VLF and LF time synchronization techniques
by Terry Richard Donich

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
Montana State University

Abstract:
A time synchronization-technique with an error tolerance of ± 1 μsec is presented. The technique involves the use of both a VLF and a IF channel.

Tests were made to determine the phase stability of the locally received 20 kHz transmission (WWVL) from the National Bureau of Standards. The phase comparison data between the WWVL signal and the local frequency standard was recorded from a VLF receiver by the Montana State University Data Acquisition System. Statistical regression techniques were used to analyze the phase comparison data. A second order polynomial which is a characteristic of the local crystal controlled frequency standard compared to an absolute reference, was subtracted from the data. The resulting error had a standard deviation of less than 1.5 μsec. Thus an axis crossing of the received 20 kHz signal could be used to time synchronize two clocks with an error tolerance of ± 1 or 2 μsec. In order to identify the correct axis crossing of the 20 kHz signal, the local clock would have to be previously time synchronized with an error tolerance of ± 23 μsec and the actual channel time delay would have to be measured.

The polynomial computed from the phase comparison data can be used to correct the shifting of the local clock's "tick pulse" relative to the NBS reference clock. As a result a basis was formed to test time synchronization techniques between the National Bureau of Standards at Fort Collins, Colorado, and the Electronics Research Laboratory at Bozeman, Montana.

To time synchronize with an error tolerance ± 23 μsec the pulse amplitude modulated 60 kHz signal (WWVB) was tested using repeatability. The falling edge of this pulse, which has a fall time of 1.5 millisecond's, was used as a time marker. A comparator circuit was used to develop a fast rise time receiver output pulse that would not shift in time as the received signal level changes. Tests of this system indicated that time synchronization with an error tolerance of ± 23 μsec was possible if 232 time interval measurements were averaged. However, the system as it existed, was not completely reliable. An analysis of the possible causes of the occasional errors lead to the transmitting antenna. This device set the transmitting system bandwidth and therefore determined the shape of the received pulse. Small changes in the antenna bandwidth could have distorted the received pulse and caused the errors. A possible solution to this problem is to reduce the width of the frequency spectrum fed from the transmitter to the antenna by wave shaping the modulator signal.

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ABSTRACT

A time synchronization technique with an error tolerance of $\pm$ 1 μsec is presented. The technique involves the use of both a VLF and a LF channel.

Tests were made to determine the phase stability of the locally received 20 kHz transmission (WWVL) from the National Bureau of Standards. The phase comparison data between the WWVL signal and the local frequency standard was recorded from a VLF receiver by the Montana State University Data Acquisition System. Statistical regression techniques were used to analyze the phase comparison data. A second order polynomial which is a characteristic of the local crystal controlled frequency standard compared to an absolute reference, was subtracted from the data. The resulting error had a standard deviation of less than 1.5 μsec. Thus an axis crossing of the received 20 kHz signal could be used to time synchronize two clocks with an error tolerance of $\pm$ 1 or 2 μsec. In order to identify the correct axis crossing of the 20 kHz signal, the local clock would have to be previously time synchronized with an error tolerance of $\pm$ 23 μsec and the actual channel time delay would have to be measured.

The polynomial computed from the phase comparison data can be used to correct the shifting of the local clock's "tick pulse" relative to the NBS reference clock. As a result a basis was formed to test time synchronization techniques between the National Bureau of Standards at Fort Collins, Colorado, and the Electronics Research Laboratory at Bozeman, Montana.

To time synchronize with an error tolerance $\pm$ 23 μsec the pulse amplitude modulated 60 kHz signal (WWVB) was tested using repeatability. The falling edge of this pulse, which has a fall time of 1.5 milliseconds, was used as a time marker. A comparator circuit was used to develop a fast rise time receiver output pulse that would not shift in time as the received signal level changes. Tests of this system indicated that time synchronization with an error tolerance of $\pm$ 23 μsec was possible if 232 time interval measurements were averaged. However, the system as it existed, was not completely reliable. An analysis of the possible causes of the occasional errors lead to the transmitting antenna. This device set the transmitting system bandwidth and therefore determined the shape of the received pulse. Small changes in the antenna bandwidth could have distorted the received pulse and caused the errors. A possible solution to this problem is to reduce the width of the frequency spectrum fed from the transmitter to the antenna by wave shaping the modulator signal.

1.0 INTRODUCTION

1.1 Time Synchronization Problem

Time synchronization is the act of adjusting a clock so that the clock can be used to answer the question "What time is it?" As with any physical measurement this process can be accomplished within a certain error tolerance set by the equipment and methods used.

For example, suppose several people equipped with portable transceivers were on a hike and became separated. After hours of hiking, one person noticed his watch had stopped. To determine what time it was his first approach would be to observe the position of the sun. He may be able to determine the time to $\pm$ 1 hour of true time and he could restart his watch and set it to the time estimated. However, if it was important that he arrive at his destination at the correct time or he would miss his ride, he would need to know the time to a smaller error tolerance. If he was able to use the transceiver to talk to a second person of the hiking party, assuming the second person's watch as a reference, he could easily set his watch to $\pm$ 1 minute of this reference. By using a method to identify each second, the error tolerance could be reduced to $\pm$ 1 second of the reference.

The channel time delay is the time interval between the encoding of the information from the reference clock and the arrival of the information at the clock to be time synchronized. If the time delay in the communication channel employed is large compared to the error tolerance, a timing error can occur. In the example above, if the second person had chosen to write the time that his reference watch read on a piece of paper, and to send it by another member of the party to the person whose watch had stopped, the timing error could have been larger than the desired error tolerance due to the time delay in sending the note. If the time delay were known, it could be added to the reading on the note to achieve time synchronization with a smaller error tolerance. When the transceivers were used for time synchronization, the time delay was small compared to the error tolerance and no correction was needed.

In many applications of geodesy and radar networks a large error tolerance is unacceptable. A common goal of many experimental time synchronization systems has been ± 1 microsecond (μsec). The upper limit of velocity that energy or matter can obtain is the speed of light which is approximately 0.3 km per microsecond. Therefore with an error tolerance of ± 1 μsec the channel time delay will have to be evaluated for all channels used to time synchronize clocks that are separated by a fraction of a kilometer or more.

To maintain a given error tolerance it is necessary to adjust the rate setting part of the clock correctly. In the example above the rate was probably set by a hairspring or tuning fork. If the person whose watch stopped knew that his watch lost 5 minutes per hour relative to a reference, he could account for this loss by knowing when his watch was last time synchronized with the reference. The same is true for precision clocks which use frequency standards as the rate setter. If the characteristics of the frequency standard are known, the offset of the clock can be computed and corrected.

The difference in the readings of two clocks (a local and a reference clock) can be observed by a time interval measurement. The time interval measurement is made by observing the number of time units between the time that two clocks have the same reading. This measurement can be used to adjust the local clock so that it reads within some error tolerance the same time as the reference clock.

Several systems are available to time synchronize geographically separated clocks. The use of High Frequency (HF; 3-30 MHz) radio propagation is probably the oldest technique using a radio frequency channel. The accuracy of this type of system is low because the channel time delay changes from one time to the next. The minimum error tolerance of this technique using complex equipment has been stated to be ± 500 μsec (1). Meteor trails (2) and satellites (3,4) have also been used for time synchronization. The accuracy of both of these techniques has been stated to be better than ± 1 μsec. However, both techniques require complex equipment installations with the satellite system the most complex.

The transportation of a portable clock for time synchronization is another technique. Accuracies of ± 1 μsec using atomic resonance controlled clocks have been stated (5). The difficulty encountered with this technique is the transportation to remote areas and to mobile users. This method is the most widely used at the present time to check experimental time synchronization systems and measure channel time delays.

All the techniques mentioned above have limitations which handicap their usability. The simplicity of a centralized single transmitter with each user having his own receiver is desirable. When using a method such as this the channel time delay has to be a constant so that it can be measured once and assumed a constant from then on. This narrows the interest to radio propagation in the Very Low Frequency and Low Frequency channels because of the observed propagation stability in these channels.

1.2 Historical Background

Radio wave propagation in the Very Low Frequency (VLF; 3-30 kHz) and Low Frequency (LF; 30-300 kHz) bands is well suited to the task of standard frequency and time transmissions. J. A. Pierce at Harvard University in 1954 was the first to note the stable phase properties of radio propagation in the VLF band (6). At Cambridge, Massachusetts, he was comparing the frequency of the received 16 kHz transmission of station GBR (Rugby, England) with his crystal frequency standard.

By 1960 Pierce had conducted an experiment using cesium beam frequency standards at each end of a VLF channel. This experiment indicated that VLF propagation could be used to make frequency comparisons with an accuracy of a few parts in 10^{11} within a 24 hour period (7). In the past ten years several similar experiments have confirmed this capability (8,9,10).

In 1955 it was determined that the use of a modulated LF carrier frequency of 60 kHz could be used for time synchronization with an error tolerance of $\pm$ 30 μsec (11). By 1960 it was shown that ground wave propagation from a Loran C station could be used for time syncrhonization with an accuracy of ± 1.0 μsec (12). The Loran C stations use transmitters with a carrier frequency of 100 kHz. The very high power output capability of these transmitters is mostly lost by the use of low efficiency wide bandwidth antennas. This type of antenna is necessary to transmit the fast rise time pulses used for timing markers. The technique used was particular cycle identification of the fast rise time pulses.

This technique was also used at a distance greater than 1500 km from the transmitting station where the groundwave had been attenuated and the majority of the energy received was due to a skywave propagation. This experiment indicated that the accuracy would be reduced to ± 10 μsec (12). However, the Loran C stations in the Continental United States are located on the east coast. The use of these stations in the Western United States is limited by the long distances and consequently low signal-to-noise ratios.

To circumvent the problem of low efficiency antennas the National Bureau of Standards (NBS) in conjunction with the National Aeronautics and Space Administration implemented the Dual Frequency VLF Timing System (13,14). In this approach carrier cycle identification was achieved by adjusting the phase of the transmitted signals so that a point of zero voltage (axis crossing) and of positive slope on the sine waves occurred simultaneously. After the initial alignment, this event occurred with a frequency equal to the difference of the carrier frequencies. Thus a sequence of timing markers was generated. The frequencies used for this system were 20 kHz and either 20.5 kHz or 19.9 kHz. The timing markers were 2 milliseconds (msec) or 10 milliseconds apart respectively. Thus it was assumed that some other system would have to be used to reduce the error tolerance to ± 1 msec or ± 5 msec.

The major problem encountered with this approach was the identification of the timing markers at the receiving end. For example if the 20 kHz and 19.9 kHz sine waves were being transmitted and one assumes that the axis crossings of the sine waves were aligned at a certain time, one period of the sine waves before or after this time the axis crossings were offset by only 251 nanoseconds. Thus a small phase offset in one signal of 126 nanoseconds relative to the other signal, which could easily occur in a VLF channel, would cause a large timing error of 50 μsec. To alleviate this problem and the problem of assuming a second system to perform a preliminary time synchronization, the researchers have suggested the use of a multiple frequency system. However, if a particular axis crossing could be identified, the expected error tolerance of a time synchronization system using this axis crossing of the received NBS 20 kHz signal would be ± 2 μsec on a world wide basis (13).

1.3 Time Synchronization Using WWVL and WWVB

The use of an axis crossing of 20 kHz carrier frequency broadcasted from the NBS station WWVL for time synchronization is intriguing. In the Continental United States an error tolerance of ± 1 μsec might be achieved.

The problem of identifying the correct axis crossing could be approached in a different manner. The technique used would have to be capable of time synchronizing a remote clock to at least one half the period of the 20 kHz signal. The effects of noise and propagation instability will reduce this requirement to approximately $\pm$ 23 μsec.

In 1964 pulse amplitude modulation was added to the NBS station WWVB broadcasting with a carrier frequency of 60 kHz. The pulse rate of the modulation is one pulse per second and the falling edge of this pulse is designated as the marker for each second. However, the fall time of this pulse is approximately 1.5 msec and a new technique will have to be developed to use it as a timing marker for time synchronization to an accuracy of $\pm$ 23 μsec.

The length of each pulse of the 60 kHz carrier is used to transmit day of the year, hour of the day, and minute of the hour information (15). Thus, complete information is available by decoding the pulse length modulation to restart a clock that has stopped and time synchronize it to an error tolerance of $\pm$ 23 μsec. The error tolerance could be reduced to $\pm$ 1 or 2 μsec by then employing a time synchronization technique using a 20 kHz axis crossing technique.

1.4 Hypothesis

The hypothesis is twofold as stated below. First the phase stability (and consequently channel time delay stability) of the locally received 20 kHz (WWVL) may be used to time synchronize with an error tolerance of $\pm$ 1 μsec. This also gives a measure of the performance of the local frequency standard. Second, the 60 kHz (WWVB) amplitude modulated signal may be used to time synchronize two clocks to ± 23 μsec so that the correct axis crossing of the locally received 20 kHz signal may be identified.

1.5 Scope of the Thesis

Chapter 2 is devoted to the topic of phase comparison between the received 20 kHz signal and the local frequency standard. The model for phase measurements is constructed and applied to data analysis on the computer. The results of a data run are shown.

In Chapter 3 a general model for time synchronization that demonstrates the use of frequency synchronization is presented. The important parameters of a time synchronization channel model are discussed with emphasis on the channels used for HF, VLF, and LF frequency standard broadcasts by the National Bureau of Standards. A possible method of using both the WWVB and WWVL signals for time synchronization is described.

Chapter 4 presents the details of the receiving system constructed to use the WWVB signal for time synchronization. The accuracy of time synchronization using this signal is derived from the signal-to-noise ratio and the system parameters. The results of data taken with this system are discussed.

Finally, Chapter 5 contains a summary of the results of this thesis and some suggestions for improved time synchronization methods using the basic concepts of this thesis.

2.0 PHASE COMPARISON TECHNIQUES

2.1 Frequency Standards Laboratory

The Montana State University Frequency Standards Laboratory is located in the Electronics Research Laboratory (ERL). The laboratory has two frequency standards, a Hewlett-Packard (HP) 103AR and a General Radio (GR) 1113A. Both standards are quartz crystal controlled. The HP frequency standard has an associated frequency divider and clock, HP Model 113BR and Model 724BR emergency power supply. The GR frequency standard has a 1114-A frequency divider, a 1103-B clock, and 1116-B emergency power supply. The available output frequencies of the two systems are shown in Table I. The specifications may be found in the manuals (16, 17, 18).

TABLE I

Frequencies Available From Standards

HP Standard	GR Standard
1 MHz	5 MHz
100 kHz	1 MHz
10 kHz	100 kHz
1 kHz	10 kHz
	1 kHz
	100 Hz

The Frequency Standards Laboratory has a twelve channel Montronics distribution amplifier with two 5 MHz channels, six 1 MHz channels, and four 100 kHz channels. This amplifier provides an output for local service that is isolated from the frequency standards (19).

A Montronics Model 100 frequency comparator is used to compare the two frequency standards. Measurements of the relative offset frequency of the standards to a few parts in 10^{11} can be made in minutes by using the quadrature function outputs (20). Long term changes in frequency between the standards can be measured by observing a recorded output of the quadrature functions.

The GR clock has a contact closure circuit which is coincident with the secondhand as it passes over the second markers (called a "tick pulse"). However, this contact closure has a jitter greater than a millisecond. Therefore, the "tick pulse" of the GR clock is used only for course timing markers. The HP clock has a tick pulse that is derived from a photo electric cell. The jitter of this "tick pulse" is specified by the manufacturer to be less than 1.0 μsec. The jitter has been measured by triggering an oscilloscope with the tick pulse and observing a 100 kHz sine wave from the frequency divider of the clock. A polaroid camera was attached to the oscilloscope so that several traces could be recorded. Assuming that the trigger circuit of the oscilloscope did not degrade the measurement, the tick pulse caused the sine wave to jitter less than 0.2 μsec for 60 samples. The rise time of the tick pulse was 0.5 μsec.

A tick pulse distribution amplifier with 6 outputs was added to the Frequency Standards Laboratory in 1966. Each output will drive a 50 ohm load with a positive 10 volt pulse. The jitter of the output pulse was less than 0.3 μsec for 60 samples by the technique described above. The rise time with all outputs loaded with 50 ohms was 0.1 μsec. If any 5 outputs were short circuited the remaining output had the same jitter specification, but the rise time changed to less than 0.5 μsec.

2.2 VLF-LF Phase Comparison System

2.2.1 Phase Measurements

Two frequency standards can be represented as in Figure 2.1.

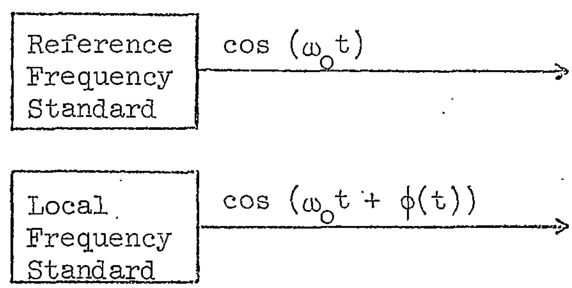

Figure 2.1

Reference and Local Frequency Standards Model

One standard, called the reference, has a radian frequency that is exactly ω_o. The second standard, called the local standard, has a radian frequency close to ω_o, but the $\phi(t)$ term causes the output phase to deviate from the phase of the reference.

The effect of the phase function $\phi(t)$, can be analyzed as an instantaneous frequency by using the derivative of the angle argument of the cosine function. Then

$$d/dt\ (\omega_o t + \phi(t)) = \omega_o + \dot{\phi}(t) \qquad (2.1)$$

where $\dot{\phi}(t) = d(\phi(t))/dt$.

The instantaneous radian frequency of the local standard at any time T is

$$\omega_L = \omega_o + \dot{\phi}\ (T)$$

If the reference standard signal is used to trigger an oscilloscope with the local standard signal connected to the vertical input, the function $\phi(t)$ can be observed by watching the shifting of the points on the cosine signal where

$$\omega_o t + \phi(t) = n\ \pi/2 \qquad n = 1,\ 3,\ 5, \cdots \tag{2.3}$$

These points are called the axis crossing points of a cosine signal. If these points move to the right then $\phi(t)$ is negative and the net effect is to lower the radian frequency of the local standard such that

$$\omega_o + \dot{\phi}(t) < \omega_o \tag{2.4}$$

The time base on an oscilloscope is calibrated in units of time. If the radian frequency ω_o is known, the phase offset relative to a reference can be discussed in terms of seconds by using the relationship

$$\Phi(t) = \frac{\phi(t)}{\omega_o} \tag{2.5}$$

where $\Phi(t)$ is in seconds, $\phi(t)$ is in radians, and ω_o is in radians per second. The phase of a sine wave relative to a reference may be measured in units of time or radians and the units may be interchanged if the radian frequency of the sine wave is known.

If the reference standard is remote from the local standard a

VLF or LF communication system can be used to compare the standards. The remote reference standard in this case is the group of oscillators at the National Bureau of Standards which is used to control the broadcasts described below.

2.2.2 NBS Transmissions and Specifications

The National Bureau of Standards started frequency standard broadcasts of WWVL (20 kHz) and WWVB (60 kHz) at the Fort Collins, Colorado, site in 1963. Several experimental stations were operated by NBS at these frequencies from 1956 to 1963 (21).

Two diamond shaped antennas, 1900 feet by 750 feet, each supported by four 400 foot masts radiate the power from two 50 kw transmitters, one operating at 20 kHz and the other operating at 60 kHz. The radiated power of the antennas is 1.8 kw at 20 kHz and 13 kw at 60 kHz (1).

The NBS Primary Frequency Standard consists of two cesium beam frequency standards called the United States Frequency Standard. The Primary Frequency Standard is used to calibrate five quartz crystal oscillators. The details of the calibration have been explained by Andrews (22). The five oscillators are called the United States Working Frequency Standard. The weighted mean frequency of these oscillators controls the transmission of the frequency standards broadcasts (23,24,21).

WWVB and WWVL transmitted frequencies are normally stable to 0.2 parts in 10^{11} relative to the NBS Primary Frequency Standard (25). The stability of the propagation medium is adequate to maintain these accuracies at any receiving site within the Continental United States (26).

2.2.3 VLF Receiver

A Montronics Model 205 VLF receiver is used to monitor the NBS frequency standard broadcasts. This receiver is a digital phase tracking type. Specifications can be found in the manual (27).

The relative phase difference between the local frequency standard connected to the receiver and received VLF signal is plotted on a strip chart recorder. The full scale reading of the recorder is 100 μsec and the resolution is approximately 1 μsec. The receiver also has an optically read digital phase difference accumulator. This accumulator has five digits with the last digit being in steps of 0.1 μsec.

The usual method of reducing the phase data from the VLF receiver is to manually fit a straight line to the plotted output of the VLF receiver during the time period of 1 hour after sunrise to 1 hour before sunset in Bozeman, Montana, (daytime data). This time period eliminates the diurnal shift (transitional propagation phase shift due to a path going from sunlight to dark or vice versa) and the night time data which is less accurate due to higher ambient noise levels (28,29). The frequency offset of the local standard can be evaluated by using the slope of this line as an approximation to the derivative of the phase function in Equation C.10 from Appendix C;

$$P_{10} = \frac{1}{8.64} \dot{\Phi}(t) \tag{2.6}$$

where $\dot{\Phi}(t) = d\,\Phi(t)/dt$ is the rate of change of the phase difference in μsec per day and P_{10} is the frequency offset in parts in 10^{10}.

If over a 6 hour period the phase of the frequency standard were to shift 2 μsec with respect to the received VLF signal, the frequency offset of the standard would be

$$P_{10} = \frac{1}{8.64} \frac{(2)}{(.25)}$$

$$P_{10} = .926$$

With a resolution of 1 μsec the possible error would be $\pm$ .463 parts in 10^{10} and

$$P_{10} = .926 \pm .463$$

The maximum capability of the received signal which is 0.2 parts in 10^{10} is not being realized. Also the line fitted to the plotted data is a guess. A better criterion should be used such as the method of Linear Regression which is based on the least squared error statistical method (31). To implement this method the phase difference accumulator could be read out digitally. Then the computer could be used to fit a straight line to these data.

2.2.4 Computer Interface with the VLF Receiver

In order to have an electrical output from the VLF receiver, a mechanical counter with a 10 lines per digit output was driven in synchronism with the accumulative phase difference indicator. The 10 lines per digit output was read out by the Montana State University Data Acquisition System. If the second counter were to change while being read out by the Data Acquisition System an error would occur. A three count buffer storage device was built to eliminate this problem.

The Data Acquisition System stored the readings on paper tape. The paper tape data were converted to cards for batch processing on an IBM 1620-II computer. The Data Acquisition System was controlled by an internal clock which interrogated the counter on 5 minute intervals 24 hours a day. Thus 288 readings were taken each day.

2.2.5 Statistical Approach To Record Analysis

The conventional quartz crystal oscillator may be idealized as having a frequency offset which is a linear function of time relative to a reference standard (30). In Equation 2.2 the instantaneous frequency term due to $\phi(t)$ can be represented by

$$\dot{\phi}(t) = a_1 + a_2 t \qquad (2.7)$$

where a_1 is the radian frequency offset from ω_o at time zero, a_2 is the time rate of change of frequency or drift trend, and t is in units of time. Most high quality quartz crystal oscillators that have been aged at least one year will exhibit the linear drift trend. To obtain the phase difference function Equation 2.7 must be integrated

$$\begin{aligned}\phi(t) &= \int \dot{\phi}(t)\, dt \qquad (2.8)\\ &= \int (a_1 + a_2 t)\, dt\\ &= a_o + a_1 t + a_2 t^2/2\end{aligned}$$

where a_o is some initial phase offset in radians.

The phase of the frequency standard compared to an absolute frequency standard (such as the cesium beam controlled NBS transmissions) is a second order polynomial as shown in Equation 2.8. However, over a short time period (such as one day) the second order curve can be approximated with a straight line by letting a_2 equal zero.

The first approach, using the computer, was to fit a straight line to the digitally recorded daytime phase data from the VLF receiver for each day by the method of Linear Regression (31). This method gave an equation for the phase difference in μsec as a function of time in days of the form

$$\bar{\Phi}(t) = a_o + a_1 t \qquad (2.9)$$

By substituting Equation 2.9 into Equation 2.6

$$P_{10} = \frac{a_1}{8.64} \qquad (2.10)$$

where P_{10} is the frequency offset of the local standard relative to the received VLF signal for that day.

The Linear Regression method gave the same type of results as the straight edge method discussed in Section 2.2.3. The Linear Regression computer program used is listed in Appendix D. Table II shows the computed frequency offset of the HP frequency standard compared to the 20 kHz NBS transmission for a 9 day period by both methods. A combination of noise and frequency standard drift caused an RMS error of less than 0.5 μsec about the linear function fit to the phase data for each day. The

TABLE II

Frequency Offset in Parts in 10^{10} of HP Frequency Standard at 1200 Hours Compared to 20 kHz NBS Transmission by Straight Edge and Least Square Error Methods.

Date	Straight Edge Frequency Offset	Least Squared Error Frequency Offset	Difference, Least Squared Minus Straight Edge
June 2, 1966	3.89	4.08	+0.19
3	4.59	4.43	-0.16
4	5.00	5.51	+0.51
5	6.53	6.25	-0.28
6	6.67	6.86	+0.19
7	7.51	7.77	+0.26
8	7.22	7.32	+0.10
9	8.90	9.17	+0.27
10	8.20	8.04	-0.16

low value of RMS error implies that a straight line is a good approximation to the phase data over the 14 hour periods used here.

The drift trend of a frequency standard is the change in frequency offset per unit time (usually per day). The average drift trend can be found by using the Linear Regression Program on the data shown in Table II. The curve computed for the least squared error data was

$$P_{10} = (3.96 \pm 0.50) + (0.59 \pm 0.09)\ t \qquad (2.11)$$

where P_{10} is the frequency offset in parts in 10^{10} and t is the time in days (t = 0 at June 2, 1966 0000 MST). Thus, the average drift trend of the HP frequency standard was 0.59 ± 0.09 parts in 10^{10} per day and the estimated frequency offset on June 2, 1966 at 0000 Mountain Standard Time was 3.96 ± 0.56 parts in 10^{10}. The confidence intervals shown are at the 80% level for an assumed normally distributed error. The same technique was used on the straight edge data. The polynomial computed was

$$P_{10} = (3.83 \pm 0.43) + (0.59 \pm 0.09)\ t \qquad (2.12)$$

Another approach used to look at long term stability was to compute the coefficients of a second order polynomial as in Equation 2.8 from the phase data for a several day period. The Combined Data Analysis Program listed in Appendix D was used to compute this polynomial. This program used Linear Regression on the daytime and nightime data during each day. The difference between these lines at 12 noon each day was