# Some of the characteristics of steady and oscillatory blood flow

by Bharat Ochhavlal Shah

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Chemical Engineering
Montana State University


Abstract:
An estimate of the difference between the Poiseuillian viscosity and the apparent viscosity at a given shear rate was made for blood under steady flow conditions. Shear stress-shear rate (viscometric) data for blood under steady flow conditions were used along with some numerical methods to obtain the difference between these two viscosities. The difference is significant at low flow rates and high values of the apparent viscosity.

Experimental work was done on oscillatory flow of blood at different frequencies of oscillation in rigid circular tubes. An apparatus was built so that oscillatory pressure and flow could be measured. The apparatus was tested with Newtonian fluids, such as glycerol solution, water, saline and plasma because the theory of oscillatory flow for a Newtonian fluid is understood very well.

Different red cell suspensions such as red blood cells in plasma, red blood cells in Dextran solutions, red blood cells in albumin-saline and hardened red blood cells in 0.5% Dextran 40 solution were used for the experimental work. Flow in two rigid circular glass tubes 400 and 776 microns in diameter was investigated. The experimental suspension hematocrit was usually about 45%, although 36% hematocrit was also used.

Experimental results showed that all pressure-time curves were sinusoidal for frequencies of oscillation of 0.5 hertz through 3 hertz. (The flow-time curves were always sinusoidal because of the inherent nature of the apparatus.) The pressure-flow data are summarized in this thesis and they are explained in terms of the known properties of blood and other red cell suspensions. These properties are the rheological data of blood, the visco-elasticity of blood, the aggregation of red blood cells and the deformation of red cells.

Red blood cells in high molecular weight Dextran solutions aggregate more strongly than they do in plasma. Hence, suspensions consisting of red blood cells in such Dextran solutions need higher pressure gradients to maintain the same flow compared to pressure gradients needed by red cells in plasma. From the experimental work with red cells in saline solution and hardened red cells in saline, it was concluded that red blood cell aggregation is significant process in oscillatory blood flow when the tube diameter is 400 microns.

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Chairperson, Graduate Committee

Head, Major Department

Graduate Dean

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# ABSTRACT

An estimate of the difference between the Poiseuillian viscosity and the apparent viscosity at a given shear rate was made for blood under steady flow conditions. Shear stress-shear rate (viscometric) data for blood under steady flow conditions were used along with some numerical methods to obtain the difference between these two viscosities. The difference is significant at low flow rates and high values of the apparent viscosity.

Experimental work was done on oscillatory flow of blood at different frequencies of oscillation in rigid circular tubes. An apparatus was built so that oscillatory pressure and flow could be measured. The apparatus was tested with Newtonian fluids, such as glycerol solution, water, saline and plasma because the theory of oscillatory flow for a Newtonian fluid is understood very well.

Different red cell suspensions such as red blood cells in plasma, red blood cells in Dextran solutions, red blood cells in albumin-saline and hardened red blood cells in 0.5% Dextran 40 solution were used for the experimental work. Flow in two rigid circular glass tubes 400 and 776 microns in diameter was investigated. The experimental suspension hematocrit was usually about 45%, although 36% hematocrit was also used.

Experimental results showed that all pressure-time curves were sinusoidal for frequencies of oscillation of 0.5 hertz through 3 hertz. (The flow-time curves were always sinusoidal because of the inherent nature of the apparatus.) The pressure-flow data are summarized in this thesis and they are explained in terms of the known properties of blood and other red cell suspensions. These properties are the rheological data of blood, the visco-elasticity of blood, the aggregation of red blood cells and the deformation of red cells.

Red blood cells in high molecular weight Dextran solutions aggregate more strongly than they do in plasma. Hence, suspensions consisting of red blood cells in such Dextran solutions need higher pressure gradients to maintain the same flow compared to pressure gradients needed by red cells in plasma. From the experimental work with red cells in saline solution and hardened red cells in saline, it was concluded that red blood cell aggregation is significant process in oscillatory blood flow when the tube diameter is 400 microns.

# INTRODUCTION

This study is an attempt to work towards the solutions of some of the problems involved in blood flow. The entire thesis is divided into two sections. Section A deals with steady flow and Section B deals with oscillatory flow. When the pressure and the flow are independent of time, the flow is termed as steady flow. When the pressure and the flow are some periodic functions of time, the flow is termed as oscillatory flow. The work shown in this thesis deals with such flows in rigid circular tubes.

## General Discussion

The basic function of the human circulatory system is to provide nourishment to various parts of the body and to remove waste materials. Blood, the fluid filling the circulatory system, is a suspension of various types of non-spherical, deformable particles (red blood cells, white cells, platelets) in an aqueous solution (plasma). While the suspending medium (plasma) has Newtonian rheological properties, the suspension is a non-Newtonian fluid (5). The parameter which has the greatest influence on the suspension flow properties is the hematocrit which is the volume of red blood cells per unit volume of whole blood.

$$\text{Hematocrit} = \frac{\text{Volume of red cells}}{\text{Volume of whole blood}} \times 100$$

Normally, the hematocrit is about 42-46%, but it can vary considerably from normal values in pathological situations.

In the human circulation system, blood is pumped from the heart through arteries, arterioles, capillaries, venules and veins, which return the blood to the heart. The microcirculation consists of the arterioles, capillaries and venules. The vessels are interconnected, forming networks (5,19). The size of the vessels through which blood flows vary from about 2-10 microns to 25,000 microns (5,19). The pressure drop across the human circulation is about 100 mm of Hg but about 80% of it exists across arterioles and capillaries. In-vivo experiments have shown that there are substantial regions of flow oscillations throughout the microcirculation. Hence oscillatory flow exists in the entire circulatory system.

In-vitro, steady flow experiments have been used to determine the rheological properties of blood. It is found that under low shear rates, blood acts like a Casson fluid (which has a yield stress, and a shear-dependent viscosity) (3). At low shear rates, red blood cells tend to reversibly aggregate. The aggregates are formed only by joining together the faces of the red cells. The length of the aggregates (rouleaux) varies inversely with the shear rate (5). This kind of phenomenon can occur in the living circulatory system and also in the experimental work. This phenomenon is reversible (5). At high shear rates red blood cells are dispersed and tend to deform. Red blood cell aggregation at low shear rates and red blood cell deformation at high shear rates may exert their rheological effects through a common

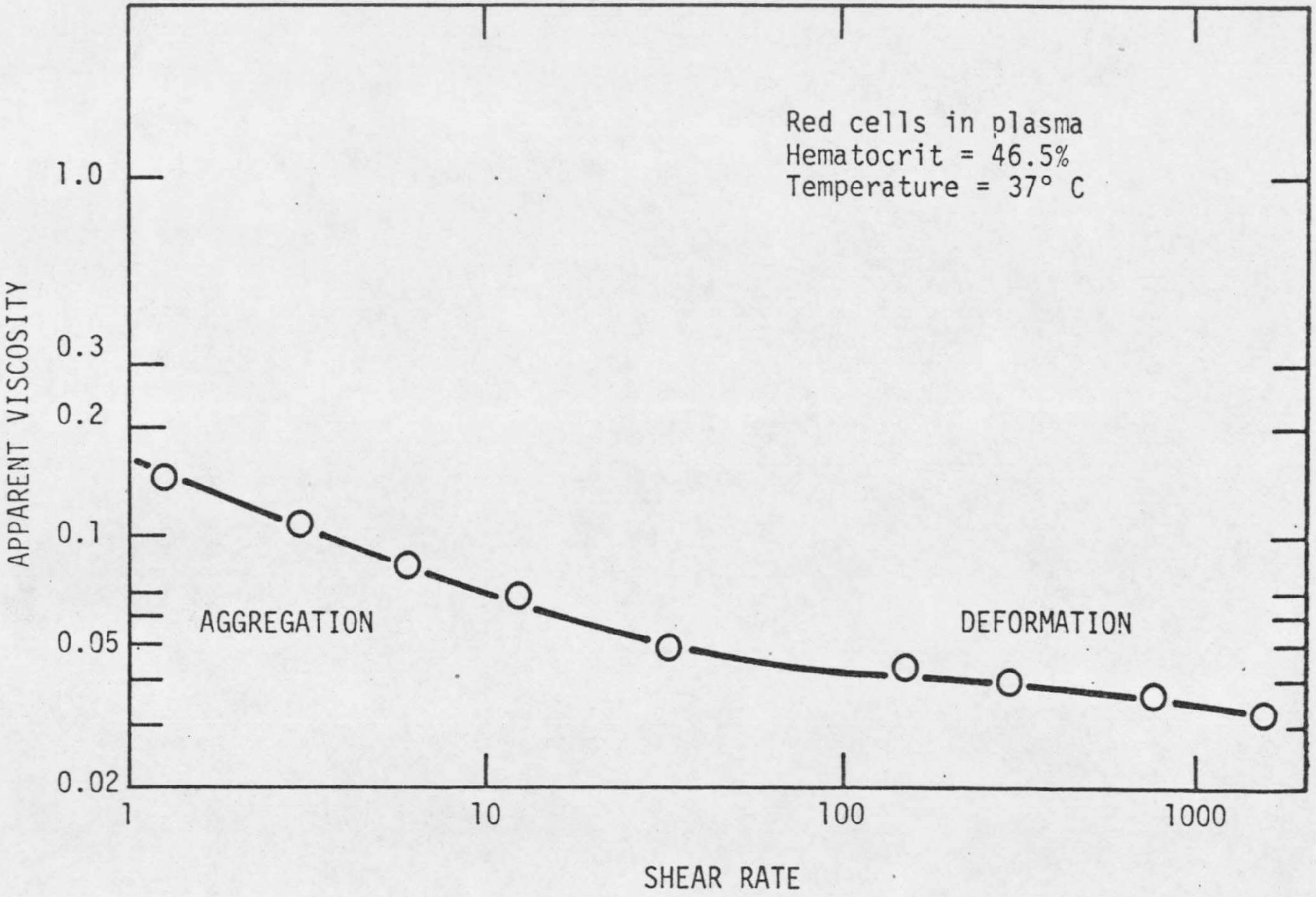


FIGURE I-1. APPARENT VISCOSITY VERSUS SHEAR RATE mechanism, shear-dependent changes in the effective cell volume. The effective cell volume is the fundamental determinant of blood viscosity (7). Apparent viscosity of blood is thus a function of shear rate and Figure I-1 shows the relation between apparent viscosity and shear rate.

Some in-vitro, oscillatory flow studies have been performed, mostly in large diameter tubes and oscillatory viscometers. The results from these studies have not been related to flow mechanisms of blood, and the interpretation of the data is often clouded by prior assumptions about the flow nature of the blood (e.g., the flow response component in phase with the flow driving function is assumed to be purely viscous while the component 90° out of phase with the driving function is assumed to be purely elastic).

Steady flow is discussed in Chapters II through VI. Oscillatory flow is discussed in Chapters VII through XII.

SECTION A

STEADY CONDITIONS

## II. STATEMENT OF THE PROBLEM

This section deals with steady flow. The rheological data (the relationship between the shear stress and the shear rate) can be obtained for blood at different hematocrits using viscometers such as the concentric cylinder viscometer and the cone and plate viscometer. Such data can be used to obtain the relationship between the bulk average velocity and the pressure drop for steady flow of blood in rigid circular tubes.

In reporting the results of blood flow studies, the data are sometimes given in terms of the viscosity calculated from the Poiseuillian equation for steady, uniform laminar flow of a Newtonian fluid through a tube. The viscosity calculated by this equation is called the Poiseuillian viscosity.

$$N_p = \frac{R^2}{8U}\left(-\frac{dp}{dx}\right)$$

Where $N_p$ = the Poiseuillian Viscosity

$R$ = outside radius of tube

$U$ = bulk average velocity

$\frac{dp}{dx}$ = pressure gradient per unit length of tube

This equation is valid only for Newtonian fluids and consequently its use with blood flow data is invalid, especially at lower flow rates.

This Poiseuillian Viscosity is sometimes compared to the apparent viscosity obtained from viscometric data. This comparison will not be valid, even for practical questions, if the blood flow is low enough so that the blood is very non-Newtonian. However, no measure of error due to the use of the Poiseuillian Viscosity has been reported, and the purpose of this work was to provide values of this error under various steady flow conditions. The difference between the Poiseuillian Viscosity and the apparent viscosity, for a given steady blood flow, reflects the magnitude of the non-Newtonian behavior of blood.

# III. REVIEW OF PREVIOUS WORK

Poiseuille may have done the first work on blood (5). He himself found that complex fluids such as blood do not necessarily obey the Poiseuillian equation which is discussed on P. 6. Other workers also found that deviations exist between the pressure drop calculated from the Poiseuillian equation and that measured by the experimental work on blood. Fahraeus and Lindquist performed some of the classical studies of blood flow in tubes. Scott-Blair and Merrill also worked with blood using capillary viscometers. The details of such individual works are not summarized in this thesis. Fung (12) gives a review of the work done through 1969 (5).

Work published in recent years by Agarwal (1) and Lipowsky (15) shows that they used the Poiseuillian equation for calculating the apparent viscosity of blood. Lipowsky (15) indicates that the velocity of blood, the blood vessel size and the pressure drop across the blood vessel were measured and the Poiseuillian equation was used to calculate the apparent viscosity. Agarwal (1) reports the relative viscosity of blood at varying hematocrits in pulmonary circulation. He used the Poiseuillian equation in his work. Additional examples of such usage of the Poiseuillian equation can also be found in literature.

An attempt has been made here to show the magnitude of the error between viscosity calculated from the Poiseuillian equation and the apparent viscosity under the same flow conditions.

# IV. THEORY AND CALCULATIONAL PROCEDURE

For a Newtonian fluid, the relationship between the pressure and the flow can be derived from the law of Conservation of Momentum for steady, laminar flow through a circular tube. The relationship obtained can be expressed by the Poiseuillian equation. This is discussed on P. 6. The viscosity derived from such equation is termed the Poiseuillian viscosity. This section describes the calculational procedure for calculating the difference between the Poiseuillian viscosity and the apparent viscosity.

The relationship between the shear stress and the shear rate (the viscometric or rheological data) at three different blood hematocrits were obtained with the help of the concentric cylinder viscometer and the cone and plate viscometer. The data are plotted and shown in Figures V-1 through V-3. The details of the viscometric apparatus and the procedures are not described here. The data for a given blood hematocrit were divided into several shear rate ranges and an analytical expression for each range was obtained. Either the linear or the non-linear regression analysis was generally used in this process. The data are represented as

$$G = f(T)$$

where G = shear rate

T = shear stress

or $$-\frac{du}{dr} = f(T) \qquad (1)$$

where u = velocity of fluid at radius r

Table V-2 summarizes the analytical expressions for different ranges at different blood hematocrits.

The relationship between the shear stress and the pressure drop per unit length of the tube for steady, uniform laminar flow of any fluid through a circular tube can be expressed as

$$T = -\frac{dp}{dx}\ \frac{r}{2} \qquad (2)$$

where $\frac{dp}{dx}$ = pressure gradient per unit length of tube

$r$ = radial coordinate

$x$ = longitudinal coordinate

This equation is derived by simplification of the momentum equations for such flow situation.

The bulk average velocity U is given by

$$U = \frac{\text{volumetric flow rate}}{\text{cross-sectional area}}$$

or $$U = \frac{\int_0^R 2\pi ur\ dr}{\int_0^R 2\pi r\ dr}$$

or $$U = \frac{2}{R^2} \int_0^R ur\ dr$$

where R = outside radius of tube

Integrating by parts and using the boundary conditions,

$$u = 0 \qquad \text{at } r = R$$

$$u = u_{max} \qquad \text{at } r = 0$$

$$U = \frac{1}{R^2} \int_0^R \left(-\frac{du}{dr}\right) r^2 \, dr \qquad (3)$$

Using r as an independent variable, the equations (1), (2) and (3) were solved simultaneously by numerical methods and the relationship between U and the pressure drop was obtained for different blood hematocrits. The computer program of the numerical method is shown in Appendix A.

The Poiseuillian equation was used to calculate the Poiseuillian Viscosity,

$$N_p = \frac{R^2}{8U} \left(-\frac{dp}{dx}\right)$$

where $N_p$ = Poiseuillian Viscosity

Apparent viscosity was simultaneously calculated by the definition,

$$N_a = \frac{T}{G}$$

$N_p$ and $N_a$ were calculated at a given wall shear stress and the difference between the two was calculated simultaneously by computer.

It may be remarked that for a Newtonion fluid,

$$-\frac{du}{dr} = \frac{1}{\mu} T$$

where $\mu$ = viscosity, a constant.

Using equations (2) and (3), it can be shown that

$$-\frac{dp}{dx} = \frac{8\mu U}{R^2}$$

The difference between $N_p$ and $N_a$ is plotted against $N_a$ and the graph is shown in Figures V-4 and V-5. $\overline{U}$ and the hematocrit are the parameters on the graph.

$$\overline{U} = \frac{U}{2R}$$

where $\overline{U}$ = reduced velocity

## V. RESULTS AND DISCUSSION

Figures V-1 through V-3 show the viscometric (rheological) data. Table V-2 summarizes the fitting of the rheological data into the analytical expressions. Figures V-4 and V-5 show the results of the calculations. It is seen that the difference between the Poiseuillian Viscosity and the apparent viscosity is significant at low bulk average velocity and at high values of hematocrits. When the apparent viscosity is about 6 centipoise and hematocrit is about 42% and reduced velocity is about 1 $sec^{-1}$, the difference between the Poiseuillian Viscosity and the apparent viscosity is about 0.5 centipoise (which is about 8% of the actual value of the apparent viscosity).

Table V-1 shows the comparison of results obtained from literature (15) and those obtained by this work. It should be noted that results obtained by Lipowsky (15) represent in-vivo experimental data and results shown in this thesis represent in-vitro experimental data. In Table V-1, the tube hematocrit is estimated from the results of Barbee (5). These tube hematocrits are not the experimental results of Lipowsky (15). The last two columns show the corresponding results from the graphs shown in Figures V-4 and V-5. Although the error between the Poiseuillian Viscosity and the apparent viscosity is not very significant, the estimated apparent viscosity is much lower than that obtained by Lipowsky (15).

Some additional examples of the use of the Poiseuillian