Optimal control of linear distributed-parameter systems
by Robert Clifford Kolb

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
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Abstract:
The thesis research is centered on the development of optimal design concepts and techniques for linear distributed-parameter systems with outputs which are functions of both spatial variables and the time variable these systems are denoted as class-II-type distributed-parameter systems. Such systems are characterized by partial differential equations in one or more spatial variables and the time variable. In this thesis, only linear, time-invariant, distributed-parameter systems which are defined on a one-dimensional spatial domain are given detailed attention.

The primary purpose of the thesis project is to develop a general body of theory for effecting the optimal control of class-II-type distributed-parameter systems, and in so doing, to make significant advances in certain individual areas of distributed-parameter control theory.

The content of the thesis is summarized as follows: First, a review of the literature concerning the analysis, synthesis and optimization of distributed-parameter control systems is given which enfolds over 200 references. Second, performance measures are developed and interpreted for both deterministic and stochastic class-II-type, distributed-parameter control systems. The concepts of e-reachability and e-maintainability are introduced in this regard. Third, several forms of block-diagram models are developed for class-II-type distributed-parameter systems by using finite integral transforms in conjunction with the infinite (one-sided) Laplace transform. These models form a basis for the application of optimal design techniques for a wide class of linear distributed-parameter systems. Fourth, a theoretical basis is laid for the statistical analysis of class-II-type distributed-parameter systems which are subject to stationary random inputs. Distributed correlation functions and distributed spectral-density functions are obtained for a general second-order partial differential equation; three basic types of boundary conditions are considered. Fifth, a procedure is developed for minimizing the spatial-averaged-integral-square error of class-II-type distributed-parameter systems which are subject to integral constraints. An optimal open-loop control law is derived by use of the Wiener-Hopf spectrum-factorization technique. A procedure of rational fraction approximation is formulated in order to realize the optimal control law. Sixth, a procedure is developed for minimizing the integral-square error of class-II-type, distributed-parameter, feedback control systems. The Wiener-Hopf spectrum-factorization technique is used to obtain the optimal compensator for the feedback system.

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Approved:

Paul E. Uhlrich
Head, Major Department

Donald A. Pierre
Chairman, Examining Committee

[signature]
Graduate Dean

MONTANA STATE UNIVERSITY
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TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

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CHAPTER 1

INTRODUCTION

1.1 PURPOSE OF THE THESIS PROJECT

The purpose of the thesis project is fivefold:

1. To review and evaluate in an organized manner the published work on the analysis, synthesis, and optimization of control systems enfolding distributed-parameter elements.

2. To develop and interpret performance measures for use as design criteria for the optimal design of distributed-parameter control systems which have outputs and/or inputs that are distributed in space as well as in time --- these systems are denoted herein as class-II-type distributed-parameter systems.

3. To develop block-diagram models for linear class-II-type distributed-parameter systems to form a basis for the application of optimal design techniques to these systems.

4. To develop a general body of theory, based on distributed correlation functions, for the analysis of stochastic class-II-type, distributed-parameter control systems.

5. To develop optimal design techniques, based on minimization of integral-square error, for class-II-type, distributed-parameter control systems.

1.2 VALUE OF THE THESIS RESEARCH

In reality, all physical systems are distributed-parameter systems; in some cases, however, the energy of a system can be considered to be concentrated at various "points" throughout the system. In this case, the dynamics of the system are adequately characterized, at least over a limited range of operation, by a set of ordinary differential equations. Hence, a lumped-parameter system, which is characterized by a set of ordinary differential equations, is merely a special case of a distributed-parameter system in which the spatial variables are discretized to reduce the partial differential equations to ordinary differential equations.

If the energy of a system is distributed throughout the system, however, a lumped-parameter mathematical model seldom gives a satisfactory characterization of dynamic performance. When this is the case, it is generally necessary to describe the system dynamics by partial differential equations, i.e., to use a distributed-parameter model of the system.

As the control of more physical processes is automated and as more accurate control is required, the availability of a body of control theory which is applicable to distributed-parameter systems is of ever-increasing importance. The following account gives several typical areas wherein distributed-parameter elements appear in control loops.

When a control loop contains transmission lines (electrical, hydraulic, or pneumatic), the control system designer must necessarily concern himself with the distributed-parameter characteristics of these lines (59, 75, 188, 207, 212, 254, 275). In aircraft and missile control and in many industrial process control systems, transmission lines are often an integral part of control loops.

In industrial process control, many diverse forms of distributed-parameter elements appear in control loops; for example, heat exchangers (95, 219, 250, 255), chemical distillation columns (168, 177), nuclear reactors (201, 286), chemical reactors (225, 283), and continuous strip processes (130, 170).

In electronics, the development of distributed-RC networks has posed some formidable distributed-parameter analysis and design problems (70, 112, 122, 123, 159, 196, 220, 247, 258, 278, 279).

A relatively new area is the application of control theory to the study and simulation of biological systems (134, 173, 176, 177, 239); most biological system models involve distributed parameters or at least the special case of time delay. Also, when a human being is an integral part of a control system, his distributed-parameter characteristics must necessarily be considered in determining the over-all performance of the system (133, 172, 178).

When the control of a distributed-parameter system is automated, it is desirable to control the process in a suitably defined optimal fashion. Thus, the purpose of an optimal design technique is to generate optimal control laws from which the variables of the system are determined with the result that a performance measure is minimized (or maximized in some cases) subject to constraints on these variables.

As evidenced in the literature review presented in Section 2.4, certain theoretical aspects of the optimal control problem for class-II-type distributed-parameter systems have been investigated by a number of authors. The following comments indicate, however, that these authors feel that additional research is necessary. A. G. Butkovskii and A. Ya. Lerner (111) state: that the known works on the theory of optimal control do not give any more or less general method of solution for systems which contain distributed parameters.

Thus far, no easily implemented methods exist for effecting the optimal design of general distributed-parameter systems. P. K. C. Wang and F. Tung (167, 209, 256) recommend: that future investigations concerning distributed-parameter systems should be directed toward establishing control theories for particular classes of systems, and at the same time, developing practical computational procedures for solving the functional equations derived by use of optimization theory.

Also, Pierre (135) states: that a topic worthy of additional research is the development of a general body of theory for effecting the design of optimal distributed-parameter control systems, as based on minimization of integral-square or mean-square error, subject to stated constraints. Therefore, the primary contribution of this thesis research consists of the development of a body of theory which satisfies this need to a large extent.

In order to form a basis for the application of optimal design techniques, several forms of block-diagram models are developed for class-II-type distributed-parameter systems; and because the choice of the performance measure is an important aspect of the optimal design problem, the development and interpretation of performance measures for class-II-type distributed-parameter systems is an integral part of the theory developed in this thesis. By using high-order rational fraction approximations to realize the optimal design, the bulk of the design problem can be performed on a general-purpose digital computer, with the result that design time and effort are reduced considerably.

1.3 SCOPE OF THE THESIS

Chapter 2 comprises a concise review of literature relative to the analysis, synthesis, and optimization of distributed-parameter control systems. This review enfolds over 200 papers, reports and books which are ordered topically to enable easy location of particular aspects of interest.

Chapter 3 embodies a development and interpretation of performance measures for class-II-type, distributed-parameter control systems. Performance measures are formulated for both deterministic and stochastic class-II-type systems.

In Chapter 4, a general body of theory is advanced for obtaining block-diagram models of linear class-II-type distributed-parameter systems. Several forms of models are considered, and in each case emphasis is placed upon simplicity and upon the ease with which the model lends itself to the application of optimal design techniques. Chapter 4 is complemented by Appendix A. Appendix A contains a brief review of the theory of the finite Laplace and finite Fourier transforms. In

Chapter 4, these finite integral transforms are used in conjunction with the infinite (one-sided) Laplace transform to derive Laplace-transform transfer functions for the block-diagram models.

Chapter 5 comprises a theory for the statistical analysis of linear class-II-type, distributed-parameter control systems. The theory of the distributed correlation function is developed for the analysis of distributed-parameter systems which are excited by stationary random inputs and/or are subject to stationary random disturbances.

Chapter 6 is devoted to developing the use of the Wiener-Hopf spectrum-factorization technique for the design of linear class-II-type, distributed-parameter control systems. The spatial-averaged-integral-square-error metric (introduced in Chapter 3) is used to obtain open-loop control laws for class-II-type distributed-parameter systems.

In Chapter 7, a technique is advanced for the minimum-integral-square-error design of class-II-type, distributed-parameter feedback control systems. A feedback signal is formed by using values of the output measured at equally spaced points along the spatial domain of the system. The Wiener-Hopf spectrum-factorization technique is used to obtain the optimal lumped-parameter compensator for the feedback system.

Chapters 6 and 7 are complemented by Appendices B, C, and D. In Appendix B a necessary condition for optimality is derived. Appendix C contains a brief account of the rational fraction approximations used to realize the optimal designs developed in Chapters 6 and 7. And Appendix D contains a digital computer program for performing multiplications, additions, and scalar multiplications of polynomials.

Finally, the concluding Chapter 8 provides a summary of the essential values of the thesis research and a statement of some aspects of distributed-parameter control theory which merit additional research.

CHAPTER 2

REVIEW AND EVALUATION OF OPTIMAL DESIGN TECHNIQUES FOR DISTRIBUTED-PARAMETER CONTROL SYSTEMS

2.1 DEFINITIONS AND PROPERTIES OF DISTRIBUTED-PARAMETER SYSTEMS

The purpose of this chapter is to present a systematic review and evaluation of the literature pertaining to distributed-parameter control systems. Before a meaningful literature review can be presented, however, some pertinent definitions and properties of distributed-parameter systems must necessarily be formulated. With this purpose in mind, a distributed-parameter element is defined herein as an element which can be characterized by one or more partial differential equations in two or more independent variables (usually time and one or more space variables). Furthermore, a distributed-parameter system is defined as one having one or more distributed-parameter elements in addition, possibly, to lumped-parameter elements. Because distributed-parameter systems contain distributed-parameter elements, the dynamics of such systems are usually characterized by partial differential equations. In some cases, it is convenient to reduce the partial differential equations to integral equations before proceeding with the analysis; and in special cases, differential-difference equations and other equation forms may be useful. In general, distributed-parameter systems can be nonlinear, time varying, and/or stochastic in nature.

In the control of distributed-parameter systems, interest is usually centered on certain distinct "input" or "output" quantities; it is convenient, therefore, both conceptually and analytically, to classify systems according to the nature of these quantities. Such a classification is formulated in reference 222, wherein distributed-parameter systems are divided into two major classes: Class I, those distributed-parameter systems which ostensibly contain no space-distributed inputs or outputs; and Class II, those distributed-parameter systems which contain space-distributed inputs or space-distributed outputs or both. A simple example will serve to illustrate this method of classification. Consider a control system containing a uniform transmission line; if the time relationship between the input voltage and the output voltage at the corresponding terminals of the line is of interest, the system is a class-I-type distributed-parameter system: e.g., pure time delay for a distortionless transmission line. In this case, the actual distributed nature of the problem is somewhat disguised in that time is the only independent variable present; the space variable appears only as a constant, the length of the line. However, if the relationship between the input voltage applied at the input terminals of the line and the potential distribution along the line as a function of both distance and time is of interest, the system is a class-II-type distributed-parameter system.

As indicated above, a system in which the output response is a scaled replica of the input --- delayed in time --- is a special form of a class-I-type distributed-parameter system. This type of behavior is herein designated by the term time-delay behavior. Many other terms describing this behavior are found in the literature --- "dead-time" (33, 34, 36, 77, 98, 146, 169, 189, 237), "time-lag" (3, 4, 10, 90, 92, 94, 151, 182), "transport-delay or -lag" (69, 102, 105, 147, 152), "retarded control or system" (8, 16, 68, 93, 119), "distance-velocity lag" (12), "lagging argument" (141), and "delayed argument" (235).

Quite often, the dynamics of a linear distributed-parameter element can be represented by a single linear partial differential equation of the form

$$\frac{\partial^m v}{\partial t^m} = H(t, x, f, v_x, v_t, \ldots) \tag{2.1}$$

where $v \equiv v(x,t)$ represents a state variable of the system, $f \equiv f(x,t)$ represents a distributed input function, v_x equals $\frac{\partial v(x,t)}{\partial x}$, v_t equals $\frac{\partial v(x,t)}{\partial t}$, and $H(t, x, f, v_x, v_t, \ldots)$ represents a given function of its arguments (the dots denote higher-order partial derivatives with respect to t and x). The left-hand member of equation (2.1) is assumed to be the highest-order derivative with respect to t that appears in (2.1). In order to complete the description of the element, the initial and boundary conditions for the particular element considered must necessarily be specified.

While equation (2.1) gives a description of a general distributed-parameter element, systems which enfold time delay are characterized more directly by ordinary or partial differential-difference equations. Thus, the dynamics of many time-delay systems can be characterized by one or more mixed ordinary differential-difference equations of the form

$$\sum_{n=0}^{k} a_n \frac{d^n v(t-\tau_n)}{dt^n} = f(t) \tag{2.2}$$

where τ_n is a constant, f(t) is an input function, and v(t) is a state variable of the system.

In the remaining sections of this chapter, the literature on distributed-parameter control systems is reviewed and evaluated. In view of the varied nature of this literature, it is convenient to divide this review into three major sections, as follows: in Section 2.2, the works pertaining to the analysis and synthesis of distributed-parameter control systems, including time-delay systems; in Section 2.3, the works pertaining to the optimal design of class-I-type, distributed-parameter control systems; and in Section 2.4, the works pertaining to the optimal design of class-II-type, distributed-parameter control systems.

2.2 METHODS OF ANALYSIS AND SYNTHESIS OF DISTRIBUTED-PARAMETER SYSTEMS

The material in this section is presented in two parts: first, the literature devoted to the analysis and synthesis of control systems enfolding time delay, hereafter designated as CSETD; and second, the literature devoted to the analysis and synthesis of more general types of distributed-parameter control systems. In this way, the methods which have been proposed for the two types of systems are easily distinguished from one another.

The literature on distributed-parameter control systems published prior to 1962 is well epitomized by Pierre (135). The all-encompassing review presented by Pierre includes over four hundred references related to the analysis, synthesis and optimization of distributed-parameter systems. Proir to the review presented by Pierre, Weiss (73) in 1959 and Choksy (92) in 1960 gave extensive bibliographies on time-delay systems. Because these extensive bibliographies on CSETD are already available, only a representative sample of the literature concerning CSETD is considered here. Major emphasis is placed on the work which has appeared since 1962.

2.2.1 Control Systems Enfolding Time Delay

As indicated in Section 2.1, the dynamics of many time-delay systems can be described by ordinary differential-difference equations of the form given by equation (2.2). The study of equations of this type was begun in the eighteenth century and since that time, hundreds of papers have appeared on this subject (see, for example, the extensive bibliography in reference 44). The classical methods of solution of ordinary differential-difference equations are presented in the recent books by Bellman and Cooke (153) and by Pinney (44).

In addition to the classical methods of solution, the operational calculus has been applied to obtain the solution of differential-difference equations. In 1934, Neufeld (2) developed the use of Heaviside's operational calculus for mixed differential-difference equations; and in 1936, Hartree, et al. (3, 4) developed an analysis technique for CSETD using Heaviside's operational calculus. Heins (5) in 1940 formalized the use of the Laplace transform for the solution of differential-difference equations. The solution of linear differential-difference equations by Laplace transform methods or by Heaviside's operational calculus leads to equations involving nonrational terms of the form e^{-sT} and e^{-pT}, respectively. Because of this, many of the conventional methods of finding the inverse Laplace transform are not applicable or are at least more difficult to apply.

Because the nonrational term e^{-sT} appears when Laplace transform methods are applied to CSETD, many novel techniques have been developed for obtaining the transient response of CSETD. Pipes (8) and later Elgerd (113) obtained the time response c(t) for a single-loop CSETD by use of the following form of inversion integral:

$$c(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{A(s)\, e^{st}}{1 + B(s)\, e^{-sT}}\, ds \tag{2.3}$$

where A(s) and B(s) are proper rational fraction functions of s. The procedure is to expand the denominator of the integrand in a power series in e^{-sT} and then to invert the series term-by-term using only a finite number of terms. Although this method is straightforward for simple systems, it is quite difficult to apply to more complex systems.

Many of the standard methods of frequency domain analysis, which are available for lumped-parameter systems, can be applied to time-delay systems. Thus, analytic methods for obtaining transient response from frequency response, as typified by the work of Solodovnik (46), Levadi (74), and Levy (72), are directly applicable. In addition, Blokh (85) developed a method for estimating the overshoot and the control time of CSETD from frequency-response data.

Because CSETD are characterized, in general, by characteristic equations which have an infinite number of roots, effecting the proper partial fraction expansion may require the use of Mittag-Leffler's theorem. Pierre (135) and Pierre and Higgins (164) applied Mittag-Leffler's theorem in finding the transient response of CSETD.

The root-locus method of analysis was first applied to single-loop CSETD by Chu (13) and later by Powell (36) and Tyner (237). But because the characteristic equation of CSETD has an infinite number of roots, the root-locus method is much more difficult to apply than in the lumped-parameter case. In order to avoid difficulties, Chu (13) used a dominant root analysis to obtain the approximate response of a system; however, McAvoy (213) showed that dominant root analysis is not valid in many instances.

Smith (61) considered some general properties of systems which have both polynomials and transcendental terms in their transfer functions and presents various transform techniques which are useful in the analysis of such systems. The z-transform was used by Boxer and Thaler (27) to obtain the approximate response of CSETD. When the z-transform method is used, however, the sampling period is restricted to an integral multiple of the time delay. This limitation is overcome by using the modified z-transform, as formulated by Schroeder (26), Siljak (105), and Pai (149). For sampled-data CSETD, Siljak (105) combined the use of the modified z-transform and Mitrovic's algebraic-graphic method (49).

The stability of CSETD is a very important aspect of control system design; and as is indicated in the next few paragraphs, it is a subject which has received considerable attention. Just as for lumped-parameter systems, the location of the roots of the characteristic equation is important in the determination of the stability of linear CSETD. In order to insure stability, all of the roots of the characteristic equation must lie in the left-half of the s plane, Re $s<0$. Although the characteristic equations of CSETD contain the transcendental term e^{-sT}, various stability criteria which are useful in determining the stability of lumped-parameter systems have been modified to render them applicable to CSETD. Satche's modification of the Nyquist criterion (11) and Chu's work on root-locus techniques (13) are typical examples of the modifications which have been accomplished.

In recent years, many papers have appeared, especially in Soviet literature, in which the stability problem for CSETD has been investigated. The Mikhailov stability criterion, which is similar to the Nyquist criterion, was used by Popov (129) and Sakawa and Sugata (190) to investigate the stability of CSETD. Pyatnikskii (142) determined necessary conditions for the structural stability of linear, single-loop CSETD --- structural stability is that property of a system which insures that the system can be made stable by choosing its parameters, without changing its structure. In another paper, Pyatnikskii (182) determined the upper bound on the degree of stability for a single-loop CSETD; that is, he determined the limit on the degree of stability when some of the parameters of the system are fixed. The stability of a time-varying CSETD has been investigated by Krasovskii and Osipov (192) using Lyapunov's stability theory and the theory of controllability. In addition to the works mentioned above, many other papers have been published on various aspects of the stability problem for CSETD, among which are references 15, 21, 22, 24, 48, 51, 67, 78, 87, 93, 94, 98, 141, 191, 230, 234, and 235.

Numerous papers are devoted to the presentation of a variety of techniques for approximating and/or simulating time delay. In many cases, a simplified model of the system is used in order to avoid treating the actual time-delay behavior of the system. This procedure is, in general, unsatisfactory unless the time delay is quite small; and even then, it may lead to erroneous results.

An often-used approach is the approximation of the nonrational term e^{-sT} by some form of rational function in s. If a suitable approximation --- one which is valid over the frequency range of interest --- is obtained, the usual methods of Laplace transform analysis for lumped-parameter systems can be applied to the approximate model. One of the first approximations to be used was the truncated Maclaurin series for e^{-sT}; however, this method may lead to erroneous results because the approximation does not converge to the actual function along the Laplace-transform inversion contour.

As compared to the truncated Maclaurin series, the Padé approximations for e^{-sT} (7, 54, 89, 91, 97) yield a more accurate characterization; consequently, they are frequently used in practice (20, 30, 102, 118, 225). Many papers are devoted to finding methods of simulating time delay for analog computer studies of CSETD, as typified by references 19, 35, 62, 102, 117, 121, 152, and 188. And, in yet others, various electrical or electromechanical methods for simulating time delay are presented (34, 69, 71, 137 138, 146, 156, 183, 187).

Some of the methods of nonlinear analysis have been modified for use in the analysis of both linear and nonlinear CSETD. For example, Cunningham (42), Kuwahara, et al. (189), and Strakhov (234) have applied phase-plane methods of analysis to CSETD. A sampled-data CSETD was analyzed by Strakhov (234) using a multisheet phase plane, the first sheet of which is valid for $0 \leq t \leq T$, the second for $T \leq t \leq 2T$, etc. Uhlrich (216) developed several finite difference techniques for the analysis of nonlinear sampled-data CSETD. The analysis of linear and nonlinear CSETD by describing function techniques (157) was effected by Kislyakov (88) and Sakawa (190), and the steady-state behavior of CSETD was also investigated by Kislyakov (83, 140) using the Krylov-Bogolybov method (157). Some additional references pertaining to the nonlinear analysis of CSETD are 86, 119, 120, and 136.

The synthesis of controllers for CSETD and/or the introduction of a time-delay element into the controller of a system has been investigated by a number of authors (16, 18, 23, 33, 37, 38, 44, 45, 52, 77, 82, 106). A novel method of block diagram manipulation was developed by Smith (33, 38, 77) for synthesizing a controller for a CSETD: minor feedback loops around the controller are used which appropriately include an analog of the time-delay and a model of the plant. In a somewhat different approach, Cohen and Coon (16) studied the effects of various parameters of proportional, proportional-plus-rate, proportional-plus-integral, and proportional-plus-integral-plus-rate controllers for CSETD; and Pinney (44) presented a systematic procedure for setting system parameters to obtain maximum damping in a particular system.

The use of "state-variable" techniques for CSETD was discussed by Kalman and Bertram (50); they state that their technique is applicable to systems with time delay if the state of the system can be predicted T seconds in advance, where T is the time delay constant. In a more recent paper, Koepcke (285) developed a synthesis procedure, using state variable techniques, by reducing the differential-difference equations to an equivalent infinite dimensional difference equation which can be solved using a general-purpose digital computer.

2.2.2 Distributed-Parameter Systems --- Other Than Time-Delay Systems

In the remaining portion of this section, a review of the literature pertaining to the analysis and synthesis of distributed-parameter systems of a more general nature is presented. As indicated in Section 2.1, the dynamics of these systems are, in general, described by partial differential equations of the form given by equation (2.1). The classical approach to partial differential equations is given in several recent books (56, 126, 202). However, methods of solving partial differential equations are not as well developed as those which exist for ordinary differential equations; this is one of the major difficulties encountered in the analysis and synthesis of distributed-parameter systems.

In a somewhat more applied approach, Fisher (227) derived the equations which describe the vibrations occuring in flexible airframes and discussed the classical mathematical techniques which are applicable to the solution of such problems. In a similar manner, Murray (217) investigated two methods for determining the eigenvalues of linear distributed-parameter systems. For electrical engineers, the wave and diffusion analogies given by Moore (81, 207) are helpful, both conceptually and analytically, because they relate several types of diffusive processes to the commonly-known electrical transmission line.

In addition to the classical solution of partial differential equations, the numerical solution of these equations has received considerable attention since the advent of high-speed digital computers which make such solutions feasible (see the extensive bibliography in reference 43). Several recent books (29, 43, 79, 202) contain numerical techniques which are applicable to the solution of partial differential equations. These computer-oriented techniques are quite useful in determining the time response of open-loop distributed-parameter systems.

In addition, several integral transforms can be used to obtain the solution of partial differential equations, as demonstrated in references 31, 40, 58, and 154. Although Laplace-transform methods yield transcendental equations in s (the Laplace variable) when applied to typical partial differential equations, they have been applied with varying degrees of success by several authors (154, 249, 259, 262). The solution of boundary-value problems by Laplace transform methods was demonstrated by Speigel (262) and Smyth (249). However, Smyth's total transform method --- which entails taking the Laplace transform with respect to both t and x, among other things --- is restricted to the solution of homogeneous partial-differential equations with boundary conditions specified for specific values of only one of the independent variables.

In a recent paper, Higgins and Oesterlei (221) presented the general theory of the finite Laplace transform (see also Appendix A). This transform technique can be applied to either ordinary or partial linear differential equations which are defined on finite domains. In Chapter 4, the utility of this method is demonstrated for linear distributed-parameter systems defined on a finite spatial domain.

Because the Laplace-transform transfer functions of distributed-parameter systems contain transcendental functions of s, the determination of the stability of such systems in the s-domain requires the use of some novel techniques. For example, Pierre (135), Pierre and Higgins (260), and Ball (132, 223) used a Riemann surface to represent the two-valued, distributed-lag transfer function $\exp[-(sD)^{1/2}]$. Using this Riemann surface representation, Pierre (253) developed an electric potential analog for obtaining frequency-response data. On the other hand, Ball used the transformation $w=s^2$ to map the s-domain Riemann surface for $\exp[-(sD)^{1/2}]$ into a w plane, wherein the root-locus technique can be applied to determine the stability of the system. In addition to the above work, the root-locus method has been applied to distributed-lag systems by Chu (13) and Rekoff (145) and to other forms of distributed-parameter systems by Stone (59) and Radant (25). However, Chu and Rekoff did not use a Riemann surface to represent the two-valued, distributed-lag transfer function.

The Nyquist criterion was applied to some simple distributed-parameter systems by Radant (25) and Leggett (17). Although Radant and Leggett gave no justification for its use, Desoer (284) recently examined the Nyquist criterion in a more rigorous manner and justified its use for linear, time-invariant distributed-parameter systems.

Using the Nyquist and related criteria, Kadymov (64) considered the stability of a system characterized by equations of the form $(1 - e^{-kv})/W(s) = 0$ where $v = (cs^2 + bs + a)^{1/2}$. In yet another paper, Papoulis (104) formulated several stability theorems for some single-loop distributed-parameter control systems. And, the stability of some nonlinear control systems with distributed parameters was investigated by Geleg (274).

Brin (143) developed a graphical and an analytical criterion --- similar to the Mikhailov (Nyquist) and Hurwitz criterion, respectively --- for transfer functions of the form $F(\sqrt{s}) = A(\sqrt{s})/B(\sqrt{s})$, where $A(\sqrt{s})$ and $B(\sqrt{s})$ are polynomials in $\sqrt{s}$. In a recent paper, Kolomeitseva and Netushil (273) have investigated the stability and transient response of systems of a similar form.

In several recent papers, Wang (195, 197, 209, 246) and Friedly (163) developed the use of Lyapunov's stability theory for certain classes of distributed-parameter systems. Using Lyapunov's direct method, Wang (209) formulated several stability theorems for linear distributed-parameter systems; and in references 195, 197, and 246, Wang considered the stability of a time-delayed-diffusive system which is described by partial differential-difference equations. For systems described by linear partial differential equations with linear boundary conditions, Friedly (163) developed a set of transformations that reduces the equations to the canonic form of Lurè (157); hence, the simplified criterion of Lurè can be applied.

To date, most of the analysis and synthesis of distributed-parameter systems has been accomplished by using some form of approximate model of the system. The approximations which have been used are varied in nature, but one common approach is the discretizing or "lumping" of the spatial variables which reduces the partial differential equations to ordinary differential equations (see, for example, 109, 161, 198, 215, or 290). A similar approach requires lumping both the space and time variables to reduce the partial differential equations to difference equations (130, 170). The difficulty with each of the above approaches is that as yet there is no general method for determining the error introduced by the lumping process; however, the errors introduced in specific examples or in problems of a specific class have been investigated. For example, Landon (200) derived bounds on the steady-state error due to lumping the space variable in the special class of problems characterized by the heat-conduction equation defined on an infinite or semi-infinite, one-dimensional spatial domain.

An alternate approach for linear systems is to approximate in the s-domain; that is, to approximate the nonrational functions of the Laplace variable s by a rational fraction function of s. If rational fraction approximations are available for the particular distributed-parameter characteristic considered, this type of approximation is especially helpful because it allows the use of many of the standard Laplace-transform methods of analysis that have been developed for lumped-parameter systems.

Rational fraction approximations have been developed for some common forms of nonrational distributed-parameter characteristics: $\exp[-(sD)^{1/2}]$ in (135, 257); $(s)^{1/2}$ in (60, 103, 210, 248, 277); and [cosh Ts + B sinh Ts] in (254). However, the infinite product approximation for [cosh Ts + B sinh Ts], when B is a function of s, is difficult to obtain because it requires finding the roots of [cosh Ts + B(s) sinh Ts] = 0.

These rational fraction approximations are especially useful for the analog computer simulation of nonrational Laplace-transform transfer functions (see, for example, 103 and 248). In Chapters 6 and 7, the rational fraction approximations for $\exp[-(sD)^{1/2}]$ and $(s)^{1/2}$ --- developed by Pierre (135, 257) and Kopal (60), respectively --- are used to demonstrate the optimal design techniques developed herein for class-II-type distributed-parameter systems.

In an attempt to determine the error introduced when lumped-parameter models are used to describe the distributed characteristics of the actual system, Ball (250) plotted dimensionless-parameter Bode diagrams of the ratio of the approximate distribution to the "exact" distribution as a function of normalized frequency (the "exact" distribution corresponds to the solution to the partial differential equation). For the two process-control systems considered, both the magnitude and the phase of the error introduced by several forms of lumped-parameter models were determined as a function of the normalized frequency.

Yet other authors (55, 174, 265, 270) have developed methods whereby simulation can be accomplished using an analog computer. Hong and Larson (174) developed a method for simulating a distributed-parameter system which can be characterized by a first-order partial differential equation defined on a finite spatial domain. And for distributed-parameter systems defined on a finite, one-dimensional, spatial domain, Meredith and Freeman (55) presented an analog computer simulation technique which consists of finding a solution that is valid for $0 \leq x \leq L$, in the time increment $0 \leq t \leq T$. The procedure is then iterated, with the now-known new initial conditions, to find the solution for the time increments $T \leq t \leq 2T$, $2T \leq t \leq 3T$, ..., until the desired solution is generated.

Interest in the availability of analysis techniques for distributed-parameter systems increased considerably after the development of distributed-RC networks. In 1959, Hager (70) published one of the first papers dealing with the analysis of distributed-RC networks; and since then, many papers have appeared in which this subject is treated with varying degrees of rigor, as typified by the following: 112, 122, 123 159, 196, 214, 220, 247, 258, 263, 278, and 279. Because the four-terminal parameters of distributed-RC networks are usually transcendental functions in the Laplace variable s, the exact model is difficult to treat analytically. Therefore, the majority of the papers which have appeared thus far contain low-order lumped-parameter approximations for the actual distributed network; and the more or less standard frequency response techniques, such as Bode diagrams, Nyquist diagrams, etc., are employed (70, 112, 122, 123, 196, 247, 263, 278).

Similar approximate methods of analysis have been developed for process control systems: Lamb and Simpkins (171) investigated the frequency response of some practical process-control systems using dimensionless-parameter Bode diagrams; and Thal-Larsen and Loscutoff (255) and Schuder and Binder (75) developed some approximate methods for finding the transient response of particular distributed-parameter systems.

In a somewhat different vein, Murray (218) used an eigenfunction expansion corresponding to the distributed-parameter plant to obtain an approximate model; subsequently, this approximate model was implemented in a closed-loop configuration, thereby allowing experimental verification of his mathematical analysis.

As an aid in the design and analysis of solenoid and transformer cores, Cooper (283) obtained a formal solution for the transient response of a rectangular region subject to Joulean heating.

For class-I-type distributed-parameter systems of a general nature, Pierre (135) and Pierre and Higgins (252) developed a sampled-data representative system in which the distributed-parameter elements of the control system are represented by an approximately equivalent sampled-data model. After the sampled-data representative system of the distributed-parameter element is formed, the analysis and/or synthesis of the resulting sampled-data system can be effected by use of the available techniques in sampled-data theory (57, 158).

For stochastic distributed-parameter systems, Wang and Tung (199) and Senin (233, 238, 271) developed methods of analysis. Wang and Tung (199) analyzed a distributed-parameter system that can be characterized by a first-order, linear partial differential equation which has <u>either</u> a random coefficient or a random input. In reference 238, Senin considered the passage of stationary random signals through linear distributed-parameter systems. And in two subsequent papers, Senin outlined the use of the statistical techniques, developed in his first paper (238), for the extrapolation of a random, distributed signal (233) and for the statistical design of class-I-type distributed-parameter systems (271). Although Senin's papers are extremely brief, the techniques appear to have considerable merit for the analysis and/or design of stochastic, distributed-parameter systems; therefore, in Chapter 5 of this thesis, his work is extended and clarified to render it more readily applicable to actual systems design and/or analysis.

The synthesis of a compensator for linear class-I-type, distributed-parameter systems was considered by Pierre (135) and Pierre and Higgins (282); a novel method of time-domain synthesis was developed which incorporates the use of a sampled-data representative system for the distributed-parameter elements (135, 252). Yet other methods of time domain synthesis (150, 194, 240) have been developed which might possibly be extended to render them applicable to synthesis of distributed-parameter systems.

Using frequency domain transformations, Wyndrum (220) and O'Shea (280) developed synthesis techniques for certain types of distributed-RC networks. Wyndrum (220) used the hyperbolic transform $w = \tanh[a(s)^{1/2}/2]$ in order to develop a synthesis procedure in the w plane; the procedure may be used to synthesize a prescribed immitance magnitude function with distributed-RC networks. In a similar manner, O'Shea (280) developed a transfer-function synthesis procedure in the $p = \cosh[(sRC)^{1/2}]$ plane.

2.3 OPTIMAL DESIGN TECHNIQUES APPLICABLE TO CLASS-I-TYPE, DISTRIBUTED-PARAMETER CONTROL SYSTEMS

In this section, the optimal design techniques which have been applied to class-I-type distributed-parameter systems are summarized; herein, in general, the phrase "optimal design technique" indicates that the application of the design technique results in a minimum ---, or, in some cases, a maximum --- of a given performance measure, i.e., a mathematical measure of system performance. Many novel techniques have been applied to effect the optimal design of class-I-type distributed-parameter systems; however, the majority of these techniques are applicable only to CSETD, and the literature devoted to the optimization of more general forms of class-I-type distributed-parameter systems is quite limited.

In one of the first papers on the optimization of CSETD, Oldenbourg (12) minimized the integral of the nonoscillatory error of a system controlled by a "step-by-step" controller by determining the optimal location of the roots of the characteristic equation. However, in this method, the length of the control interval is limited to being greater than, or equal to, the time delay of the system if the error is to remain nonoscillatory.

The use of nonlinear switching controllers for CSETD has been investigated by several authors (65, 68, 236). In reference 68, Mitsumskii indicated that self-oscillations will occur in certain systems if the time-delay value shifts from a specified design value. Mitsumakii suggested the use of a linear controller in the region of small error for such systems.

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