An analytical study of the behavior of composite girder bridges subjected to loads applied parallel to the plane of the slab
by Jagannath Kishanchand Khanna

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Civil Engineering and Engineering Mechanics
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Abstract:
The behavior of composite girder "bridges subjected to loads applied parallel to the plane of the slab was investigated. The finite element method of analysis which treats the slab and the longitudinal girders as an assemblage of plate elements was used. The force displacement relationship for the rectangular plate element was developed with six degrees of freedom, three translations and three rotations.

The series method of substructures is explained for solving a large number of simultaneous equations.

The influence of three kinds of diaphragms, beam, bar and plate diaphragms, on the behavior of the bridge was examined. From the analysis it was noted that the bridge undergoes considerable warping in the absence of diaphragms. The beam diaphragms , whose nodes coincide with the nodes of the slab, do not prevent the distortions of the bridge cross section. The bar and plate diaphragms are of great significance in reducing the transverse deflections of the bridge.

The transverse bending moments resulting from the vertical deflections of the girders are sizeable in the absence of bar or plate diaphragms.

The intermediate diaphragms are seen to have great importance in transferring the load from the loaded exterior girder to the unloaded girders when the loads are applied on the bottom edge of the exterior girder.

AN ANALYTICAL STUDY OF THE BEHAVIOR OF COMPOSITE GIRDER BRIDGES SUBJECTED TO LOADS APPLIED PARALLEL TO THE PLANE OF THE SLAB

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To

my wife Kaushal

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ABSTRACT

The behavior of composite girder bridges subjected to loads applied parallel to the plane of the slab was investigated. The finite element method of analysis which treats the slab and the longitudinal girders as an assemblage of plate elements was used. The force displacement relationship for the rectangular plate element was developed with six degrees of freedom, three translations and three rotations.

The series method of substructures is explained for solving a large number of simultaneous equations.

The influence of three kinds of diaphragms, beam, bar and plate diaphragms, on the behavior of the bridge was examined. From the analysis it was noted that the bridge undergoes considerable warping in the absence of diaphragms. The beam diaphragms, whose nodes coincide with the nodes of the slab, do not prevent the distortions of the bridge cross section. The bar and plate diaphragms are of great significance in reducing the transverse deflections of the bridge.

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NOTATION

a, b	plate dimensions in the x and y directions, respectively
A	cross sectional area of bar diaphragms
[A]	coordinate transformation matrix relating element displacements to displacements of the overall structure
A_s	cross sectional area of eccentric beam diaphragms
c	a constant = $(1+\gamma)/2$
[C]	matrix relating dimensionless degrees of freedom with generalized coordinates
C_m	moment coefficient
[D]	matrix of Hooke's coefficients
E	modulus of elasticity
{f}	displacement field described in terms of generalized coordinates
{F}	force vector
G	modulus of rigidity
I	total potential energy
I_x	moment of inertia of beam cross section with respect to the reference surface
J	St. Venant's torsional constant
[k]	element stiffness matrix
[$\underline{k}$]	dimensionless stiffness matrix
[K]	stiffness matrix for the overall structure
L	span length of a bridge
M_x, M_y	moments about x and y axes, respectively

N_x, N_y	axial forces per unit length of plate
{P}	equivalent nodal forces
{P(x,y)}	matrix of displacement patterns in cartesian coordinates
{P(ξ,η)}	matrix of displacement patterns in dimensionless coordinates
{Q}	matrix of forces acting on an element
R_x, R_y, R_z	longitudinal, transverse and vertical reactions, respectively
S_x	first moment of the stiffner area with respect to the reference surface
[t]	transfer matrix relating dimensionless degrees of freedom $\{\underline{\delta}_i\}$ at a node i of the plate element with dimensional degrees of freedom $\{\delta_i\}$
[T]	strain transfer matrix relating dimensional strains to dimensionless strains
u,v,w	displacement patterns in x, y and z directions, respectively
$\underline{u}, \underline{v}, \underline{w}$	displacement patterns in ξ,η and z directions, respectively
u_x, v_x, w_x	partial derivatives of u, v and w with respect to x
u_ξ, v_ξ, w_ξ	partial derivatives of u, v and w with respect to ξ
u_y, v_y, w_y	partial derivatives of u, v and w with respect to y
u_η, v_η, w_η	partial derivatives of u, v and w with respect to η
U,V,W	translational degrees of freedom
x,y,z	cartesian coordinate axes
{α},{β}	generalized coordinates
{δ}	nodal degrees of freedom
$\{\underline{\delta}\}$	dimensionless degrees of freedom
ΔL	length of beam element

$\{\varepsilon\}$	strain vector $(\varepsilon_x,\varepsilon_y,\varepsilon_{xy})$
$\{\underline{\varepsilon}\}$	dimensionless strain vector $(\varepsilon_\xi,\varepsilon_\eta,\varepsilon_{\xi\eta})$
ξ,η	dimensionless coordinate axes
$\theta_x,\theta_y,\theta_z$	rotations about x,y and z axes, respectively
γ	Poisson's ratio
$[\Phi(\xi,\eta)]$	displacement patterns in terms of degrees of freedom
$\{\sigma\}$	column vector of stresses $(\sigma_x,\sigma_y,\tau_{xy})$

INTRODUCTION

1.1 GENERAL

Highway bridges consisting of stiffened steel plate or reinforced concrete slab decks supported by and acting compositely with girders have been used extensively during recent years. These bridges are subjected to loads applied perpendicular and parallel to the plane of the slab. The forces parallel to the plane of the slab include wind loads, earthquake loads and centrifugal forces. These lateral forces are normally ignored in the design of the deck slab and the main girders since their effects are thought to be minor compared to the effects of vertical loads. However, they are considered in the design of diaphragms and other lateral bracing between girders.

Existing composite girder bridges have proved satisfactory in their performance. However, it appears that a detailed analytical study of the spatial behavior of the composite girder bridges subjected to horizontal loads has never been undertaken in the past. The complete three-dimensional analysis of the bridge, therefore, can not be carried out using present day techniques.

A satisfactory analysis of a composite girder system can be accomplished by using the orthotropic plate theory provided the loads are applied vertically. When the girder system is subjected to loads applied parallel to the plane of the slab, the use of the orthotropic plate theory is highly questionable, since this type of loading produces torsion, and causes warping of the cross section.

1.2 OBJECT AND SCOPE

The purpose of this investigation is to develop an analytical model that can be used to study the response of composite girder systems subjected to loads applied parallel to the plane of the slab. In addition to studying the effects of these loads on the slab and main girders, it is planned to investigate the effect of various types of diaphragms on the structural behavior and load distribution characteristics of the bridge.

A finite element approach is used to study the behavior of composite girder bridges when subjected to lateral loads. The slab and web of the main girders are treated as an assemblage of plate elements. The force displacement relationship for the rectangular plate element having six degrees of freedom at each node (three translations and three rotations), is developed. Flanges of girders are treated as beams lying in the horizontal plane.

The influence coefficients for the deflections, shears and moments, etc., for composite girder bridges are obtained for the unit horizontal load. The loads are applied at the node points of the top and bottom flanges of the exterior girders. In order to analyze the role of diaphragms and to study the changes in the behavior of the bridges due to the inclusion of the diaphragms, the bridges are solved without diaphragms in one case and with diaphragms at the end sections of the bridges and at an interval of one-quarter span.

The study includes three kinds of diaphragms; beams, bars and plate diaphragms. The beam diaphragms are treated as an assemblage of beam elements whose nodes coincide with the nodes of the middle plane of the slab. Plate and bar diaphragms are assumed to have nodes which coincide with the nodes of the top and the bottom flanges of the longitudinal girders.

The study also considers two different support conditions for the bridge. In one case the girders of the bridge are pinned at one end and are supported on rollers at the other end, whereas, in the second case both ends of the girders are pinned.

Various other parameters that affect the behavior of bridges, e.g., the relative stiffness of slab and longitudinal girders, number and spacing of longitudinal girders and diaphragms, are not included in the present investigation. Similarly the investigation does not consider the stability analysis of the bridge structures.

1.3 BACKGROUND

There is no literature available for analyzing the effects of horizontal loads on the composite girder bridges. A few techniques of analyses for the distribution of the vertical loads to the various longitudinal members forming the bridge are well known.

One method divides the structure into individual and longitudinal and transverse members, each possessing an appropriate flexural and torsional stiffness. For each point of intersection of the members, equations of deflections and slope compatibility are set up and a set of governing simultaneous differential and/or algebraic equations is solved. Here, one could distinguish between the longitudinal or primary members of the structure from the secondary or transverse members by modifying the stiffness properties of various members. The works of Lightfoot and Sawko (1,2)*, Hendry and Jaegar (3), Hetenyi (4), Pippard and De Waele (5) are examples of the above method.

As is indicated by Davis, et al, (6), Newmark (7) developed a distribution procedure for application to slab on steel I-beams, wherein he assumed negligible shear transfer of longitudinal shear at the beam slab interface. He used the moment distribution method modified to include slab elements to distribute the transverse slab moments and shears. In this technique the flexural rigidity of the girders could be adjusted to compensate for composite action of the slab with the supporting girders.

A more rigorous method of analysis considers a composite beam bridge as an elastically equivalent slab system, an "orthotropic plate", whose structural properties in the two orthogonal directions are uniformly distributed along their length. Analyses of the orthotropic plates are generally based on Huber's (8) theory of anisotropic plates. Such a simplification of the bridge may be justified if (a) the ratios of stiffener spacing to slab boundary dimensions are very small to ensure approximate homogenity of stiffness, (b) flexural and torsional rigidities are independent of the boundary conditions of the slab and the distribution of the load, (c) perfect bond exists between the slab and eccentric stiffeners, and

* Numbers in parentheses refer to references.

(d) there is no warping of the cross section of the bridge structure.

The idea of applying the theory of orthotropic plates to a grid system of a bridge deck by treating it as an idealized plate was proposed by Guyan (9) in 1946. Later, Massonnet (10, 11, 12), Cornelius (13), Pfluger (14, 15), Trenks (16), Glencke (17), and others extended and generalized the use of this method. Pfluger developed a system of three fourth order differential equations in order to include in-plane motions of the plate. Trenks showed that three simultaneous differential equations expressing three components of deformation of the deck plate may be transformed into one differential equation of eighth order. Most of the research published in recent years involves theoretical studies of orthotropic deck plates and deals with mathematical methods for analysing such structures (18 - 24).

A primary drawback of the orthotropic plate method is that the stiffnesses of the slab and beam are to be 'smeared' into an equivalent plate. The flexural and torsional rigidities of the plate can, at best, be approximations and are often difficult to evaluate. Secondly, once the solution of the plate problem is obtained, it is difficult to isolate the moments, shears and displacements for the longitudinal and transverse girders, which are of primary importance.

FINITE ELEMENT METHOD OF ANALYSIS

2.1 GENERAL

In the present investigation the finite element idealization is used as the basic numerical technique to solve a set of differential equations of a continuum with regard to appropriate boundary conditions. Engineering structures built up of bars, beams and plates are generally too complex to be analyzed by the theory of elasticity. The problem becomes tractable if the fundamental conditions of equilibrium and compatibility are expressed in such a manner that the mathematical formulation is given in terms of algebraic equations. The finite element technique is a convenient scheme for obtaining these equations.

The finite element method is very well described in the literature (25,26) and hence only a brief description of the general features of the method are given here. In addition, certain features of the present study which have not been presented before are discussed in detail.

The finite element method is divided into three steps:

Step 1: Structural Idealization:

A structural system is considered as an assemblage of discrete structural elements interconnected at a limited number of node points, usually the corners of the elements. Each element is assumed to have only a finite number of degrees of freedom. The formulation of such a model, referred to as the structural idealization, reduces the infinite degrees of freedom of the continuum to finite degrees of freedom suitable for the matrix method of analysis. An engineering judgement is vital at this stage since an exact analysis is performed on this substitute structure and hence the results are valid only to the extent the substitute structure represents the original structure.

Step 2: Evaluation of Element Properties:

The force-displacement relationship - stiffness or flexibility matrix - which must be obtained now is the critical phase of the method. An element which has infinite degrees of freedom is restricted to limited degrees of freedom. This in general implies violation of the continuity conditions or the equilibrium conditions or both. Both compatibility and equilibrium conditions are satisfied only under certain special situations, like the case of bending of beams with two degrees of freedom (slope and deflection) at either end. Most of the elements reported in the literature are based on the displacement method (use of stiffness matrix) of analysis and may be subdivided into the following catagories:

(1) Elements satisfying displacement compatibility,
(2) Elements satisfying equilibrium, or
(3) Elements violating both equilibrium and compatibility

(1) Elements satisfying displacement compatibility:

A set of deformation patterns are chosen to define, uniquely the state of displacement within each element. The nodal displacements act as the undetermined parameters. Selection of the displacement patterns which will explicitly specify continuity of deformations and their first derivatives between adjacent elements is difficult. However, elements satisfying continuity of displacements and their first derivatives and those which satisfy compatibility of displacements only, along the whole interface between adjacent elements are used in the literature with a great success.

Once the displacement patterns are chosen in terms of nodal degrees of freedom, the principle of virtual displacement or the principle of minimum total potential energy is employed to obtain force displacement relationship for the element. Hence the stiffness matrix for the element is obtained.

The process guarantees equilibrium of nodal forces, but does not ensure the stress equilibrium within the element or along the boundaries of the element unless the displacement functions are chosen so that they identically satisfy the differential equation(s) of equilibrium. The elements satisfying the displacement compatibility along the edges of the adjacent elements provide a lower bound to the correct solution. The value of such a bound is not very great since the underestimation of displacements and stresses is true only in an average sense over the entire continuum and is not necessarily true at every point of the continuum. It may be noted that the deformation patterns are invented rather than derived. Great care is required in choosing functions so that the necessary rigid body displacements are satisfied to ensure convergence to the correct solution.

(2) Elements satisfying equilibrium:

In this technique one assumes stress patterns which are in equilibrium at the outset instead of assuming displacement distribution as in the previous method (27, 28). Now the virtual force approach or the principle of minimum total complementary potential energy is employed to obtain the stiffness matrix of the element. Alternately the displacement distribution, in terms of nodal degrees of freedom, may be derived from the assumed stress distribution and the method of virtual displacement may be employed for the calculation of the stiffness matrix.

The displacement distribution so obtained, in general, violates the compatibility of boundary displacements on adjacent elements. The method gives an over-estimate of the total strain energy and therefore provides an upper bound on the average displacements. Any distribution of strain can be approximated by uniform strains by reducing the element size. Hence, for convergence to the correct solutions it is essential to include stress patterns which produce uniform strain. The method is, in general, more difficult to derive than the previous method.

(3) Elements violating both equilibrium and compatibility:

Here, the displacement pattern is prescribed along the edges of the element in terms of its nodal values in such a way that complete compatibility between adjacent elements is established. The elasticity problem of the element subjected to these boundary displacements is solved, exactly or approximately. If an exact solution is obtained, an element which satisfies both compatibility and equilibrium conditions is obtained. For an approximate solution the complementary strain energy, defined in terms of the internally equilibrating stress field, may be minimized. This violates the compatibility conditions within the element and the equilibrium conditions are satisfied only approximately on the boundaries. Experience with this technique has indicated good convergence (29).

Step 3: Analysis of the Element Assemblage:

Once the stiffness matrix for each element of the substitute structure is computed in its local coordinate system, it is modified into a global (entire structure) coordinate system. The elements of the modified stiffness matrix are placed in their correct positions in the larger framework of the stiffness of an entire structure. The overlapping terms are superimposed. This process is equivalent to carrying out the matrix multiplication $[A^T]$ $[K][A]$. Where [A] is a coordinate transformation matrix and [K] is a square matrix with the stiffness matrix of each element listed on its main diagonal (26). In practice, the matrix multiplication is seldom carried out since it is time consuming and takes considerable computer core storage.

A necessary criterion of the assembly is that the degrees of freedom for the node of an element be equal to the degrees of freedom of the node of the structure. This may require expansion of the element stiffness matrix by inserting an appropriate number of zeroes. The general process of assembly for these stiffnesses are identical, irrespective of number of nodes an element possesses.

The stiffness matrix for the entire structure is a singular matrix because the system is free to move as a rigid body when external loads are applied. The order of singularity of the matrix is equal to the number of possible rigid body motions. If the order of singularity is greater than this, then the structure is internally unstable or collapsible. A nonsingular matrix is now obtained by imposing sufficient boundary restraints on the structure.

It may be noted that forces acting on the structure are limited only to the nodes. If any other type of loads are applied to the structure then they must be reduced to "equivalent nodal forces" in the finite element analysis.

After the stiffness matrix for the structure is assembled, the simultaneous linear equations are ready for solution. Any standard method of solution of simultaneous equations may be employed. Certain special techniques may be used to solve a large number of simultaneous equations (see Chapter 3).

2.2 FINITE ELEMENT MODEL FOR BRIDGE STRUCTURE

The finite element approach is employed to treat a typical composite floor system (Figure 1) as a three dimensional space structure. The slab and girder elements are treated as an assemblage of rectangular plate elements each of its nodes having six degrees of freedom. Three degrees of freedom describe the transverse bending of the plate element and the remaining