



Active RC networks utilizing the voltage follower
by James Llafet Hogin

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Abstract:

The (RCVF) is defined as an active network whose passive portion consists of a three-port grounded RC structure and whose active portion consists of the three-terminal grounded voltage follower. The necessary and sufficient conditions on a voltage transfer function for its realization by the (RCVF) are developed. Synthesis methods for the passive portion of the (RCVF) are reviewed, developed, and discussed. A digital computer program is presented which demonstrates the feasibility of optimizing (RCVF) networks by the method of steepest descent. A discussion and analysis of practical voltage followers and the practical considerations involved with their use is presented. In addition, a useful three stage voltage follower is developed which has very high input impedance and a gain very close to unity.

Illustrative examples and experimental verification are given. It is concluded that the (RCVF) is a highly useful and practical network.

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TABLE OF CONTENTS

	Page
Vita	ii
Acknowledgments	iii
Table of Contents	iv
List of Tables	vi
List of Figures	vii
List of Symbols	x
Abstract	xii
I. Introduction	1
A. The Active Network	2
B. The Voltage Follower	4
C. Comparison of Active and Passive Transfer Function Synthesis	5
II. Statement of Problem	11
III. Necessary and Sufficient Conditions for the Realization of the Voltage Transfer Function T_v	13
A. Surplus Factor Utilization	21
IV. (RCVF) Realization Techniques	26
A. A Pictorial Representation of the Passive Synthesis Problem	26
B. Review of Previously Proposed Configurations	30

	Page
C. A Realization Technique for the Biquadratic Transfer Function	31
D. Ladder Network Realization for Low-pass and High-pass Transfer Functions	36
E. A Network Configuration for Fourth-order Band-pass Transfer Functions	47
F. Examples	51
V. Network Optimization	59
A. Optimization Parameters	59
B. Computer Program Description	67
C. Example Network	83
VI. Practical Considerations	90
A. The Non-ideal Voltage Follower	90
B. Voltage Follower Circuits	91
C. Experimental Results	96
VII. Conclusion and Recommendations	107
A. Conclusion	107
B. Recommendations	107
Appendix I. Optimization Computer Program Listing	111
Appendix II. Optimization Computer Program Input and Output Data for Example Problem	121
Appendix III. Numerical Computations	125
Literature Consulted	130

LIST OF TABLES

- 6.1 Frequency-attenuation tabulation of experimental and theoretical values for network of Figure 6.8.
- 6.2 Frequency-attenuation tabulation of experimental and theoretical values for network of Figure 6.9.
- 6.3 Frequency-attenuation tabulation of experimental and theoretical values for network of Figure 6.10.

LIST OF FIGURES

- 1.1 The active RC network utilizing the voltage follower.
- 1.2 Realization of $E_{out}/I_{in} = \frac{1}{4} \frac{s^2 + 2.828s + 4}{s^2 + 1.414s + 1}$ with a lossless network terminated by resistors.
- 1.3 Realization of $E_{out}/E_{in} = 0.2 \frac{s^2 + 2.828s + 4}{s^2 + 1.414s + 1}$ by Pantell's method.
- 1.4 Simple realization of $E_{out}/I_{in} = \frac{s^2 + 2.828s + 4}{s^2 + 1.414s + 1}$.
- 1.5 Realization of $E_{out}/E_{in} = \frac{1}{4} \frac{s^2 + 2.828s + 4}{s^2 + 1.414s + 1}$ with the (RCVF).
- 3.1 The "A" and "B" two-port unbalanced networks.
- 3.2 Interconnection of "A" and "B" networks.
- 3.3 The "A" and "B" networks connected to the voltage follower.
- 4.1 A possible current distribution.
- 4.2 Current distribution of passive portion of (RCVF) network to realize $T_v = \frac{a_2 s^2 + a_1 s + a_0}{p_2 s^2 + p_1 s + p_0}$.
- 4.3 Network configuration with input current distribution of Figure 4.2.
- 4.4 Current distribution for case of $a_1/a_0 \leq r_1/r_0$.
- 4.5 Current distribution for case of $r_1/r_0 \leq a_1/a_0$.
- 4.6 The basic ladder network.

- 4.7 The basic decomposed ladder network.
- 4.8 Decomposed ladder network with shunt arm capacitors and series arm resistors.
- 4.9 Current distribution for basic ladder network realization of y_{11} .
- 4.10 Pole-zero distributions for $y_{11}^{(0)}$ to $y_{11}^{(6)}$.
- 4.11 Three basic building blocks for the fourth-order band-pass transfer function realization.
- 4.12 Current distribution for network corresponding to transfer function of Eq. (4.23).
- 4.13 Network realization for $T_V = \frac{1}{4} \frac{s^2 + 2.828s + 4}{s^2 + 1.414s + 1}$.
- 4.14 Network realization for $T_V = 1/s^4 + 2.6132s^3 + 3.4143s^2 + 2.6132s + 1$.
- 4.15 Network realization for $T_V = 2.172s^2/s^4 + 1.414s^3 + 3s^2 + 1.414s + 1$.
- 5.1 Flow chart for network optimization computer program.
- 5.2 Configuration of example network.
- 5.3 Optimized network for realization of $T_V = 1/s^4 + 2.6132s^3 + 3.4143s^2 + 2.6132s + 1$.
- 6.1 Current distribution diagram for nonunity voltage follower gain k .
- 6.2 Voltage follower of Reference 32.
- 6.3 Altered network.

- 6.4 System diagram for networks of Figures 6.2 and 6.3.
- 6.5 Three stage voltage follower network.
- 6.6 Current distribution for twin-T network.
- 6.7 Network for realization of $T_v = 0.1s/s^2 + .1s + 1$.
- 6.8 (RCVF) for realization of second-order band-pass transfer function denormalized to 300,000 ohms and 150 cps.
- 6.9 (RCVF) for realization of fourth-order low-pass transfer function denormalized to 1 megohm and 85 cps.
- 6.10 (RCVF) for realization of fourth-order band-pass transfer function denormalized to 500,000 ohms and 39.8 cps.

LIST OF SYMBOLS

RC	Abbreviation for the term "resistance-capacitance".
RCVF	Abbreviation for the active network composed of resistors, capacitors, and a single grounded voltage follower.
E_i	Voltage parameter.
I_i	Current parameter.
y_{ij}	Short-circuit admittance parameters.
T_v	Voltage transfer function.
s	Complex frequency variable.
$N_{ij}(s)$	Numerator polynomial of y_{ij} for $i=j$. Negative of numerator polynomial for $i \neq j$.
$D(s)$	Denominator polynomial of y_{ij} .
$A(s)$	Numerator polynomial of T_v .
$P(s)$	Denominator polynomial of T_v .
$R(s)$	Polynomial equal to $P(s)-A(s)$.
$E(s)$	Augmenting polynomial such that $P(s)+E(s)$ has negative real zeros.
a_i	Coefficients of $A(s)$.
p_i	Coefficients of $P(s)$.
r_i	Coefficients of $R(s)$.
e_i	Coefficients of $E(s)$.
d_i	Coefficients of $D(s)$.
Y_i	Admittance parameter.

Z_i	Impedance parameter.
R_i	Resistance parameter.
C_i	Capacitance parameter.
η_i	Zeros of admittance.
δ_i	Poles of admittance.
Q	Optimization parameter.
S_k^T	Sensitivity of T with respect to k .
\tilde{S}_k^P	Polynomial root sensitivity (p = any polynomial root).
t_i	Coefficients of T_v .
C_D	Capacitance distribution parameter.
R_D	Resistance distribution parameter.
C_S	Capacitance summation parameter.
C_R	Capacitance ratio parameter.
R_R	Resistance ratio parameter.
w_i	Optimization parameter weighting functions.
$F(s)$	Second-order polynomial with zeros lying on unit circle in the RHP.
ϕ_i	Angle zeros of $F(s)$ make with the positive real axis.
$B(s)$	Augmenting polynomial for $F(s)$.
β_i	Zeros of $B(s)$.
b_i	Coefficients of $B(s)$.
G_i	Function of the coefficients of $B(s)$.

ABSTRACT

The (RCVF) is defined as an active network whose passive portion consists of a three-port grounded RC structure and whose active portion consists of the three-terminal grounded voltage follower. The necessary and sufficient conditions on a voltage transfer function for its realization by the (RCVF) are developed. Synthesis methods for the passive portion of the (RCVF) are reviewed, developed, and discussed. A digital computer program is presented which demonstrates the feasibility of optimizing (RCVF) networks by the method of steepest descent. A discussion and analysis of practical voltage followers and the practical considerations involved with their use is presented. In addition, a useful three stage voltage follower is developed which has very high input impedance and a gain very close to unity.

Illustrative examples and experimental verification are given. It is concluded that the (RCVF) is a highly useful and practical network.

I. INTRODUCTION

Active RC networks are networks containing only resistors, capacitors and active elements. These networks are attractive at the lower frequencies where magnetic elements are large, expensive and difficult or impossible to realize. In addition, the advent of microcircuit technology, where a stable inductance parameter is difficult to attain, has resulted in an increased interest in this type of network. The need for a power supply for the active elements is often unimportant if, as frequently happens, the network is a part of a system for which power is required regardless of the network.

The purpose of this study is to develop synthesis procedures concerned with a particular type of active RC network: an active RC network whose active element consists of a single ideal voltage follower. The ideal voltage follower is defined as a 2-port network whose input admittance and output impedance are zero and whose ratio of output to input voltage is unity. The cathode, emitter, or source follower circuit may be considered a practical approximation to the ideal voltage follower.

In addition to developing synthesis procedures, this study is concerned with the optimization of the synthesized

networks by means of the method of steepest descent as applied by the digital computer. Finally, the study is concerned with the practical demonstration of the synthesis methods and with the problem of obtaining a voltage follower which closely approximates the ideal.

A. THE ACTIVE NETWORK

The active RC network configuration of concern in this research is shown in Figure 1.1. The passive portion of the network consists of a 3-port grounded RC structure. The active portion consists of the 3-terminal grounded voltage follower defined above. For convenience, this type of active network will be referred to as the (RCVF).

An important consideration in the design of active networks is the sensitivity of the desired network function to variations of the active element portion of the active network. This consideration is in addition to that of the sensitivity associated with variations of the passive network elements. It should be pointed out, however, that from a practical point of view, the gain of the voltage follower proposed as the active element in this study can be made more stable than alternate types of active devices which require a gain other than unity. This observation is based

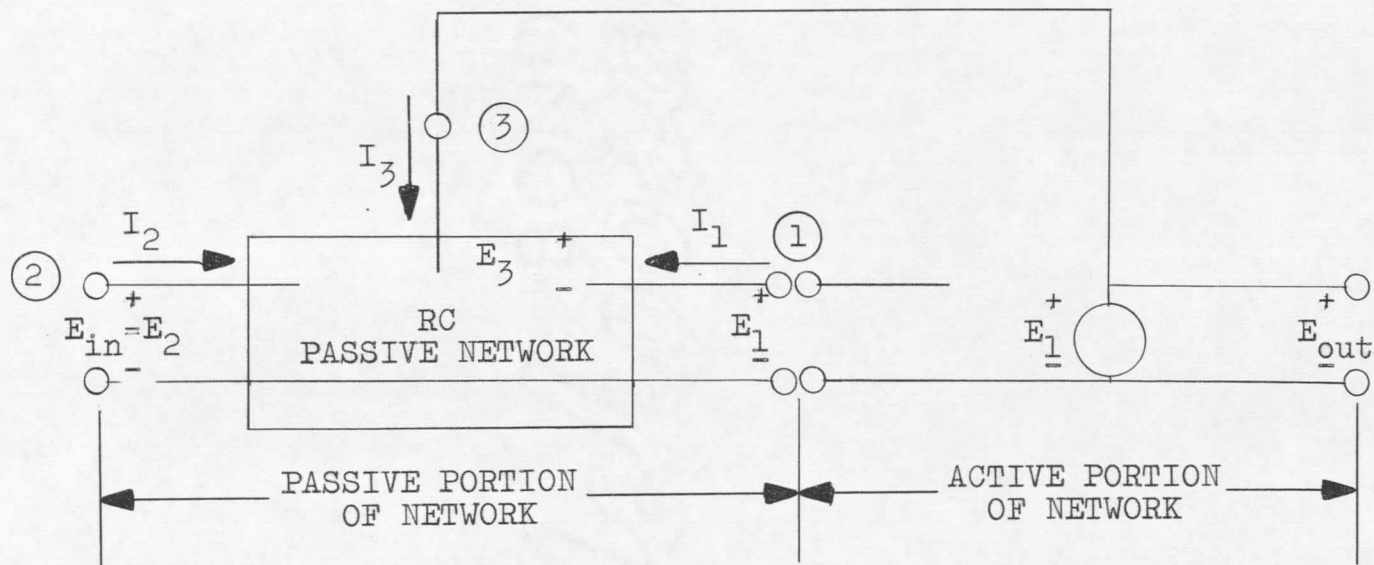


Figure 1.1. The active RC network utilizing the voltage follower (RCVF).

on the fact that the voltage follower is concerned with the direct comparison of input voltage and output voltage rather than with their indirect comparison. Thus, a direct comparison of active networks on the basis of the magnitude of the sensitivity function associated with the active element could be misleading. It is more informative to specify network function variations caused by expected variations in the active device. In conclusion, the active element proposed for this study may be made highly stable and at least partially compensate for the fact that the networks are quite sensitive to changes in the active element.

An additional reason for considering the voltage follower as an active device is that, in its elementary forms, it is an extremely simple and widely used device.

B. THE VOLTAGE TRANSFER FUNCTION

The development of the voltage transfer function for the (RCVF) proceeds as follows. Referring to the passive portion of the network shown in Figure 1.1 and observing the conventions shown, the first of the short-circuit admittance equations is

$$I_1 = y_{11}E_1 + y_{12}E_2 + y_{13}E_3. \quad (1.1)$$

For the particular configuration shown in Figure 1.1, $I_1 = 0$

and $E_3 = E_1$. Equation (1.1) then becomes

$$y_{11}E_1 + y_{12}E_2 + y_{13}E_1 = 0 \quad (1.2)$$

and the voltage transfer function T_v may be solved as

$$T_v = \frac{E_{out}}{E_{in}} = \frac{E_1}{E_2} = \frac{-y_{12}}{y_{11} + y_{13}} \quad (1.3)$$

Since the poles of the short circuit admittance parameters are identical (provided common factors have not been cancelled), we may conveniently define

$$y_{ij} = \begin{cases} \frac{N_{ij}}{D} & \text{for } i=j \\ \frac{-N_{ij}}{D} & \text{for } i \neq j \end{cases} \quad (1.4)$$

where the N 's and D are polynomials, and write the voltage transfer function as

$$T_v = \frac{E_{out}}{E_{in}} = \frac{N_{12}}{N_{11} - N_{13}} = \frac{A(s)}{P(s)} \quad (1.5)$$

This manner of defining the y_{ij} allows us to consider all of the coefficients of the N_{ij} as having the same sign (which we arbitrarily choose positive).

C. COMPARISON OF ACTIVE AND PASSIVE TRANSFER FUNCTION SYNTHESIS

The main difference between active and passive transfer function synthesis is that passive synthesis is generally

concerned with the direct realization of a given transfer quantity by means of one of the transfer immittances. Active synthesis is generally concerned with the indirect realization of the same transfer quantity by means of the sum or difference of two transfer immittances. The active elements of the active network serve as sum or difference taking devices.

As an example, let us consider a given transfer function T_v and examine some of the ways this function may be realized with passive circuitry. Then it will be shown how this same function may be realized with the (RCVF) active network.

Consider the transfer function $T_v = K \frac{s^2 + 2.828s + 4}{s^2 + 1.414s + 1}$ where K is an arbitrary positive constant. This function may be realized by a passive network since its poles lie in the left-half of the complex frequency plane (LHP). The function is a minimum phase function since its zeros also lie in the LHP. Guillemin (25) shows that a minimum phase transfer function cannot be realized by a lossless network terminated in a single resistor. He shows, however, a realization consisting of a lossless network terminated by more than one resistor. Applying this method to the present case results in the network of Figure 1.2. (See Appendix III-A

