Huygens principle applied to the cylindrical antenna boundary value problem
by Orville Kenneth Nyhus

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY in Electrical Engineering
Montana State University

Abstract:
The subject of this thesis is the application of Huygens' principle to the cylindrical antenna boundary value problem, The research described herein yields an explicit expression for the impedance of a cylindrical antenna.

The content of the thesis may be summarized as follows: First, the principles involved in applying the mathematical form of Huygens' principle to the cylindrical antenna boundary value problem are presented,, Second, a Green's function is derived which satisfies the conditions required by the mathematical form of Huygens5 principle, Two forms of Green'S function are obtained. Third, expressions for the current distri-bution and impedance of the cylindrical antenna are obtained, Fourth, numerical results are presented and compared with published measured data and with other theoretical results. The numerical results demonstrate that the impedance characteristics of a cylindrical antenna are correctly described through the application of Huygens' principle,

HUYGENS' PRINCIPLE APPLIED TO THE CYLINDRICAL ANTENNA BOUNDARY VALUE PROBLEM

by

ORVILLE KENNETH NYHUS

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Electrical Engineering

Approved:

Head, Major Department

Chairman, Examining Committee

Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

June, 1969

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Dr. J. W. Christie and Dr. J. L. Knox for their guidance and constructive criticism during this graduate research program. He also wishes to thank Dr. John Taylor for his very helpful discussions and suggestions regarding this work.

The financial support afforded by the Electrical Engineering Department of Montana State University and by a National Science Foundation Graduate Fellowship is gratefully acknowledged. Without this support, this graduate work would not have been possible at this time.

Last, but not least in importance, the author wishes to express his appreciation to his wife, Kathryn, for sharing the rigors of graduate school and for typing the manuscript, with the help of a five year old son, Daniel, and a two month daughter, Jennifer. To his family, the author wishes to express his regret at their loss of companionship during the tenure of graduate study.

TABLE OF CONTENTS

LIST OF FIGURES

Page

ABSTRACT

The subject of this thesis is the application of Huygens' principle to the cylindrical antenna boundary value problem. The research described herein yields an explicit expression for the impedance of a cylindrical antenna.

The content of the thesis may be summarized as follows: First, the principles involved in applying the mathematical form of Huygens' principle to the cylindrical antenna boundary value problem are presented. Second, a Green's function is derived which satisfies the conditions required by the mathematical form of Huygens' principle. Two forms of Green's function are obtained. Third, expressions for the current distribution and impedance of the cylindrical antenna are obtained. Fourth, numerical results are presented and compared with published measured data and with other theoretical results. The numerical results demonstrate that the impedance characteristics of a cylindrical antenna are correctly described through the application of Huygens' principle.

CHAPTER 1

INTRODUCTION

1.1 DESCRIPTION OF THE PROBLEM

An extremely important characteristic of an antenna is its driving point impedance. The value of this impedance must be known before the design of input or terminating networks can be accomplished. It is desirable to be able to calculate this impedance based on the construction and physical dimensions of the antenna. In most cases, this is a very complex problem.

The particular antenna to be dealt with in this thesis is the cylindrical antenna, shown in Figure 1.1. The terminals may be considered to be at any point along the conductor. Probably the most common case considers the terminals to be at the center. This is often called a dipole antenna and it may appear in the form of a suspended wire, or rigid rod.

A vertical monopole above a ground plane has nearly the same characteristics as a dipole, if one assumes an ideal ground plane. The radiation pattern is the same as a dipole in the half-space above the ground plane. That is, the monopole and its image in the ground plane essentially form a dipole insofar as the radiation pattern is concerned. The impedance of a monopole is half the dipole impedance.

The simplicity of the dipole and the monopole, together with their effective performance in practice, give them great utility. Consequently, the cylindrical antenna has been the subject of much research. One of the most difficult and most often attacked problems has been the analytic

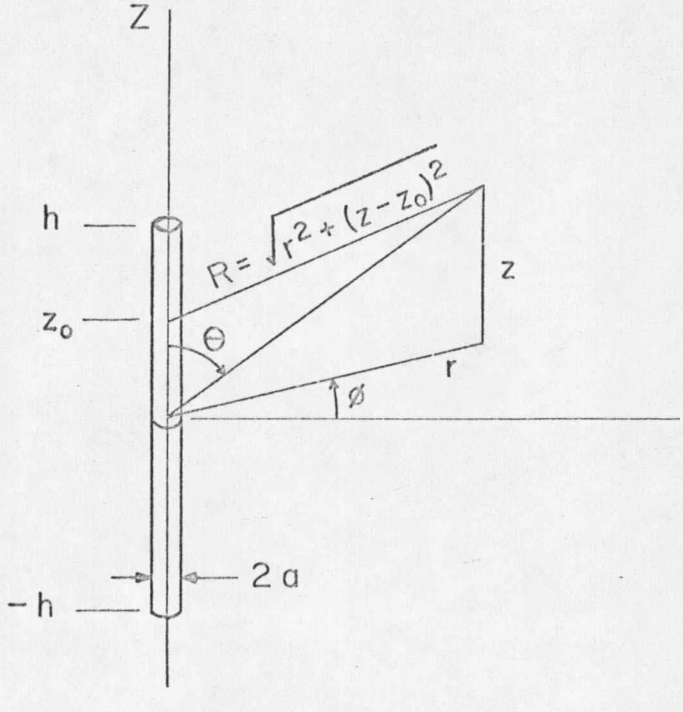

Fig. 1.1 Cylindrical antenna.

determination of the impedance based on the dimensions of the cylinder. This thesis describes further research into this problem.

1.2 HISTORICAL BACKGROUND

Work with antennas dates from the time of Hertz in the late 1880's. A review of progress in linear antenna analysis up to 1967 is given by Ronald W. P. King [35]. As King states, early antenna analysis was concerned with radiation properties at distances relatively far from the antenna. For this purpose the infinitesimal Hertzian dipole [26], [33], [35] was used as a convenient model. No attempt was made to determine the actual current distribution for finite linear antenna elements.

Solutions appeared as early as 1898 to boundary value problems for geometic shapes closely related to finite cylinders. These include prolate spheroids, or ellipsoids, [1], [45], [54], [58], [67], and the biconical antenna [33], [45], [58], [61]. The prolate spheroid approaches a line of finite length as its eccentricity approaches unity. Similarly, as the cone angle of the biconical antenna approaches zero, it becomes a finite line. The reason for choosing these geometric figures is that their surfaces may be represented in terms of a single variable in appropriate coordinate systems (Spheroidal coordinates for the spheroid and spherical coordinates for the biconical antenna are appropriate). This simplifies the analysis somewhat. However, these geometric figures are inadequate for representing actual cylinders, especially cylinders of relatively large radius compared to their length.

The late 1920's saw the beginning of analysis of finite linear antenna elements with assumed current distributions. No attempt was made to analytically determine the actual current distribution, but a sinusoidal distribution of the form

$I\ (z) = I\ (0)\ \sin k\ (h - |z|)$

was assumed. As King points out [35], this was based partly on measured values, partly on its adequacy in special cases, and partly on the mistaken idea that a section of two-wire transmission line bent outward at the end to form a dipole has the same spatial waveform on the dipole as on the lossless tranmission line. More recent measurements show that the sinusoidal distribution is a good first approximation for antennas near on odd number of half wavelengths long and whose length to diameter ratio 2h/2a is large, but is considerably in error for antennas whose diameter is near the same order of magnitude as its length.

The assumed sinusoidal current distribution yielded much more accurate radiation patterns, especially for elements more than a half wavelength long, than were obtained with the infinitesimal dipole. It also opened the way for impedance calculations, particularly the EMF method [9], [12], [17], [33], [45]. This method yields accurate impedance values if the actual current distribution is known. With the assumed sinusoidal distribution, reasonable results are obtained for antenna lengths near an odd number of half wavelengths. However, for center-fed dipoles near an integral number of wavelengths long the sinusoidal function vanishes at the driving point. This would indicate that the impedance is infinite, which is not realistic. Therefore more accurate representation is needed for the actual current distribution in order to obtain accurate impedance values for dipoles of arbitrary length.

In 1938 Hallén formulated the boundary value problem for the cylindrical antenna itself [23], [33], [35], [36], [45]. Many papers relating to this problem have appeared since, prominent among them being papers by King and Harrison [39] and by King and Middleton [41].

Hallén's formulation of the problem gives an integral equation in which the unknown current distribution function appears inside an integral. Techinques used to solve the integral equation include iteration [36], [39], [41], Fourier series [18], [65], variational methods [36], and numerical integration [49]. A review of some early methods of solving the cylindrical antenna problem is given by King and Harrison [40], and a comparison is given by Middleton and King [50].

Results obtained by these various methods for certain configurations are in good agreement. However, there are certain limitations on the dimensions of antennas to which these methods can be applied.

For example: In the iterative solution the numerical accuracy improves as higher order iterations are performed. But the complexity of the expressions involved limits the number of iterations which are conveniently performed. The highest order of iteration published is second order. A parameter used in the iterative solution is $\Omega = 2 \ln (2h/a)$ [39], [41], [45]. As Ω decreases (the antenna becomes thicker) more iterations are needed to maintain a certain level of numerical accuracy. Since the number of iterations is limited, the antenna dimensions for which meaningful calculations can be obtained are also limited. The thickest antennas for which results are published have $\Omega = 10$ (i. e., $2h/a = 150$).

Attempts have been made to apply these techniques to multi-element arrays with cylindrical elements [36], [38]. Progress is limited, being confined to rather special two element arrays. As with the single element, the boundary value problem yields integral equations for the current in each element.

The EMF method, using assumed sinusoidal current distributions, has been applied to the two element array [9], [17], [33]. Reasonable results are obtained when the elements are near a half wavelength long. For center-fed elements near a wavelength long the method does not yield reasonable results because the sinusoidal distribution is not a good approximation for the actual current distribution in this case.

1.3 SCOPE OF THE THESIS

This thesis research is concerned with a method for the solution of the cylindrical antenna boundary value problem which apparently has not been investigated before. The previous formulations [36], [39], [41] of the problem have used known boundary values for the magnetic vector potential to form an integral equation for the current. It is shown in this thesis that it is possible, using the mathematical formulation of Huygens' principle, to express the current in terms of an integral over the known boundary values of the magnetic vector potential. This is an explicit, closed form expression for the current distribution function. It should be expected that this will give numerical results which are not restricted by antenna dimensions.

Evaluation of the integral derived from Huygens' principle is a substantial problem in itself. Two approaches to evaluation of the integral are presented.

The scope of the thesis is fourfold: First, the application of Huygens' principle to the cylindrical antenna is presented. Second, techniques for evaluation of the integral expression obtained are described. Third, numerical values for the current distribution and impedance for the cylindrical antenna are presented which have been obtained using Huygens' principle. Fourth, aspects meriting additional study are discussed briefly.

The purpose of the thesis is to establish the fact that Huygens' principle can be applied to the cylindrical antenna boundary value problem. With this fact established, further research into this problem and into the possible application of Huygens' principle to other similar problems will be justified.

A comprehensive survey of the literature will not be presented because such a large number of publications deal with the cylindrical antenna. The reader is referred to reference [36] for an extensive bibliography.

CHAPTER 2

THE CYLINDRICAL ANTENNA BOUNDARY VALUE PROBLEM

2.1 INTRODUCTION

This chapter describes the model used for the cylindrical antenna. Consideration of the boundary conditions lead to an expression for the vector potential on the surface of the antenna.

The model, of necessity, contains some idealizations to simplify the problem. However, these idealizations are the ones most commonly used in the literature [41] and are not expected to appreciably affect the end results.

The presentation of boundary conditions is essentially the same as that presented elsewhere [39], but is included for review and completeness of the thesis.

2.2 THE MODEL

An illustration of the cylindrical antenna is shown in Figure 1.1. Such an antenna is usually made of a highly conducting material such as copper or aluminum. Although a term can be included in the analysis to account for finite conductivity [39], infinite conductivity will be assumed. For highly conductive materials, the antenna impedance depends almost entirely upon the radiation and induction fields surrounding the antenna rather than upon the conductivity of the antenna elements.

The excitation of the antenna is taken to be a voltage applied uniformly across an infinitesimal gap at the center of the antenna. No attempt will be made to describe how this might be accomplished in practice since the attachment of any real transmission line can be expected to perturb the current distribution on the cylinder. The theoretical analysis to follow assumes the antenna to be completely isolated in free space.

The infinitesimal gap at the driving point can be expected to exhibit a large capacitance. This capacitive reactance will not be considered as part of the antenna impedance. The antenna impedance of interest is considered to be due entirely to the radiation and induction fields. The gap capacitance could be approximated as a parallel plate capacitor in parallel with the rest of the antenna. This matter has been discussed by Schelkunoff [62].

An exact formulation for the current distribution on the cylindrical antenna would have to include the fact that current flows radially on the ends of a solid cylinder or over the ends and inside a hollow cylinder. A hollow cylinder of typical size can be expected to be operating in a cut off waveguide mode internally so that the current flowing on the inside surface would be small. Solid cylinders of relatively small radii also have little current on the ends since the current must vanish at the centers of the end faces. A comparison of tubular and solid cylinders has been presented by Einarsson [19]. In the analysis to follow, it will be assumed that the current vanishes at the edges of the end faces rather than at the centers, that is, only axial current flow is assumed to exist on the antenna. This eliminates the need for solving a boundary value problem for the end faces of the cylinder. To neglect the current on the antenna ends is common and has been discussed in the literature [41].