



An intrusive slotted cylinder antenna array for subsurface moisture profiling
by David Leo Herrick

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF
PHILOSOPHY in ELECTRICAL ENGINEERING
Montana State University
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Abstract:

Theoretical analysis and measured performance of an intrusive microwave probe for sensing the distribution of soil moisture from near surface to root zone depths (1-2 m) are presented. The probe consists of a linear array of slot antennas mounted along the axis of a metal cylinder and is intended for borehole installation.

Measurements of self and mutual slot antenna admittances are used to derive a discrete depth profile of the soil's complex permittivity. The moisture distribution is then deduced from this permittivity profile by empirical relations.

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FOR SUBSURFACE MOISTURE PROFILING

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A thesis submitted in partial fulfillment
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ABSTRACT

Theoretical analysis and measured performance of an intrusive microwave probe for sensing the distribution of soil moisture from near surface to root zone depths (1-2 m) are presented. The probe consists of a linear array of slot antennas mounted along the axis of a metal cylinder and is intended for borehole installation. Measurements of self and mutual slot antenna admittances are used to derive a discrete depth profile of the soil's complex permittivity. The moisture distribution is then deduced from this permittivity profile by empirical relations.

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Knowledge of soil moisture conditions at root zone depth is important for a variety of reasons. In some instances the particular strain of crop to be planted is chosen based on soil moisture estimates. For example, winter wheat is normally planted if moisture appears plentiful in the fall. Otherwise winter snows are allowed to replenish soil water and then spring wheat is planted. Soil moisture conditions are also an important factor in the design and efficient utilization of irrigation systems.

Although soil moisture is an important parameter in crop management decisions, it is not an easy quantity to measure. The distribution of soil moisture varies as a function of time and position due to such factors as percolation, evaporation, irrigation, rainfall, and plant use. Due to the nonhomogeneous nature of soil, there may be great differences in the distribution of soil moisture in adjacent areas. Traditional field

measurements of soil water have depended upon sampling given locations at specified times and depths to determine profiles of the moisture distribution. More recently, remote sensing satellites have made it possible to monitor large tracts of land but the attainable profile depth has been limited to a few centimeters [1].

There are two basic approaches to measuring soil moisture conditions:

- 1) those which determine the potential energy of the moisture;
- 2) those which measure moisture content per unit mass or per unit volume of soil.

The potential energy measurement determines the ease with which a plant may remove a given quantity of water from the soil. This is a fundamental factor governing a plant's response to moisture and may be measured by tensiometers, vapor pressure determination, freezing point depression, etc. However, the potential energy of soil water is, in principle, related to the soil's gravimetric (weight basis) and volumetric (volume basis) moisture content. Thus an indirect estimate of crop performance can be made from measurements of soil

moisture content. Since moisture content is more easily determined than potential energy, the latter approach is commonly taken.

Some Techniques for Measuring Soil Moisture Content

Moisture content can be measured in a variety of ways. A standard laboratory technique is the oven drying of soil samples. The moisture content is expressed as the difference in the sample's wet and dry weights normalized to either the sample volume or the dry weight. In equation form,

$$\%W_g = \frac{\text{Wet Weight} - \text{Dry Weight}}{\text{Dry Weight}} \times 100\% \quad (1-1)$$

or

$$\%W_v = \frac{\text{Wet Weight} - \text{Dry Weight}}{\text{Volume of Soil Sample}} \times 100\% \quad (1-2)$$

where $\%W_g$ is the gravimetric percentage and $\%W_v$ is the volumetric percentage.

Oven drying is probably the most accurate way to measure $\%W$, but this method has several disadvantages:

- 1) it is difficult to automate;
- 2) it is very slow (sample drying often takes 48 hours);

3) continuous real time measurements are impossible. A newer method which overcomes these limitations is the neutron probe [2]. This instrument consists primarily of a fast neutron source and a detector/counter unit. When inserted in a bore hole, fast neutrons collide with hydrogen atoms in the soil, lose energy, and are scattered back to the detector as slow neutrons. The particle count per second as observed by the detector is thus proportional to the number of hydrogen atoms in the surrounding soil. Since there is more hydrogen associated with soil water than with clay, organic matter, or other soil particles, the slow neutron flux is also proportional to the amount of water in a bulk volume of soil. The relationship between particle count and %W is empirical and the operator must rely on calibration curves. Although less accurate than oven drying, the neutron probe is especially useful when repeated sampling must be performed in a limited area where numerous sampling holes are undesirable. One disadvantage of the neutron probe is that the sensing volume is essentially a sphere 18 inches across. This makes it difficult to resolve fine layers in the vertical

soil profile. Also, deployment of the probe is strictly regulated since radioactive materials are involved.

As an alternative to the neutron probe, the possibility of a microwave based measurement system arises. Considerable attention, both theoretical and experimental, has been given to electromagnetic sensing of soil properties. It is convenient to organize the existing literature into two major categories:

- 1) efforts to relate a soil's electromagnetic properties (e.g. complex permittivity) to its other physical properties (e.g. soil moisture);

- 2) efforts to find practical field techniques for measuring soil electromagnetic properties.

Literature Review of Electromagnetic Properties of Soil

The electromagnetic behavior of soil may be generally described in terms of complex permittivity and permeability tensors. In practice, however, permittivity and permeability are generally treated as scalars. This assumption implies that soil is essentially isotropic. Furthermore, soil permeability is usually assigned a free space value of $\mu_0 = 4\pi \times 10^{-7}$ H/m. This

approximation is reasonable provided the soil has a negligible iron content. Thus, most electromagnetic sensing of soil consists of measuring its complex permittivity $\hat{\epsilon}$.

The permittivity of soils is influenced by many factors such as soil type, moisture content, temperature, and density. Many authors have attempted to derive dielectric mixing formulas for predicting $\hat{\epsilon}_m$, the permittivity of a mixture, in terms of the permittivity of its constituents and the nature of their dispersion. Bruggeman [3] derived expressions which set upper and lower bounds on $\hat{\epsilon}_m$ for a porphyritic two phase mixture. His expressions were experimentally verified by Pierce [4]. De Loor [5] presents a mixing formula for solutions having ellipsoidal, spherical, or disc shaped inclusions. He also discusses various dielectric loss mechanisms and the frequencies at which they become important. Tinga and Voss [6] review a host of dielectric mixture formulas from the literature and then proceed to treat multiphase inclusions of confocal ellipsoids in a uniform host medium. These theoretical results are then used

to predict the complex permittivity of Douglas Fir versus moisture content.

One problem in applying dielectric mixing formulas is that the permittivities of some constituents are highly variable. Thus it is not clear which values to use in the mixing formulas. In particular, the variable properties of water are well known. Windle and Shaw [7] studied the dielectric properties of wool/water systems and found that the permittivity of localized or "bound" water was substantially less than that of bulk water. The so-called bound water consists of a thin layer of water molecules intimately bonded to polar sites in the host medium. This bonding reduces the rotational polarizability of the water molecules and thus lowers their relaxation frequency. This effect has also been reported by Schwan [8] on measurements of water/hemoglobin mixtures from .01 to 1 Ghz. De Loor [9] states that the relaxation frequency of bound water is typically between .3 and 1 Ghz, compared to about 22 Ghz for bulk water. The contribution of bound water to the dielectric properties of a mixture is significant at the lower microwave frequencies and can lead to

errors when applying dielectric mixture formulas. This fact was noted by Tinga and Voss [6].

It is not necessary to rely on dielectric mixture formulas to relate soil permittivity to moisture content. Instead one may choose to generate a data base from experimental measurements on actual soils and then use curve fitting techniques to establish empirical relations. This approach has been used successfully with the neutron probe [2]. Hoekstra and Delaney [10] measured the permittivity of several clay, silt and sandy soils as a function of moisture content and temperature at frequencies ranging from 0.1 Ghz to 26 Ghz. They found that soil type had a strong effect on dielectric properties below about 100 Mhz and that the relaxation frequency of bound soil water was lower than that of bulk water. At frequencies above 3 Ghz they observed that the relation between volumetric water content and complex permittivity is relatively independent of soil type. Unfortunately penetration of more than a few centimeters at these frequencies is difficult because of the high attenuation offered by moist soil.

Literature Review of Field Techniques for Measuring
Soil Permittivity

Most field measurement techniques may be classified as either intrusive or nonintrusive. Intrusive probes include buried transmission lines [11] and antennas [12]. Nonintrusive probes include various radar [13-16] and radiometry [17-20] techniques. The Journal of Microwave Power [21] contains a bibliography of 186 publications regarding nonintrusive microwave sensing. Lytle [22] gives an excellent survey of intrusive techniques.

It would be convenient to use nonintrusive (remote sensing) techniques for soil moisture profiling. Large surface areas could be conveniently scanned and no disturbance of the soil would be necessary. Unfortunately such methods appear incapable of sensing to root zone depth. To show this, consider the penetration of plane waves into a lossy medium. Assuming normal incidence and an $e^{j\omega t}$ time dependence, the magnitude of the electric field as a function of depth (z) can be written [23]

$$|E_x| = |E_0| e^{-\alpha z} \quad \text{volts/m} \quad (1-3)$$

where

$$E_0 = \text{magnitude of electric field at surface } (z=0)$$

$$\alpha = \text{Im}\{k\} = \text{Im}\{w \sqrt{\epsilon_0} \sqrt{\hat{\epsilon}_r}\} = \text{attenuation factor}$$

(nepers/m).

The attenuation in the lossy medium may be expressed in decibels as

$$A = 20 \log_{10} e^{-\alpha z} \text{ decibels.} \quad (1-4)$$

Typical $\hat{\epsilon}_r$ for a dry soil (5% $\text{gH}_2\text{O}/\text{cm}^3$) at 4 Ghz is $2.6 - j0.3$ [10]. This is equivalent to an attenuation of 68 dB/m, or a round trip attenuation of 136 dB/m. Even with efficient coupling of the plane wave into the soil, this excessive attenuation precludes remotely sensing moisture at significant depths.

This penetration problem is acknowledged in the literature. Ulaby and Batalivala [13] using frequencies between 2 Ghz and 8 Ghz in a radar sensing system obtained useful correlation against measured moisture profiles to a depth of only 15 cm. Deeper penetration may be achieved by using lower frequencies at the cost of reduced resolution and increased dependence on soil type [10].

Another difficulty encountered with surface based measurements is uniquely resolving the nonuniform vertical moisture profile of soils. This problem has been treated theoretically for the case of sensing natural microwave emissions (i.e. radiometry) by Stogryn [20].

If depth of penetration is a prime consideration, some form of intrusive probe is necessary. Many of the intrusive systems described by Lytle [22] involve two or more boreholes. These would be adequate for low frequency applications where attenuation between boreholes is small, but are not very useful at microwave frequencies.

Examples of single hole probes include buried transmission lines and monopole antennas. A transmission line system described by Chudobiak et al [24] employed time domain reflectometry (TDR) techniques. A balanced open wire transmission line was inserted vertically into the soil to a depth of several meters. A fast rise time pulse was then propagated down the line and all reflections were monitored at the surface. The amplitudes of the reflections are proportional to the change in permittivity between soil layers, and the

arrival times are proportional to the depth of the layers. To calibrate the system in terms of depth, marker reflections were established by designing step impedance changes into the transmission line at preset intervals. Unfortunately the lossiness of the soil (discussed above) causes reflections to die out as rapidly as for a nonintrusive probe. Scott and King [25] used a buried monopole to measure the complex permittivity of lake water at VHF frequencies. This procedure worked well for homogeneous media. However, resolution in a layered medium such as soil would be poor because the permittivity is averaged over the length of the monopole.

The Slotted Cylinder Geometry for Soil Moisture Sensing

As was pointed out in the literature review, effects of soil type on permittivity can be minimized by increasing the operating frequency of the sensor. However, increasing the operating frequency seriously restricts attainable depth of penetration due to propagation losses in the soil. One solution to these combined problems is to employ a slotted cylinder geometry of the form shown

in Figure 1-1. This probe consists of a conducting cylinder with an array of slot antennas (also called aperture radiators) situated along its axis. The array is installed intrusively so that each slot antenna senses a different soil depth. The antennas are individually fed from the surface by coaxial transmission lines running through the center of the cylinder. Measurements of slot antenna properties (e.g. VSWR and mutual coupling) provide a discrete (histogram) profile of the soil's electromagnetic characteristics along the length of the cylinder. Experimentally derived curves relating soil moisture and complex permittivity, such as those of Hoekstra and Delaney [10], could then be used to convert the permittivity profile to a moisture profile.

Root zone moisture measurement at arbitrarily high frequencies are possible with this geometry because the sensing antenna is located at the desired depth. In fact the total depth of the profile is governed by the length of the array, and the resolution of the profile is determined by the spacing and beam pattern of the

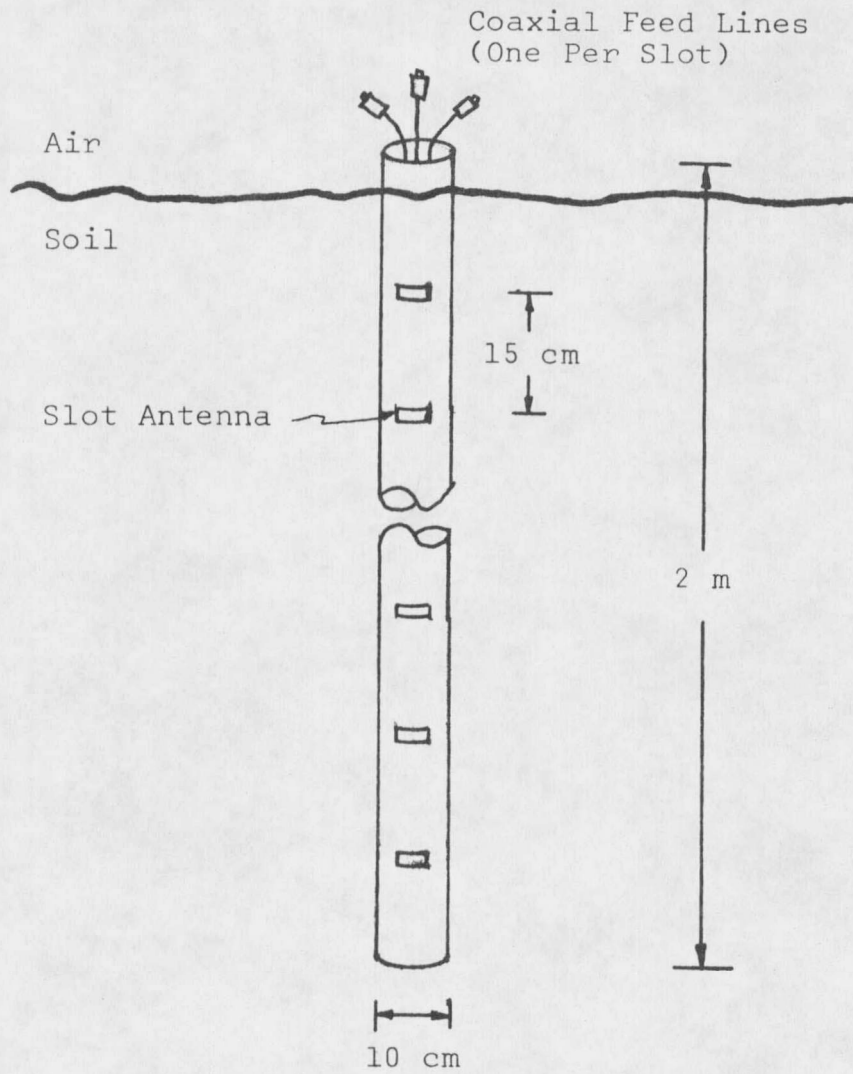


Figure 1-1. A Slotted Cylinder Array for Soil Moisture Sensing.

individual radiators. These two very important parameters are thus under designer control.

It should be pointed out that a circular cylinder geometry was chosen for the probe so that it could be easily installed in a borehole. There is nothing especially convenient about the electrical properties of a slotted circular cylinder. A square cylinder would suffice just as well. Aperture radiators were chosen for array elements so that the array could be flush with the cylinder's surface. The array elements are fed separately from the surface to maximize the possible array configurations. This allows the user to design redundancy into his measurement strategy since the array can be reconfigured at will from the surface.

In effect the slotted cylinder array comprises a multipoint microwave network which can be characterized by a scattering matrix. As will be shown in Chapter 2, it is possible to relate the scattering matrix of the array elements to the soil's effective complex permittivity. These parameter calculations prove costly for a slotted cylinder geometry but it is shown that less costly slotted plane results may often be

substituted with little error. Also since accurate phase measurements are difficult to obtain in the field and require relatively expensive instrumentation, the usefulness of s-parameter magnitudes only is discussed. Chapter 3 presents comparisons between theory and experiment for both slotted cylinder and slotted plane arrays. Chapter 4 summarizes the potential usefulness of the slotted cylinder as a moisture probe.

CHAPTER 2
ANALYSIS OF THE SLOTTED CYLINDER AND
SLOTTED GROUND PLANE ARRAYS

Introduction

The primary purpose of this chapter is to develop theoretical expressions for the self admittance, mutual admittance, and scattering parameters of a slotted cylinder array in terms of the complex permittivity of the medium in which it is immersed. These expressions are useful for two reasons:

1) if the expected variation of permittivity with soil moisture is known, then the sensitivity of the array to small changes in moisture may be predicted;

2) unknown soil permittivity may be measured by solving the inverse expression, i.e. a value of permittivity is selected which fits the measured admittance data over a range of frequencies.

The admittance versus permittivity expressions are also presented for a slotted ground plane geometry because they are considerably easier to evaluate and, under certain conditions, may be substituted for the slotted cylinder equations with negligible error. A

numerical comparison between the self admittance and mutual coupling of a slotted plane and a 6 inch (15.2 cm) diameter slotted cylinder is included.

Previous Work

The analysis of a slotted cylinder array radiating into a lossy environment has been treated previously [26-32]. Much of this work concerned the study of spacecraft aperture antennas subjected to thin lossy plasmas during reentry.

Silver and Saunders [30] gave a general method of analysis for the fields due to a single slot with an arbitrary two dimensional aperture distribution. Then expressions for the far field of a uniformly excited circumferential slot and an infinite axial slot were derived. Knop [29] presents integral expressions for the admittance of a finite axial slot on a dielectric clad cylinder. He assumes a TE_{10} aperture source distribution. Golden and Stewart [27,28] compute the self and mutual admittance of TE_{10} excited axial slots radiating into lossy plasmas. Tetenbaum [32] compares the properties of slotted cylinders with those of slotted ground planes.

Assumptions

The theoretical analysis of the slotted cylinder array as a moisture probe is based on the following major assumptions:

- 1) soil can be represented for moisture sensing purposes as a homogeneous isotropic linear medium;
- 2) the slot antennas are excited by a dominant mode TE_{10} aperture distribution.

The basis for the first assumption is that soil should appear locally homogeneous to the side-looking slot antennas because of the shallow outward penetration of the microwaves (i.e. small skin depth) and the natural horizontal layering of the soil. Also the soil should be isotropic if ferrous compounds are not abundant and should behave linearly for small field intensities. A TE_{10} aperture distribution is assumed because it is the dominant mode of the waveguide feed which excites the aperture. Although some distortion of this distribution is likely, Compton [33] has shown that a first order error in assumed aperture distribution results in only a second order error in the calculated slot admittance.

External Self Admittance of Rectangular Slots on an
Infinitely Long Conducting Cylinder

The steps required to formulate the external self admittance of a slot antenna may be summarized as follows:

- 1) determine the Fourier expansion for the tangential components of electric field over the cylinder's surface;
- 2) invoke Poynting's theorem and Parseval's theorem to calculate the complex power flow through the aperture;
- 3) express the self admittance as the ratio of the complex power leaving the aperture to the square of the effective aperture voltage.

Fourier Expansions for the External Fields of the
Slotted Cylinder

Consider the single slot geometry shown in Figure 2-1. Let the tangential components of electric field at the cylinder's surface ($r = r_0$) be

$$\vec{E}_\phi = E_\phi(\phi, z, r_0) \vec{a}_\phi$$

and

$$\vec{E}_z = E_z(\phi, z, r_0) \vec{a}_z$$

