An analysis of interdependence in the Montana economy : an input-output study
by Theodore Austin Hoff

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of '
DOCTOR OF PHILOSOPHY in Agricultural Economics
Montana State University

Abstract:
Scope and Method of Study The overall objective of this study was to analyze the interdependence that exists among and between Montana's economic sectors. The Leontief input-output model was used to carry out the analysis. The Montana model consisted of 12 endogenous sectors and 7 exogenous sectors. Secondary data was used to formulate the transactions matrix of the model. The transactions matrix presents the flow of goods and services, measured in dollar terms, between the sectors of the model during the calendar year 1963. The technical coefficients matrix of the endogenous sectors was then computed from the flow table. The technical coefficients matrix was inverted to get the matrix of interdependence coefficients. Sector output and income multipliers were derived from the interdependence coefficients of the model. Methods of adjusting the multipliers to handle special cases of output distribution were developed. A method of isolating the expansionary effects of an increase in personal income which results from increased production levels of the sectors was developed. A method of appraising the direction and degree of interdependence was also developed.

Findings and Conclusions Montana is dependent upon external markets for the sale of agricultural and manufacturing output. These sectors account for 30 percent of the total output of the state and 60 percent of their output is exported. These sectors also have the greatest economic impact with respect to a change in sector production, as indicated by the relative sizes of the sector output multipliers.

The Montana economy exhibits a very low degree of interdependence as compared to the national economy. The triangularized Montana transactions matrix shows that the remaining sectors of the economy are technically independent of the Livestock and Livestock Products sector and the Food and Kindred Products sector. One-way interdependence is also evident in the relationships of the remaining sectors. The amount of one-way interdependence shows that Montana is relatively much less developed than the national economy.

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of

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ABSTRACT

Scope and Method of Study

The overall objective of this study was to analyze the interdependence that exists among and between Montana's economic sectors. The Leontief input-output model was used to carry out the analysis. The Montana model consisted of 12 endogenous sectors and 7 exogenous sectors. Secondary data was used to formulate the transactions matrix of the model. The transactions matrix presents the flow of goods and services, measured in dollar terms, between the sectors of the model during the calendar year 1963. The technical coefficients matrix of the endogenous sectors was then computed from the flow table. The technical coefficients matrix was inverted to get the matrix of interdependence coefficients. Sector output and income multipliers were derived from the interdependence coefficients of the model. Methods of adjusting the multipliers to handle special cases of output distribution were developed. A method of isolating the expansionary effects of an increase in personal income which results from increased production levels of the sectors was developed. A method of appraising the direction and degree of interdependence was also developed.

Findings and Conclusions

Montana is dependent upon external markets for the sale of agricultural and manufacturing output. These sectors account for 30 percent of the total output of the state and 60 percent of their output is exported. These sectors also have the greatest economic impact with respect to a change in sector production, as indicated by the relative sizes of the sector output multipliers.

The Montana economy exhibits a very low degree of interdependence as compared to the national economy. The triangularized Montana transactions matrix shows that the remaining sectors of the economy are technically independent of the Livestock and Livestock Products sector and the Food and Kindred Products sector. One-way interdependence is also evident in the relationships of the remaining sectors. The amount of one-way interdependence shows that Montana is relatively much less developed than the national economy.

CHAPTER I--INTRODUCTION

Taylor has said that Montana is in the process of transition from a pre-industrial state to an industrial state. 1/ He describes the pre-industrial state as follows. It is characterized by a concentration of resources in "primary" industries such as agriculture and mining. The pre-industrial state's population is predominantly rural, with local government units designed to service the rural population. Interdependence of economic activities is also very low. He describes the industrial state as follows. It is characterized by a concentration of resources in secondary and tertiary industries, particularly manufacturing. The population is predominantly urban, and service industries develop to supply the needs of the urban population. Interdependence of economic activities reaches a high level, and government units are designed to serve the needs of an urban population.

The transitional economy lies somewhere between the pre-industrial and industrial economies. The primary industries are declining relative to the secondary and service industries, the population is changing from rural to urban centered, and government units are experiencing a change in the kind of government activity that must be provided.

Business and government leaders in Montana have recognized that Montana is in this transitional period of development. 2/ Organizations

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1/ Maurice C. Taylor, "Montana in Transition," Montana Business Quarterly, Fall 1967, Bureau of Business and Economic Research, University of Montana, Missoula, 1967, p. 9.

2/ Montana Industrial Horizons, Vol. 5, No. 4, 1960; Vol. 5, No. 6, 1960; Vol. 8, No. 4, 1963; Vol. 9, No. 4, 1964, State Department of Planning and Economic Development.

have been established by government and businessmen with the goal of promoting economic development in an orderly and desirable pattern. For example, the Montana State Planning Board was originally set up in 1935, but remained in a state of limbo for many years. It was reactivated again on July 1, 1955, for the express purpose of promoting economic development. The State Planning Board was reorganized by action of the 1967 Montana Legislature, and the name changed to the "State Department of Planning and Economic Development," the name reflecting the increased interest in economic development.

City and county Planning Boards have been set up by local government units for the purpose of guiding the economic growth patterns of their localities. Planning institutes 3/ have been held so that the local planning boards can get together with the State Department of Planning and Economic Development to exchange viewpoints and ideas for promoting orderly growth.

Businessmen, through their local Chambers of Commerce, have expressed their opinions and desires concerning economic development. 4/ Montana Interests, Inc., was organized by Montana businessmen for the specific purpose of promoting economic growth in the state. 5/

3/ Montana Planning Institutes, co-sponsored by the Association of Montana Planning Boards and the Bureau of Government Research at the University of Montana. Proceedings have been published by the Bureau of Government Research, University of Montana, Missoula.

4/ "Chamber of Commerce Promotes Economic Development," Montana Industrial Horizons, Vol. 6, No. 1, 1961.

5/ Montana Industrial Horizons, Vol. 4, No. 1, 1968.

They have recognized that economic growth is vitally needed in Montana.

The Problem

Montana's rate of economic growth has been very low compared to the national average since World War II. 6/ Several people have expressed their opinions as to the reasons for the low growth rate and what actions the state can and should take to promote economic growth. 7/ Research projects have been undertaken for the purpose of analyzing the economy of the state to determine the reasons for the slow growth rate and to propose means of improving it. One of these projects, the Upper Midwest Economic Study, 8/ encompasses the entire area in the Ninth Federal Reserve District. The procedure of the Upper Midwest Economic Study was first to make a detailed quantitative description of the regions economy, followed by an analysis of change in the economic structure and relationships in the region. 9/

6/ Taylor, op. cit.

7/ Paul B. Blomgren, "The Montana Economy in Perspective," Montana Business Quarterly, Fall 1962; Maxine Johnson, "Problems of Economic Growth in Montana," Montana Business Quarterly, Winter 1963; and Maurice C. Taylor, op. cit.

8/ James M. Henderson and Anne O. Krueger, National Growth and Economic Change in the Upper Midwest, University of Minnesota Press, Minneapolis, 1965.

9/ John R. Borchert, "Regional Development and Local Planning," Proceedings of the Second Montana Planning Institute, Missoula, Montana, September 16 and 17, 1960, Bureau of Government Research, University of Montana, Missoula.

In August of 1967, the Governor and the State Department of Planning and Economic Development of Montana appointed the Bureau of Business and Economic Research, University of Montana, as the primary contractors for the state planning research and development program. 10/ In establishing this program, the state implicitly recognized that the problems of development that Montana is experiencing can be alleviated through proper use of planning. Planning, however, if it is to be of any use, must be based on knowledge of the economic, demographic, and sociological structure of the state. Thus, research in these areas is a prerequisite to formulation of effective development policy.

The Need for the Study

The model used in this study is designed to show the interdependence of economic activities within the state. It is concerned with the aggregate or macro-economic aspects of the economy, not with the micro-economic aspects of the firms that comprise the economy. The model is useful for indicating the present economic relationships and can be used to predict the effects of some types of economic changes in the economy, subject to the framework and assumptions of the model.

10/ State of Montana Planning Design Study, Bureau of Business and Economic Research, University of Montana, Missoula, 1968, p. 103.

The types of economic change that can be analyzed by the model are changes in the level of economic activity of the state's industries. As the level of activity changes, either by expansion or contraction of an industry's output, the effects of the change spread throughout the economy to a larger or smaller degree, depending upon the interdependency of the industries within the economy. The total effects of change can be divided into two categories: (1) the initial or direct effects; and (2) the secondary or indirect effects. For example, assume that a manufacturing firm decides to build a new plant in the state. The direct effects of this action include such things as the increase in employment and personal income, an increase in the tax base, and the new demand for intermediate inputs used by the plant and purchased from other firms in the economy. The indirect effects arise as the economy responds and adjusts to the direct effects. An increase in consumer spending will occur, due to the increase in personal income. Firms supplying intermediate inputs to the new plant will expand their activity, again increasing employment, intermediate input demand, etc. Thus, the initial impact of the change is multiplied as the economy adjusts to the new plant.

Objectives of This Study

The over-all goal of this study is to quantify the structural interdependence of the economic elements in the Montana economy, thus providing a means of analyzing the economic structure of the state.

The specific objectives of the study are:

(1) To formulate an input-output model which will illustrate the structural interdependence of the economic enterprises within the State of Montana;

(2) To estimate direct and indirect effects of changes in the economy under the assumptions of the model;

(3) To appraise the degree of economic development exhibited by the state's economy as compared to the economy of the United States;

(4) To compute the output and income multipliers for the economy;

(5) To illustrate the usefulness of the model for evaluating economic change.

The Montana model in this study was constructed entirely from secondary data. The statistical appendix to this study, Appendix A, describes the methodology used to determined the elements of the transactions table along with the sources in which data was available for each sector. The model was then used to attain the objectives stated above.

Chapter II presents the review of the literature of input-output models, the theoretical explanation of the model, and the basic assumptions of the model. Chapter III describes the economy of Montana, the resources available to it, and the physical and demographic characteristics of the state. Chapter IV contains the empirical model of the state's economy and a discussion of the effects of changes in output of the state's industries. Chapter V compares the economy of Montana with the economy of the United States, indicating the differences and similarities of the two economies. Chapter VI presents the summary, implications, and conclusions.

CHAPTER II--THE LEONTIEF INPUT-OUTPUT MODEL

Review of the Literature

Input-output type analysis was first conceived by Francois Quesnay with the publication of his Tableau Economique. Quesnay's Tableau stressed the economic interdependence of economic activities within the firm. 1/

About 100 years later, Leon Walras developed a system of simultaneous equations of an input-output nature. Equations representing the supply of and demand for the factors of production and commodity demand and supply were included. Prices were the variables in the system, thus solution of the system would give the equilibrium price of each and every item. The sheer size of the model precluded finding the solution.

Wassily Leontief is the principal developer of aggregate input-output analysis as we know it today. He formulated the first empirical interindustry model of the United States economy in 1936 2/ and subsequently published the first transactions table for the United States. 3/ This model consisted of 44 sectors based on the year 1929.

1/ William H. Miernyk, The Elements of Input-Output Analysis, Random House, New York, 1965, p. 4.

2/ Wassily Leontief, "Quantitative Input-Output Relations in the Economic System of the United States," The Review of Economics and Statistics, XVIII, August 1936, pp. 105-125.

3/ Wassily Leontief, The Structure of the American Economy, 1919-1939, Harvard University Press, Cambridge, Massachusetts, 1941.

The lack of high speed electronic computers made the mathematical calculations required to determine the inverse matrix impossible to complete, so the model was aggregated to a workable size, allowing the computation of the inverse matrix. As a check on the accuracy and reliability of the model, "reverse" projections were made to 1919, estimating the 1919 output. This estimate and the actual 1919 output compared very favorably, with the differences attributed mainly to resource changes.

The advent of the high speed electronic computer enabled input-output researchers to construct models that were disaggregated to a much larger extent. For example, a 450 sector model was constructed for 1947. 4/ This study was used as the basis for many regional studies. The Bureau of Labor Statistics constructed an 87 sector model for 1958. 5/ This is the most recently completed model of the United States economy. Present government plans call for construction of an input-output model every five years. 6/

Input-output analysis has been used extensively by many countries, principally by national planning and economic development committees. 7/

4/ Duane M. Evans and Marvin Hoffenberg, "The Interindustry Relations Study for 1947," The Review of Economics and Statistics, XXXIV, May 1952, pp. 99-142.

5/ M. R. Goldman, M. L. Marimont, and B. N. Vaccara, "The Interindustry Structure of the United States--A Report on the 1958 Input-Output Study," Survey of Current Business, Vol. 44, No. 11.

6/ Ibid.

7/ Miernyk, op. cit., p. 79.

Its use has been generally limited to the developed economies where data is more readily available from secondary sources. Input-output analysis could be used to great benefit in analyzing under-developed economies to determine proper economic development policy. Lack of adequate data, however, usually prevents construction of empirical models.

Input-output analysis has become widely used for regional economic analysis since about 1951, starting with Walter Isards study. 8/ Studies have been conducted on county, multi-county, state, and multi-state areas. The Colorado River Basin Input-Output study 9/ is an example of the latter type, and is the largest regional study undertaken so far. At the present time, at least 54 regional studies have been completed with an additional 33 currently in progress. 10/

There are two avenues of approach when utilizing an input-output model in a regional study. The first entails applying the national

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8/ Walter Isard, "Inter-Regional and Regional Input-Output Analysis: A Model of a Space Economy," Review of Economics and Statistics, XXXIII, November 1951, pp. 318-328.

9/ William H. Miernyk, Bernard Udis, and Clyde Stewart, The Colorado River Basin Input-Output Study, Sponsored by the U. S. Public Health Service.

10/ Philip J. Bourque and Gerald Hansen, An Inventory of Regional Input-Output Studies in the United States, Occasional Paper 17, Graduate School of Business, University of Washington, Seattle, 1967.

coefficients to the region, adjusting them to the economy of the region. Miernyk 11/ discusses one method, that of adjustment according to a location quotient. Other methods have also been used, some explained in rather vague terms. 12/

The other approach to formulating a regional model is to generate a transactions table from regional primary and secondary data. In some studies the data has been obtained via survey of businesses within the region. 13/ In others, data has been collected from secondary sources. 14/ Data collection via survey will likely yield the most accurate information with the added advantage of the data being in a directly usable form. The disadvantage of the survey method is that, considering the large number of business establishments involved, it is extremely costly and time consuming.

Utilizing secondary sources of data, on the other hand, is much less expensive. Bureau of the Census publications provide a wealth

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11/ William H. Miernyk, "Long-Range Forecasting with a Regional Input-Output Model," Western Economic Journal, Vol. VI, No. 3, June 1968, pp. 165-176.

12/ Ibid., p. 166.

13/ Werner A Hirsh, "Interindustry Relations of a Metropolitan Area," The Review of Economics and Statistics, XLI, November 1959, pp. 360-369.

14/ Frederick T. Moore and James W. Peterson, "Regional Analysis: An Interindustry Model of Utah," The Review of Economics and Statistics, XXXVIII, November 1955, pp. 368-381.

of information. There are also many other government publications of a statistical nature that will provide useful information. This method of data collection is relatively inexpensive, but there is a disadvantage in that the data is not given in exactly the form needed for that model, therefore it must sometimes be manipulated in order to put it in usable form. 15/

Input-output models can be used to analyze a particular industry in depth. This type of model would contain several sectors for the industry under study with a high degree of aggregation of the remainder of the economy. This has been done in several studies which emphasized agriculture. For example, Martin and Carter 16/ emphasized the role of agriculture in the California economy. Their model contained 10 agricultural sectors and 27 non-agricultural sectors. On the other hand, the national model of 1958 contained only 2 agricultural sectors, but had 53 manufacturing sectors out of a total of 87 sectors. Other examples of input-output studies which emphasized agriculture are listed in the Literature Cited, references (59), (63), and (65).

15/ There has been some discussion among leading proponents of input-output analysis about government data publications. Government data collection agencies could go far toward alleviating data problems by collecting data in a form that would lend itself to input-output analysis, particularly in the various census publications.

16/ William E. Martin and Harold O. Carter, A California Interindustry Analysis Emphasizing Agriculture; Part I: The Input-Output Model; Part II: Statistical Supplement, University of California, Giannini Foundation Research Report No. 250, Berkeley, 1962.

Theoretical Explanation for the Input-Output Model

The input-output model consists of three basic parts: (1) the transactions or flow matrix; (2) the matrix of technical coefficients; and (3) the matrix of direct and indirect coefficients, called the matrix of interdependence coefficients. The transactions matrix is the heart of the model. The matrix of technical coefficients is derived from the transactions matrix, and the matrix of direct and indirect coefficients is derived from the matrix of technical coefficients.

The Transactions Matrix

Goods and services are produced for one of two purposes; they are either consumed by the public or they serve as an input for the production of other goods and services. The transactions matrix shows this distribution of goods and services among the sectors defined in the model.

As an example, assume a very simple economy with three producing sectors and one primary input. The transactions are expressed in dollar terms. The output of any one sector will either be used by that sector to produce more goods, serve as an input to one of the other producing sectors, or be purchased as a consumption item. These transactions can be represented by a system of equations:

$$X_1 = x_{11} + x_{12} + x_{13} + y_1$$

$$X_2 = x_{21} + x_{22} + x_{23} + y_2$$

$$X_3 = x_{31} + x_{32} + x_{33} + y_3$$

$$L_0 = L_{01} + L_{02} + L_{03} + y_0$$

where: X_i = gross output of the i^{th} sector;

L_0 = total primary input;

x_{ij} = output of the i^{th} sector purchased by the j^{th} sector and used to produce X_j;

L_{0j} = primary resource purchased by the j^{th} sector to produce X_j;

y_i = output of the i^{th} sector used to satisfy final demand;

y_0 = the amount of the primary resource purchased by final demand.

Figure 1 represents this system of equations in the form of a transactions table.

		Purchasing Sectors			Final Demand	Total Output
		1	2	3		
Producing Sectors	1	x_{11}	x_{12}	x_{13}	y_1	X_1
	2	x_{21}	x_{22}	x_{23}	y_2	X_2
	3	x_{31}	x_{32}	x_{33}	y_3	X_3
Primary Input		L_{01}	L_{02}	L_{03}	y_0	L_0
Total Input		X_1	X_2	X_3		

Figure 1. Simple Transactions Table.

Sectors 1, 2, and 3 comprise the processing or endogenous sectors, producing the goods and services for the economy. There are an equal number of rows and columns in the processing portion, with row i showing sector i sales and column i showing sector i purchases. The transactions table is similar to a double entry bookkeeping system inasmuch as the sum across the row will equal the sum across the column for each of the producing sectors. Elements within the processing sector show how much each sector purchases from every other sectors in order to produce its output. Alternatively, they show sales of each sector to the other sectors.

The primary input row shows the amount of primary inputs purchased by each sector. In this simple model, the primary input can be thought of as the human input. In an empirical model, primary inputs could be broken down into as many rows as desired. Typically, households, government, and imports comprise the primary input sectors.

The final demand sector shows how much output is purchased as a final demand item from each of the producing sectors. As with the primary input sector, the final demand sector can be expanded to include as much detail as desired. Typically, the final demand sectors include household or consumer demand, government demand, and exports.

The processing sectors are represented by both a row and a column, thus showing both input and output distribution. This need not be the case with the non-processing or exogenous sectors. The final demand sectors are independent of primary inputs in this model. They are related inasmuch as payment received for primary inputs represents income which in turn is used to purchase final demand items, but there is not the direct relationship of the rows and columns as in the endogenous sectors.

When applying the input-output model to a specific research problem, the placement of each sector either in the endogenous or exogenous partition of the transaction matrix is determined by the researcher. In some cases he may wish to include a given sector in the endogenous partition. In others, he may put it in the exogenous section. The placement will be determined by the information the researcher wants to generate.

Endogenous sectors are in essence intermediate production activities responding to the demands of the final demand sector. As such, their production levels change only in response to stimuli from external sources. Exogenous sectors, on the other hand, are autonomous. They do not react to, but instead provide, the economic stimuli that causes change in the system. Thus the level of the production activities of the endogenous sectors are determined by the demands placed upon them by the exogenous sectors.

There are two types of input-output models, the "open" model and the "closed" model. The "open" model includes both endogenous and exogenous sectors while the "closed" model has only endogenous sectors. The "closed" model would imply a closed economic system with no external economic forces acting on it. The system is assumed to be in equilibrium and will remain in this state. An "open" model, however, equilibrates to the exogenous final demand sectors. The exogenous sectors therefore determine the economic activity that takes place in the system in accordance with the assumptions of the model. 17/

Technical Coefficients

A technical coefficient a_{ij} is defined as the dollar amount of output of sector i required to produce one dollar of output in sector j. They are derived from the transactions table under the assumption that the output of a sector i is a linear function of the inputs purchased by sector i. Using the notation on page 8, the production function of the j^{th} sector can be written as

$$X_j = \sum_i x_{ij} + L_{0j}$$

The technical coefficients a_{ij} are derived by dividing both sides of the equation by X_j. Thus,

$$1 = \frac{X_j}{X_j} = \sum_i \frac{x_{ij}}{X_j} + \frac{L_{0j}}{X_j} = \sum_i a_{ij} + A^*_{0j}$$

where: $a_{ij} = \frac{x_{ij}}{x_j}$ and $A^*_{0j} = \frac{L_{0j}}{X_j}$. Also, $x_{ij} = a_{ij}X_j$.

Now substituting for x_{ij} in the system of equations on page 14, we can rewrite the equations as

$$X_i = \sum_j x_{ij} + y_i = \sum_j a_{ij}X_j + y_i$$

17/ The assumptions of the model are discussed on page 24 of this chapter.

The system of equations can be written in matrix form as

$$X = AX + Y$$

Where: X = the vector of gross output with elements X_i, $i = 1, \ldots n$

A = the n x n matrix of a_{ij}, $i,j = 1, \ldots n$

Y = the vector of final demand with elements y_i, $i = 1, \ldots n$

The equation $X = AX + Y$ can be rewritten in the form

$$Y = X - AX$$

The term by term expansion of this equation is

$$y_i = X_i - \sum_j a_{ij} X_j$$

In words, the output of sector i available for final demand y_i is equal to gross output (X_i) minus the output of sector i required by other sectors as intermediate inputs ($\sum_j a_{ij} X_j$).

Factoring the matrix equation $Y = X - AX$ results in the equation $Y = (I-A)X$. The square matrix $(I-A)$ is known as the "Leontief Matrix" and has some special properties. Recall that $a_{ij} = \frac{x_{ij}}{X_j}$, hence the a_{ij}'s are $0 \leq a_{ij} < 1$. Therefore, the elements of $(I-A)$, which we will denote as a'_{ij}, have the following properties:

(1) $0 \leq a'_{ij} < 1$ for all $i = j$

(2) $-1 < a'_{ij} \leq 0$ for all $i \neq j$

(3) $0 \leq \left|\sum_i a'_{ij}\right| < 1$ for all i

these properties are sufficient to insure that the matrix $(I-A)^{-1}$ exists. 18/ Therefore the equation $Y = (I-A)\ X$ can be premultiplied by $(I-A)^{-1}$ to get the equation

$$X = (I-A)^{-1}Y$$

The matrix $(I-A)^{-1}$ is the matrix of interdependence coefficients. To aid the exposition, this matrix will later be referred to as the matrix B.

Interdependence Coefficients

The interdependence coefficients indicate the total change in input requirements of all sectors due to a unit change in demand for the output of any sector or group of sectors. Thus they indicate both direct and indirect effects. Direct effects are defined as the initial changes in gross output of each sector required to meet the change in final demand of a given sector. Indirect effects are the remaining changes that take place as the system adjusts to the initial change.

To illustrate the use of the interdependence matrix, consider the equation of the model in the form

$$X = (I-A)^{-1}Y$$

Assume a vector ΔY which shows a change in one or more of the elements of the final demand vector Y. The resultant system will then be

18/ G. Hadley, Linear Algebra, Addison-Wesley, Reading, Massachusetts, 1961, p. 118.

$$(X + \Delta X) = (I-A)^{-1} (Y + \Delta Y)$$

where ΔX is the vector of change in gross output due to the change in the final demand vector. Distributing the right side of the equation yields

$$X + \Delta X = (I-A)^{-1}Y + (I-A)^{-1}\Delta Y$$

Subtracting the original equation $X = (I-A)^{-1}Y$ from the above, we get $\Delta X = (I-A)^{-1}\Delta Y$. The elements of the vector ΔX are the changes that occur in the output of each sector of the model when there is a change in final demand in one or more sectors.

The $(I-A)^{-1}$ matrix is convergent, therefore it can be derived from a series expansion of $I-A^{\infty}$. 19/ The definition of convergence is $\lim_{n\to\infty} A^n = 0$, A is a square matrix. If A is convergent, which is true of the matrix of a_{ij}'s of the Leontief input-output model, then the following equation is also true.

$$I = I-A^{\infty} = (I-A)(I + A + A^2 + A^3 + \ldots)$$

Premultiply both sides of this equation to get

$$(I-A)^{-1} = I + A + A^2 + A^3 + \ldots$$

The series expansion can be substituted for the $(I-A)^{-1}$ matrix in the equation $\Delta X = (I-A)^{-1}\Delta Y$ to obtain the equation

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19/ A detailed discussion of matrix convergence and the calculation of $(I-A)^{-1}$ via a series expansion is presented in Literature Cited references numbered 142, 146, 147, and 25.

$$\Delta X = (I + A + A^2 + A^3 + \ldots + A^{\infty})\Delta Y$$

Partially distributing the right side of this equation results in

$$\Delta X = (I + A)\Delta Y + (A^2 + A^3 + \ldots + A^{\infty})\Delta Y$$

In this equation, ΔX is shown as the sum of two components. The component $(I + A)\Delta Y - \Delta Y + A\Delta Y$ contains the initial change in final demand ΔY plus the initial adjustment required in the form of increased inputs from the remaining sectors. The remaining component $(A^2 + A^3 + \ldots A^{\infty})\Delta Y$ contains the remaining rounds of adjustment before the system again reaches an equilibrium position. The changes represented by

$$\Delta Y + A\Delta Y$$

are called the direct effects, and the remainder are called the indirect effects. 20/

Output Multipliers

An output multiplier is defined as the ratio of change in gross output per dollar change in final demand. An output multiplier can be calculated for each sector, and an aggregate output multiplier would be a weighted sum of the sector multipliers.

20/ Hollis B. Chenery and Paul B. Clark, Interindustry Economics, John Wiley and Sons, New York, 1959, p. 52.

The output multipliers are derived from the $(I-A)^{-1}$ matrix. In order to facilitate the discussion of the derivation, the following notation will be used:

(1) ΔX is the vector showing change in gross output of each sector;

(2) ΔY is the vector of change in final demand;

(3) B is the matrix $(I-A)^{-1}$ with elements b_{ij};

(4) B^i is the i^{th} column vector of B.

From the preceding section, we have the equation

(5) $\Delta X = (I-A)^{-1}\Delta Y = B\Delta Y$

The term by term expansion of (5) is

(6) $\Delta X_i = \sum_j b_{ij}\Delta y_j$

Assume that all y_j in Y are zero except $y_k \neq 0$. Then (6) becomes

(7) $\Delta X_i = b_{ik}\Delta y_k$

since the remaining elements of the summation are zero. Using vector notation, this becomes

(8) $\Delta X = B^k\Delta Y = B^k\Delta y_k$

The total change in gross output is the column sum of the vector X. Therefore, summing the vectors of both sides of equation (8) we get

(9) $\sum_i \Delta X_i = \Delta y_k \sum_i b_{ik}$

Letting $y_k = \$1$, the k^{th} sector output multiplier is $\sum_i b_{ik}$. To illustrate the use of the output multiplier, assume $\Delta y_k = \$15$, and $\sum_i b_{ik} = 2.3$. Then gross output, $\sum_i \Delta X_i$, would increase by 2.3 x 15 = \$34.50. The elements of ΔX show the increase in output of each sector.

Income Multipliers

Personal income is all income received by individuals in Montana. An income multiplier is defined as the ratio of change in total personal income per \$1 change in final demand for each sector. As in the case of the output multipliers, an income multiplier can be calculated for each sector. The derivation of the income multiplier is similar to the derivation of the output multiplier but involves one more matrix, P.

P is a square matrix where a diagonal element p_{ii} is the proportion of personal income derived per dollar of output of sector i. All off-diagonal elements are zero. Using the notation of the preceding section, we can set up the equations:

(10) $\Delta X = B\Delta Y$

(11) $P\Delta X = PB\Delta Y$

matrices P and B being of the same order. Let all elements $\Delta Y = 0$ except for the element $\Delta y_k \neq 0$. Following the procedure of (6), (7), (8), and (9), equation (11) reduces to

(12) $P\Delta X = y_k(PB)^k$

where $(PB)^k$ is the k^{th} column vector of (PB). Summing both sides of equation (12) we get

$$(13) \quad \sum_i (P\Delta X)_i = \Delta y_k \sum_i (Pb)_{ik}$$

The left side of equation (13) is the total change in personal income. Thus, the sum of the elements of $(PB)^k$ is the k^{th} sector personal income multiplier.

Resource Multipliers

Resources are utilized as inputs for production of each sector in combination with the inputs purchased from the remaining sectors. They may be explicitly stated in the model as in the case of labor, or they may be implicitly assumed. Water requirements are an example of the latter.

Resource multipliers are defined as the ratio of change in total resource requirements per $1 change in final demand. The mathematics of the derivation of resource multipliers is exactly the same as for income multipliers, with the matrix P being replaced by the Matrix R, where r_{ii} is the amount of the resource required to produce a dollar of output in sector i, and the off-diagonal elements of R are zero.

Assumptions of the Leontief Input-Output Model

There are three basic assumptions of the Leontief model: 21/

(1) Each commodity is supplied by a single industry of sector production;

21/ Ibid., p. 22.

(2) The inputs purchased by each sector are a function only of the level of output of that sector;

(3) The total effect of carrying on several types of production is the sum of the separate effects.

The first assumption rules out the possibility of purchasing a particular product from more than one sector. It also assumes that the output of a sector, when it produces two different types of goods, is always produced in the same ratio of the two goods. Thus each unit of output always contains the same proportion of the particular products produced by that sector.

The second assumption implies that a unit of output always requires the same proportions of inputs, thus eliminating the possibility of substitution of one input for another. The input coefficients are constant, and any expansion of output of a particular sector requires a proportionate increase in the inputs to that sector.

The third assumption rules out the possibility of external economies and diseconomies of scale. The second and third assumptions taken together insure that the production function is also linear.

The result of assumptions 2 and 3 above is the further assumption that the technical coefficients of the model are constant. The validity and reasonableness of this assumption has been argued extensively. Chenery and Clark discuss the pros and cons of this assumption and conclude that it is not unrealistic in the short run, periods of perhaps five years or less. 22/ Forecasts for more than

22/ Ibid., pp. 33-42.

five years ahead made on the basis of the model should be treated with caution. However, this is a fault common to most, if not all, predictive models.

The application of an input-output analysis for the purpose of analyzing the structure of an economy also requires the assumption that there are no errors of aggregation. That is, the correct inter-sectoral transactions are recorded in the transactions table. Whether there are errors or not is extremely difficult to evaluate. However, a testing procedure could be devised to determine the probable accuracy of the estimates. Quandt 23/ discusses the possibility of treating the a_{ij}'s as point estimates of the mean of a distribution, using standard statistical techniques to test the estimates. As yet there have not been any empirical input-output studies undertaken using this approach.

23/ Richard E. Quandt, "On the Solution of Probabilistic Leontief Systems," Naval Logistics Quarterly 6, No. 4, December 1959, and "Probabilistic Errors in Leontief Systems," Naval Logistics Quarterly 5, No. 2, June 1958.

CHAPTER III--A SURVEY OF THE MONTANA ECONOMY

An input-output model is a tabular description of the economic activities of the region under study. The degree of aggregation of the model and the composition of the sectors of the model will depend upon the characteristics of the economy. Thus, an overview of the Montana economy is needed to properly design the Montana input-output model and to evaluate the results.

Geographic and Economic Characteristics

There are approximately 93,600,000 acres in Montana. Table I shows the ownership distribution of the land resource as it was in 1960.

TABLE I. DISTRIBUTION OF LAND OWNERSHIP IN MONTANA, 1960.*

Ownership	Land	
	Acres	Percent
1. Private ownership	61,166,000	65.4
2. State ownership		
Grazing land	4,195,000	4.5
Agricultural land	487,000	.5
Timber & special grants	496,000	.5
Total State Land	5,178,000	5.5
3. Federal ownership		
Forest Service, forest	16,635,000	17.8
Forest Service, L. U.	1,936,000	2.1
Park Service	1,141,000	1.2
Bureau of Land Management	6,659,000	7.1
Other	885,000	.9
Total Federal Land	27,256,000	29.1
4. Total All Land Area	93,600,000	100.0

*Source: Montana Agriculture Basic Facts, Cooperative Extension Service and Agr. Expt. Sta., Bulletin 293, Montana State University, Bozeman, 1962, p. 13.

The topography of the land resource varies from the flatlands and rolling hills of eastern Montana to the tree-covered mountains of western Montana. Agricultural land includes about 13.5 million acres classed as tillable land with about 40 million acres of grazing land. 1/

The major share of the tillable land is used in dryland crop farming in the Great Plains Area of eastern and central Montana. Annual precipitation averages 14 inches, but has varied from a low of 8 inches to as much as 20 inches. 2/ The growing season, considered to run from the last frost in spring to the first frost in fall, is about 110 days on the average, but, like average precipitation, varies considerably from year to year and from location to location. 3/ Because of these climatic conditions, farmers in Montana are rather restricted as to the species of crops that can be produced economically. Wheat and barley are the principal cash crops, owing to their adaptability to the limiting climatic conditions.

The major livestock enterprise in the state is the range-based cow-calf operation, producing feeder calves for export. At the present

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1/ State of Montana, Twenty-First Biennial Report of the Montana State Board of Equalization, Tribune Printing, Great Falls.

2/ Montana Agriculture Basic Facts, Cooperative Extension Service and Agr. Expt. Sta., Bulletin 293, Montana State University, Bozeman, 1962, p. 11.

3/ Montana State Department of Agriculture and the U. S. Department of Agriculture, Statistical Reporting Service, Montana Agricultural Statistics, Vol. X, Tribune Printing, Great Falls, 1965, p. 16.

time, there is relatively little cattle feeding in the state, with an average of 50,000 head on feed in 1963. 4/ Sheep and lamb enterprises provide the second largest share of livestock products, generating about 10 percent of livestock products cash receipts. As in the case of cattle, the sheep operations are range-based with the major product being feeder lambs.

Commercial forestland in the state is located mainly in western Montana. Altogether, commercial forest area amounts to 17.3 million acres of softwood saw timber with virtually no hardwood timber. 5/ Montana ranks second in value of lumber production in the Rocky Mountain States. 6/ The lumber industries provide considerable employment in Montana, and lumber products are one of the principal exports from the state.

There are extensive mineral operations in the state, with copper mining and petroleum extraction the main industries. Mineral industries, both mining and manufacturing, play very important roles in the economy

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4/ U. S. Department of Agriculture, Statistical Reporting Service, Livestock and Meat Statistics, 1963, Supplement to Stat. Bul. 333, Washington, D. C., p. 14.

5/ Alvin K. Wilson and John S. Spencer, Jr., Timber Resources and Industries in the Rocky Mountain States, U. S. Forest Service Sta. Resources Bul. INT-7, 1967. Intermountain Forest and Range Expt. Sta., Ogden, Utah, p. 36.

6/ Ibid., p. 23.

of the state. 7/ Mineral leases on public lands produce in excess of $5 million per year in Montana. 8/

Industrial activity is located primarily around Great Falls and Billings, with the major industrial activities consisting of primary metals refining and petroleum refining. 9/ Wood products plants are scattered throughout western Montana with the largest operations in Missoula County. 10/

Table II shows the total output of the state by sector. Manufacturing produces the greatest share of the total output, followed by agriculture and trade. Agriculture, mining, and manufacturing are the primary industries in the state, with the remaining sectors acting in supporting roles.

The Human Resource

The 1963 population in Montana was estimated to be 701,000 people, increasing at an average annual rate of .9 percent per year. 11/ The

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7/ U. S. Department of Commerce, Bureau of the Census, Census of Mineral Industries, 1963, Area Statistics: Montana, U. S. Government Printing Office, Washington, D. C., 1967.

8/ U. S. Department of Interior, Bureau of Land Management, Public Land Statistics, 1963, Washington, D. C., 1964.

9/ U. S. Department of Commerce, Bureau of the Census, Census of Manufactures, 1963, Vol. III: Area Statistics.

10/ Ibid.

11/ U. S. Department of Commerce, Bureau of the Census, Statistical Abstract of the U. S., 1966, Washington, D. C., 1966.

TABLE II. TOTAL OUTPUT OF MONTANA, 1963, BY SECTOR.

Sector	Total Output a/ Thousand Dollars	Percent
Agriculture		
Livestock and livestock products	206,856	8.0
Crops	307,985	12.0
Manufacturing		
Food and kindred products	140,885	5.4
Lumber and wood products	168,504	6.6
Other manufacturing	365,533	14.3
Transportation and Public Warehousing	156,056	6.1
Communications and Public Utilities	132,840	5.2
Real Estate, Finance and Insurance	143,398	5.6
Mining	173,121	6.7
Services	206,777	8.1
Trade, Wholesale and Retail	374,734	14.6
Construction	189,503	7.4
Total	2,566,192	100.0

a/ Total output of each sector by product class is discussed in Appendix A.

population has shifted from predominantly rural in 1935 to over 50 percent urban in 1963. 12/ Along with the location shift, there has been a change in the age distribution of the population. In 1930, nearly 60 percent of the population was in the 18-64 year old age group. By 1960, this had decreased to 52 percent. During this period, total population increased by 25 percent, but the 18-64 age group increased by only 10 percent. 13/ This shift in age composition has increased the need for public facilities (schools, hospitals, etc.) beyond the need due simply to population growth.

Employment and Personal Income

Table III shows the distribution of employment and personal income in Montana in 1963. The trade sector, which includes all wholesale and retail establishments, employs the greatest number of people and also provides the greatest amount of personal income. There were 9,387 trade establishments in Montana in 1963, of which 1,864 had no payroll. The average number of employees on a payroll was five. Total wages and salaries in the trade sector amounted to $151,716,000, proprietor income comprising the remainder of personal income earned in this sector.

Agriculture ranked second with respect to personal income but ranked seventh with respect to employment. The average size farm in

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12/ Montana Agriculture Basic Facts, op. cit., p. 5.

13/ Ibid.

TABLE III. EMPLOYMENT AND PERSONAL INCOME, 1963.*

Sector	Employment a/ (Thousandths)	Personal Income b/ Thousand Dollars	Percent
Agriculture	13.0	201,008	12.7
Manufacturing	22.4	153,512	9.7
Transportation	12.1	92,148	5.8
Communications & Public Utilities	5.8	45,808	2.9
Real Estate, Finance & Insurance	6.8	49,565	3.1
Mining	7.1	58,377	3.7
Services	24.1	134,073	8.4
Trade	40.3	227,980	14.3
Construction	12.3	89,781	5.7
State and Local Government	23.4	131,000	8.2
Federal Government	10.0 c/	141,619 d/	8.9
Other	.6	108,147 e/	6.8
Total	163.6	1,433,118 f/	90.2
Federal and State Transfers g/		154,982	9.8
Total Personal Income		1,588,000	100.0

a/ Average number employed during the calendar year.
b/ Includes wages and salaries, properitor income, property income, and transfers.
c/ Excludes military employment.
d/ Includes military payments.
e/ Includes rent payments, and miscellaneous income.
f/ Total earned income in Montana.
g/ Social Security, welfare, GI Bill, Farm Programs, etc.

*Source: Ninth Federal Reserve District Statistical Review

Montana was 2,436 acres in 1964. 14/ Since 1935 the trend has been toward larger and larger farms with fewer farm numbers. This has been particularly the case in the Great Plains dryland farming region of the state. The 1964 Census of Agriculture indicates a total of 27,020 farms, but only 21,044 had sales of farm products of $2,500 or more. The size of farms and the type of crops and livestock produced lead to rather capital intensive farm operations with correspondingly low labor requirements.

Manufacturing industries ranked in third place both in employment and personal income. The lumber and wood products industry employed the greatest number of people, accounting for 36 percent of total manufacturing employment and 25 percent of total manufacturing output. 15/ The food and kindred products industry was second in manufacturing employment with 25 percent of the total, and food products output was about 20 percent of total manufacturing output. 16/ The primary metals industry, principally non-ferrous metal refining, ranked third in manufacturing employment with 16 percent of the total employment, but produced over 30 percent of the total output of the manufacturing sector. 17/ Similarly, the petroleum refining industry accounted for

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14/ U. S. Department of Commerce, Bureau of the Census, Census of Agriculture, 1964, Statistics of the State and Counties.

15/ Census of Manufactures, op. cit.

16/ Ibid.

17/ Ibid.