Active RC networks utilizing the voltage follower
by James Llafet Hogin

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
Montana State University

Abstract:
The (RCVF) is defined as an active network whose passive portion consists of a three-port grounded RC structure and whose active portion consists of the three-terminal grounded voltage follower. The necessary and sufficient conditions on a voltage transfer function for its realization by the (RCVF) are developed. Synthesis methods for the passive portion of the (RCVF) are reviewed, developed, and discussed. A digital computer program is presented which demonstrates the feasibility of optimizing (RCVF) networks by the method of steepest descent. A discussion and analysis of practical voltage followers and the practical considerations involved with their use is presented. In addition, a useful three stage voltage follower is developed which has very high input impedance and a gain very close to unity.

Illustrative examples and experimental verification are given. It is concluded that the (RCVF) is a highly useful and practical network.

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LIST OF SYMBOLS

RC	Abbreviation for the term "resistance-capacitance".
RCVF	Abbreviation for the active network composed of resistors, capacitors, and a single grounded voltage follower.
E_i	Voltage parameter.
I_i	Current parameter.
y_{ij}	Short-circuit admittance parameters.
T_v	Voltage transfer function.
s	Complex frequency variable.
$N_{ij}(s)$	Numerator polynomial of y_{ij} for $i=j$. Negative of numerator polynomial for $i \neq j$.
D(s)	Denominator polynomial of y_{ij}.
A(s)	Numerator polynomial of T_v.
P(s)	Denominator polynomial of T_v.
R(s)	Polynomial equal to P(s)-A(s).
E(s)	Augmenting polynomial such that P(s)+E(s) has negative real zeros.
a_i	Coefficients of A(s).
p_i	Coefficients of P(s).
r_i	Coefficients of R(s).
e_i	Coefficients of E(s).
d_i	Coefficients of D(s).
Y_i	Admittance parameter.

Z_i	Impedance parameter.
R_i	Resistance parameter.
C_i	Capacitance parameter.
η_i	Zeros of admittance.
δ_i	Poles of admittance.
Q	Optimization parameter.
S_k^T	Sensitivity of T with respect to k.
$\tilde{S}_k^p$	Polynomial root sensitivity (p = any polynomial root).
t_i	Coefficients of T_v.
C_D	Capacitance distribution parameter.
R_D	Resistance distribution parameter.
C_S	Capacitance summation parameter.
C_R	Capacitance ratio parameter.
R_R	Resistance ratio parameter.
w_i	Optimization parameter weighting functions.
F(s)	Second-order polynomial with zeros lying on unit circle in the RHP.
ϕ_i	Angle zeros of F(s) make with the positive real axis.
B(s)	Augmenting polynomial for F(s).
β_i	Zeros of B(s).
b_i	Coefficients of B(s).
G_i	Function of the coefficients of B(s).

ABSTRACT

The (RCVF) is defined as an active network whose passive portion consists of a three-port grounded RC structure and whose active portion consists of the three-terminal grounded voltage follower. The necessary and sufficient conditions on a voltage transfer function for its realization by the (RCVF) are developed. Synthesis methods for the passive portion of the (RCVF) are reviewed, developed, and discussed. A digital computer program is presented which demonstrates the feasibility of optimizing (RCVF) networks by the method of steepest descent. A discussion and analysis of practical voltage followers and the practical considerations involved with their use is presented. In addition, a useful three stage voltage follower is developed which has very high input impedance and a gain very close to unity.

Illustrative examples and experimental verification are given. It is concluded that the (RCVF) is a highly useful and practical network.

I. INTRODUCTION

Active RC networks are networks containing only resistors, capacitors and active elements. These networks are attractive at the lower frequencies where magnetic elements are large, expensive and difficult or impossible to realize. In addition, the advent of microcircuit technology, where a stable inductance parameter is difficult to attain, has resulted in an increased interest in this type of network. The need for a power supply for the active elements is often unimportant if, as frequently happens, the network is a part of a system for which power is required regardless of the network.

The purpose of this study is to develop synthesis procedures concerned with a particular type of active RC network: an active RC network whose active element consists of a single ideal voltage follower. The ideal voltage follower is defined as a 2-port network whose input admittance and output impedance are zero and whose ratio of output to input voltage is unity. The cathode, emitter, or source follower circuit may be considered a practical approximation to the ideal voltage follower.

In addition to developing synthesis procedures, this study is concerned with the optimization of the synthesized networks by means of the method of steepest descent as applied by the digital computer. Finally, the study is concerned with the practical demonstration of the synthesis methods and with the problem of obtaining a voltage follower which closely approximates the ideal.

A. THE ACTIVE NETWORK

The active RC network configuration of concern in this research is shown in Figure 1.1. The passive portion of the network consists of a 3-port grounded RC structure. The active portion consists of the 3-terminal grounded voltage follower defined above. For convenience, this type of active network will be referred to as the (RCVF).

An important consideration in the design of active networks is the sensitivity of the desired network function to variations of the active element portion of the active network. This consideration is in addition to that of the sensitivity associated with variations of the passive network elements. It should be pointed out, however, that from a practical point of view, the gain of the voltage follower proposed as the active element in this study can be made more stable than alternate types of active devices which require a gain other than unity. This observation is based

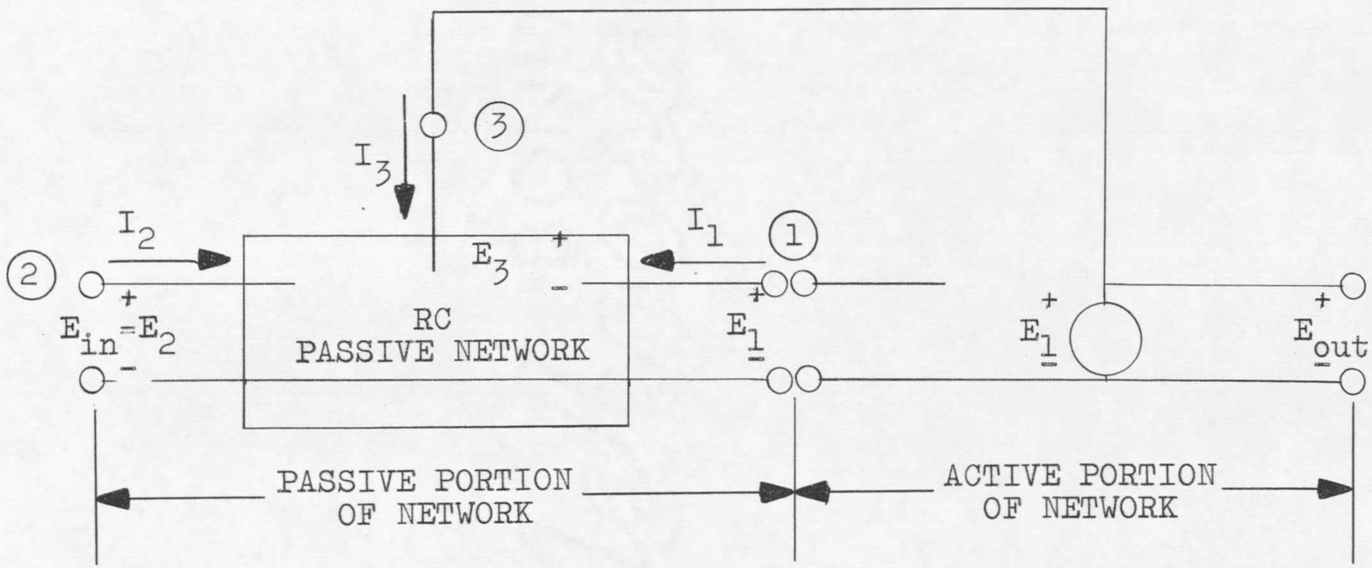

Figure 1.1. The active RC network utilizing the voltage follower (RCVF).

on the fact that the voltage follower is concerned with the direct comparison of input voltage and output voltage rather than with their indirect comparison. Thus, a direct comparison of active networks on the basis of the magnitude of the sensitivity function associated with the active element could be misleading. It is more informative to specify network function variations caused by expected variations in the active device. In conclusion, the active element proposed for this study may be made highly stable and at least partially compensate for the fact that the networks are quite sensitive to changes in the active element.

An additional reason for considering the voltage follower as an active device is that, in its elementary forms, it is an extremely simple and widely used device.

B. THE VOLTAGE TRANSFER FUNCTION

The development of the voltage transfer function for the (RCVF) proceeds as follows. Referring to the passive portion of the network shown in Figure 1.1 and observing the conventions shown, the first of the short-circuit admittance equations is

$$I_1=y_{11}E_1+y_{12}E_2+y_{13}E_3. \tag{1.1}$$

For the particular configuration shown in Figure 1.1, $I_1=0$

and $E_3=E_1$. Equation (1.1) then becomes

$$y_{11}E_1+y_{12}E_2+y_{13}E_1=0 \tag{1.2}$$

and the voltage transfer function T_v may be solved as

$$T_v=\frac{E_{out}}{E_{in}}=\frac{E_1}{E_2}=\frac{-y_{12}}{y_{11}+y_{13}} \tag{1.3}$$

Since the poles of the short circuit admittance parameters are identical (provided common factors have not been cancelled), we may conveniently define

$$y_{ij}\equiv\begin{cases}\dfrac{N_{ij}}{D} & \text{for } i=j\\[2ex] \dfrac{-N_{ij}}{D} & \text{for } i\neq j\end{cases} \tag{1.4}$$

where the N's and D are polynomials, and write the voltage transfer function as

$$T_v=\frac{E_{out}}{E_{in}}=\frac{N_{12}}{N_{11}-N_{13}}\equiv\frac{A(s)}{P(s)} \tag{1.5}$$

This manner of defining the y_{ij} allows us to consider all of the coefficients of the N_{ij} as having the same sign (which we arbitrarily choose positive).

C. COMPARISON OF ACTIVE AND PASSIVE TRANSFER FUNCTION SYNTHESIS

The main difference between active and passive transfer function synthesis is that passive synthesis is generally concerned with the direct realization of a given transfer quantity by means of one of the transfer immittances. Active synthesis is generally concerned with the indirect realization of the same transfer quantity by means of the sum or difference of two transfer immittances. The active elements of the active network serve as sum or difference taking devices.

As an example, let us consider a given transfer function T_V and examine some of the ways this function may be realized with passive circuitry. Then it will be shown how this same function may be realized with the (RCVF) active network.

Consider the transfer function $T_V = K \frac{s^2+2.828s+4}{s^2+1.414s+1}$ where K is an arbitrary positive constant. This function may be realized by a passive network since its poles lie in the left-half of the complex frequency plane (LHP). The function is a minimum phase function since its zeros also lie in the LHP. Guillemin (25) shows that a minimum phase transfer function cannot be realized by a lossless network terminated in a single resistor. He shows, however, a realization consisting of a lossless network terminated by more than one resistor. Applying this method to the present case results in the network of Figure 1.2. (See Appendix III-A