Economic analysis of farm firm growth in a semi-arid area of Montana
by Donald Keith Larson

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Agricultural Economics
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Abstract:
Economic uncertainty is one c£ the foremost problems in Montana's agricultural areas because production, price, and technological. uncertainties are recognized both, as elements of cost and as factors contributing to the instability of farm incomes. These risks and 'uncertainties introduce many complexities into the decision making process and influence the firm's growth process.

A large number of mathematical models of various types have been utilized to analyze growth problems in a dynamic framework and have contained features designed to aid decision making. The growth model developed in this study focuses on the effects of uncertainty, where the uncertain state of nature is production of winter wheat. The criterion used for decision making under uncertainty is maximization of expected utility as a function of net worth at the end of the planning horizon, with due consideration to the risk of bankruptcy which would destroy the firm.

The stochastic firm growth model was formulated within a dynamic programming framework in. which the stochastic nature of winter wheat yields and selected decision alternatives determine the transition of the process from period-to-period. The stochastic dynamic programming model was applied to a typical cash-grain farm in northcentral Montana. The only decision variable is rate of expansion while two state variables, capacity and ratio of debt to capacity,' are includ.d In the "model. Optimal expansion policies were derived for specified assumptions concerning various parameters in the stochastic model. These parameters include the cost for land, bankruptcy threshold, inability to borrow threshold, rates of interest for borrowed funds, levels of family consumption , the nature of government farm programs, and the form of the decision maker's utility function for wealth (net worthy.

The optimal policy for the basic model was to buy 320 acres of cropland during any year in which the constraints of the model did not prohibit the purchase. This policy was insensitive to reasonable changes in the consumpuion function, borrowing rate of interest, and land valves. However, the optimal policy was affected by a sufficient increase in the cost of land and when the deterministic income fron wheat certificates was removed. Also, as the risk-aversion coefficient was increased within the decision maker's utility function, the optimal policy became less expansionary in nature.

It is concluded that stochastic dynamic programming provides an effective means to study firm growth processes. The evidence indicates that this type of model adds to the nody of theoretical knowledge of firm growth and can also yield practical results which ere suitable to guide managers of growing agricultural firms especially in a stochastic environment.

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CHAPTER I

INTRODUCTION

I. The General Problem of Firm Growth

Growth of farms and other agricultural firms has been one of the significant phenomena in United States agriculture for several decades, i.e., the upward trend in acres per farm, output per farm, and capital requirements per farm. Enlargement may have stemmed from intensification of the farm firm on an existing land base or from operating more land. Among the forces that have encouraged growth, the most significant have been the continued technological advances, specialization, improvement in management's ability, and changes in the overall economic environment [50,pp.17-20; 8,p.1536]. Growth is dynamic in nature and renders the neo-classical, static concept of the firm somewhat unrealistic. With the development of high-speed computers and improved analytical techniques, an economic evaluation of firm growth and associated problems has taken on an increasing importance in recent years. Firm growth has attracted the interest of many economists, and resulted in the formulation of at least two regional research committees which have held symposiums on firm growth.

The agricultural firm consists of the means of production, land, labor, capital, and management; generally, land represents most of the total value of capital invested in the firm. Growth is some measure of the total sum of all the production means which are under the firm's command; these means of production are measured in units, such as acres of land, hours of labor, or dollars of assets. The growth process includes a choice of growth directions, among other things a choice that involves changes in the various means of production specified by a vector P, where $P = (P_1, P_2, \ldots, P_n)$ represents the amounts of each means of production. Also, the growth directions of the firm not only include elements in P but also a vector X, where $X = (X_1, X_2, \ldots, X_n)$ represents the production possibilities that the firm may select.

The problem of growth involves analysis of the decisions which affect the growth process over time. This process is influenced by the goals of the decision maker, e.g., maximization of the rate of growth, capital accumulation, leisure time, or minimization of risk and uncertainty to assure the firm's survival in the future. Growth problems involve identification of growth opportunities by the decision maker. Also, growth opportunities arise in other ways, both internal and external to the firm, that are not directly related to the decision maker. Differences in characteristics of the resources create growth problems in that the various means of production cannot be combined into one resource and various resources have different contributions to offer toward fulfillment of the various goals. Also, there is the problem of accessibility of the resources and means of control. Acquisition of financial capital creates growth problems not only in terms of the choice of how to finance, but by other factors affecting the growth processes such as capital withdrawals for consumption, social security, and Federal and state income taxes. If capital is borrowed, the amount that can be obtained in this manner depends on the type of resource involved.

Besides the above problems, external forces occur over time in conjunction with the growth process. Yields, prices, and other variables are subject to random variations, while technological and general economic changes produce risks and uncertainties that are important in decision making processes for firm growth.

Firm growth in agriculture connotes problems which are broader in scope than those usually associated with firm growth, i.e., the social costs and benefits of "orderly" growth, if in fact, growth is "orderly". There are the problems of income distribution in all farming sectors, total financial well-being of farm people compared with other sectors, and the impact growth has on the structure of financial markets serving agriculture. The macro-economic problems are important, but this study focuses on decision problems confronted by the farm firm.

II. Risk and Uncertainty in the Growth Process

Dryland grain farming in the semi-arid areas of Montana is recognized as a high-risk venture because variation in production is a function of annual precipitation and natural perils such as hail, fire, insect infestation, and plant diseases. A short growing season limits the choice of crops to those that are better adapted to the short season and comparatively dry climate. The external forces place farmers in a position of uncertainty with respect to prices and technology. In combination, these risks and uncertainties are elements of cost and are factors contributing to the instability of farm incomes.

Random yields in the planning environment of the agricultural firm have considerable importance in the growth process. The internally generated cash flow of the firm over time, together with borrowed capital funds, creates the financial capital for the growth process. This cash flow is subject to considerable variation due to the external forces affecting yields. Repeated decisions based on unlikely high yields are sure to lead to economic disaster. To base plans on average yields does not account for variations that could occur, in particular, the economic impact if several low yield years should occur sequentially. Consequently, a farmer would like to effectively evaluate the highly variable cash flow under alternative production and/or investment decisions. For example, he might focus on the decision alternative which returns the highest average amount of reinvestable surplus. This reinvestable surplus (or savings) can be of importance to the firm's repayment ability if external acquisition of capital funds is necessary in the growth process. In general, management's problem is to establish limits within which planning can be more effective for the growth of the firm and attainment of goals, in which growth can be a goal itself. Therefore, the farm operator needs knowledge about the relationship between random climatic variables and crop yields to establish limits within which he can manage and to make effective decisions within these limits.

The growth problem of the farm firm is dynamic. Traditional neo-classical static theory of the firm is unrealistic mainly because of its high level of abstraction and that perfect knowledge or certainty is assumed. Complexity of the dynamics can be overwhelming, and therefore, models of firm growth must be carefully formulated to capture the essential aspects without inclusion of any relatively unnecessary variables or relationships.

Moreover, the economic and physical parameters may need to be varied temporally for a given planning horizon, since stock and flow characteristics are fundamental to short- and long-run planning periods in economics. Consequently, fundamental to firm growth is a systematic analysis involving stocks and flows in a dynamic production-investment framework that involves both physical and financial accounts along with parameters for estimating the risk and uncertainty of returns. The temporal variation of economic and physical parameters is basic to production-investment decision making under risk and uncertainty. Thus, a production-investment analysis must be dynamic to reflect expected growth; if not, these decisions will not be based on correct expectations [55]. In paraphrasing Boulding [7,p.27] and Vickers [58,p.42], the firm has a balance sheet which describes its position as a static point in time. However, the balance sheet is a tool of dynamic analysis when it is recognized that the structure (financial and physical) it describes is subject to change, along with changes in the internal and external constraints, as the economic environment changes. Therefore, the balance sheet can serve as a basis for analyzing the stock and flow characteristics of the growth process, given that certain production-investment actions are carried out by the operator.

Another complexity arises in connection with the balance sheet. Firm sizes and financial situations are heterogeneous in nature. Size and financial status describe but a point in time with respect to the growth process; consequently, the model should be able to handle changes from known or relevant size-financial situations to others in a systematic manner over some given planning horizon. Financial survival and rate of growth are important economic considerations to the decision making process in the selection of an optimal policy for growth. Also, the incorporation of internal or management barriers to growth and measurement of their economic consequences are also important to an economic analysis.

Fundamental to analyzing the growth process is the selection of an appropriate goal. The growth process is considered to be influenced by the goals of the decision maker, i.e., the growth process may be different and involve different problems for various goals. Once a goal is selected, it can generally be formulated within the objective function along with a set of constraints. A worthwhile addition to the criterion function, in the growth process under uncertainty, is the inclusion of the expected utility hypothesis. 1/ Interesting and difficult problems arise when estimating the utility of, for example, surplus capital in the capital accumulation process and also estimating the utility of capital leakages for family consumption expenditures, social security, state and Federal income taxes. Preference for or aversion to risk is implicit in the utility function which further complicates the analysis. In general, a utility function describes the psychological nature of the operator, i.e., whether he is a gambler or is conservative, or lies somewhere in between these extremes. Measurement of risk aversion is a complex problem due to the diversity of behavior among farmers.

III. The Research Problem

The problem of this research effort is to employ the use of a dynamic growth model which will focus on the effects of production uncertainty in making an economic evaluation of the feasibility of farm firm growth in a semi-arid, grain crop producing area of Montana. Due to limited data and time, this research effort is limited to the winter

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1/ Maximization of the expected utility of an alternative, where each outcome is transformed into a utility value, weighted by a set of probabilities, and summed to give an expected value.

wheat area; solutions to this problem should have some application to other dryland agricultural areas in Montana and the Great Plains. The conclusions from this study should be of interest (1) to farmers, (2) to farm credit lenders, (3) to farm policy makers, and (4) provide a basis for research dealing with macro-economic problems of farm firm growth.

The specific objectives of this study will be:

1. To develop a general methodology or procedure for analyzing firm growth where the selection of an optimal decision over the planning period is dependent on stochastic variables.

2. Apply the general methodology to a representative winter wheat-cash grain farm in Northcentral Montana where the optimal decision for firm growth is based on the stochastic nature of the winter wheat yields. Various parameters in the model will be changed to determine their influence on the optimal decision and growth rates that can be achieved over a defined planning horizon.

3. Associated directly with objective 2 is to determine the nature of the probability distribution for a sample of winter wheat yields and estimate the parameters which characterize the probability density function.

4. Incorporate into the stochastic firm growth model a nonlinear utility function, which has a risk-aversion parameter that will be varied, to determine the effect on the optimal decision and on the

economic consequences of the firm's ability to grow as measured by a defined growth criterion function.

IV. Stochastic Sequential Decision Process

The nature of agricultural production is periodic, i.e., discrete time rather than continuous time best describes the agricultural production process. Thus the stochastic process specified to represent firm growth is a discrete time process. Stochastic means that for a decision, k, there is not one outcome, but a set of possible outcomes given by a stochastic vector with a known or given probability distribution function.

The decision process begins with the data and ends with recommendation of a course of action, which constitutes what Bross calls the decision maker [10,p.29], shown in Figure 1.

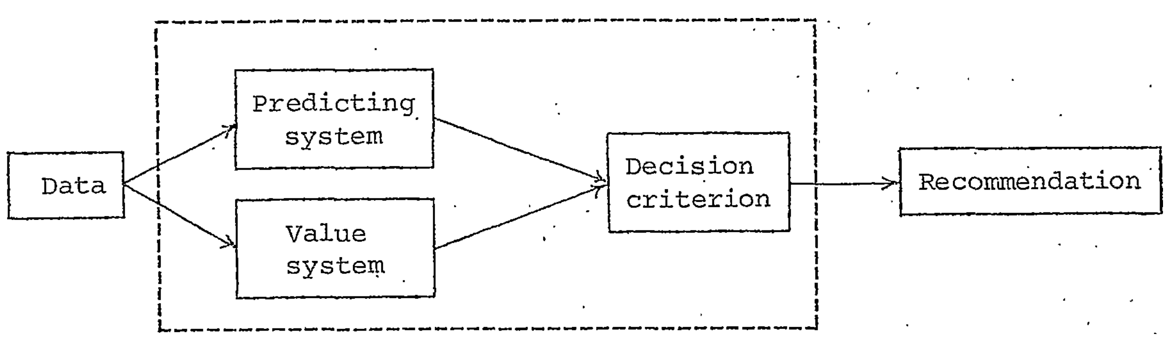

Figure 1. The Decision Maker.

The information flows in two ways: (1) into the predicting system where possible outcomes and courses of action are formulated, and (2) into the value system where the individual's utility or desirability associated with each outcome is formulated. Then by applying the decision criterion a course of action is selected.

For firm growth, the "data" would constitute returns or crop yields associated with stochastic variables (climatic variables, for example) from which conditional expectations of net returns can be derived from the "predicting system". Estimation of conditional expectations can usually be made statistically from a time series by regression analysis. These conditional expectations are some measure of value relevant to growth of the firm. Involved in the "predicting system" is a set of actions

$$(k_\ell \in K) \quad , \qquad k_\ell, \ \ell = 1, 2, 3, \ldots$$

open to the decision maker, which have a direct influence on the conditional probabilities associated with each k_ℓ.

Substituting the information from the "predicting system" into the "decision criterion", then each course of action is evaluated by some criterion such as

$$\begin{aligned} V_i &= \underset{k_\ell}{\text{Max}}\ E\ [\text{outcome}] \quad \text{or} \\ V_i^* &= \underset{k_\ell}{\text{Min}}\ E\ [\text{outcome}] \quad , \quad i = 1, 2, 3, \ldots, M. \end{aligned} \tag{1.1}$$

According to Penrose [45,p.88], "Growth is a process, size is a state." Consequently, the decision maker is faced with a problem of economic dynamics, and the decision process cannot ignore the interrelations in the time dimension, where the choice of outcome needs to be viewed in the context of a policy or strategy to be followed over the firm's planning horizon. Thus, to equation (1.1), the time dimension must be added which means that a decision made in the given year affects the optimal decision in future years.

The "value system" of the decision maker is complex in that it involves the individual's utility function, which in turn reflects a preference for (aversion to) risk and debt, as well as many other factors. Because of the complex nature of the value system, further discussion is postponed until later.

V. Review of the Literature

Because of the voluminous nature of the literature on firm growth that has appeared since 1965, it is impossible to do justice to those studies in the limited space and scope of this study. In light of the above, this section will be limited to the discussion of an article which itself constitutes a good review of past studies dealing with and related to the problems of firm growth.

An article by Renborg [48,pp.51-102] contains over 124 references related to the problems, theories, and methodology for firm growth. The purpose in Renborg's work was to analyze existing growth theories of the firm in terms of "their possibilities for controlling and directing the growth process of an agricultural firm over time," without discussing the various theories' abilities to describe or predict the firm's behavior and indicate structural changes. However, he does measure the theories against a set of ideas or problems that are involved in the growth process; although the set is arbitrary, it is worth restating. Renborg indicates that "good firm growth theories" should account for:

1. the entrepreneur's goals;
2. the available opportunities for growth;
3. the differences in characteristics of various resources;
4. the differences in characteristics of various sources of financial capital;
5. the fact that larger farms often require new management abilities by the farmer;
6. the fact that growth is a process over time and that growth costs are associated with this process; and
7. the fact that risk and uncertainty influence growth over time.

Problems associated closely with growth have traditionally been treated by economists in the past by using the neo-classical theory of the firm which gives some clues to the firm size problem but must be looked upon as an insufficient planning instrument for management. The neo-classical theory covers only profit maximization as a goal and assumes that all outcomes are known with certainty.

In light of the problems outlined by Renborg, this study would seem to be a contribution to the overall theory of firm growth, particularly since it focuses on item (7) of his list of important items to be included in any firm growth study. In fact, that is a primary feature of this research endeavor on firm growth--to focus on the stochastic aspects of the problem.

Items (1), (2), and (6) of Renborg's list of desirable features in growth theories are included in this study. The entrepreneur's goal is specified by assumption and is reflected in the criterion function of the growth model. The opportunities for growth are specified by a limited set of decision alternatives. The growth process is dynamic, and the interrelations in the time dimension are accounted for because production and/or investment choices made by the entrepreneur affects the future outcomes of the farm firm. The planning horizon is defined as opposed to solving for the optimal length of the planning period. The growth process has associated costs in terms of the resource cost and the fixed interest expense if the operator has to borrow funds.

Rate of growth and consequently, future value of the firm, is highly dependent on annual net income of the firm from which savings can be retained and reinvested. Borrowing of capital funds is allowed as if the funds came from one source, and consequently, the differences in characteristics of various sources of financial capital are not considered (see item (4) on page 12). Production costs, both fixed and variable, as well as consumption expenses for the firm manager and family, and personal income taxes are deducted from the firm's gross income to arrive at savings. Capital constraints are imposed in the model to reflect the inability to borrow and the associated economic consequences.

The farm management abilities of the operator in this study are treated as a constant, i.e., the operator of the small farm has the same abilities as those for the larger farm. There are no costs associated with the obtainment of new abilities as the firm increases in size. Thus, item (5) of Renborg's list is not a part of this study.

The behavior of the decision maker is a complex situation due to the multi-dimensional nature of risk taking attributed to many subjective components and susceptibility of a variety of motivational and other influences. Actual behavior is not depicted in this study, but a specified nonlinear utility function is assumed to describe the operator's attitude toward risk and uncertainty through the criterion function.

CHAPTER II

A STOCHASTIC FARM FIRM GROWTH MODEL

The study of firm growth has involved a whole series of analyses using mathematical programming methods. These methods have included production, investment, and financial alternatives in conjunction with a wide and flexible range of possibilities in formulating the goals of the entrepreneur. The goals were generally formulated as a combination of the objective function with a set of constraints. All of these mathematical programming models are flexible tools for handling multi-goal and decision making situations. The mathematical programming models used to solve production, investment, finance, and growth problems have been multi-period linear programming, integer linear programming, simulation, and recursive programming. The optimizing or programming approach of these methods have, for some studies, attempted to incorporate the existence of risk and uncertainty in the planning environment. Johnson, Tefertiller and Moore [31,pp.908-919], for example, used "a stochastic linear programming" model with the "distribution method of solution" to bring the effects of important stochastic variables into the analytic framework. However, their approach suffers from a rather serious restriction in that the decision maker knows with probability one (with certainty) the outcome of stochastic events a period in advance.

Therefore, in this study another mathematical programming model is developed that focuses on the effects of risk and uncertainty where the outcome of future events is not known in advance and the events are random. The choice of alternative actions is formulated in a multi-stage decision process in which a set of decisions is sought that will maximize (or minimize) an appropriate objection function, where the stochastic nature of the problem is taken into account. Dynamic programming, not to be confused with dynamic linear programming, is the mathematical model to be used. Boehlje and White [6] indicated that dynamic programming has not been applied specifically to firm growth problems. Minden [40] developed a conceptualized firm growth model using dynamic programming but did not apply the framework to an empirical problem.

Dynamic Programming Background

About 1951, Bellman became interested in the recurrence relation in decision theory which was spawned in the 1940's by a group of statisticians interested in statistical decision theory. Bellman [3,4,5] has been instrumental in the development of the theory of sequential, or multi-stage, decision processes, both of deterministic and stochastic types. From the theory of multi-stage decision processes, Bellman coined the term "dynamic programming" to describe the subject matter. It involves a certain conceptual framework which provides a mathematical tool that can be applied to many kinds of problems. There are many examples of multi-stage decision processes in the economic life of the real world such as investment programs, insurance policies, card games like poker or contract bridge, and the design of many types of experiments.

Dynamic programming is not described by a set of equations in a particular form nor do pre-programmed computational algorithms exist, as is the case in linear programming [20,p.213]. The use of dynamic programming (DP as it will be referred to in the remainder of this study) has not been as readily accepted as linear programming due partly to computational problems which arise in its use, and partly to a lack of full understanding of how it can be used in decision analysis. There seems to be more difficulty with model formulation in applying DP than is the case with linear programming. DP is not restricted to linear relationships and consequently, meaningful lumpiness of production and cost inputs, such as land, machinery, equipment, and buildings can readily be incorporated into the DP analysis. Also, DP constitutes a formal optimization process rather than the "if this, what then" type of approach characterized by most simulation studies.

Recent books by Howard [29] and White [59] have supplemented the work of Bellman and co-workers by attempting to set forth DP in a general, somewhat descriptive framework and yet indicate its computationally feasibility. Some of the initial formulation and computational obstacles of DP have been overcome by Burt and co-workers [11,12,13,14, 15,16,17] in applying the tool to a number of widely different practical farm management and other research problems in economics. In these studies, the economic consequences of risk and uncertainty, associated with certain defined random variables, was an important consideration.

Therefore, the work of Bellman, Burt and respective co-workers, Howard and White provides a launching pad for this study on farm firm growth, particularly the unpublished manuscript by Burt [12].

Definitions and Concepts of DP

In DP a sequence of decisions, the multi-stage decision process, is sought which will maximize (or minimize) an appropriately defined objective or criterion function. The optimal policy will depend on (1) the length of the process which can be divided into time periods or intervals called "stages", and (2) on the current "state" of the system. "State" refers to the sum total of all relevant information about the process at a given stage. In other words, the "state" of the process in farm firm growth describes the firm's debt position as well as the firm's resource base, such as total cropland, machinery, equipment, and buildings.

In firm growth, a set of decision alternatives or relevant actions exist that will change the state of the process in the passage from one stage to the next. This implied change is referred to as the "state transition". Transition from one state to another can be made deterministically (with certainty) or stochastically (according to a probability distribution); the latter is referred to as a stochastic decision process. In farm firm growth, the transition will be stochastic because of the uncertain nature of climatic variables that influence crop yields.

In dynamic programming, the objective or criterion function must be one of"Markovian nature" [4,p.54]:

> After any number of decisions, say k, we wish the effect of the remaining N-k stages of the decision process upon the total return to depend only upon the state of the system at the end of the k-th decision and the subsequent decisions.

Given the state of the process at a given stage, the decision process is to be independent of previous stages. In other words, the set of state variables must be sufficiently complete so that the optimal policy derived for a given state and stage depends only on that state and not upon how one got to that state. Thus, for DP to be applicable, the state-description variable(s) must include all necessary information to accomplish this separation. Consequently, "state" is a relative term depending on the analysis, and it is the job of the researcher to achieve adequate realism of state description in a manner consistent with the Markovian requirement [20,p.215].

In the context of farm firm growth, given that the firm now occupies some state i, which for example, could define the firm's debt or equity, the transition to state j in the following stage or year is dependent only on i and the decision made and not any prior history. 1/ This transition is what Howard [29] refers to as a recursive structure.

Under this recursive structure, optimization of the multi-stage decision process can be characterized by an optimal policy, or what is

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1/ See references [17,p.122;15,p.35] for an excellent description of the Markov requirement used in two practical problems.

more descriptively called a conditional decision rule. Optimization implies a rule which specifies a particular decision to be made at each stage for any given state that might occur in that stage, thus the name conditional decision rule. The policy or conditional decision rule is to be chosen such that a criterion function is optimized.

When dealing with multi-stage decision processes that meet the Markovian property, the basic property of optimal policies is stated [4,p.57]:

> An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

This principle, known as the "principle of optimality", justifies the linking of the (N)-stage decision process to the (N-1)-stage process, and suggests the inductive property of dynamic programming which leads to a recurrence relation.

Hopefully, the definitions and concepts of DP will become clearer as a general growth model is formulated and then applied to the specific firm growth problem.

The General Model

The general stochastic firm growth model which focuses on the effects of risk and uncertainty logically requires two stages. The first step is that of selecting efficient 2/ combinations of stochastic returns. The second part is the development of the stochastic firm growth model where the stochastic variables determine the transition of the process from period-to-period in estimation of an optimal decision rule.

For selection of efficient combinations of enterprises, the discussion is kept brief because the delineation of an efficient set of enterprise alternatives from which the decision unit can choose was not made in this first attempt to apply DP to a representative farm firm. The major reason will become clearer in the discussion of the general and empirical models. The objective is to call the reader's attention to a relevant, when applicable, part of growth analysis by presenting some sources which can be used as a basis in the analysis of efficient alternatives. An application should clarify any problems and details associated with an efficient set of alternatives and the estimation of the mean, variance, and covariance. The second part is the general model; but before proceeding, the firm's objective function should be discussed.

Measurement of growth is a problem for which one of several criterion functions can be used which incorporate the farm operator's goal

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2/ Efficient is used in the sense of a trade-off between mean and variance of returns rather than the economic sense--efficient combinations of enterprises are associated with highest expected return for a given variance of returns, or conversely the smallest variance for a given expected value.