The effect on final achievement in solid geometry of an introductory unit based upon developing visualization and understanding through the use of models
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A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF EDUCATION
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Abstract:
The belief that early concrete experiences in the construction of three-dimensional models in the study of solid geometry would result in better understanding and improved final achievement led to this investigation. An examination of the literature concerning the teaching of solid geometry revealed a need for improved techniques of teaching to help overcome the difficulties of spatial visualization and understanding often encountered by pupils studying solid geometry. No research had been done to determine the value of the concentrated use of models at the beginning of the course; however, the use of models, which was widely recommended, was in accord with generally accepted theories of learning reported in the literature.

A 10-day introductory unit on model construction was designed and an investigation conducted in which eight teachers and 219 pupils cooperated. In teaching their classes, the teachers of the experimental groups used the introductory unit on the construction of the five regular solids and the five stellated solids based upon the regular solids. The teachers of the control groups did not use introductory exercises in model construction. Pupils of the experimental and control groups were tested at the beginning of the course to measure initial space perception abilities and at the conclusion of the course to measure final achievement in solid geometry. It was found that there was no statistically significant difference in final achievement between pupils who studied solid geometry in the traditional manner and pupils who completed an introductory unit consisting of the drawing of one-piece patterns, the cutting, and the assembling of tagboard models of the five regular solids plus the stellated polyhedra based upon these five solids prior to proceeding with the conventional study of the course.

The following conclusions concerning the teaching of solid geometry were made on the basis of the review of literature and the findings of this study: (1) there is a need for improved teaching techniques in the field of solid geometry; (2) learning theorists and educators are generally agreed that the use of models in the study of solid geometry will improve learning in solid geometry; (3) the age of the high school pupil and the mathematics courses taken prior to enrollment in high school solid geometry do not affect final achievement in solid geometry; (4) there is little or no correlation between space perception abilities and final achievement in solid geometry; (5) a reduction in time spent in formal study of solid geometry does not affect final achievement in solid geometry; (6) the belief that final achievement in solid geometry, as measured by conventional tests, is improved by the use of models has no statistical justification; and (7) the use of models in developing spatial concepts does not affect achievement on topics involving these spatial concepts.

THE EFFECT ON FINAL ACHIEVEMENT IN SOLID GEOMETRY OF AN INTRODUCTORY UNIT BASED UPON DEVELOPING VISUALIZATION AND UNDERSTANDING THROUGH THE USE OF MODELS

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J.O.P.

TABLE OF CONTENTS

Chapter | Page

LIST OF TABLES

ABSTRACT

The belief that early concrete experiences in the construction of three-dimensional models in the study of solid geometry would result in better understanding and improved final achievement led to this investigation. An examination of the literature concerning the teaching of solid geometry revealed a need for improved techniques of teaching to help overcome the difficulties of spatial visualization and understanding often encountered by pupils studying solid geometry. No research had been done to determine the value of the concentrated use of models at the beginning of the course; however, the use of models, which was widely recommended, was in accord with generally accepted theories of learning reported in the literature.

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The following conclusions concerning the teaching of solid geometry were made on the basis of the review of literature and the findings of this study: (1) there is a need for improved teaching techniques in the field of solid geometry; (2) learning theorists and educators are generally agreed that the use of models in the study of solid geometry will improve learning in solid geometry; (3) the age of the high school pupil and the mathematics courses taken prior to enrollment in high school solid geometry do not affect final achievement in solid geometry; (4) there is little or no correlation between space perception abilities and final achievement in solid geometry; (5) a reduction in time spent in formal study of solid geometry does not affect final achievement in solid geometry; (6) the belief that final achievement in solid geometry, as measured by conventional tests, is improved by the use of models has no statistical justification; and (7) the use of models in developing spatial concepts does not affect achievement on topics involving these spatial concepts.

CHAPTER I

INTRODUCTION

The fact that we live in a multidimensional world has become much more evident in recent years. Problems of national defense are becoming increasingly concerned with the multidimensional aspects of the oceans, the atmosphere, the globe, and even of outer space. The time is rapidly approaching when a person will have to be able to visualize happenings in many dimensions if he is to understand and to appreciate the events of the day.

The concepts of space which are studied in solid geometry also have important applications in many other fields necessary to the maintenance of civilization. The successful carpenter must be able to apply many of the principles normally studied in solid geometry. This is also true of the plumber, the electrician, the mechanic, the machinist, and the electronic technician. Such diverse professions as architecture, engineering, geology, oceanography, metallurgy, dentistry, surgery, and art demand the use of the three-dimensional thinking and visualization which is the concern of solid geometry. A serious study of the life processes requires spatial perception ability. Many recent advances in both inorganic and organic chemistry have resulted from the direct application of the concepts of space studied in solid geometry.

The objectives of the teaching of solid geometry are certainly in keeping with the demands of the vocations which involve spatial concepts. These objectives, which have been listed by a number of different writers,[1] include the extension of ability in postulational thinking, improved skill in mensuration of surfaces and solids in space, the development of skill in applying three-dimensional geometric principles to practical problems, and the development of the pupil's ability in spatial perception and visualization.

The development of the pupil's proficiency in spatial perception and visualization has for many years been considered one of the problems which confronts the teacher of solid geometry.[2] Because many pupils have difficulty in working with the complex geometric figures, there is a need for improved techniques in the teaching of three-dimensional concepts.

The use of models acquaints the pupil with the concrete forms of solid figures and enables him to more readily understand the abstractions of solid geometry. The construction of models provides an even greater opportunity for understanding the nature of solid figures. In his experiences in teaching solid geometry, the writer found that the construction of models at the beginning of the course appeared to improve the pupil's ability in spatial perception. For example, one of the first abstractions presented in solid geometry states that if two planes intersect, the intersection is a straight line. Somewhat later in the course,

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[1]Butler, C. H., and Wren, F. L., The Teaching of Secondary Mathematics, pp. 399-411; Morgan, F. M., and Breckenridge, W. E., Solid Geometry, p. 1; Reeve, W. D., editor, The Place of Mathematics in Secondary Education, Fifteenth Yearbook of The National Council of Teachers of Mathematics, pp. 115-116; Shute, W. G.; Shirk, W. W.; and Porter, G. F., Solid Geometry, pp. 1-2; Strader, W. W., and Rhoads, L. D., Solid Geometry, p. 1.

[2]Hartley, M. C., "Models in Solid Geometry," The Mathematics Teacher 35:5, January, 1942.

the pupils encounter the fact that three planes meet in a point. The visualization of these simple concepts often causes difficulty for beginning pupils of solid geometry. If the pupils have made models of the geometric solids, these propositions often become clear and meaningful to them. In fact, the propositions sometimes become so clear and obvious that some of the pupils feel they do not even require stating.

It was felt that providing the pupil with concrete experiences in spatial visualization prior to the consideration of the abstractions found in the traditional solid geometry course aided the development of the pupil's ability in spatial perception and served to raise his level of achievement in the course. This feeling that final achievement in solid geometry was improved by early concrete experiences in the construction of three-dimensional models led to this investigation.

Statement of the Problem

It was the problem of this study to determine the effect on final achievement in solid geometry of an introductory unit based upon developing visualization and understanding through the construction of tagboard models of certain geometric solids prior to the study of the usual topics of the course.

The hypothesis (H_1) to be tested was that pupils who completed an introductory unit consisting of the drawing of one-piece patterns, the cutting, and the assembling of tagboard models of the five regular solids plus the stellated polyhedra based upon these five solids prior to proceeding with the conventional study of solid geometry would show a significant improvement in final achievement over pupils who studied solid geometry in the traditional manner.

Procedures

To determine the effect on final achievement in solid geometry of an introductory unit based upon developing visualization and understanding through the use of models, the following procedures were used.

An examination of the literature relating to the problem of the study was made to determine (1) the status of the teaching of solid geometry, (2) the difficulties encountered in the study of solid geometry, (3) the place of models in the teaching of solid geometry, and (4) what objective data had been collected relative to the problem of the study.

An introductory unit on the construction of models of geometric solids was devised. The writer used two of his classes in solid geometry as pilot groups to determine the time required for the unit and pupil reaction to the unit.

In order to obtain data for testing the hypothesis, it was necessary to enlist the aid of teachers of semester courses in high school solid geometry who were then assigned to experimental and control groups. A questionnaire was prepared which was used in obtaining information from the cooperating teachers concerning such things as the course length, duration of the class periods, prerequisites for solid geometry, and grade level of pupils normally enrolled in solid geometry.

The teachers of the control groups taught their classes in their usual manner while those of the experimental groups taught in a specified manner. This specified manner consisted of having each pupil construct tagboard models of the five regular solids and the five stellated polyhedra based upon the five regular solids. Following this 10-day introductory unit in construction, which occupied approximately 12 per cent of the semester course, the textbooks were issued to the pupils and the remainder of the course proceeded as usual.

In order to provide additional information to be used in equating the pupils of the experimental and control groups, a test designed to measure ability in spatial perception was selected. The test selected for this purpose was the "Differential Aptitude Space Relations Test, Form A."[3] This test was administered to the pupils in the experimental and control groups on the first day of class.

The "Cooperative Solid Geometry Test, Form P,"[4] was selected for use in measuring the final achievement in solid geometry of the experimental and the control group pupils. This test was administered to the pupils of the experimental and control groups on the last day of the semester course. The results were then examined statistically to determine the validity of the hypothesis.

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[3]Bennett, G. K.; Seashore, H. G.; and Wesman, A. G., Differential Aptitude Tests: Space Relations: Form A, The Psychological Corporation, New York, 1947, 11 pp. See Appendix A.

[4]Lundholm, H. T.; Long, J. A.; and Siceloff, L. P., Cooperative Solid Geometry Test: Form P, Cooperative Test Service, Educational Testing Service, Princeton, N.J., 1938, 4 pp. See Appendix B.

Limitations

Certain limitations which were placed upon the study are given in the following paragraphs:

The experimental portion of the study was limited to a period of 10 days at the beginning of the semester course in solid geometry because it was felt that this was sufficient time to permit the pupils to complete the construction of the models and for the additional reason that any longer period of time would probably have discouraged teachers from participating in the study. The 10 days of class time devoted to this study, on the part of the experimental groups, represented about 12 per cent of the entire semester course.

The level of significance for the statistical tests was set at .05. Since the hypothesis (H_1) stated the direction of the predicted difference, the region of rejection was one-tailed.

In order that the statistical tests have the desired power, the sample size was set at about 200 pupils. A portion of the approximately 200 pupils were assigned to experimental groups and the remaining portion to control groups. The random assignment of the cooperating teachers to the experimental or control groups determined in which group a pupil was placed.

To obtain the assistance of competent and experienced teachers in conducting the investigation, volunteers were secured among the teachers attending a summer National Science Foundation Mathematics Institute at Montana State College. The aid of additional mathematics teachers was secured by contacting them individually. These teachers, who had not attended the National Science Foundations Mathematics Institute at Montana State College, were likewise experienced and competent.

Because of the necessity of conserving the time of the participating teachers and pupils, the measurement of space perception abilities demonstrated by the pupils at the beginning of the experiment was confined to one test and the measurement of final achievement was confined to one final test.

The feeling that providing the pupil with early concrete experiences in the construction of three-dimensional models was a technique of teaching solid geometry which would improve final achievement in the course led to this investigation. To determine the effect on final achievement in solid geometry of an introductory unit based upon developing visualization and understanding through the use of models, an experiment was conducted in which the final achievement of solid geometry pupils who used the specified introductory unit on model construction was compared with the final achievement of solid geometry pupils who did not use introductory exercises in model construction. A review of the literature relative to the problem of the study is reported next.

CHAPTER II

REVIEW OF LITERATURE

The purpose of the review of literature was to provide a background of information concerning the history and present status of the teaching of solid geometry as related to the effect of the use of models on final achievement and to present a brief summary of previous investigations of this and closely related problems. Four phases pertinent to the problem of this study were considered in the review: (1) the status of the teaching of solid geometry, (2) the reported difficulties encountered in the study of solid geometry, (3) the place of models in the teaching of solid geometry, and (4) the studies related to space perception and the use of models in the teaching of solid geometry. These four phases are presented in the following sections.

Status of the Teaching of Solid Geometry

A search of the literature relating to the status of the teaching of solid geometry was made to determine if there was a need for improved teaching techniques such as the introductory unit on model construction used in this study. The investigation of the literature relating to the status of the teaching of solid geometry disclosed an apparent dissatisfaction in that area. There was a wealth of evidence in the literature that this dissatisfaction extended back to before the turn of the present century. In February, 1932, The National Council of Teachers of Mathematics appointed a Committee on Geometry which was directed to study the entire question of geometry in the schools.[1] The committee which was established proceeded with a study of the periodicals, reports, books, and textbooks concerning the teaching of geometry which had been published since about 1900. This survey included foreign as well as domestic publications. That there had been an apparent lack of satisfaction with the teaching of geometry for the previous 50 or more years and that although many attempts had been made to correct the situation, no solution had yet been reached, was pointed out by the committee in its statement:

> One receives the impression that the teaching of geometry for the last fifty years or more has not been entirely satisfactory, that numerous efforts have constantly been made to improve the situation, and that often these methods are really not new but have been tried in one form or another before.[2]

The fact that dissatisfaction with the teaching of solid geometry continued over subsequent years was indicated by Hartley in 1945. In commenting on the lack of advances in teaching techniques, courses of study, and textbooks in solid geometry as compared to plane geometry, he stated that ". . . solid geometry as now taught in our secondary schools has been called the most uninteresting subject of the curriculum. . . ."[3]

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[1] Beatley, Ralph, "Preliminary Report of the Committee on Geometry," The Mathematics Teacher 25:427, November, 1932.

[2] Beatley, "Second Report of the Committee on Geometry," The Mathematics Teacher 26:369, October, 1933.

[3] Hartley, M. C., "A Laboratory Approach to Solid Geometry," in Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of the National Council of Teachers of Mathematics, p. 42.

The present controversy among both mathematicians and teachers of mathematics concerning whether or not to retain solid geometry as a distinct course in the curriculum certainly indicates that a basic dissatisfaction with the teaching of solid geometry continues to exist. In the spring of 1960, the writer participated in a regional conference which was sponsored by the National Association of State Directors of Teacher Education and Certification in cooperation with the American Association for the Advancement of Science. This conference was concerned, in part, with the training of teachers of high school mathematics. During the portion of the discussion pertaining to geometry, a number of the participants showed a concern over the present status of the teaching of solid geometry. They indicated that there appeared to be a continuing general dissatisfaction with the course as presently conceived, not only from the standpoint of content, but, more pointedly, from the standpoint of the ineffectiveness of the teaching of the topics normally contained in the course.

A study of the literature relative to the problem of this study revealed that for the past 80 years or more there has been dissatisfaction with the teaching of solid geometry. The 1933 report by the National Council of Teachers of Mathematics Committee on Geometry indicated that for at least 50 years prior to 1933 the teaching of solid geometry had not been satisfactory. A continuing dissatisfaction was reported in 1945 by Hartley[4] and in 1960 at a meeting of educators who

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[4] Ibid., p. 42.

also pointed out a need for improving the effectiveness of the teaching and overcoming the difficulties encountered by pupils when studying solid geometry. A search of the literature was made to find out what difficulties were encountered by pupils in the study of solid geometry.

Reported Difficulties Encountered in the Study of Solid Geometry

The difficulties encountered in the study of solid geometry which were reported in the literature were reviewed to determine which difficulties could be overcome by the use of the introductory unit on model construction employed in this study. The areas in which pupils experience their greatest difficulties in the study of geometry have been reported in a study conducted by Smith.[5] The three areas of difficulty which he listed were: unfamiliarity with geometric figures, lack of understanding of the "if-then" relationship, and lack of understanding of the meaning of proof. The unfamiliarity with geometric figures which pupils encounter in the study of solid geometry was found to be complicated by difficulties of spatial visualization and understanding. These difficulties of spatial visualization and understanding were closely related to this study since the introductory unit used in this study was based upon the visualization and understanding of geometric figures through the use of models.

In the literature reviewed, there was a general consensus by

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[5]Smith, R. R., "Three Major Difficulties in the Learning of Demonstrative Geometry," The Mathematics Teacher 33:100, March, 1940.

several writers[6] that pupils of solid geometry often experience considerable difficulty in visualizing three-dimensional relationships. Wiseman[7] considered the analysis of complex figures to be one of the major difficulties experienced by high school geometry pupils. In speaking of the three-dimensional visualization of the points, lines, and planes represented in two-dimensional drawings, Breslich[8] also recognized the difficulty of spatial visualization as a major difficulty in the study of solid geometry. That the average pupil encounters difficulty in working with three-dimensional figures was pointed out by Bernardo when he wrote, "It has been my experience, as it has been undoubtedly that of many who teach solid geometry, to find that the three dimensional concepts are not easily conceived by the average student."[9]

An examination of the literature indicated that although other areas of difficulty in the study of solid geometry existed, the areas

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[6]Bernardo, J. V., "A Helpful Technique in Teaching Solid Geometry," The Mathematics Teacher 33:39, January, 1940; Brown, E. I., "A Different Approach to Solid Geometry," The Mathematics Teacher 29:145, March, 1936; Drake, Richard, and Johnson, Donovan, "Vitalizing Geometry with Visual Aids," The Mathematics Teacher 33:56-57, February, 1940; Hartley, "Models in Solid Geometry," The Mathematics Teacher 35:5, January, 1942; Morris, D. L., "Inverted Geometry," The Mathematics Teacher 31:79, February, 1938; Nicholson, G. H., "The Teaching of Solid Geometry at the University of Vermont," The Mathematics Teacher 30:326, November, 1937; Wiseman, J. D., Jr., "Complex Figures in Geometry," The Mathematics Teacher 52:91, February, 1959.

[7]Wiseman, op. cit., p. 91.

[8]Breslich, E. R., "Visual Aids in Teaching," in Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of The National Council of Teachers of Mathematics, p. 38.

[9]Bernardo, op. cit., p. 39.

which caused major difficulties concerned the pupil's unfamiliarity with complex geometric figures and the visualization of three-dimensional objects and relationships. Models were suggested as teaching devices for overcoming the difficulties in spatial visualization which the pupil encounters when studying solid geometry.

Place of Models in the Teaching of Solid Geometry

Since the introductory unit used in this study involved the construction of models, a search of the literature concerning the place of models in the teaching of solid geometry was made. The search revealed two phases related to this study: (1) the practice concerning the use of models and (2) the psychology of learning as related to the use of models.

Practice concerning the use of models. The practice of using models of geometric solids and of various theorems to assist in developing the concept of space was widely recommended throughout the literature reviewed. The literature was replete with statements by many writers[10]

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[10] Ibid., p. 39; Brown, op. cit., p. 145; Butler, C. H., and Wren, F. L., The Teaching of Secondary Mathematics, pp. 116-117; Drake and Johnson, op. cit., pp. 56-57; Hartley, "A Laboratory Approach to Solid Geometry," in Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of The National Council of Teachers of Mathematics, pp. 42, 49; Hartley, "Models in Solid Geometry," The Mathematics Teacher 35: 5, January, 1942; Morris, op. cit., p. 79; Nicholson, op. cit., p. 326; Reeve, W. D., editor, The Place of Mathematics in Secondary Education, Fifteenth Yearbook of The National Council of Teachers of Mathematics, pp. 96-97; The Mathematical Association of America, Inc., National Committee on Mathematical Requirements, The Reorganization of Mathematics in Secondary Education, Part I, pp. 9, 52; Simons, L. G., "Historical Material on Models and Other Teaching Aids in Mathematics," in Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of The National Council of Teachers of Mathematics, pp. 253-265; Wiseman, op. cit., p. 91.

regarding the value of multi-sensory aids and pointing out the interrelationship of spatial perception and solid geometry. Hartley pointed out the value of using models and indicated the desirability of their more extensive use in the teaching of solid geometry when he wrote:

> Models have been employed for many, many years, but they have not been used extensively in solid geometry because their value in the development of the concept of space has been woefully underestimated. In the early stages of a course in solid geometry, models are essential.[11]

That the importance of sensory learning as applied to mathematics was a generally accepted fact was indicated in the following statement by Syer: "It hardly seems necessary to justify the importance of sensory learning--it is an essential part of all learning. . . ."[12] The writers of the literature reveiwed were agreed that solid geometry requires multi-sensory aids in one form or another to provide a foundation of sensory learning experiences which can help to improve the pupil's ability in spatial perception and understanding. Drake and Johnson agreed with the other writers in the field that much more should be done to develop visual aids and to improve the techniques of their use when they stated:

> Geometry more than any other subject is dependent upon visual aids. Every theorem and every exercise requires a diagram in a book, on the blackboard or in one's mind. Yet probably in no

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[11] Hartley, "A Laboratory Approach to Solid Geometry," in Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook of The National Council of Teachers of Mathematics, p. 49.

[12] Syer, H. W., "Sensory Learning Applied to Mathematics," in The Learning of Mathematics: Its Theory and Practice, Twenty-First Yearbook of The National Council of Teachers of Mathematics, p. 99.

> subject have teachers shown less progress in developing visual aids and improved visual techniques. . . . much more visual material is needed to bring geometry closer to everyday things.[13]

The practice of using models and other multi-sensory aids in the teaching of solid geometry was widely recommended in the literature which was reviewed. The second phase of the literature concerning the place of models in the teaching of solid geometry involved the psychology of learning as related to the use of models.

Psychology of learning as related to the use of models. The use of models and other sensory aids to learning has a very secure foundation in the psychology of learning.[14] A principle of learning which was quite generally accepted by the writers of the literature which was reviewed was that learning proceeds most effectively from the concrete to the abstract, from the particular to the general, and from the known to the unknown.[15] Most pupils need experiences in working with simple, concrete examples of an idea before they can understand a more complex generalization.[16] The use of multi-sensory aids in concept formation provides for

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[13]Drake and Johnson, op. cit., p. 56.

[14]Van Engen, Henry, "The Formation of Concepts," in The Learning of Mathematics: Its Theory and Practice, Twenty-First Yearbook of The National Council of Teachers of Mathematics, pp. 86-87.

[15]Christian, R. R., "A Role for High School Geometry," The Mathematics Teacher 53:433, October, 1960; Manheimer, Wallace, "Some Heretical Thoughts from an Orthodox Teacher," The Mathematics Teacher 53:23-24, January, 1960.

[16]Christian, op. cit., p. 433; Peak, Philip, "What Contributions to Mathematics Instruction Can We Expect in the Last One Half of the Twentieth Century?" School Science and Mathematics 51:175, March, 1951.

these concrete experiences which are necessary before pupils can be expected to abstract and generalize.

In his book, Theories of Learning, Hilgard[17] reviewed 10 theories of learning and developed 14 points of general agreement among psychologists writing in the field of learning theory. Three of these points of agreement have a particular bearing upon the use of models in the teaching of solid geometry: (1) that learning which is under the control of reward is generally to be more desired than learning which is under the control of punishment--that success as a motivator of learning is preferable to failure as a motivator; (2) that active participation on the part of the learner is to be preferred to passive reception; and (3) that if the learner discovers relationships for himself and if, during the period of learning, he has experience in applying the principles to a variety of situations, there will be improved transfer to new tasks.

The first of the three points of agreement named in the preceding paragraph, which recognized the desirability of success as a motivator, is related to the use of models in the teaching of solid geometry. Both Hartung[18] and Lankford[19] stated that the use of multi-sensory aids improves the motivation for the learning of mathematics because it helps

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[17] Hilgard, E. R., Theories of Learning, pp. 485-487.

[18] Hartung, M. L., "Motivation for Education in Mathematics," in The Learning of Mathematics: Its Theory and Practice, Twenty-First Yearbook of The National Council of Teachers of Mathematics, pp. 54-57.

[19] Lankford, F. G., Jr., "Implications of the Psychology of Learning for the Teaching of Mathematics," in The Growth of Mathematical Ideas: Grades K-12, Twenty-Fourth Yearbook of The National Council of Teachers of Mathematics, p. 416.

the pupil to gain insights and understandings which result in successful learning. An introductory unit in the construction of models, such as that proposed in this study, would provide opportunities for learning motivated by success. The pupil should be able to have more initial success in making models than in grasping an abstract mathematical idea. Also, providing the pupil with concrete experiences in the construction of models should furnish more opportunity for success in the abstractions which will be encountered later than could be expected if the pupil had not had these experiences with concrete objects.

Learning in which the pupil is an active participant, which was considered desirable according the the second point of agreement among psychologists, as listed on page 16, could be provided for through the use of pupil constructed models since active participation on the part of the pupil is more readily assured in making models than in listening to a lecture. If, in addition to a verbal description of an object, a pupil is permitted to see, touch, and manipulate the object, a greater understanding will result than if only the verbal description is provided.

Lankford[20] stated that visual aids in the teaching of mathematics provide the pupil with assistance in gaining insights, discovering generalizations, or formulating hypotheses. The value of experiences which permit the pupil to discover relationships for himself, which was given on page 16 as the third point of agreement having a bearing on the use of

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[20] Ibid., p. 416.

models in the teaching of solid geometry, has been recognized by several authorities.[21] The self-discovery of principles stimulates interest, memory, and the will to investigate further. The construction and study of models of the geometric solids by the pupils, as proposed in this experiment, certainly would provide an opportunity for the discovery of relationships and for the application of the principles to a variety of situations.

A study of the literature concerning the teaching of solid geometry disclosed that the solution to the problem of improving the pupil's ability in spatial perception, which was most commonly given, involved the use of multi-sensory aids. These recommended aids most frequently took the form of models of geometric figures and models of various theorems. The use of this type of multi-sensory aid was in accord with the generally accepted theories of learning which were reported in the literature that was reviewed. A search was made to find reports of previous investigations concerning the use of models in the teaching of solid geometry.

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[21] Berger, E. J., "Principles Guiding the Use of Teacher- and Pupil-Made Learning Aids," in Emerging Practices in Mathematics Education, Twenty-Second Yearbook of The National Council of Teachers of Mathematics, p. 169; Brune, I. H., "Language in Mathematics," in The Learning of Mathematics: Its Theory and Practice, Twenty-First Yearbook of The National Council of Teachers of Mathematics, p. 163; Henderson, K. B., and Pingry, R. E., "Problem-Solving in Mathematics," in The Learning of Mathematics: Its Theory and Practice, Twenty-First Yearbook of The National Council of Teachers of Mathematics, p. 260; Hildebrandt, E. H. C., "Mathematical Modes of Thought," in The Growth of Mathematical Ideas: Grades K-12, Twenty-Fourth Yearbook of The National Council of Teachers of Mathematics, p. 371; Lankford, op. cit., pp. 405, 416, 429.

Studies Related to Space Perception and the Use of Models in the Teaching of Solid Geometry

An examination of the literature related to the problem of this study was made to determine what research had been done concerning space perception and the use of models in the teaching of solid geometry. Four recent studies in the field were located. Although teachers of solid geometry who use models to develop space perception abilities contend that learning is improved, none of these four related studies furnished any objective data to substantiate the belief that learning is improved by the use of models. One recent study by Johnson[22] reported some data concerning such claims. It dealt with the effectiveness of the use of certain audio-visual aids on the achievement of pupils in geometry and presented objective data concerning the effectiveness of the use of models on final achievement in solid geometry.

The three other studies reviewed took a different approach to the relationship between solid geometry and spatial visualization. All three studies were concerned with the effect of the study of solid geometry on space perception abilities. Ranucci[23] investigated the effect of the study of solid geometry on space perception abilities while Brown[24]

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[22] Johnson, D. A., An Experimental Study of the Relative Effectiveness of Certain Visual Aids in Teaching Geometry, Doctoral dissertation, 389 pp.

[23] Ranucci, R. R., Effect of the Study of Solid Geometry on Certain Aspects of Space Perception Abilities, Doctoral dissertation, 75 pp.

[24] Brown, F. R., The Effect of an Experimental Course in Geometry on Ability to Visualize in Three Dimensions, Doctoral dissertation, 82 pp.

reported on experimental courses in plane and solid geometry and their effect on three-dimensional visualization. Cohen[25] evaluated a technique to improve space perception abilities through the construction of models by pupils in solid geometry. It was felt that although these three studies by Ranucci, Brown, and Cohen were not directly related to the problem of this study, they did involve closely related problems and provided additional information concerning the possible interrelationship of spatial visualization and solid geometry. For this reason, they were included in this review.

Effect of the use of models on final achievement in solid geometry. One study which presented objective data concerning the effectiveness of the use of models on final achievement in solid geometry was reported by Johnson.[26] The first portion of the report by Johnson was concerned with experiments in plane geometry and is reported here for information only. The plane geometry experiments were conducted in 27 plane geometry classes in 12 Minnesota high schools. Experiments using experimental and control classes from the same high school were performed to compare the effectiveness of the use of films and filmstrips on the achievement of the pupils. Some experimental classes used only filmstrips, some only sound motion pictures, and some used both films and filmstrips. The data which were obtained supported the acceptance of the hypothesis that there

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[25] Cohen, Louis, An Evaluation of a Technique to Improve Space Perception Abilities Through the Construction of Models by Students in a Course in Solid Geometry, Doctoral dissertation, 72 pp.

[26] Johnson, op. cit., 389 pp.

was no significant difference in informational learning between the experimental groups which used either films or filmstrips, or both, and the control groups which did not use the aids.

The second portion of Johnson's report dealt with an experiment to determine the effectiveness of the use of models on final achievement in solid geometry. The experiment was conducted in a Minneapolis high school using two solid geometry classes taught by the same instructor. The 34 pupils of one class, which was designated as the experimental group, had their instruction supplemented by the use of the "Multi-Model Geometric Construction Set." This was a device which could be manipulated by the pupil to construct models of almost all the theorems, postulates, or exercises of a typical solid geometry textbook. The 35 pupils of the other class, the control group, did not use the construction set to build models. Both groups had the same time devoted to instruction, the same assignments, and the same instruction except for the use of the construction set. The study covered the entire semester course of solid geometry. Achievement was measured by the "Survey of Object Visualization Test" and the "Cooperative Solid Geometry Test." The conclusion reached was that there was no significant difference in achievement between pupils who used a construction set to build models of theorems, postulates, and exercises in solid geometry and pupils who did not use a construction set in their study of solid geometry.

Effect of the study of solid geometry on space perception abilities. Three experiments were found which dealt with the effect of the study of solid geometry on space perception abilities. The problem of the study by Ranucci[27] was to determine if the study of solid geometry affects space perception abilities. Pupils in 11 New Jersey high schools were used in an experiment which extended over a period of five months. There were 225 pupils in the experimental group and 90 in the control group. The experimental group consisted of pupils studying solid geometry; the control group consisted of pupils who were not studying solid geometry. The tests used to measure the space perception abilities were "Thurstone's Lozenge Test A," "The Revised Minnesota Paper Form Board Test, Form MA," the "Army General Classification Test, Form AH" (Block Counting Section only), and the "Space Relations Test, Differential Aptitude Battery." Because the two groups were found to be statistically different initially, Ranucci used matched groups. He matched on the basis of school attended, mathematical background, sex, I.Q., and whether or not the pupil had taken mechanical drawing. The statistical tests showed that there was no significant difference in space perception abilities between those who studied solid geometry and those who did not.

The second study which dealt with the effect of the study of solid geometry upon ability to visualize in three dimensions was reported by Brown.[28] This study, which involved a total of 281 pupils in three Illinois schools, was concerned with three hypotheses. One was that there was no significant difference in ability in spatial visualization between pupils who completed a two-year sequence composed of plane

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[27]Ranucci, op. cit., 75 pp.

[28]Brown, op. cit., 82 pp.

geometry, algebra, and solid geometry and those who completed a one-year course in plane geometry fused with solid geometry. The second hypothesis involved a comparison between the spatial perception abilities of those who had completed a one-year conventional course in plane geometry and those who had completed a one-year fused plane-solid geometry course. The third hypothesis dealt with the relative achievements in the subject matter commonly taught in plane geometry between those who completed the one-year plane geometry course and those who completed the one-year fused course. The instruments used in the evaluation were the "Differential Aptitude Space Relations Test" and the "Cooperative Plane Geometry Test."

The conclusions arrived at by Brown were that the pupils in both groups learned the subject matter commonly taught in plane geometry; that there was no significant difference in spatial visualization between those who studied a one-year course in plane geometry and those who studied a one-year fused course of plane and solid geometry; and that those taking the conventional two-year sequence composed of plane geometry, algebra, and solid geometry developed greater ability in spatial visualization than those taking the one-year fused course. Brown pointed out that the difference in ability in spatial perception could very well have been due to the natural maturing of the pupils in the two-year course over those aging only one year and, therefore, was not necessarily due to any particular influence of the solid geometry course.

Cohen[29] conducted the third study dealing with the improvement of

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[29]Cohen, op. cit., 72 pp.

space perception abilities through the study of solid geometry. Three experimental and three control solid geometry classes in a New York high school were used in the study. The experimental classes constructed models of 22 previously selected theorems throughout the entire semester course. The problem was to determine if the pupils' abilities in spatial perception would be improved by such training. From the 240 pupils involved in the study, 63 matched pairs were secured. Space perception abilities were measured by the "Minnesota Paper Form Board Test" and the "Space Relations Test, Differential Aptitude Tests." Although Cohen found that the experimental group showed a greater gain than did the control group, the difference was not significant at either the 1 per cent or 5 per cent levels. Cohen concluded that the construction of models of solid geometry theorems by pupils during their study of solid geometry did not improve their visualization abilities as measured by the two tests.

A search of the literature related to the problem of this study revealed four recent studies which were concerned with space perception and the use of models in the teaching of solid geometry. None of the four related studies which were examined furnished any objective data to substantiate the claim that training in space perception will improve achievement in solid geometry or the claim that the study of solid geometry will increase ability in spatial perception.

Summary

The reports in the literature reviewed indicated that, generally speaking, the teaching of solid geometry has not been considered entirely satisfactory for at least the past 80 years. The reasons for this dissatisfaction appeared to lie in the difficulties which the pupil encounters in the study of the subject. Although other areas of difficulty exist, the ones which cause the greatest difficulties concern the pupil's unfamiliarity with complex geometric figures and the visualization and understanding of three-dimensional figures.

The use of models of geometric solids and of various theorems has been advocated and tried to rectify these difficulties which the pupil encounters in working with three-dimensional figures. Psychologists and others writing in the field of learning theory are generally agreed that learning is aided by the proper use of multi-sensory aids.

Until very recently, there was little evidence of anyone's questioning the belief that training in space perception will improve achievement in solid geometry or the belief that the study of solid geometry will increase ability in spatial perception. There was one study reported by Johnson[30] which attempted to measure the effect on final achievement in solid geometry of pupil manipulation of models of the theorems and problems. Johnson reported no significant difference in final achievement between pupils who manipulated models and pupils who did not manipulate models.

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[30] Johnson, op. cit., 389 pp.

Three studies involving the effect of the study of solid geometry upon the ability of spatial visualization were reported by Ranucci,[31] by Brown,[32] and by Cohen.[33] Ranucci reported no significant difference in spatial perception between a group which had studied solid geometry and a group which had not. Brown reported a significant difference in spatial perception abilities in favor of pupils who had taken a two-year sequence involving solid geometry over those who took only a one-year course in plane geometry fused with solid geometry. It is very possible that this difference was due to natural maturation rather than to the study of solid geometry. Cohen found no significant difference in space perception abilities between pupils who made models of theorems during their study of solid geometry and those who studied solid geometry without making models.

There was no statistical evidence in the literature reviewed which supported either the contention that improved spatial perception will increase achievement in solid geometry or the contention that a study of solid geometry will improve space perception abilities.

To determine the effect on final achievement in solid geometry of an introductory unit on model construction, an experiment was designed and conducted. A detailed description of the design of the experiment and the investigational procedures followed are presented next.

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[31] Ranucci, op. cit., 75 pp.

[32] Brown, op. cit., 82 pp.

[33] Cohen, op. cit., 72 pp.

CHAPTER III

EXPERIMENTAL DESIGN AND INVESTIGATIONAL PROCEDURES

Because it was felt that introductory exercises in the construction of models would improve final achievement in solid geometry, an experiment involving the cooperation of teachers and pupils in several schools was designed and an investigation conducted to determine if there was any significant difference in the final achievement in solid geometry between pupils who studied solid geometry in the traditional manner and pupils who completed an introductory unit consisting of the drawing of one-piece patterns, the cutting, and the assembling of tagboard models of certain geometric solids prior to the study of the usual topics of the course. The following items were considered in designing the experiment and conducting the investigation: (1) introductory unit on model construction, (2) selection and assignment of teachers, (3) information used in comparing the experimental and control groups, (4) teaching procedures, and (5) statistical procedures. These five items are presented in the following sections.

Introductory Unit on Model Construction

A unit on model construction was selected for use at the beginning of the solid geometry course rather than later in the course because of the need for aids to the imagination which will help the pupil to make a transition from the two-dimensional thinking of plane geometry to the three-dimensional thinking necessary in solid geometry. The use of models to help the imagination during the early stages of the course was advocated by Nyberg,[1] but he recommended that later in the course models be used less and that more emphasis be placed on mental images of the solid figures. Working with models at the beginning of the course provides opportunities for a study of the relationships between points, lines, and planes in space, which Breslich[2] recommended as one of the first steps in the study of solid geometry. The use of model construction at the beginning of the solid geometry course not only acquaints the pupil with concrete forms of solid figures and provides opportunities for understanding the nature of solids, but also furnishes a background of experiences to serve as a basis for future discussions. Devising the introductory unit on model construction to influence final achievement in solid geometry required the consideration of three phases: (1) criteria for the selection of exercises to be included in the unit, (2) the determination of the time required for completion of the unit, and (3) the content of the unit.

Criteria for selection of exercises to be included in the unit. In selecting the exercises to be included in the introductory unit on model construction, it was felt that certain criteria should be met. The group of exercises selected should follow generally accepted principles of learning;[3] that is, they should: proceed from the known to the

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[1]Nyberg, J. A., Fundamentals of Solid Geometry, p. 35.

[2]Breslich, E. R., Purposeful Mathematics: Solid Geometry, p. 11.

[3]See Chapter II, pages 15-18.

unknown, provide opportunities for successful learning, require the active participation of the pupils, and provide opportunities for the self-discovery of principles. The exercises should be directly related to the course content of solid geometry, which includes the following sequence of topics usually studied in the course: points, lines, and planes; polyhedral angles; prisms, pyramids, and polyhedrons; cylinders and cones; and spheres and spherical polygons. A group of related exercises proceeding from the simple to the complex which would illustrate some of the properties of lines and planes in space, polyhedral angles, and figures enclosed in planes were desired to introduce the basic properties of three-dimensional figures.

Since the plan of the experiment involved the cooperation of teachers and pupils in various schools, it was deemed advisable that the introductory unit on model construction occupy as short a time as possible so as not to discourage participation in the study. However, the time allotted to the exercises should be sufficient to permit the pupils to gain insights and understandings which could have an effect on their final achievement in the course and should be sufficient to permit an adequate measure of the effect on final achievement in solid geometry of the introductory unit on model construction. The exercises selected for inclusion in the introductory unit should be of such a nature that the directions accompanying them could be kept comparatively simple to insure uniform content when presented by the several teachers cooperating in the experimental portion of the study. The exercises selected should require no special references or resources, and the materials and equipment used