 35, (Chicago, Illinois: Portland Cement Association, 1960), pp. 1-8.



SURFACE DEFINITIONS

FIGURE 2

moving a line GHA, which is parallel to the XZ plane, over a line ABC, which passes through the X axis and is parallel to the YZ plane, but non-parallel to the Y axis. Similarly, the same surface is generated by moving line ABC over line AHG and the X axis. Points O, A, G, E and C are common to both portions of Figure 2.



GEOMETRY

FIGURE 3

Using the expanded quadrant ABOH of Figure 2 in Figure 3a, the surface may be defined in terms of x, y and z. By similar triangles:

$$\frac{c}{h} = \frac{x}{a} \qquad c = \frac{xh}{a}$$

$$\frac{z}{c} = \frac{y}{b} \qquad z = \frac{yc}{b} = \frac{y}{b}\,\frac{xh}{a} = xy\,\frac{h}{ab}$$

$$\text{(using } k = \frac{h}{ab}\text{)} \qquad z = kxy \qquad (1)$$

To simplify the analysis, the axes are rotated 45 degrees in Figure 3b. This gives

$$x = x' \cos\phi - y' \sin\phi = 0.707\,(x' - y')$$

$$y = y' \cos\phi + x' \sin\phi = 0.707\,(x' + y')$$

Substituting into equation (1),

$$z = 0.5k\ (x' + y')\ (x' - y') = 0.5k\left[(x')^2 - (y')^2\right] \qquad (2)$$

When x' is constant,

$$z - 0.5k\ (x')^2 = z - k_1 = z' = -0.5k\ (y')^2 \qquad (3)$$

When y' is constant,

$$z + 0.5k\ (y')^2 = z + k_2 = z'' = 0.5k\ (x')^2 \qquad (4)$$

Equations (3) and (4) are general expressions for parabolas lying in or parallel to the Y'Z plane and X'Z plane respectively. The vertex of these parabolas will lie in the X'Z plane and Y'Z plane respectively. Thus, the parabola in the Y'Z plane and the one in the X'Z plane have the origin of the axes as a common vertex. Hereafter these two parabolas will be referred to as the main parabolas. The negative sign of equation (3) indicates concave parabolas, whereas the positive sign of equation (4) indicates convex parabolas.

If z is held constant in equation (2),

$$1 = k_3\left[(x')^2 - (y')^2\right] \qquad (5)$$

Thus, any horizontal plane cutting the warped surface forms two hyperbolas. For this reason the surface has been named hyperbolic paraboloid.

The bending moment equals zero throughout a two-hinged parabolic arch supporting only a uniform load. The moment in any two-hinged arch is equal to the simple beam bending moment minus the moment due to the horizontal reaction H. If it is assumed that the convex and concave parabolic arches forming the surface of a hyperbolic paraboloid each carry 1/2 an