VLF and LF time synchronization techniques
by Terry Richard Donich

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
Montana State University
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Abstract:
A time synchronization-technique with an error tolerance of ± 1 μsec is presented. The technique involves the use of both a VLF and a IF channel.

Tests were made to determine the phase stability of the locally received 20 kHz transmission (WWVL) from the National Bureau of Standards. The phase comparison data between the WWVL signal and the local frequency standard was recorded from a VLF receiver by the Montana State University Data Acquisition System. Statistical regression techniques were used to analyze the phase comparison data. A second order polynomial which is a characteristic of the local crystal controlled frequency standard compared to an absolute reference, was subtracted from the data. The resulting error had a standard deviation of less than 1.5 μsec. Thus an axis crossing of the received 20 kHz signal could be used to time synchronize two clocks with an error tolerance of ± 1 or 2 μsec. In order to identify the correct axis crossing of the 20 kHz signal, the local clock would have to be previously time synchronized with an error tolerance of ± 23 μsec and the actual channel time delay would have to be measured.

The polynomial computed from the phase comparison data can be used to correct the shifting of the local clock’s "tick pulse" relative to the NBS reference clock. As a result a basis was formed to test time synchronization techniques between the National Bureau of Standards at Fort Collins, Colorado, and the Electronics Research Laboratory at Bozeman, Montana.

To time synchronize with an error tolerance ± 23 μsec the pulse amplitude modulated 60 kHz signal (WWVB) was tested using repeatability. The falling edge of this pulse, which has a fall time of 1.5 millisecond's, was used as a time marker. A comparator circuit was used to develop a fast rise time receiver output pulse that would not shift in time as the received signal level changes. Tests of this system indicated that time synchronization with an error tolerance of ± 23 μsec was possible if 232 time interval measurements were averaged. However, the system as it existed, was not completely reliable. An analysis of the possible causes of the occasional errors lead to the transmitting antenna. This device set the transmitting system bandwidth and therefore determined the shape of the received pulse. Small changes in the antenna bandwidth could have distorted the received pulse and caused the errors. A possible solution to this problem is to reduce the width of the frequency spectrum fed from the transmitter to the antenna by wave shaping the modulator signal.
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A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of

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ABSTRACT

A time synchronization technique with an error tolerance of ±1 μsec is presented. The technique involves the use of both a VIF and an IF channel.

Tests were made to determine the phase stability of the locally received 20 kHz transmission (WWVL) from the National Bureau of Standards. The phase comparison data between the WWVL signal and the local frequency standard was recorded from a VIF receiver by the Montana State University Data Acquisition System. Statistical regression techniques were used to analyze the phase comparison data. A second order polynomial which is a characteristic of the local crystal controlled frequency standard compared to an absolute reference, was subtracted from the data. The resulting error had a standard deviation of less than 1.5 μsec. Thus an axis crossing of the received 20 kHz signal could be used to time synchronize two clocks with an error tolerance of ±1 or 2 μsec. In order to identify the correct axis crossing of the 20 kHz signal, the local clock would have to be previously time synchronized with an error tolerance of ±23 μsec and the actual channel time delay would have to be measured.

The polynomial computed from the phase comparison data can be used to correct the shifting of the local clock's "tick pulse" relative to the NBS reference clock. As a result a basis was formed to test time synchronization techniques between the National Bureau of Standards at Fort Collins, Colorado, and the Electronics Research Laboratory at Bozeman, Montana.

To time synchronize with an error tolerance ±23 μsec the pulse amplitude modulated 60 kHz signal (WWVB) was tested using repeatability. The falling edge of this pulse, which has a fall time of 1.5 milliseconds, was used as a time marker. A comparator circuit was used to develop a fast rise time receiver output pulse that would not shift in time as the received signal level changes. Tests of this system indicated that time synchronization with an error tolerance of ±23 μsec was possible if 232 time interval measurements were averaged. However, the system as it existed, was not completely reliable. An analysis of the possible causes of the occasional errors lead to the transmitting antenna. This device set the transmitting system bandwidth and therefore determined the shape of the received pulse. Small changes in the antenna bandwidth could have distorted the received pulse and caused the errors. A possible solution to this problem is to reduce the width of the frequency spectrum fed from the transmitter to the antenna by wave shaping the modulator signal.
1.0 INTRODUCTION
1.1 Time Synchronization Problem

Time synchronization is the act of adjusting a clock so that the clock can be used to answer the question "What time is it?" As with any physical measurement this process can be accomplished within a certain error tolerance set by the equipment and methods used.

For example, suppose several people equipped with portable transceivers were on a hike and became separated. After hours of hiking, one person noticed his watch had stopped. To determine what time it was his first approach would be to observe the position of the sun. He may be able to determine the time to ± 1 hour of true time and he could restart his watch and set it to the time estimated. However, if it was important that he arrive at his destination at the correct time or he would miss his ride, he would need to know the time to a smaller error tolerance. If he was able to use the transceiver to talk to a second person of the hiking party, assuming the second person’s watch as a reference, he could easily set his watch to ± 1 minute of this reference. By using a method to identify each second, the error tolerance could be reduced to ± 1 second of the reference.

The channel time delay is the time interval between the encoding of the information from the reference clock and the arrival of the information at the clock to be time synchronized. If the time delay in the communication channel employed is large compared to the error tolerance, a timing error can occur. In the example above, if the second person had chosen to write the time that his reference watch read on a piece of paper,
and to send it by another member of the party to the person whose watch had stopped, the timing error could have been larger than the desired error tolerance due to the time delay in sending the note. If the time delay were known, it could be added to the reading on the note to achieve time synchronization with a smaller error tolerance. When the transceivers were used for time synchronization, the time delay was small compared to the error tolerance and no correction was needed.

In many applications of geodesy and radar networks a large error tolerance is unacceptable. A common goal of many experimental time synchronization systems has been $\pm 1$ microsecond ($\mu$sec). The upper limit of velocity that energy or matter can obtain is the speed of light which is approximately $0.3$ km per microsecond. Therefore with an error tolerance of $\pm 1 \mu$sec the channel time delay will have to be evaluated for all channels used to time synchronize clocks that are separated by a fraction of a kilometer or more.

To maintain a given error tolerance it is necessary to adjust the rate setting part of the clock correctly. In the example above the rate was probably set by a hairspring or tuning fork. If the person whose watch stopped knew that his watch lost 5 minutes per hour relative to a reference, he could account for this loss by knowing when his watch was last time synchronized with the reference. The same is true for precision clocks which use frequency standards as the rate setter. If the characteristics of the frequency standard are known, the offset of the clock can be computed and corrected.
The difference in the readings of two clocks (a local and a
reference clock) can be observed by a time interval measurement. The time
interval measurement is made by observing the number of time units between
the time that two clocks have the same reading. This measurement can be
used to adjust the local clock so that it reads within some error
tolerance the same time as the reference clock.

Several systems are available to time synchronize geographically
separated clocks. The use of High Frequency (HF; 3-30 MHz) radio propa-
gation is probably the oldest technique using a radio frequency channel.
The accuracy of this type of system is low because the channel time delay
changes from one time to the next. The minimum error tolerance of this
technique using complex equipment has been stated to be $\pm 500 \mu\text{sec}$ (1).
Meteor trails (2) and satellites (3,4) have also been used for time
synchronization. The accuracy of both of these techniques has been
stated to be better than $\pm 1 \mu\text{sec}$. However, both techniques require
complex equipment installations with the satellite system the most complex.

The transportation of a portable clock for time synchronization is
another technique. Accuracies of $\pm 1 \mu\text{sec}$ using atomic resonance
controlled clocks have been stated (5). The difficulty encountered with
this technique is the transportation to remote areas and to mobile users.
This method is the most widely used at the present time to check experi-
mental time synchronization systems and measure channel time delays.

All the techniques mentioned above have limitations which
handicap their usability. The simplicity of a centralized single
transmitter with each user having his own receiver is desirable. When using a method such as this the channel time delay has to be a constant so that it can be measured once and assumed a constant from then on. This narrows the interest to radio propagation in the Very Low Frequency and Low Frequency channels because of the observed propagation stability in these channels.

1.2 Historical Background

Radio wave propagation in the Very Low Frequency (VLF; 3-30 kHz) and Low Frequency (LF; 30-300 kHz) bands is well suited to the task of standard frequency and time transmissions. J. A. Pierce at Harvard University in 1954 was the first to note the stable phase properties of radio propagation in the VLF band (6). At Cambridge, Massachusetts, he was comparing the frequency of the received 16 kHz transmission of station GBR (Rugby, England) with his crystal frequency standard.

By 1960 Pierce had conducted an experiment using cesium beam frequency standards at each end of a VLF channel. This experiment indicated that VLF propagation could be used to make frequency comparisons with an accuracy of a few parts in $10^{11}$ within a 24 hour period (7). In the past ten years several similar experiments have confirmed this capability (8,9,10).

In 1955 it was determined that the use of a modulated LF carrier frequency of 60 kHz could be used for time synchronization with an error tolerance of $\pm 30$ μsec (11). By 1960 it was shown that ground wave propagation from a Loran C station could be used for time synchronizations.
with an accuracy of $\pm 1.0 \mu\text{sec}$ (12). The Loran C stations use transmitters with a carrier frequency of 100 kHz. The very high power output capability of these transmitters is mostly lost by the use of low efficiency wide bandwidth antennas. This type of antenna is necessary to transmit the fast rise time pulses used for timing markers. The technique used was particular cycle identification of the fast rise time pulses.

This technique was also used at a distance greater than 1500 km from the transmitting station where the groundwave had been attenuated and the majority of the energy received was due to a skywave propagation. This experiment indicated that the accuracy would be reduced to $\pm 10 \mu\text{sec}$ (12). However, the Loran C stations in the Continental United States are located on the east coast. The use of these stations in the Western United States is limited by the long distances and consequently low signal-to-noise ratios.

To circumvent the problem of low efficiency antennas the National Bureau of Standards (NBS) in conjunction with the National Aeronautics and Space Administration implemented the Dual Frequency VLF Timing System (13,14). In this approach carrier cycle identification was achieved by adjusting the phase of the transmitted signals so that a point of zero voltage (axis crossing) and of positive slope on the sine waves occurred simultaneously. After the initial alignment, this event occurred with a frequency equal to the difference of the carrier frequencies. Thus a sequence of timing markers was generated. The frequencies used for this system were 20 kHz and either 20.5 kHz or 19.9 kHz. The timing markers
were 2 milliseconds (msec) or 10 milliseconds apart respectively. Thus it was assumed that some other system would have to be used to reduce the error tolerance to ±1 msec or ±5 msec.

The major problem encountered with this approach was the identification of the timing markers at the receiving end. For example if the 20 kHz and 19.9 kHz sine waves were being transmitted and one assumes that the axis crossings of the sine waves were aligned at a certain time, one period of the sine waves before or after this time the axis crossings were offset by only 251 nanoseconds. Thus a small phase offset in one signal of 126 nanoseconds relative to the other signal, which could easily occur in a VLF channel, would cause a large timing error of 50 μsec. To alleviate this problem and the problem of assuming a second system to perform a preliminary time synchronization, the researchers have suggested the use of a multiple frequency system. However, if a particular axis crossing could be identified, the expected error tolerance of a time synchronization system using this axis crossing of the received NBS 20 kHz signal would be ±2 μsec on a world wide basis (13).

1.3 Time Synchronization Using WWVL and WWVB

The use of an axis crossing of 20 kHz carrier frequency broadcasted from the NBS station WWVL for time synchronization is intriguing. In the Continental United States an error tolerance of ±1 μsec might be achieved.

The problem of identifying the correct axis crossing could be
approached in a different manner. The technique used would have to be capable of time synchronizing a remote clock to at least one half the period of the 20 kHz signal. The effects of noise and propagation instability will reduce this requirement to approximately $\pm 23 \mu\text{sec}$.

In 1964 pulse amplitude modulation was added to the NBS station WWVB broadcasting with a carrier frequency of 60 kHz. The pulse rate of the modulation is one pulse per second and the falling edge of this pulse is designated as the marker for each second. However, the fall time of this pulse is approximately 1.5 msec and a new technique will have to be developed to use it as a timing marker for time synchronization to an accuracy of $\pm 23 \mu\text{sec}$.

The length of each pulse of the 60 kHz carrier is used to transmit day of the year, hour of the day, and minute of the hour information (15). Thus, complete information is available by decoding the pulse length modulation to restart a clock that has stopped and time synchronize it to an error tolerance of $\pm 23 \mu\text{sec}$. The error tolerance could be reduced to $\pm 1$ or $2 \mu\text{sec}$ by then employing a time synchronization technique using a 20 kHz axis crossing technique.

1.4 Hypothesis

The hypothesis is twofold as stated below. First the phase stability (and consequently channel time delay stability) of the locally received 20 kHz (WWVL) may be used to time synchronize with an error tolerance of $\pm 1 \mu\text{sec}$. This also gives a measure of the performance of the local frequency standard. Second, the 60 kHz (WWVB) amplitude
modulated signal may be used to time synchronize two clocks to ± 23 μsec so that the correct axis crossing of the locally received 20 kHz signal may be identified.

1.5 Scope of the Thesis

Chapter 2 is devoted to the topic of phase comparison between the received 20 kHz signal and the local frequency standard. The model for phase measurements is constructed and applied to data analysis on the computer. The results of a data run are shown.

In Chapter 3 a general model for time synchronization that demonstrates the use of frequency synchronization is presented. The important parameters of a time synchronization channel model are discussed with emphasis on the channels used for HF, VLF, and LF frequency standard broadcasts by the National Bureau of Standards. A possible method of using both the WWVB and WWVL signals for time synchronization is described.

Chapter 4 presents the details of the receiving system constructed to use the WWVB signal for time synchronization. The accuracy of time synchronization using this signal is derived from the signal-to-noise ratio and the system parameters. The results of data taken with this system are discussed.

Finally, Chapter 5 contains a summary of the results of this thesis and some suggestions for improved time synchronization methods using the basic concepts of this thesis.
2.0 PHASE COMPARISON TECHNIQUES
2.1 Frequency Standards Laboratory

The Montana State University Frequency Standards Laboratory is located in the Electronics Research Laboratory (ERL). The laboratory has two frequency standards, a Hewlett-Packard (HP) 103AR and a General Radio (GR) 1113A. Both standards are quartz crystal controlled. The HP frequency standard has an associated frequency divider and clock, HP Model 113BR and Model 724BR emergency power supply. The GR frequency standard has a 1114-A frequency divider, a 1103-B clock, and 1116-B emergency power supply. The available output frequencies of the two systems are shown in Table I. The specifications may be found in the manuals (16, 17, 18).

| TABLE I |
| Frequencies Available From Standards |

<table>
<thead>
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<th>HP Standard</th>
<th>GR Standard</th>
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<tr>
<td>1 MHz</td>
<td>5 MHz</td>
</tr>
<tr>
<td>100 kHz</td>
<td>1 MHz</td>
</tr>
<tr>
<td>10 kHz</td>
<td>100 kHz</td>
</tr>
<tr>
<td>1 kHz</td>
<td>10 kHz</td>
</tr>
<tr>
<td>100 Hz</td>
<td>100 Hz</td>
</tr>
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</table>

The Frequency Standards Laboratory has a twelve channel Montronics distribution amplifier with two 5 MHz channels, six 1 MHz channels, and four 100 kHz channels. This amplifier provides an output for local service that is isolated from the frequency standards (19).
A Montronics Model 100 frequency comparator is used to compare the two frequency standards. Measurements of the relative offset frequency of the standards to a few parts in $10^{-11}$ can be made in minutes by using the quadrature function outputs (20). Long term changes in frequency between the standards can be measured by observing a recorded output of the quadrature functions.

The GR clock has a contact closure circuit which is coincident with the secondhand as it passes over the second markers (called a "tick pulse"). However, this contact closure has a jitter greater than a millisecond. Therefore, the "tick pulse" of the GR clock is used only for course timing markers. The HP clock has a tick pulse that is derived from a photo electric cell. The jitter of this "tick pulse" is specified by the manufacturer to be less than 1.0 μsec. The jitter has been measured by triggering an oscilloscope with the tick pulse and observing a 100 kHz sine wave from the frequency divider of the clock. A Polaroid camera was attached to the oscilloscope so that several traces could be recorded. Assuming that the trigger circuit of the oscilloscope did not degrade the measurement, the tick pulse caused the sine wave to jitter less than 0.2 μsec for 60 samples. The rise time of the tick pulse was 0.5 μsec.

A tick pulse distribution amplifier with 6 outputs was added to the Frequency Standards Laboratory in 1966. Each output will drive a 50 ohm load with a positive 10 volt pulse. The jitter of the output pulse was less than 0.3 μsec for 60 samples by the technique described above. The rise time with all outputs loaded with 50 ohms was 0.1 μsec. If any 5
outputs were short circuited the remaining output had the same jitter specification, but the rise time changed to less than 0.5 μsec.

2.2 VLF-LF Phase Comparison System

2.2.1 Phase Measurements

Two frequency standards can be represented as in Figure 2.1.

\[ \begin{align*}
\text{Reference Frequency Standard} & : \cos(\omega_o t) \\
\text{Local Frequency Standard} & : \cos(\omega_o t + \phi(t))
\end{align*} \]

Figure 2.1
Reference and Local Frequency Standards Model

One standard, called the reference, has a radian frequency that is exactly \( \omega_o \). The second standard, called the local standard, has a radian frequency close to \( \omega_o \), but the \( \phi(t) \) term causes the output phase to deviate from the phase of the reference.

The effect of the phase function \( \phi(t) \), can be analyzed as an instantaneous frequency by using the derivative of the angle argument of the cosine function. Then

\[
\frac{d}{dt} (\omega_o t + \phi(t)) = \omega_o + \phi'(t)
\]  

(2.1)

where \( \phi'(t) = \frac{d(\phi(t))}{dt} \).
The instantaneous radian frequency of the local standard at any time \( T \) is

\[
\omega_L = \omega_0 + \dot{\phi}(T)
\]

If the reference standard signal is used to trigger an oscilloscope with the local standard signal connected to the vertical input, the function \( \phi(t) \) can be observed by watching the shifting of the points on the cosine signal where

\[
\omega_0 t + \phi(t) = n \pi/2 \quad n = 1, 3, 5, \ldots
\]

These points are called the axis crossing points of a cosine signal. If these points move to the right then \( \phi(t) \) is negative and the net effect is to lower the radian frequency of the local standard such that

\[
\omega_0 + \dot{\phi}(t) < \omega_0
\]

The time base on an oscilloscope is calibrated in units of time. If the radian frequency \( \omega_0 \) is known, the phase offset relative to a reference can be discussed in terms of seconds by using the relationship

\[
\bar{\omega}(t) = \frac{\phi(t)}{\omega_0}
\]

where \( \bar{\omega}(t) \) is in seconds, \( \phi(t) \) is in radians, and \( \omega_0 \) is in radians per second. The phase of a sine wave relative to a reference may be measured in units of time or radians and the units may be interchanged if the radian frequency of the sine wave is known.

If the reference standard is remote from the local standard a
VLF or LF communication system can be used to compare the standards. The remote reference standard in this case is the group of oscillators at the National Bureau of Standards which is used to control the broadcasts described below.

2.2.2 NBS Transmissions and Specifications

The National Bureau of Standards started frequency standard broadcasts of WWVL (20 kHz) and WWVB (60 kHz) at the Fort Collins, Colorado, site in 1963. Several experimental stations were operated by NBS at these frequencies from 1956 to 1963 (21).

Two diamond shaped antennas, 1900 feet by 750 feet, each supported by four 400 foot masts radiate the power from two 50 kw transmitters, one operating at 20 kHz and the other operating at 60 kHz. The radiated power of the antennas is 1.8 kw at 20 kHz and 13 kw at 60 kHz (1).

The NBS Primary Frequency Standard consists of two cesium beam frequency standards called the United States Frequency Standard. The Primary Frequency Standard is used to calibrate five quartz crystal oscillators. The details of the calibration have been explained by Andrews (22). The five oscillators are called the United States Working Frequency Standard. The weighted mean frequency of these oscillators controls the transmission of the frequency standards broadcasts (23,24,21).

WWVB and WWVL transmitted frequencies are normally stable to 0.2 parts in $10^{11}$ relative to the NBS Primary Frequency Standard (25). The stability of the propagation medium is adequate to maintain these accuracies at any receiving site within the Continental United States (26).
2.2.3 VLF Receiver

A Montronics Model 205 VLF receiver is used to monitor the NBS frequency standard broadcasts. This receiver is a digital phase tracking type. Specifications can be found in the manual (27).

The relative phase difference between the local frequency standard connected to the receiver and received VLF signal is plotted on a strip chart recorder. The full scale reading of the recorder is 100 μsec and the resolution is approximately 1 μsec. The receiver also has an optically read digital phase difference accumulator. This accumulator has five digits with the last digit being in steps of 0.1 μsec.

The usual method of reducing the phase data from the VLF receiver is to manually fit a straight line to the plotted output of the VLF receiver during the time period of 1 hour after sunrise to 1 hour before sunset in Bozeman, Montana, (daytime data). This time period eliminates the diurnal shift (transitional propagation phase shift due to a path going from sunlight to dark or vice versa) and the nighttime data which is less accurate due to higher ambient noise levels (28,29). The frequency offset of the local standard can be evaluated by using the slope of this line as an approximation to the derivative of the phase function in Equation C.10 from Appendix C;

\[ P_{10} = \frac{1}{8.64} \dot{\phi}(t) \]  

(2.6)

where \( \dot{\phi}(t) = \frac{d \phi(t)}{dt} \) is the rate of change of the phase difference in μsec per day and \( P_{10} \) is the frequency offset in parts in \( 10^{-10} \).

If over a 6 hour period the phase of the frequency standard were
to shift 2 μsec with respect to the received VLF signal, the frequency offset of the standard would be

\[ P_{10} = \frac{1}{8.64} \left( \frac{(2)}{.25} \right) \]

\[ P_{10} = .926 \]

With a resolution of 1 μsec the possible error would be ± .463 parts in \(10^{10}\) and

\[ P_{10} = .926 ± .463 \]

The maximum capability of the received signal which is 0.2 parts in \(10^{10}\) is not being realized. Also the line fitted to the plotted data is a guess. A better criterion should be used such as the method of Linear Regression which is based on the least squared error statistical method (31). To implement this method the phase difference accumulator could be read out digitally. Then the computer could be used to fit a straight line to these data.

2.2.4 Computer Interface with the VLF Receiver

In order to have an electrical output from the VLF receiver, a mechanical counter with a 10 lines per digit output was driven in synchronism with the accumulative phase difference indicator. The 10 lines per digit output was read out by the Montana State University Data Acquisition System. If the second counter were to change while being read out by the Data Acquisition System an error would occur. A three count buffer
storage device was built to eliminate this problem.

The Data Acquisition System stored the readings on paper tape. The paper tape data were converted to cards for batch processing on an IBM 1620-II computer. The Data Acquisition System was controlled by an internal clock which interrogated the counter on 5 minute intervals 24 hours a day. Thus 288 readings were taken each day.

2.2.5 Statistical Approach To Record Analysis

The conventional quartz crystal oscillator may be idealized as having a frequency offset which is a linear function of time relative to a reference standard (30). In Equation 2.2 the instantaneous frequency term due to $\phi(t)$ can be represented by

$$\phi(t) = a_1 + a_2 t$$

(2.7)

where $a_1$ is the radian frequency offset from $a_0$ at time zero, $a_2$ is the time rate of change of frequency or drift trend, and $t$ is in units of time. Most high quality quartz crystal oscillators that have been aged at least one year will exhibit the linear drift trend. To obtain the phase difference function Equation 2.7 must be integrated

$$\phi(t) = \int \phi(t) \, dt$$

(2.8)

$$= \int (a_1 + a_2 t) \, dt$$

$$= a_0 + a_1 t + a_2 t^2 / 2$$

where $a_0$ is some initial phase offset in radians.
The phase of the frequency standard compared to an absolute frequency standard (such as the cesium beam controlled NBS transmissions) is a second order polynomial as shown in Equation 2.8. However, over a short time period (such as one day) the second order curve can be approximated with a straight line by letting \( a_2 \) equal zero.

The first approach, using the computer, was to fit a straight line to the digitally recorded daytime phase data from the VLF receiver for each day by the method of Linear Regression (31). This method gave an equation for the phase difference in \( \mu \text{sec} \) as a function of time in days of the form

\[
\bar{\phi}(t) = a_0 + a_1 t \quad (2.9)
\]

By substituting Equation 2.9 into Equation 2.6

\[
P_{10} = \frac{a_1}{8.64} \quad (2.10)
\]

where \( P_{10} \) is the frequency offset of the local standard relative to the received VLF signal for that day.

The Linear Regression method gave the same type of results as the straight edge method discussed in Section 2.2.3. The Linear Regression computer program used is listed in Appendix D. Table II shows the computed frequency offset of the HP frequency standard compared to the 20 kHz NBS transmission for a 9 day period by both methods. A combination of noise and frequency standard drift caused an RMS error of less than 0.5 \( \mu \text{sec} \) about the linear function fit to the phase data for each day. The
TABLE II

Frequency Offset in Parts in $10^{-10}$ of HP Frequency Standard at 1200 Hours Compared to 20 kHz NBS Transmission by Straight Edge and Least Square Error Methods.

<table>
<thead>
<tr>
<th>Date</th>
<th>Straight Edge Frequency Offset</th>
<th>Least Squared Error Frequency Offset</th>
<th>Difference, Least Squared Minus Straight Edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 2, 1966</td>
<td>3.89</td>
<td>4.08</td>
<td>+0.19</td>
</tr>
<tr>
<td>3</td>
<td>4.59</td>
<td>4.43</td>
<td>-0.16</td>
</tr>
<tr>
<td>4</td>
<td>5.00</td>
<td>5.51</td>
<td>+0.51</td>
</tr>
<tr>
<td>5</td>
<td>6.53</td>
<td>6.25</td>
<td>-0.28</td>
</tr>
<tr>
<td>6</td>
<td>6.67</td>
<td>6.86</td>
<td>+0.19</td>
</tr>
<tr>
<td>7</td>
<td>7.51</td>
<td>7.77</td>
<td>+0.26</td>
</tr>
<tr>
<td>8</td>
<td>7.22</td>
<td>7.32</td>
<td>+0.10</td>
</tr>
<tr>
<td>9</td>
<td>8.90</td>
<td>9.17</td>
<td>+0.27</td>
</tr>
<tr>
<td>10</td>
<td>8.20</td>
<td>8.04</td>
<td>-0.16</td>
</tr>
</tbody>
</table>
low value of RMS error implies that a straight line is a good approximation to the phase data over the 14 hour periods used here.

The drift trend of a frequency standard is the change in frequency offset per unit time (usually per day). The average drift trend can be found by using the Linear Regression Program on the data shown in Table II. The curve computed for the least squared error data was

\[ P_{10} = (3.96 \pm 0.50) + (0.59 \pm 0.09) t \]  \hspace{1cm} (2.11)

where \( P_{10} \) is the frequency offset in parts in \( 10^{-10} \) and \( t \) is the time in days (\( t = 0 \) at June 2, 1966 0000 MST). Thus, the average drift trend of the HP frequency standard was \( 0.59 \pm 0.09 \) parts in \( 10^{-10} \) per day and the estimated frequency offset on June 2, 1966 at 0000 Mountain Standard Time was \( 3.96 \pm 0.56 \) parts in \( 10^{-10} \). The confidence intervals shown are at the 80% level for an assumed normally distributed error. The same technique was used on the straight edge data. The polynomial computed was

\[ P_{10} = (3.83 \pm 0.43) + (0.59 \pm 0.09) t \]  \hspace{1cm} (2.12)

Another approach used to look at long term stability was to compute the coefficients of a second order polynomial as in Equation 2.8 from the phase data for a several day period. The Combined Data Analysis Program listed in Appendix D was used to compute this polynomial. This program used Linear Regression on the daytime and nighttime data during each day. The difference between these lines at 12 noon each day was
added to the nighttime data to correct for the diurnal shift. Data taken during a diurnal shift were omitted. The second order curve computed from the phase data taken during the dates shown in Table II was

$$B(t) = -76.173 + 33.299t + 2.646t^2$$  \hspace{1cm} (2.13)

where $B(t)$ is in $\mu$sec and $t$ is in days. By substituting Equation 2.13 into Equation 2.6 the frequency offset was

$$P_{10} = 3.85 + .61t$$  \hspace{1cm} (2.14)

This equation agrees with Equations 2.11 and 2.12 quite closely. The phase data had an RMS error about the second order curve of $1.5 \mu$sec for the 9 day period. The error was less than $\pm 2 \mu$sec 85 percent or more of the time. This technique has been extended up to 15 days continuous data with the same result. Therefore, the drift of the local clock's "tick pulse" which was controlled by the phase of the frequency standard* can be approximated by the drift of an idealized crystal oscillator with an RMS error less than $\pm 1.5 \mu$sec.

An interesting fact is brought out in Figure 2.2. In Figure 2.2 the iterative estimate of the drift trend for the HP frequency standard is shown as a function of the total number of days of data used. The drift trend can be established within 2.0 parts in $10^{11}$ with the use of four days' data.

---

*See Chapter 3.0
Figure 2.2 Iterative Drift Trend Analysis of the HP 103AR Frequency Standard
2.3 Characteristic of the Frequency Standards

The HP frequency standard has been operating continuously for three years. The average drift trend, as of January 1968, was +0.3 parts in $10^{10}$ per day. The GR frequency standard has a drift of -2.0 parts in $10^{10}$ per day. The GR frequency standard has a on-off snap action type of oven which makes the short term stability (stability based on measurement made during a period of less than 5 minutes) much worse than the HP frequency standard which has a slowly varying proportional type of oven. For these reasons the HP frequency standard is used for all precision frequency and time synchronization research.

The drift trend of the HP frequency standard since it was rebuilt in 1965 is shown in Figure 2.3. The points shown in this figure are based on 30 day averages of data reduced as discussed in Section 2.2.3.

When the HP frequency standard is reset, it has a tendency to drift more than its normal drift trend the first 48 hours. This was first observed during the experiments with the Data Acquisition System. Therefore it was necessary to adjust the HP frequency standard at least 48 hours before a precision experiment is conducted.

The HP frequency standard should not be reset more often than necessary because of drift trend changes after a reset. Therefore a maximum tolerance of frequency offset of $\pm 5$ parts in $10^{10}$ was established. If the HP frequency standard was reset 5 parts in $10^{10}$ low with respect to the received VLF signal, it was approximately 32 days before its frequency was 5 parts in $10^{10}$ high and a reset was needed. The reset
Figure 2.3 Drift Trend for HP 103AR Frequency Standard
was performed by adjusting the fine frequency control which drives a mechanical counter calibrated in parts in $10^{-10}$. 
3.0 TIME SYNCHRONIZATION
3.1. Introduction

Two clocks could be added to the frequency standards in Figure 2.1 as shown in Figure 3.1.

![Figure 3.1 Local and Reference Clock Models](image)

A clock is defined for our purpose as a device that counts a number of cycles equal to the frequency of the standard used to drive it and generates an electrical pulse called a "tick pulse" when this number is reached. For example, if the frequency of the reference standard in Figure 3.1 was exactly $10^6$ Hz, the clock would count $10^6$ cycles of the cosine signal and $P_r(t)$, the clock output, would change from zero volts to $V_o$ volts as shown in Equation 3.1,

$$P_r(t) = \begin{cases} V_o & \text{when } \omega_o T_n = n \omega_o \\ 0 & \text{otherwise} \end{cases} \quad (3.1)$$

at a rate of once per second where $T_n$ is the time of occurrence of the "tick pulse".

The equation that describes the operation of the local clock is

$$P_L(t) = \begin{cases} V_o & \text{when } \omega_o T_n + \phi(t) = n \omega_o \\ 0 & \text{otherwise} \end{cases} \quad (3.2)$$
The $\phi(t)$ term in equation 3.2 affects the location in time of the local clock's "tick pulse". Suppose $\phi(t) = K$, a positive constant number of radians. This implies that the cosine signal from the local standard has a positive phase shift. The condition for $P_L(t) = V_0$ is

$$\phi_0 T_n + K = n\omega_0 \quad n = 1, 2, 3, \ldots$$

$$T_n = n - \frac{K}{\omega_0} \quad n = 1, 2, 3, \ldots \quad (3.3)$$

or the "tick pulse" of the local clock occurs $K/\omega_0$ seconds earlier than the "tick pulse" of the reference clock. A time interval measurement between the clocks would show that the local clock is ahead of the reference clock by $K/\omega_0$ seconds.

If the $\phi(t)$ function was not a constant but depended on time, a time interval measurement would change each time a measurement was taken. Equation 2.13 in Chapter 2.0 was a polynomial computed from the phase data of the Hewlett-Packard frequency standard when compared to the received VLF signal. If the variable $t$ in this equation is changed by five minutes ($\Delta t = 5/1440$) the argument $\phi(t)$ changes by less than 1 μsec. This short term effect of $\phi(t)$ is small and can be neglected. The long term shift due to the frequency standard can be corrected by calculating $\phi(T_2) - \phi(T_1)$ where $T_1$ and $T_2$ are the time in days of the first and second measurements respectively to the nearest five minutes. The correction for the frequency offset and drift of the local standard can then be made by subtracting this quantity from the time interval measurement taken at $T_2$. The time interval measurements between the local and
reference clocks should agree within some error tolerance after this correction. The error tolerance is set by the measurement techniques used. The VLF phase tracking techniques discussed in Chapter 2.0 could be used to correct the time interval measurements between a clock at NBS and the local clock with an RMS error of less than 1.5 μsec.

However, the correction for the \( \phi(t) \) term in the local clock system is only part of the time synchronization problem. The second part is to answer the question, "What time is it?" If two clocks are in the same laboratory it is quite easy to set the local clock so that it reads the same time as the reference clock. The "tick pulses" could also be set so that they agree within a small error tolerance.

If the reference clock is remote from the local clock the problem is much more complex. Any communication channel used to transfer the reference clock's time markers has a time delay; i.e., the time interval from when the information is put into the channel until it is received at the opposite end. The channel time delay must be accounted for if the received signal is to be used for time synchronization.

The stability of the channel time delay is another important factor. Stability as used here means the maximum change possible from one time of usage to the next. The stability of a channel will be one of the factors that sets the accuracy of time synchronization using the channel. It should be noted that by using a transponding technique the channel time delay can be determined each time it is used. It is assumed that the channel time delay is stable for a short time interval each time the
channel is used. An example of this type of channel usage is the meteor burst time synchronization system developed at the Electronics Research Laboratory (2).

The channel also adds noise to the received signal. The noise tends to obscure the received time marker. Time interval measurements between the local clock and the received signal will change in a random manner due to the noise. This implies that statistical techniques may be used to bound the errors due to noise.

Thus, the channel that has been described can be represented as shown in Figure 3.2.

![Figure 3.2 Channel Model](image)

The channel matching devices could include such devices as transmitters, receivers, antennas, telephone line amplifiers, filters, etc. The time delay of these devices can usually be determined by time interval measurements between the input and output. Analytical tools such as using the derivative of the transfer function phase response are described by Papoulis (32) and Brown (33).
The HF channels used for time synchronization at the Electronics Research Laboratory are described in the next section. The last sections of the chapter are devoted to the use of the VLF and LF channels for time synchronization.

3.2 HF Time Services of NBS

The National Bureau of Standards operates two HF transmitters, WWV at Fort Collins, Colorado, and WWVH at Maui, Hawaii. Station WWV broadcasts on frequencies of 2.5, 5, 10, 15, 20, and 25 MHz. Station WWVH broadcasts on frequencies of 2.5, 5, 10, and 15 MHz.

The time markers generated by WWV are 5 cycle bursts of a 1 kHz sine wave and by WWVH are 6 cycle bursts of a 1.2 kHz sine wave. The leading edge of the first cycle is coincident with the tick pulse of each station's clock (24). The clocks of both stations are kept within ± 25 µsec of the NBS Master Clock (22).

If the HP clock in the Frequency Standards Laboratory is stopped for any reason it must be resynchronized. The WWV broadcast at 15 MHz is used to do this. The hands of the clock are easily set within ± 0.5 seconds by listening to the WWV transmission. Time synchronization to ± 10 milliseconds can be accomplished with the use of an oscilloscope to measure the time interval between the local clock and the received tone burst of WWV. If the time delay of the propagation mode between Fort Collins, Colorado, and Bozeman, Montana, were accurately known it would be possible to time synchronize clocks at these locations to within ± 0.5 milliseconds (1). However, ± 0.5 milliseconds is the limit of time synchro-
nization using HF transmissions due to the limitations of time delay stability of HF propagation.

3.3 VLF & LF Time Services of NBS

The NBS station WWVL (20 kHz) adjusts the phase of the transmitter signal so that an axis crossing with positive slope occurs simultaneously with the master clock's "tick pulse" (14). A positive axis crossing of 20 kHz repeats every 50 μsec (the period of 20 kHz). Thus, to identify the correct axis crossing, the local clock must be time synchronized within ± 25 μsec of that particular axis crossing.

In section 2.2.5 it was shown that after the second order phase function of an idealized quartz oscillator was subtracted from the phase comparison data between the HP oscillator and the received WWVL standard frequency transmissions the resulting error had a 1.5 μsec standard deviation and 85% or more of the time the error was less than ± 2 μsec. Thus, in order to permit identification of the particular axis crossing in question at least 85% of the time the local clock's specification should be reduced by 2.0 μsec or ± 23 μsec. Thus, preliminary time synchronization to ± 23 μsec would permit identification of a specific axis crossing of 20 kHz. This is assuming the channel time delay can be determined by calculation and measurement (14). The ultimate time synchronization capability of this technique would be better than ± 1 μsec. This limitation would be imposed by the stability of propagation medium and the effects of noise (26).

In order to time synchronize the clocks to ± 23 μsec it may be
possible to use the NBS station WWVB (60 kHz). The carrier level at the transmitter output is shifted down 10 dB in coincidence with the station's master clock. The falling edge of this pulse could be used for time synchronization. The expected channel time delay stability of the 60 kHz signal is better than ± 4 μsec (34). This would suggest that the channel could be used for time synchronization to ± 23 μsec.

The WWVB antenna is tuned. This is done to decrease the ratio of R/r where R is the equivalent resistor for the power absorbed in the antenna circuit and r is the equivalent resistor for the power radiated by the antenna. Thus, the lower the value of R/r the more efficient the antenna is as a radiator (35,36). However, as R decreases in a tuned circuit the bandwidth decreases and the pulse rise and fall times increase. The fall time of the received WWVB signal is approximately 1.5 milliseconds (See Figure 4.1). The slow fall time of this pulse creates a problem of how to use it for time synchronization to ± 23 μsec. The equipment designed to use this transmitted pulse will be discussed in Chapter 4.0.

The method for comparing the local clock to the receiver output is a time interval measurement. However, due to the relatively high atmospheric noise levels at this frequency (29) and the consequently low signal to noise ratios, the individual time interval measurements vary in a random manner. A statistical approach can be used to average the time interval measurements. The number of time interval measurements needed to reduce the effect of noise for a given signal to noise ratio
may be calculated by using the Central Limit Theorem as shown in Chapter 4.0.
4.0 60 kHz TIME SYNCHRONIZATION SYSTEM
4.1 System Approach

A block diagram of the proposed 60 kHz time synchronization system is shown in Figure 4.1. The equipment other than the 60 kHz time synchronization receiver and the time interval counter was discussed in Chapter 2.0. The counter measures the time interval between the "tick pulse" of the HP clock and the receiver output pulse.

The received WWVB amplitude modulated pulse is shown in Figure 4.2. The modulation index for the 10 dB level shift is 52%. The 90% to 10% fall time is 1.5 milliseconds. This fall time implies that the radio frequency (RF) amplifier should have a 3 db composite bandwidth of 666 Hz centered at 60 kHz. In order to reduce distortion of the pulse, the bandwidth was designed to be 790 Hz. The wider bandwidth also allowed for some detuning as a function of temperature without changing the time delay characteristics of the amplifier. The maximum gain at 60 kHz was 86 db. The schematic diagram of the RF section is shown in Appendix E.

The IF signal is usually very amplitude stable during the daytime. From day to day the maximum signal level change observed was less than 8 dB. This signal level change can be corrected by adjusting the gain control each day. The gain of the RF amplifier must be constant regardless of the signal level to reduce the distortion of the amplitude modulating waveform. Linearization of the gain was accomplished by placing the D-C operational point of the transistors as far from the cutoff and saturation regions as possible and by using emitter feedback. The RF section did not vary by more than .05 dB from being linear over the range used.
Emergency Power Supply

Tick Distribution Amplifier

HP103AR Frequency Standard

HP113BR Clock

60 kHz Phase Tracking Receiver

60 kHz Receiver

HP523B Time Interval Counter

HP560A Printer

Figure 4.1 60 kHz System
Figure 4.2 WWB Received Signal
40

The antenna used was in the geometrical shape of an isosceles triangle with a 36 foot horizontal base and a 30 foot altitude. The loop antenna had 7 turns and was tuned to 60 kHz. The 3 db bandwidth was approximately 2 kHz. A matching network was used to interface to 100 feet of 50 ohm lead in coaxial cable.

A threshold type of device was needed to define a point on the slow fall time received pulse (Figure 4.2). A normal threshold device such as a Schmidt Trigger seeks a certain voltage level and changes state. If the signal changes level between two successive readings the relative time that the Schmidt Trigger changes state will change accordingly. Thus, a signal level adaptive threshold device was needed. This function is performed by using two detectors and a comparator. The first detector (D1) is a normal detector with sufficient bandwidth to follow the pulse. The second detector (D2) has a long time constant to hold a sample of the peak signal level. The second detector output is scaled by a constant K. Operational amplifiers were used to amplify and isolate the detector signals. The comparator changes state when the two signals are at the same voltage as shown in Figure 4.3. The schematic diagram of this circuit is shown in Appendix E.

4.2 Channel Description

The 60 kHz electromagnetic wave propagates between two concentric spheres. The inner sphere is the earth and the outer sphere is the D layer of the ionosphere. The portion of the D layer of the ionosphere that is sunlit has an approximate altitude of 70 kilometers from the
Figure 4.3 Outputs of D1, D2 and the Comparator
surface of the earth. The portion of the D layer that is not sunlit has an approximate altitude of 90 kilometers. Two theories have been used to describe the electromagnetic wave propagation inside this geometry, namely: The Geometrical Optics Theory \((39,40)\) and The Waveguide Theory \((41,42)\). Both of the approaches will give the same result \((43)\). The Geometrical Optics Theory will be used to estimate the channel time delay.

Experiments have shown that the propagation at 60 kHz has very little dispersion. Therefore, the velocity of light may be used for the phase and group velocity \((37)\). The great circle distance from WWVB to the Electronics Research Laboratory (ERL) at Montana State University, is 737.17 km. A ground wave traveling at the velocity of light \((C = 2.9978 \times 10^5 \text{ km/sec})\) would arrive at the ERL 2459 \(\mu\)sec after being transmitted from WWVB. The ground wave is expected to be the dominant signal. Therefore 2459 \(\mu\)sec will be used as the approximate channel time delay. Table III shows the time delay of the skywave signal for various D layer altitudes. These time delays were computed for a single reflection from the D layer. The actual channel time delay will have to be determined by another time synchronization system such as a portable atomic clock. The stability of channel time delay is expected to be better than \(\pm 4 \mu\)sec from day to day during the daytime within a 1000 km radius from WWVB \((34)\).

4.3 System Time Delay

A measurement of the system time delay and stability was made
TABLE III

Channel Time Delay for Various Altitudes of the D Layer

<table>
<thead>
<tr>
<th>ALTITUDE (km)</th>
<th>TIME DELAY (µsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>2504.3</td>
</tr>
<tr>
<td>70</td>
<td>2511.3</td>
</tr>
<tr>
<td>75</td>
<td>2518.7</td>
</tr>
<tr>
<td>80</td>
<td>2527.5</td>
</tr>
<tr>
<td>85</td>
<td>2532.8</td>
</tr>
<tr>
<td>90</td>
<td>2546.7</td>
</tr>
<tr>
<td>95</td>
<td>2552.1</td>
</tr>
<tr>
<td>100</td>
<td>2561.6</td>
</tr>
</tbody>
</table>
using the method shown in Figure 4.4. Drifts in the modulator section of the Hewlett-Packard 606B signal generator necessitated calibration at least every 15 minutes. Calibration was accomplished by adjusting the power supply connected to the low pass filter input such that the 60 kHz signal decreased 10 db in level from the no modulation signal level. The level of the square wave generator was adjusted until its peak voltage agreed with the power supply within 50 millivolts. This insured that the amplitude modulation was a 10 db level shift within .05 db. The fall time of the pulse was set at 1.5 milliseconds by the low pass filter.

The receiver time delay as measured this way was 1143 μsec. The worst combination of signal level change (maximum change of 6 db) and temperature change (from 65°F to 85°F), caused the time delay to change by less than ± 1.5 μsec.

The time delay of the receiving system with the antenna was tested by using a whip antenna to inject the modulated generator signal into the 60 kHz antenna. The time delay measured was 1269 μsec. Accuracy of this measurement was ± 2 μsec. The signal level and temperature of the receiver were held constant during this test. Therefore, the time delay of the system was 1269 ± 3.5 μsec.

The system was sensitive to modulation index changes of the received signal. A modulation index shift from 52% to 51% caused the receiver output pulse to be delayed by 8 μsec. The data were corrected by making an oscilloscope measurement of the modulation index. This
Figure 4.4 Time Delay Measurements
method of measuring modulation index could lead to an error of $\pm 1/2\%$
or a timing error of $\pm 4 \mu\text{sec}$. Thus the system time delay when using the
correction for modulation index changes was $1269 \pm 7.5 \mu\text{sec}$.

4.4 Noise Performance of System

In Appendix A it is shown that the standard deviation of a
receiver output pulse with respect to an ideal clock's "tick pulse" is

$$
\sigma(T_0) = \frac{1}{\sqrt{2\pi C/N f_T}}
$$

where $C/N$ is signal-to-noise ratio and $f_T$ is the 3 db transmitter half
bandwidth. Standard deviations for several values of signal-to-noise
ratios are shown in Table IV.

![Table IV](image)

Standard Deviation of Receiver Output Pulse With Respect to an
Ideal Clock Tick Pulse as a Function of Signal to Noise Ratios.

<table>
<thead>
<tr>
<th>$C/N$ db</th>
<th>$\sigma(T_0)$ $\mu\text{sec}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>2.81</td>
</tr>
<tr>
<td>45</td>
<td>5.05</td>
</tr>
<tr>
<td>40</td>
<td>9.00</td>
</tr>
<tr>
<td>35</td>
<td>16.08</td>
</tr>
<tr>
<td>30</td>
<td>28.1</td>
</tr>
<tr>
<td>25</td>
<td>50.5</td>
</tr>
<tr>
<td>20</td>
<td>90.0</td>
</tr>
<tr>
<td>15</td>
<td>160.5</td>
</tr>
</tbody>
</table>

Using the Central Limit Theorem as described in Appendix B,
Table V has been constructed to show the number of samples needed for
different signal to noise ratios and error tolerances.
### TABLE V

Number of Samples Needed for a Given Signal to Noise Ratio (C/N) and Error Factor (ε) at the 99% Confidence Level.

<table>
<thead>
<tr>
<th>C/N (db)</th>
<th>ε(μsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16</td>
</tr>
<tr>
<td>50</td>
<td>0.172</td>
</tr>
<tr>
<td>45</td>
<td>0.541</td>
</tr>
<tr>
<td>40</td>
<td>1.68</td>
</tr>
<tr>
<td>35</td>
<td>5.49</td>
</tr>
<tr>
<td>30</td>
<td>17.2</td>
</tr>
<tr>
<td>25</td>
<td>54.1</td>
</tr>
<tr>
<td>20</td>
<td>168</td>
</tr>
<tr>
<td>15</td>
<td>549</td>
</tr>
</tbody>
</table>
4.5 60 kHz Time Synchronization Experiment

The total time delay of the time synchronization system and channel for any single pulse is the sum of the factors described in equation 4.1

\[ T_D = T_C + T_E + T_N \]  

(4.1)

where \( T_D \) is the total time delay from the time a pulse is transmitted until a pulse is developed at the output of the receiver, \( T_C \) is the channel time delay, \( T_E \) is the equipment time delay, and \( T_N \) is the time delay error due to noise in the measurement.

Suppose \( T_M \) is the measured time interval between a local clock's "tick pulse" and the output pulse of the receiver for any single pulse as measured by the 60 kHz time synchronization system. Then \( T_M \), called the local time interval measurement, is equal to

\[ T_M = \Delta T + T_D \]  

(4.2)

where \( \Delta T \) is the time difference between the local clock and the clock controlling the transmission. Substituting for \( T_D \) from Equation 4.1

\[ T_M = \Delta T + T_C + T_E + T_N \]  

(4.3)

If the time delay errors of noise were reduced to zero and the channel and equipment time delays were known, the time difference between the clocks \( \Delta T \) could be solved for by

\[ \Delta T = T_M - T_C - T_E \]  

(4.4)
However, it would take an infinite number of samples to reduce the time delay errors due to noise to zero. Thus a realistic value of error due to noise must be chosen. Also the exact channel and equipment time delays are not known.

The experiment conducted with the 60 kHz system was based on repeatability. This was accomplished by using the method described in Section 3.1 to subtract the shift of the local clock's "tick pulse" from the measurement made by the 60 kHz system. The data taken the first day were not corrected. The second and all other days' data were corrected for the shift of the "tick pulse".

After the data are corrected for the frequency standards offset, a sample mean can be computed for each data run. Let \( \bar{T}_i \) be the sample mean of the \( i^{th} \) data set. The problem is then how many samples are necessary so that each \( \bar{T}_i \) will be within a \( \pm 23 \) \( \mu \text{sec} \) error tolerance of the true value (i.e., the value that would be obtained with a perfect channel, perfect equipment, and no noise). An approximation to the true value is the average of each data set's \( \bar{T}_i \) for several days.

In Chapter 2.0 it was shown that the RMS phase error about an idealized oscillator's phase characteristic was 1.5 \( \mu \text{sec} \) during a 9 day period. This will be used as a measure of the ability to correct the local time interval measurements (\( T_M \)) for the local clock's "tick pulse" shifts. The channel time delay is unknown but was stated in Section 4.1 to vary less than \( \pm 4 \) \( \mu \text{sec} \). The equipment delay was evaluated to an accuracy of \( \pm 7.5 \) \( \mu \text{sec} \) when using a correction for modulation index
changes. The sum of these uncertainties is less than $\pm 13 \mu\text{sec}$. Thus, the time delay errors due to noise must be reduced to less than $\pm 10 \mu\text{sec}$.

The average signal-to-noise ratio of the received 60 kHz NBS transmission was 25 db. This value was established by measuring the signal level during normal transmission and the noise level during a time NBS was not transmitting on 60 kHz. The noise measurement was made at the output of the RF section of the 60 kHz time synchronization receiver with a true reading rms meter. A signal-to-noise ratio of 25 db in Table V implies that 139 samples are needed for a typical day to reduce the effects of noise to $\pm 10 \mu\text{sec}$.

A minimum of 240 data points was taken for each data set to insure that a signal to noise ratio change would not shift the error tolerance larger than $\pm 23 \mu\text{sec}$. Two data sets were recorded each day with a 6 minute interval between them.

4.6 Data Analysis and Interpretation

The statistical parameters of local time interval measurements ($T_m$) were computed by the Statistical Analysis Program listed in Appendix D. An occasional burst of noise would shift a datum point out of a reasonable band around the sample mean. The computer program was designed to reduce this problem by using only data points that were within a $\pm 250 \mu\text{sec}$ band around the sample mean of all of the data.

Two typical data runs each of 5 days length are shown in Table VI and VII. The standard deviation of the daily data has a minimum value of 26.80 $\mu\text{sec}$ and a maximum value of 65.38 $\mu\text{sec}$. The corresponding signal-
### TABLE VI

**Statistical Parameter of January 15th through 19th Data**

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>No. of Data Points</th>
<th>Std. Dev. $(\mu$sec)</th>
<th>Mean $\overline{T}_i$ $(\mu$sec)</th>
<th>$(\overline{T}_i - T^*)$ $(\mu$sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 15</td>
<td>13:24</td>
<td>246</td>
<td>61.54</td>
<td>3497.0</td>
<td>-20.8</td>
</tr>
<tr>
<td>1968</td>
<td>13:39</td>
<td>Local Noise Problem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan. 16</td>
<td>12:16</td>
<td>250</td>
<td>26.80</td>
<td>3506.3</td>
<td>-11.5</td>
</tr>
<tr>
<td></td>
<td>12:31</td>
<td>249</td>
<td>40.16</td>
<td>3497.5</td>
<td>-20.3</td>
</tr>
<tr>
<td>Jan. 17</td>
<td>12:18</td>
<td>248</td>
<td>43.62</td>
<td>3526.8</td>
<td>+9.0</td>
</tr>
<tr>
<td></td>
<td>12:33</td>
<td>250</td>
<td>33.88</td>
<td>3513.8</td>
<td>-4.0</td>
</tr>
<tr>
<td>Jan. 18</td>
<td>13:01</td>
<td>249</td>
<td>59.86</td>
<td>3539.9</td>
<td>+22.1</td>
</tr>
<tr>
<td></td>
<td>13:16</td>
<td>246</td>
<td>37.79</td>
<td>3538.4</td>
<td>+20.6</td>
</tr>
<tr>
<td>Jan. 19</td>
<td>12:15</td>
<td>248</td>
<td>55.08</td>
<td>3522.4</td>
<td>+4.6</td>
</tr>
<tr>
<td></td>
<td>12:30</td>
<td>237</td>
<td>56.95</td>
<td>3518.0</td>
<td>+0.2</td>
</tr>
</tbody>
</table>

Average of $\overline{T}_i = T^* = 3517.8 \mu$sec

RMS Error about $T^* = 14.9 \mu$sec
### TABLE VII

**Statistical Parameters of December 6th through 10th Data**

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>No. of Data Points</th>
<th>Std. Dev. (μsec)</th>
<th>Mean $T_1$ (μsec)</th>
<th>$(T_1 - T^*)$ (μsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 6</td>
<td>12:06</td>
<td>242</td>
<td>49.47</td>
<td>3518.5</td>
<td>16.9</td>
</tr>
<tr>
<td></td>
<td>12:16</td>
<td>260</td>
<td>44.87</td>
<td>3497.7</td>
<td>-3.9</td>
</tr>
<tr>
<td>Dec. 7</td>
<td>13:28</td>
<td>269</td>
<td>49.66</td>
<td>3580.2(^1)</td>
<td>78.6</td>
</tr>
<tr>
<td></td>
<td>13:38</td>
<td>260</td>
<td>55.36</td>
<td>3568.7(^1)</td>
<td>67.1</td>
</tr>
<tr>
<td>Dec. 8</td>
<td>12:17</td>
<td>263</td>
<td>29.56</td>
<td>3512.9</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>12:27</td>
<td>299</td>
<td>60.12</td>
<td>3503.8</td>
<td>2.6</td>
</tr>
<tr>
<td>Dec. 9</td>
<td>12:19</td>
<td>295</td>
<td>65.38</td>
<td>3479.6</td>
<td>-22</td>
</tr>
<tr>
<td></td>
<td>12:29</td>
<td>279</td>
<td>55.27</td>
<td>3485.9</td>
<td>-15.7</td>
</tr>
<tr>
<td>Dec. 10</td>
<td>14:04</td>
<td>236</td>
<td>60.29</td>
<td>3514.7</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>14:14</td>
<td>276</td>
<td>53.04</td>
<td>3499.5</td>
<td>-2.1</td>
</tr>
</tbody>
</table>

Average of $T_1 = T^* = 3501.6 \, \mu$sec

RMS Error about $T^* = 12.9 \, \mu$sec

\(^1\)Not included in $T^*$ or RMS error calculation
to-noise ratios interpolated from Table IV are 30.4 db and 22.8 db respectively. The change in the signal-to-noise ratio was due to received signal level changes and man made noise radiating from the power lines within the building. Various devices operating within the building were sources of large noise signals.

From Table V, the minimum signal-to-noise ratio of 22.8 db and an error factor $e = \pm 10 \mu sec$ implies that at least 232 datum points are needed. Thus, the 240 datum points taken were adequate to time synchronize to $\pm 23 \mu sec$ under almost all conditions based on the signal-to-noise ratio criterion. The exception to this was caused by a local man-made noise source resulting in an extremely low signal-to-noise ratio. When this condition was observed on an oscilloscope monitoring the RF output, no data were taken. This condition existed approximately 10 per cent of the time.

In Table VI, $T^*$, the average of the daily sample means ($\bar{T}_1$) was 3517.8 $\mu sec$. This value was used as an approximation to the true value described in section 4.5. The difference between the daily sample means and the average ($\bar{T}_1 - T^*$) is also shown in Table VI. All of the daily sample mean values were within the error tolerance of $\pm 23 \mu sec$ of the average $T^*$. The RMS error of the daily sample means about the average was 14.9 $\mu sec$.

The same parameters are shown in Table VII for data taken in December 1967. The data taken on December 7th were excluded from the calculations as obviously they did not meet the error tolerance of $\pm 23$
54 μsec from the average. Assuming that the receiving system was operating properly, it may be hypothesized that the large shift in this data was due to either a change in the channel time delay or a change in the transmitting system.

No Geoalerts were announced from the NBS station WWV during the time data in Table VII were taken. The phase tracking receiver discussed in Chapter 2.0 showed no phase anomalies or unusual phase shifts in the received WWVB signal. Also, the standard deviations of the December 7th data were normal in comparison to the rest of the data in Tables VI and VII. From equation 4.1 the signal-to-noise ratio is inversely proportional to the standard deviation. Thus, assuming the noise level was constant it may be concluded that the signal level did not change significantly on this day. These pieces of information imply that most likely the channel time delay did not change.

The transmitting system was listed as operational with no outages in the NBS Time and Frequency Bulletin (25). It is possible that an unmeasured modulation index change occurred. If the modulation index changed 8% it would shift the data 64 μsec. The modulation index measurements varied less than 0.5% that day.

In section 3.3 of this thesis it was stated that the transmitting antenna set the bandwidth of the transmitting system. Since the bandwidth of the local receiver and antenna was sufficient not to distort the pulse, the received pulse shape was determined by the transmitting antenna. In a telephone conversation, Mr. David Andrews, Chief of the Time and Frequency
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Services of NBS, stated that a 10% change in the "Q" of the WWVB antenna due to environmental changes was realistic. The "Q" of this antenna is approximately 129. A 10% higher "Q" would delay the one half voltage point on the detected envelope by approximately 50 µsec. A 14% rise in the "Q" of the WWVB antenna would yield a 70 µsec delay.

4.7 Additional Remarks

The 60 kHz time synchronization system as it now exists, is not completely reliable for ± 23 µsec time synchronization. This is most likely due to the transmitting antenna. If further testing at the WWVB transmitting site concludes that the "Q" of the antenna does shift, then it should be corrected. One possible method for correction would be to reduce the width of the frequency spectrum that is fed to the antenna. Then small bandwidth changes of the antenna would not distort the transmitted waveform. The usual approach in pulse communication systems is to wave shape the signal before amplitude modulating the carrier (38). This may also include a reduction in the antenna "Q".

The WWVL transmitted frequency is offset -300 parts in 10^10 at the present time from the WWVB transmitted frequency. If two clocks were driven by two frequency standards one of which had a frequency offset of -300 parts in 10^10 from the other; the readings of the clocks would disagree by 9 µsec in 5 minutes. The difference in the readings of the clocks would constantly enlarge by 9 µsec per 5 minutes or 2592 µsec per day.

One possible method of time synchronization using the WWVB and
WWVL signals would be to use two clock systems. The first clock system could be time synchronized to an error tolerance of \( \pm 23 \) μsec by the 60 kHz system as described in this thesis. The clock of the second system, controlled by a frequency standard with a \(-300\) parts in \(10^{10}\) offset, could be adjusted a known amount from the first clock. The second clock then would be synchronized to the WWVL time scale with the original error tolerance of \( \pm 23 \) μsec. If the phase delay of the 20 kHz signal were accurately known (such as by portable clock measurements) one could identify the particular axis crossing of the WWVL sine wave that was used as a timing marker.

A second approach that would eliminate the need for two clocks and frequency standard would be to synchronize the WWVL and WWVB transmitters. Either of these approaches would give ultimate time synchronization capabilities to \( \pm 1 \) μsec.
5.0 CONCLUSIONS
5.1 Summary

A method of using VHF and LF propagation to time synchronize geographically separated clocks to an error tolerance of ± 1 µsec was presented in this thesis.

The first problem was to establish a frequency synchronized base between the quartz oscillator at the local facility and the oscillators at NBS. This was necessary to evaluate the shifting of the local clock's "tick pulse" relative to the NBS clock which was assumed the reference. The data were taken by the Montana State University Data Acquisition System from a phase tracking receiver tuned to the WWVL (20 kHz) NBS signal and batch processed on a computer. The coefficients of a second order polynomial (the phase shift of an idealized quartz oscillator) were computed by regression techniques and subtracted from the phase data. The resulting error had an RMS value less than 1.5 µsec. The time shifting of the local clock relative to the NBS clock is also related by this polynomial. Thus, measurements made with the local clock could be corrected by using this polynomial. The local clock would then have an RMS error of less than 1.5 µsec relative to the NBS clock. This technique would provide a platform for testing the stability of time synchronization techniques between NBS and Montana State University.

This argument could be turned around to say that frequency synchronization could be established by comparing the local clock's "tick pulse" to an axis crossing of the received 20 kHz signal. If a particular axis crossing could be identified it could also be used for time synchronization.
with an RMS error tolerance of approximately 1.5 μsec. However, this
time synchronization technique involves knowing that the local clock is
already time synchronized to ± 23 μsec and also knowing the channel time
delay which will have to be measured.

To establish time synchronization to ± 23 μsec it may be possible
to use the NBS signal from WWVB with carrier frequency at 60 kHz. Time
information is transmitted from WWVB by pulse amplitude modulating the
carrier signal. A receiver was constructed to use the falling edge of
this pulse to generate a fast rise time pulse. Time interval measurements
were made between the local clock's "tick pulse" and the receiver output
pulse. These measurements were corrected by the use of the polynomial
discussed above, computed from simultaneously taken phase data. Thus
if the system time delay including the channel time delay were stable, the
corrected time interval data should be a constant within some error toler-
ance. It was found that 232 sample time interval measurements must be
averaged to time synchronize to ± 23 μsec under most observed conditions
using the received WWVB signal.

The results of the experiment showed that occasionally the
sample mean of a data set would shift out of the error tolerance range.
An analysis of the probable error causes suggested that the transmitting
antenna, which determines the system bandwidth, may have changed band-
width and therefore distorted the received pulse. If further tests at
the transmitter site indicate that this large structure is sensitive to
its outside environment, the bandwidth changing problem could be corrected
by wave shaping the modulator signal. The modulation index of the received signal was found to vary. It is assumed that the LF channel does not influence the modulation index. Thus the modulation index of the transmitter probably varies in the same manner as the modulation index of the received signal. The correction of the data for modulation index changes was a hindrance to data analysis. Much time could be saved by modifying the transmitter to generate a constant modulation index.

In the opinion of the author, if the necessary transmitter site modifications were made, the 60 kHz time synchronization system would provide reliable time synchronization data. However, the actual channel time delay will have to be measured.

5.2 Suggestions for Further Research

After the necessary transmitter site modifications are made it might be possible to develop the 60 kHz time synchronization system further, such that an axis crossing of 60 kHz could be identified. If the time delay of the equipment could be evaluated to within ± 1 μsec and the error due to noise reduced to ± 2 μsec, then assuming that the maximum time delay changes of the 60 kHz channel are less than ± 4 μsec, it would be possible to time synchronize to ± 7 μsec. This would be sufficient to identify a particular axis crossing of the received 60 kHz signal. The phase stability of this channel is better than ± 0.2 μsec. With the restriction of local equipment capability one would expect that time synchronization with an error tolerance of ± 1 μsec could be established. However, to reduce the errors due to noise to ± 2 μsec
with the present transmitter power level, it would take approximately
3,460 samples. If the transmitted power were increased 20 db the number
of samples needed would change to 35.
APPENDIX A

The derivative of the output voltage of an R-C low pass filter with a step input voltage is

\[ \frac{dE}{dT} \bigg|_{E = E/2} = \pi E f_T \]  \hspace{1cm} (A.1)

where \( E \) is the maximum voltage of the step and \( f_T \) is the 3 dB cut off frequency of the filter.

When a thermal type noise is present and the signal to noise ratio is much larger than 1, the detector D1 output signal has an amplitude standard deviation of \( \sigma(a) \). If \( T_0 \) is the reference time that the amplitude of the noiseless signal equals the second detector voltage (See Figure 4.3), then the standard deviation of the crossing time is defined as \( \sigma(T_0) \) and

\[ \frac{\sigma(a)}{\sigma(T_0)} = \frac{dE}{dT} = \pi E f_T \]  \hspace{1cm} (A.2)

\[ \sigma(T_0) = \frac{\sigma(a)}{\pi E f_T} \]  \hspace{1cm} (A.3)

Since \( E = \sqrt{2}C \), where \( C \) is the RMS carrier level and \( \sigma(a) = N \) where \( N \) is RMS noise level in the RF section, then

\[ \sigma(T_0) = \frac{1}{\sqrt{2} \left( \frac{C}{N} \right) \pi f_T} \]  \hspace{1cm} (A.4)
High atmospheric noise levels limit the performance of the system. The observed local signal to noise levels of the WWVB signal is 25 dB. Thus with a transmitter 3 dB half bandwidth of 250 Hz equation A.4 shows

\[
\sigma(T_0) = \frac{1}{\sqrt{2\pi} (17.8)(250)}
\]

\[= 50.5 \mu\text{sec}\]

Standard deviations for several values of signal to noise ratios are shown in Table IV.
APPENDIX B

A bound on the number of samples needed to reduce the effects of random noise is developed. The Central Limit Theorem of Statistics can be used to establish this bound. Stated mathematically, how many samples are needed such that

$$\Pr(|\bar{T}_{MN} - T_o| < \epsilon) \approx B$$

(B.1)

where $\bar{T}_{MN}$ is the sample mean of the $N$ observed values of $T$, $T_o$ is the true mean (i.e. noise less measurement), $\epsilon$ is the error tolerance and $B$ is the probability level. The Central Limit Theorem implies that the density of $\bar{T}_{MN}$ approaches a normal distribution with mean $T_o$ and variance $\sigma^2(T_o)/N$ as $N$ increases without bound. Then a random variable $X_N$ defined as

$$X_N = \frac{\bar{T}_{MN} - T_o}{\sigma(T_o)}\sqrt{N}$$

(B.2)

approaches a normally distributed random variable with mean zero and unit variance. Dividing the argument $\epsilon$ by $\sigma(T_o)/\sqrt{N}$ changes Equation B.1 to read

$$\Pr(|X_N| < \frac{\epsilon\sqrt{N}}{\sigma(T_o)}) \approx B$$

(B.3)

Let $A$ be defined by

$$A = \frac{\epsilon\sqrt{N}}{\sigma(T_o)}$$

(B.4)
Given the probability level $B$, the value of $A$ can be found from the tables of the normal density function. If $B$ is equal to .99, $A$ is equal to 2.33. In equation B.4

$$\frac{\varepsilon \sqrt{N}}{\sigma(T_o)} = 2.33$$

(B.5)

and solving for $N$

$$N = \left( \frac{2.33 \sigma(T_o)}{\varepsilon} \right)^2$$

(B.6)

The relationship of $\sigma(T_o)$ to the signal to noise ratio and the bandwidth was shown in Appendix A to be

$$\sigma(T_o) = \frac{1}{\pi \sqrt{2} C/N f_T}$$

(B.7)

where the variables are as defined in that Appendix. Substituting Equation B.7 into Equation B.6 it is shown that

$$N = \left( \frac{2.33}{\varepsilon \pi \sqrt{2} C/N f_T} \right)^2$$

(B.8)

If $N$ or a greater number of samples are taken, then the probability that the sample mean is within $\varepsilon$ units from the true mean $T_o$ is .99. Stated in equation form

$$P(-\varepsilon < \bar{T}_{MN} - T_o < \varepsilon) = .99$$

(B.9)
if the number of samples taken is greater than or equal to \( N \).
APPENDIX C

Suppose the sine wave outputs of two frequency standards were in phase at the start of some time interval. After $\Delta T$ seconds the difference in phase would be given by

$$\phi_{12} = (\omega_1 - \omega_2) \Delta T \quad (C.1)$$

where $\omega_1$ and $\omega_2$ are the radian frequencies and $\phi_{12}$ is the phase difference between the sine waves after $\Delta T$ seconds. If the first frequency standard is assumed to be the reference and the second frequency standard has an instantaneous relative offset $\delta$ from the first, then

$$\omega_1 = 2\pi F_0 \quad (C.2)$$

$$\omega_2 = 2\pi F_0 + 2\pi \delta F_0 \quad (C.3)$$

where $F_0$ is the reference frequency. Substituting these relationships into Equation C.1 shows

$$\phi_{12} = 2\pi \delta F_0 \Delta T \quad (C.4)$$

or

$$\delta_{12} = \frac{\phi_{12}}{2\pi F_0} = \delta \Delta T \quad (C.5)$$

where $\delta_{12}$ is the phase difference in seconds.

A more common measure of frequency offset is in terms of parts in $10^n$ which is related to the relative offset by

$$P_n = \delta \times 10^n \quad (C.6)$$
where $P_n$ is the frequency offset in parts in $10^n$ and $\delta$ is the relative frequency offset. Substituting Equation C.6 into Equation C.5 shows

$$\delta_{12} = P_n \times 10^{-n} \Delta T \quad (C.7)$$

If $P_n$ is a function of time one must change to differential form

$$d\delta_{12} = P_n \times 10^{-n} dt \quad (C.8)$$

Converting $\delta_{12}$ to microseconds, $T$ to days and letting $n = 10$

$$d\delta_{12} = 8.64 P_{10} dt \quad (C.9)$$

Thus

$$P_{10} = \frac{1}{8.64} \frac{d\delta_{12}}{dt} \quad (C.10)$$

or by integrating

$$\delta_{12} = 8.64 \int_0^T P_{10} dt \quad (C.11)$$

where $\delta_{12}$ is the phase function in microseconds, $P_{10}$ is the frequency offset function in parts in $10^{10}$, and $T$ is the time in days.
APPENDIX D

This appendix contains computer programs used in this Thesis. A general description of the program and input specifications is given before the listing of the actual program. All programs are in Fortran II for use with an IBM 1620-II computer.
Linear Regression Program Listing.

This program fits a best straight line to a data set in the minimum mean squared error sense. Any data taken on any intervals may be used. The inputs are tabulated below.

Card 1 -- Format (15H)

Identification card for each data set. In the first 15 columns any alphanumeric identification code may be used.

Card 2 -- Format (I10, 4F10.3)

NX -- The number of data points to be read in.

TE 995 -- From the cumulative "Student" distribution for NX-1 degrees of freedom and a probability level of 99.5%.

TE 975 -- Same as above, probability level of 97.5%.

TE 950 -- Same as above, probability level of 95.0%.

TE 900 -- Same as above, probability level of 90.0%.

Card 3 ... -- Format (2F10.3) For each card.

Y (I) -- The data points, 1 data point per card.

X (I) -- The corresponding coordinate of Y(I).

Note: The program will execute for any number of data sets but there must be 3 blank cards at the end to call exit and terminate program.
C LINEAR REGRESSION.

DIMENSION Y(200),X(200),C(4),CI(8),W(200),E(200)

500 READ 801
READ 11,NX,TE99,TE975,TE950,TE900,(Y(I),X(I),I=1,NX)
IF(NX)99,99,510

510 A=NX
LP=NX
SX=0.
SY=0.
SXY=0.
SX2=0.
SXX=0.
E2=0.
C(1)=.99
C(2)=.95
C(3)=.90
C(4)=.80
DO 50 K=1,LP
SXX=SXX+X(K)**2
SY=SY+Y(K)
50 SX=SX+X(K)
YBAR=SY/A
XBAR=SX/A
DO 60 L=1,LP
SXY=SXY+(Y(L)-YBAR)*(X(L)-XBAR)
60.
60 SX2=SX2+(X(L)-XBAR)**2
BETA=SXY/SX2
ALPHA=YBAR-BETA*XBAR
DO 70 M=1,LP
70 E2=E2+(Y(M)-ALPHA-BETA*X(M))**2
SIGMA2=E2/(A-2.)
SIGMA=SQRTF(SIGMA2)
SIGNR2=E2/A
SIGNR=SQRTF(SIGNR2)
C CONFIDENCE INTERVALS
DEGFR=LP-2
ROOT=SQRTF(DEGFR*SX2/E2)
D=LP
ROOTA=SQRTF((D*DEGFR*SX2)/(E2*SXX))
CI(1)= TE995/ROOT
CI(2)= TE995/ROOTA
CI(3)= TE975/ROOT
CI(4)= TE975/ROOTA
CI(5)= TE950/ROOT
CI(6)= TE950/ROOTA
CI(7)= TE900/ROOT
CI(8)= TE900/ROOTA
DO 80 I=1,LP
W(I)=ALPHA+BETA*X(I)
80 E(I)=Y(I)-W(I)
74

PRINT 21
PRINT 31,NX,BETA,ALPHA,SIGMA2,SIGMA
PRINT 41
PRINT 31,NX,BETA,ALPHA,SIGNR2,SIGNR
PRINT 51
DO 83 IA=1,4
83 PRINT 61,CI(IA),CI(2*IA),CI(2*IA-1)
PRINT 801
PUNCH 801
PRINT 71
DO 90 IB=1,LP
PUNCH 141,W(IB)
90 PRINT 81,X(IB),Y(IB),W(IB),E(IB)
GO TO 500
99 CONTINUE
11 FORMAT(I10,4F10.3/(2F10.3))
21 FORMAT(1H1,38HREGRESSION ASSUMING ERROR HAS UNKNOWN
112HDISTRIBUTION)
31 FORMAT(/5X,3HNX=I3,5X,5HBETA=F14.8,5X,6HALPHA=F14.7,
15X,7HSIGMA2=F16.8,5X,6HSIGMA=F14.7///)
41 FORMAT(1X,28HASSUMING NORMAL DISTRIBUTION///)
51 FORMAT (/10X,39HCONFIDENCE INTERVALS FOR ALPHA AND BETA///)
61 FORMAT (10X,12HPROB. LEVEL=F5.2,4X,F12.6,4X,F12.6///)
71 FORMAT (20X,5HTIME=,11X,3X,11HDATA VALUE=,8X,
111HREGRESSION=,11X,5HERROR)
81 FORMAT (5X, F20.2, 5X, F20.5, 2X, F17.5, 2X, F12.5)
91 FORMAT (8+10.3)
141 FORMAT (F10.3)
801 FORMAT (15H

CALL EXIT

END
Combined Data Analysis Program Listing

This program reads the VLF Receiver phase data from the Data Acquisition System on a daily basis. After the diurnal shift is removed from the nighttime data a second order regression curve is computed from the data.

Card 1 -- Format (I5, I3, I2, 4I5)

ID(1) -- The month and day in numeric form.
ID(2) -- The year in numeric form.
JF -- The day number starting from the 1st day's data as zero.
J6 -- 1st diurnal shift point (neglect data from J6 to J08).
J08 -- 2nd diurnal shift point.
J80 -- 3rd diurnal shift point (neglect data from J80 to J52).
J52 -- 4th diurnal shift point.

Card 2 -- Format (I5, I3, 12F4.0) For each card.

ID(1) -- The month and day in numeric form.
ID(2) -- The year in numeric form.
X(I) -- 12 data points, 4 digits long.

Correction Cards

Card 26 -- Format (I10, F10.0)

If the next data pass is not immediate, the first card of the deck punched on the pass before is used as the first card of the correction cards. If the next pass is immediate, neglect this card and add correction card according to schedule below.
IK -- The number of the first data point to be corrected (288 data points per day; for example, IK = 144 at 12 noon).

COR -- The value of the correction in μsec x 10.

The last correction card is always a blank card.

Iterative Sum Continuation Cards

Next to Last Card -- Format (4E14.8)

Last Card -- Format (4E14.8)

These cards are to be filled with zeros in exponential form in the first data pass. If the next data pass is immediate these cards can be neglected. If the next pass is not immediate, use the last two cards from the deck punched the pass before.

If sense switch no. 3 is on, the program will continue immediately on the next day’s data and compute a new curve for combined data. If sense switch no. 3 is off, the program will terminate.
C  COMBINED DATA ANALYSIS PROGRAM

DIMENSION ID(5), X(288), A(4,4), ST(4), SXT(3), W(288),
1E(288), TQ(180), YQ(180)

IXX=0

30 READ 111, ID(I), ID(3), IP, J6, J08, J80, J52, IABC

   IF(IABC) 99, 280, 99

280 TN=-5

   DO 10 I=1,288
       READ 121, TM, X(I)

10   TPP=TN-TM+S.

   IF(TPP ) 20, 40, 99

20   X(I+1)=X(I)

   X(I)=99999.
   TN=TN+S.
   TZ1=TN/100.
   IZ1=TZ1
   TZ3=IZ1*100
   TZ2=TN-TZ3


2000   TN=TN+40.

2010   I=I+1

   GO TO 21

40   TN=TN+S.

   TZ1=TN/100.
   IZ1=TZ1
TZ3 = IZ1*100
TZ2 = TN - TZ3
IF(TZ2 < 55.) 10, 2030, 2030
2030 TN = TN + 40.
10 CONTINUE
P = IP
IXX = IXX + 1
DO 190 I = 1, 288
IF(X(I)) 180, 190, 190
180 X(I) = 99999.
190 CONTINUE
C CORRECTIONS
ITD = 0
IK = 1
IF(IXX - 1) 99, 58, 57
58 C = 0.
90 READ 151, IK, COR
ITD = ITD + 1
IF(IK) 99, 70, 50
50 C = C + COR
57 DO 60 I = 1, 288
IF(I - IK) 60, 80, 80
80 IF(X(I) = 99999.) 100, 60, 60
100 IF(ITD) 99, 102, 103
102 X(I) = X(I) + C
GO TO 60
103 \( X(I) = X(I) + \text{COR} \)
60 CONTINUE
GO TO 90
70 \( \text{IM} = 1 \)
220 PRINT 211, ID(1), ID(3)
PRINT 231, C
DO 200 I = 1, 24
KI = (I - 1) * 12 + 1
KK = I * 12
200 PRINT 201, (X(J), J = KI, KK)
IF(IP) 99, 72, 170
C FIND IX MAX AND IX MIN
72 \( \text{XMAX} = -99999. \)
\( \text{XMIN} = 99999. \)
DO 110 I = 1, 288
IF(X(I) = 99999.) 120, 110, 110
120 IF(X(I) - XMAX) 140, 140, 130
130 \( \text{XMAX} = X(I) \)
140 IF(X(I) - XMIN) 150, 110, 110
150 \( \text{XMIN} = X(I) \)
110 CONTINUE
Q = C - XMAX
C = Q
DO 250 I = 1, 288
IF(X(I)-99999.) 257,250,250

257  X(I)=X(I)-XMAX

250  CONTINUE

PRINT 221, ID(1), ID(3)

DO 270 I=1, 24
  KI=(I-1)*12+1
  KK=I*12

270 PRINT 201, (X(J), J=KI, KK)

170 DO 1000 I=1, 288
  X(I)=X(I)/10.
  IF(X(I)-99999.9) 1000, 1001, 1001

1001  KXXL=X(I)
  X(I)=KXXL

1000  CONTINUE

  DO 260 I=1,24
    IZ=(I-1)*12
    IF(IZ-J80) 538, 538, 260

538  IF(IZ-J08) 260, 539, 539

539  IF(X(IZ)-99999.) 536, 260, 260

536  PI=IP
    ZI=I-1
    T=PI+ZI/100.

PUNCH 601, X(KI), T

260  CONTINUE

5X1=0.
SX2=0.
ST1=0.
ST2=0.
SXT1=0.
SXT2=0.
K1=0
K2=0
ST22=0.
ST21=0.
E21=0.
E22=0.
DO 350 131,288
AI=I
IF(X(I)=9999,) 580,350,350
580 IF(I-J6) 360,360,370
370 IF(I-J52) 380,360,360
360 SX1=SX1+X(I)
ST1=ST1+AI
K1=K1+1
GO TO 350
380 IF(I-J80) 390,390,350
390 IF(I-J08) 350,410,410
410 SX2=SX2+X(I)
ST2=ST2+AI
K2=K2+1
350 CONTINUE
28 AK1=K1
AK2=K2
TBAR1=ST1/AK1
XBAR1=SX1/AK1
TBAR2=ST2/AK2
XBAR2=SX2/AK2
DO 400 I=1,288
AI=I
IF(X(I)-9999.)590,400,400
590 IF(I-J6)420,420,430
430 IF(I-J52)440,420,420
420 SXT1=SXT1+(AI-TBAR1)*(X(I)-XBAR1)
ST21=ST21+(AI-TBAR1)**2
GO TO 400
440 IF(I-J80D450,450,400
450 IF(I-J08)400,470,470
470 SXT2=SXT2+(AI-TBAR2)*(X(I)-XBAR2)
ST22=ST22+(AI-TBAR2)**2
400 CONTINUE
BETAI=SXT1/ST21
BETAZ=SXT2/ST22
ALPHAI=XBAR1-BETAI*TBAR1
ALPHAZ=XBAR2-BETAZ*TBAR2
DO 480 I=1,288
AI = I
IF(X(I) - 9999.) 600, 480, 480
600 IF(I - J6) 520, 520, 530
530 IF(I - J52) 540, 520, 520
520 E21 = E21 + (X(I) - ALPHA1 - BETA1*AI)**2
   GO TO 480
540 IF(I - J80) 550, 550, 480
550 IF(I - J08) 480, 570, 570
570 E22 = E22 + (X(I) - ALPHA2 - BETA2*AI)**2
480 CONTINUE
   SIGM21 = E21/(AK1 - 2.)
   SIGM22 = E22/(AK2 - 2.)
   SIGMA1 = SQRTF(SIGM21)
   SIGMA2 = SQRTF(SIGM22)
   PRINT 801, ID(I), ID(3), IP, J6, J08, J80, J52
   PRINT 511, SIGM21, SIGMA1, K1
   PRINT 521, ALPHA1, BETA1
   PRINT 531, SIGM22, SIGMA2, K2
   PRINT 521, ALPHA2, BETA2
   DIR = ALPHA1 + BETA1*144. - ALPHA2 - BETA2*144.
   PRINT 541, DIR
   DO 660 I = 1, 288
      IF(X(I) - 9999.) 640, 660, 660
640 IF(I - J6) 610, 610, 620
620 IF(I - J52) 660, 610, 610
85

610 X(I)=X(I)-DIR
660 CONTINUE
   PRINT 211, ID(1), ID(3)
   DO 107 I=1, 24
      KI=(I-1)*12+1
      KK=I*12
107 PRINT 191, (X(J), J=KI, KK)
   IF(I*XX-1)99, 108, 109
108 READ 11, ST(I), ST(2), ST(3), ST(4)
   READ 11, SXT(I), SXT(2), SXT(3), NB
   NA=K1+K2+NB
   GO TO 104
109 NA=NA+K1+K2

ORDER OF REGRESSION
104 K=2
   KA=K*2
   IA=K+1
   KX=IA+1
   DO 15 L=1, KA
   DO 720 J=1, 288
      IF(X(J)-9999.) 650, 720, 720
      650 IF(J-J6) 330, 330, 340
      340 IF(J-J08) 720, 160, 160
      160 IF(J-J80) 330, 330, 347
      347 IF(J-J52) 720, 330, 330
330 AJ=J
   T=P+AJ/288.
   ST(L)=ST(L)+T**L
   IF(L-1)99,700,710
700 SXT(I)=SXT(I)+X(J)
710 IF(L-1)99,720,730
730 IF(L-IA) 740,740,720
740 JA=L-1
   SXT(L)=SXT(L)+X(J)*T**JA
720 CONTINUE
   IF(L-IA) 17,17,15
17 A(L,KX)=SXT(L)
15 CONTINUE
   A(1,1)=NA
   DO 25 J=1,IA
   DO 25 I=1,IA
   IF(I+J-2) 25,25,35
35 MA=I+J-2
   A(I,J)=ST(MA)
25 CONTINUE
   PRINT 61,K,NA
   PRINT 132
   DO 177 I=1,IA
177 PRINT 31,(A(I,J),J=1,KX)
C  AUGMENTED MATRIX SOLUTION
C SOLUTION TO RIGHT SIDE OF MATRIX

DO .75 L=1,IA
IX=IA-L
DO 85 J=L,KX
JX=KX+L-J
A(L,JX)=A(L,JX)/A(L,L)
IF(L-IA) 65,85,105
65 DO 95 I=I,IX
  LX=L+I
  A(LX,JX)=A(LX,JX)-A(LX,L)*A(L,JX)
85 CONTINUE
75 CONTINUE

C UPPER RIGHT MATRIX REDUCTION

105 NX=IA-1
  DO 115 L=1,NX.
  LX=IA-L+1
  MX=L+1
  DO 125 J=1,MX,L
    JX=IA-J+2
    NXX=IA-L
    DO 135 I=1,NXX
      IX=IA-I-L+1
    135 A(IX,JX) = A(IX,JX) - A(IX,LX)*A(LX,JX)
  125 CONTINUE
115 CONTINUE
C MATRIX SOLUTION COMPLETE

PRINT 91
DO 145 I=1,IA
145 PRINT 31,(A(I,J),J=1,KX)
PRINT 41
E2=0.0
DO 779 I=1,288
IF(X(I)-9999.) 760,770,770
760 IF(I-J6 ) 780,780,790
790 IF(I-J08 ) 770,780,800
800 IF(I-J80 ) 780,780,810
810 IF(I-J52 ) 770,780,780
780 W(I)=A(I,KX)
    AI=I
    T=P+AI/288.
DO 787 J=1,K
    JA=J+1
787 W(I)=W(I)+A(JA,KX)*(T**J)
    E(I)=X(I)-W(I)
PRINT 51,I,T ,X(I),W(I),E(I)
    E2=E2+E(I)**2
GO TO 779
770 E(I)=99999.
779 CONTINUE
    AI=I
SNA = K1 + K2
SIGMA2 = E2 / SNA
SIGMA = SQRT(SIGMA2)
PRINT 82
PRINT 81, E2, SIGMA2, SIGMA
IF (SENSE SWITCH 3) 30, 99
99 CONTINUE
11 FORMAT (4E14.8)
31 FORMAT (1H, 8E14.8)
41 FORMAT (1HO, 8HDATA NO. 13X, 4HTIME, 15X=10HDATA VALUE, 9X
1, 16HREGRESSION VALUE, 20X, 5HERRO)
51 FORMAT (1H, I5, F20.8, 3(5X, F20.8))
61 FORMAT (1HO, 20HORDER OF REGRESSION=95, 5X,
122HNUMBER OF DATA POINTS=I5///)
81 FORMAT (1H, 3(IIX, E14.8))
82 FORMAT (1HO, 12X, 13HERRO SQUARED, 17X, 8HVARIANCE, 8X,
118HSTANDARD DEVIATION)
91 FORMAT (///1X, 9HSOLUTIONS///)
111 FORMAT (7X, I5, I3, 3X, I2, 5I5)
121 FORMAT (12X, F4.0, 4X, F4.0)
131 FORMAT (1+0, 23HNOT SUFFICIENT DATA FOR I5, I3, 6HPHASE
112, 10HOF PROGRAM)
132 FORMAT (1HO, 16HAUGMENTED MATRIX///)
141 FORMAT (3E14.8)
151 FORMAT (110, F10.0)
90

191 FORMAT(12F10.4)
201 FORMAT(12F7.0)
211 FORMAT(1H1,22HCORRECTED VALUES FOR 15,13)
221 FORMAT(1H1,38HDATA VALUES FOR FIRST DAY NORMALIZED 115,13)
231 FORMAT(1H0,30HTOTAL CORRECTION FOR NEXT DAY F10.2)
511 FORMAT(1H0,23HNIGHT DATA, VARIANCE E14.8,7X,
120HSTANDARD DEVIATION E14.8,15HNO. OF POINTS 13)
521 FORMAT(1H0,7H ALPHA E14.8,6HBETA E14.8).
531 FORMAT(1H0,21HDAY DATA VARIANCE E14.8,7X,
120HSTANDARD DEVIATION E14.8,13HNO. OF POINTS 13)
541 FORMAT(1H0,29HNIGHT DATA DAILY CORRECTIONF10.3)
601 FORMAT(2F10.3)
120HDAILY SHIFT POINTS,4I4)
801 FORMAT(1H1,12HREGRESSION 15,13,10HDAY NUMBER15,
CALL EXIT
END
Statistical Analysis Program Listing

This program computes the sample mean, variance, and standard deviation of the data set. It also has a variable data window to discard data out of the range of interest.

Card 1 -- Format (I5)

K -- the number of data sets within the data deck. This program terminates after K data sets have been read in.

Card 2 -- Format (15H)

This is a blank identification card, one for each data set, which has space for 15 alphanumeric characters.

Card 3 -- Format (6F10.0)

A(n) -- This is the range of data points taken about the sample mean of range A(n-1). A(1) is the range of the whole data set. This sets the "window" about the preceding sample means. This range of data is then used to compute a new set of statistical parameters.

Card 4... -- Format (F10.0, 10X, I5) For each card.

X(I) -- The data points.

J -- Equal to 1 on the last card of a data set and equal to 0 for all others.

If K>1 the second data set starts with card 2.
C STAT. ANALYSIS WITH VARIABLE WINDOW TO REDUCE THE
C EFFECTS OF IMPULSE NOISE

DIMENSION X(2000), A(7)

READ 101, K
DO 150 KT=1, K
READ 131
60 READ 121, (A(N), N=2, 7)
A(1)=99999.
I=1
GO TO 20
10 I=I+1
C A NUMBER 1 IN COLUMN 25 WILL STOP DATA READING
20 READ 111, X(I), J
IF(J-1) 10, 300, 10
C LOOP WITH VARIABLE WINDOW
300 DO 70 M=1, 7
SUMX=0.0
SUMX2=0.0
IK=0
XMIN=99999.
XMAX=0
IF(M-1) 99, 30, 110
110 IF(A(M)-1.) 70, 80, 80
80 A(M)=A(M)/2.
Y=XBAR+A(M)
Z = XBAR - A(M)

30 DO 40 L = 1, I
   IF (X(L) - 99999.) 50, 40, 40

C SET WINDOW
   50 IF (M - 1) 99, 100, 240
   240 IF (X(L) - Y) 90, 100, 40
   90 IF (X(L) - Z) 40, 10, 100

C FIND MAX AND MIN
   100 IF (X(L) - XMAX) .200, 200, 210
   210 XMAX = X(L)
   200 IF (X(L) - XMIN) 220, 230, 230
   220 XMIN = X(L)

C STAT ANALYSIS
   230 SUMX = SUMX + X(L)
   .SUMX2 = SUMX2 + X(L) * X(L)
   .IK = IK + 1
   40 CONTINUE
   AK = IK
   XBAR = SUMX / AK
   SIG2 = (SUMX2 / AK) - XBAR * XBAR
   SIG = SQRTF(SIG2)
   IF (M - 1) 99, 310, 330

310 PRINT 141
   GO TO 500

330 PRINT 201
500 PRINT 131
   IF(M-1) 99,160,170
160 PRINT 151
   GO TO 180
170 PRINT 161,Z,Y
   A(M)=2.*A(M)
   PRINT 411,A(M)
180 IF(IK-I) 400,41,410
410 PRINT 171,XBAR,SIG2,SIG
   PRINT 181,XMAX,XMIN
   PRINT 191,I,IK
   GO TO 70
400 PRINT 401
70 CONTINUE
150 CONTINUE
99 CONTINUE
101 FORMAT (I5)
111 FORMAT (F10.0,10X,I5)
121 FORMAT (6F10.0)
131 FORMAT(15H.
141 FORMAT(1H1)
151 FORMAT (1H ,9HNO WINDOW)
161 FORMAT(1H ,20HTHE WINDOW IS FROM E14.8,4HTO E14.8)
171 FORMAT(1HO,5HMEAN=E14.8,9HVARIANCE=E14.8,8HSTD.DEV=E14.8)
181 FORMAT(1H ,14HMAXIMUM VALUE=E14.8,14HMINIMUM VALUE=E14.8)
191 FORMAT(1H ,22H NUMBER OF DATA POINTS=15,3X,9H ACCEPTED=15)
201 FORMAT (1HO//)
401 FORMAT (1HO,23H NO. DATA IN THIS WINDOW)
411 FORMAT (1H,14H WINDOW LENGTH=E14.8)

CALL EXIT

END
Phase Program Listing

This program computes equation C.11. Listed below are the input variables and an explanation of their use.

Card 1 -- Format (15)

  KZ -- The number of data sets within the data deck. This program terminates run after KZ data sets have been read in.

Card 2 -- Format (15H)

  Identification card for each data set. In first 15 columns any alphanumeric characters may be entered.

Card 3 -- Format (F10.3, Il; F10.3)

  DPTS -- The first data point under consideration.
  XINT -- The interval length of x-axis.

Card 4... -- Format (F10.3, Il) For each card.

  DPTS -- The new data point which is read consecutively throughout data set.
  K -- Equal to 1 on the last card of a data set and equal to zero for all others.

If KZ > 1 the second data set starts with Card 2.
C PHASE IN MICROSEC FORM OFFSET MEASUREMENTS OR LINEAR
C REGRESSION
READ 1001,KZ
DO 1000 KK=1,KZ
PRINT 151
READ 161
PRINT 161
PRINT 161
READ 10, DPTS,K,XINT
10 FORMAT(F10.3,I1,F10.3)
PT2=DPTS
PRINT 101,PT2
PRINT 121
AREA=0.0
X=0.0
70 READ 15,DPTS,K
X=X+XINT
Z=ABSF(DPTS+PT2)-ABSF(DPTS-PT2)
IF(Z<0.0)60,60,50
50 AREA=AREA+PT2*XINT+((DPTS-PT2)*XINT)/2.
GO TO 80
60 AREA=AREA+((PT2+DPTS)*XINT)/2.
80 Q=AREA*8.64
PRINT 40,X,DPTS,Q
PT2=DPTS
IF(K<1) 70,1000,70
1000 CONTINUE
  15 FORMAT(F10.3,I1)
  40 FORMAT(1H,3X,F4.0,6X,F8.3,8X,E14.8)
 101 FORMAT(1H,17HSTARTING POINT = ,F8.3)
 121 FORMAT(1H0,7HDAY NO.,4X,10HDATA POINT,4X,9HPHASE IN
          19HMICROSEC.)
 151. FORMAT(1H1)
 161 FORMAT(15H     )
 1001 FORMAT(15)
 99 CALL EXIT
END
This appendix contains the schematic for the RF section and pulse detection circuits of the 60 kHz time synchronization receiver.
FIG. E.1 RF SECTION

- ALL CAPACITOR VALUES IN RF SECTION ARE pF
- ALL RESISTOR VALUES ARE 1/4 WATT, VALUE IN OHM & SE
- ALL RESISTOR VALUES ARE IN OHM

UNLESS OTHERWISE SPECIFIED:
FIG. 5.2 PULSE DETECTION CIRCUIT

ALL CAPACITOR VALUES IN WHOLE NUMBERS ARE μF
ALL CAPACITOR VALUES IN DECIPLAS ARE μF
ALL RESISTORS ARE 1/4 WATT, VALUE IN UNITS, 2 DC
ALL TRANSISTOR VALUES ARE IN μH


Donich, Terry Richard
VLF and LF time synchronization techniques