Curvature ductility of reinforced and prestressed concrete columns
by Bruce Alan Suprenant

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering
Montana State University
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Abstract:
Engineers are concerned with the survival of reinforced and prestressed concrete columns during earthquakes. The prediction of column survival can be deduced from moment-curvature curves of the column section. An analytical approach is incorporated into a computer model. The computer program is based on assumed stress-strain relations for confined and unconfined concrete, nonprestressed and prestressing steel. The results of studies on reinforced and prestressed concrete columns indicate that reinforced concrete columns may be designed to resist earthquakes, while prestressed concrete columns may not.

The initial reduction in moment capacity, after concrete cover spalling, of a prestressed concrete column could be as much as 50%. Analyses indicate that the bond between concrete and prestressing strand after concrete cover spalling is not critical.
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A thesis submitted in partial fulfillment
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APPROVAL

of a thesis submitted by

Bruce Alan Suprenant

This thesis has been read by each member of the thesis committee and has been found to be satisfactory regarding content, English usage, format, citation, bibliographic style, and consistency, and is ready for submission to the College of Graduate Studies.

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Signature  Bruce Alan Spearman
Date  June 11, 1984
Any thesis would be difficult to finish without the support of many people. I would like to thank my parents, Jack and Betty, for their encouragement and support over many years and my wife, Susan, for her patience. Also, my little one, Ashley, for sleeping through the nights.
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ABSTRACT

Engineers are concerned with the survival of reinforced and prestressed concrete columns during earthquakes. The prediction of column survival can be deduced from moment-curvature curves of the column section. An analytical approach is incorporated into a computer model. The computer program is based on assumed stress-strain relations for confined and unconfined concrete, nonprestressed and prestressing steel. The results of studies on reinforced and prestressed concrete columns indicate that reinforced concrete columns may be designed to resist earthquakes, while prestressed concrete columns may not.

The initial reduction in moment capacity, after concrete cover spalling, of a prestressed concrete column could be as much as 50%. Analyses indicate that the bond between concrete and prestressing strand after concrete cover spalling is not critical.
CHAPTER I

BACKGROUND

Introduction

The ACI Code \([10,11]\) requires the consideration of strength and serviceability in reinforced concrete design. Ductility is considered to be of secondary importance and is detailed into a structure, rather than designed. Because the ACI Code \([10,11]\) regulates or delegates ductility to a secondary level, many engineers have been confused by the importance of ductility for a structure or member and its implication in the selection of lateral load levels for earthquake design.

Dynamic analyses of structures responding to ground motions recorded during severe earthquakes have been performed \([35,58]\). These show that the theoretical elastic response inertia loads are much greater than the lateral loads recommended by design codes \([10,11,46,59]\) for earthquake loading. It is well documented \([9,24,30,37]\) that structures designed to the lateral load levels of design codes have survived severe earthquakes. This post earthquake survival has been attributed mainly to the ability of ductile structures to dissipate energy by post-elastic deformations. Although other energy dissipaters such as damping and soil-structure interaction help provide a reduced response, ductility of reinforced concrete members is considered to be the most important energy dissipater \([35]\).

This thesis presents a theoretical study of the ductility of square confined reinforced and prestressed concrete column sections under monotonic loading. The parameters which were investigated to determine their effect on the ductility of reinforced and prestressed
concrete columns are: axial load, longitudinal steel content, confinement, bond compatibility factor, concrete cover, and non-prestressed steel.

**Ductility for Seismic Loading**

A measure of the ductility of structures with regard to seismic loading is the displacement ductility factor defined as $\Delta_u/\Delta_y$, where $\Delta_u$ is the lateral deflection at the end of the post-elastic range and $\Delta_y$ is the lateral deflection at first yield [37]. The displacement ductility factor required in design may be estimated on the basis of the ratio of elastic response load to code load and typical values for displacement ductility factors usually range from three to five [9,56].

A rotational ductility factor for members has been calculated by some dynamic analyses as $\Theta_u/\Theta_y$, where $\Theta_u =$ maximum rotation of end of member and $\Theta_y =$ rotation at end of member at first yield. The design engineer, however, would benefit by information concerning the member section behavior, expressed by the curvature ductility factor $\phi_u/\phi_y$. Where $\phi_u =$ maximum curvature at the section and $\phi_y =$ curvature of the section at first yield. For the designer, required $\phi_u/\phi_y$ values are a more important index for ductility demand than the displacement or rotational ductility factors. This is because once yielding has commenced in a structure the deformations concentrate at plastic hinge positions and further displacement occurs mainly by curvature in the plastic hinge region [37]. Thus the required $\phi_u/\phi_y$ ratio will be larger than the $\Theta_u/\Theta_y$ ratio, and the $\Theta_u/\Theta_y$ ratio will be greater than the $\Delta_u/\Delta_y$ ratio.

The relationship between curvature ductility and displacement ductility has been illustrated with reference to a cantilever column with a lateral load at the end by Park and Paulay [37]. By using well known curvature-area theorems, the lateral deflection at the
column top for first yield and ultimate deformation can be determined. Therefore, the displacement ductility factor may be related to the curvature displacement factor as shown in Figure 1.

For multistory frames, the $\phi_u/\phi_y$ ratio required of members, designed according to present code seismic loading, has not yet been clearly established. The relationship between displacement and curvature ductilities can be complex [37]. Plastic hinges forming in beams and columns do so at different levels of axial and lateral load. Park and Paulay [37] have attempted to deduce curvature ductility demand of multistory frames using static collapse mechanisms.

The sequence of plastic hinge development in structures will influence the curvature ductility demand. Nonlinear dynamic analyses [37] have indicated that ductility demand concentrates in the weak parts of structures. The weak portion may act as a plastic hinge, enabling the remaining structure to respond elastically, or requiring the first hinge, i.e., weak portion, to achieve high curvature ductilities. Park and Paulay [37] have illustrated this by examining static collapse mechanism. Frame and shear walls which can be used for seismic resistance are shown in Figure 2. Possible mechanisms which could form due to flexural yielding and formation of plastic hinges are also shown in Figure 2. A column sidesway mechanism can form, if yielding commences in the columns at only one level, this would indicate that the columns of all other levels are stronger or carry less load. Such a mechanism requires very larger curvature ductility demands on the plastic hinges of the critical level [37], particularly for tall buildings. If, however, the yielding commences in the beams before the columns, a beam sidesway mechanism will develop [37]. This mechanism makes more moderate demands on the curvature ductility at the plastic hinges in the beams and column bases. This is basically due to the greater number of hinges required to
Member with ultimate curvature reached: (a) member, (b) bending moment diagram, (c) curvature diagram.

\[ \Delta_y = \frac{\phi_y \ell}{2} \frac{2e}{3} \]

\[ \Delta_u = \left( \frac{\phi_y \ell}{2} \frac{2e}{3} \right) + (\phi_u - \phi_y) \xi p (\xi - 0.5 \xi p) \]

\[ u = \frac{\Delta_u}{\Delta_y} = 1 + \left( \frac{\phi_u - \phi_y}{\phi_y} \right) 3 \xi p \left( \xi - 0.5 \xi p \right) \]

\[ \frac{\phi_u}{\phi_y} = \frac{\ell^2 (u - 1)}{3 \xi p (\xi - 0.5 \xi p)} + 1 \]

Curvature Ductility versus Displacement Ductility

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<tr>
<th>( \xi p/\xi )</th>
<th>0.02</th>
<th>0.05</th>
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Figure 1. Relation between displacement and curvature ductility.
Figure 2. Building structures under seismic loading and possible mechanisms [37].
form in the beam mechanism. A beam sidesway mechanism is the preferred mode of inelastic deformation. Hence most codes [10,11,30,59] require a strong column-weak beam approach when designing moment resistant frames.

Higher modes of vibration in the actual dynamic situation influence the moment pattern and it has been found [37] that plastic hinges in beams move up the frame in waves involving a few levels at a time. For static collapse mechanism involving a plastic hinge at the base, the curvature ductility demand for a given displacement ductility factor depends very much on the plastic hinge length. Recent experimental information is available [17, 38,50] on plastic hinge lengths for reinforced concrete columns confined by hoop and spiral reinforcement.

The static collapse mechanisms of Figure 2 are idealized in that they involve behavior under code type static loading. Due mainly to the effects of higher modes of vibrations, the actual dynamic situation is different, but a review of Figure 2 will give the designer a reasonable perception of the collapse mechanism.

The strong column-weak beam design concept attempts to have plastic hinges form in the beams rather than the columns. The seismic provisions of ACI 318-77 [10] and ACI 318-83 [11] require that at beam-column connections the sum of the moment strengths of the column should exceed the sum of the moment strengths of the beams along each principal plane at the connections. Park and Paulay [37] have shown that this code requirement will not prevent plastic hinges formation in the columns for three reasons:

(a) The beam input moment may be considerably higher than calculated because of statistical variations in steel yield strength and the onset of steel strain-hardening.

(b) Points of inflection may occur well away from the mid-height of columns at various stages during an earthquake [19], thus requiring a column strength to prevent hinge formation which is much greater than required by ACI 318-77 and ACI 318-83.
(c) Biaxial bending resulting from an arbitrary ground motion will generally reduce the flexural strength of a column [34].

It is evident that column flexural strengths greater than the ACI requirements would be needed if plastic hinges in columns are to be avoided. However, column hinging may occur.

A column sidesway mechanism, besides requiring large curvature demands on the section, will also require an engineer with considerable ingenuity, since the straightening and repair of the columns will prove to be difficult. A recent example of damage concentrating mainly in one story of a structure is the Olive View Hospital which suffered considerable damage because of the 1971 San Fernando earthquake [24]. The permanent lateral displacement of the structure after the earthquake, which was about 2 ft, resulted almost entirely from the deformations in the first story columns. Because of the extensive damage, the Olive View Hospital was demolished.

As an example demonstrating the necessity of analyzing column curvature ductility, consider the shock-absorbing soft story concept for multistory earthquake structures proposed by Fintel and Khan [19]. This concept is based on controlling the lateral forces that occur in the structure during an earthquake by forcing all the inelastic deformation to a soft story. This concept requires extensive yielding in the columns and therefore requires the columns to sustain large post-elastic deformations. It appears that the “soft story” concept should be implemented in conjunction with appropriate consideration to future repair.

The “Guide Specifications for Seismic Design of Highway Bridges, 1983” [20] indicates that bridge piers should be designed for a required curvature ductility. This is the first code to imply that curvature ductility is an important requirement in design.

In the preceding discussion, the difference between reinforced or prestressed concrete ductility was not considered. While the ductility of reinforced concrete has been the subject of numerous experimental and theoretical studies [6,13,16,17,21,22,25,30,35,36,39,
the ductility of prestressed concrete has received little attention [9, 51]. Because of the lack of experimental and theoretical studies, prestressed concrete in primary seismic resistant elements such as shear walls and frames has not met with the same code acceptance as reinforced concrete.

There is a lack of detailed building code provisions for the seismic design of prestressed concrete. For example, ACI 318-83 [11] contains special provisions for the seismic design of reinforced concrete structures but does not have corresponding provisions for prestressed concrete. This is also true of the Uniform Building Code [59], the SEAOC Code [46], and the seismic design provisions of the Applied Technology Council [44].

Lin [30] presented results of a dynamic computer analysis of a 19 story prestressed concrete apartment building. The building was first designed using the equivalent static earthquake forces specified by the 1961 Uniform Building Code. The elastic dynamic response of the structure to the North-South component of the 1940 El Centro earthquake was calculated. The El Centro earthquake produced forces and displacements about five times the Code values. This indicates the need for prestressed structures to be capable of developing large post-elastic deformations or be designed for greater equivalent static lateral loads.

The area under moment-curvature curves for prestressed and reinforced concrete give an indication of their energy absorption capacities. Despeyroux [18] and Candy [18] established very important differences between the behavior of reinforced and prestressed concrete. Candy illustrated the differences by using Figure 3. Cyclic loading tests have given curves for the two materials approximately as drawn. The shaded area represents the energy dissipation; the area up to the dashed line represents the energy absorbed. Thus although the energy absorbed by a prestressed member and a reinforced concrete member
Figure 3. Dissipation of energy in prestressed and reinforced concrete members [18].
may be the same, much more energy would be dissipated in the reinforced concrete member [35]. Candy concluded that the reduction in response caused by plastic strain is much smaller in prestressed concrete.

Figures 4 (a), 4 (b), and 4 (c) show idealized moment-curvature hysteresis loops for cyclic loaded prestressed and reinforced concrete members [58]. Nonlinear dynamic analyses of single degree of freedom systems responding to severe earthquakes have shown that the maximum displacement of a prestressed concrete system is on the average 1.3 times that of a reinforced concrete system with the same code design strength, viscous damping ratio, and initial stiffness [37]. The moment-curvature loops of prestressed concrete members can be "fattened" to reduce the displacement response, if non-prestressed steel is added to the member to provide energy dissipation. Figure 4 (c) shows a typical moment-curvature loop for partially prestressed concrete.

Although the displacement response of prestressed and reinforced concrete members are different, the possibility of yielding occurring at column ends makes it important to ensure that the columns are capable of behaving in a ductile manner. Hence, for reinforced and prestressed concrete columns adequate transverse steel in the form of hoops should be provided at the potential plastic hinge regions.

**Existing Code Requirements for Special Transverse Steel in Columns for Seismic Loading**

The ACI [10,11], SEAOC [46], and ATC [44] codes contain recommendations for the amount of special transverse reinforcement required in the ends of columns when ductile moment-resisting frames are to be designed for seismic loading. All columns of ductile moment-resisting frames designed according to the SEAOC code are required to have transverse reinforcement. ACI 318-77 requires special transverse reinforcement only when the column load exceeds $0.4 \phi P_b$. 
Figure 4. Idealized moment-curvature hysteresis loops for structural concrete systems [58].
For circular spiral steel the ratio of volume of spiral reinforcement to total volume of core is given as:

\[ \rho_s = 0.45 \left[ \frac{A_g}{A_c} - 1 \right] \frac{f'_c}{f_y} \]  

but not less than \(0.12 \frac{f'_c}{f_y}\).

When rectangular hoop reinforcement is used without supplementary cross ties, the required area of a bar leg is given as:

\[ A_{sh} = 0.225 \phi_h S_h \left[ \frac{A_g}{A_c} - 1 \right] \frac{f'_c}{f_y} \]  

Equation (1) was derived from the requirement that the strength of an axially loaded spiral column after the concrete cover has spalled should at least equal the strength before spalling. Based on test results achieved by Richart, Brandtzaeg, and Brown [47], it was assumed that when the concrete cover spalls, the spiral steel yields and exerts a radial pressure on the concrete core. This pressure increases the compressive strength of the core by approximately 4.1 times the radial pressure. Assuming that rectangular hoops are only 50 percent as effective as circular spirals as confining reinforcement, Equation (2) can be derived.

ACI 318-83 [11] proposes two new equations for spiral and loop confinement of columns. For spiral reinforcement,

\[ \rho_s = 0.12 \frac{f'_c}{f_y} \]  

and for hoop reinforcement not less than Equations 4 or 5

\[ A_{sh} = 0.3 (\phi_h f'_c/f_y)\left((A_g/A_c) - 1\right) \]  
\[ A_{sh} = 0.12 \phi_h f'_c/f_y \]

The philosophy of preserving the ultimate strength of axially loaded columns after spalling has been discussed [37]. A more appropriate criteria would emphasize the ultimate deformations and limited amount of moment reduction of concrete columns. The
confinement afforded by the transverse steel of Equations (1), (2), (3) or (4) will result in improved column behavior but these equations may not accurately represent the amount of confining steel necessary to achieve the required ultimate curvatures under earthquake loading. Roy and Sozen [48] indicated that confinement by rectangular hoops may not cause any strength increase, thus the philosophy behind the derivation of Equation (2) may never be realized in actual practice.

The previous discussion has not been related to prestressed columns. While investigations on confining steel for prestressed columns is in progress, current recommendations [10,11,46,59] for prestressed columns are the same as those mentioned for reinforced concrete columns due to a lack of information.

**Determining Transverse Steel Content**

An assessment of the quantity of special transverse steel required in columns for seismic design should take into account:

1. Required curvature ductility factor, $\gamma_u / \gamma_y$
2. Column axial load level, $P/P_b$
3. Rate of loading, $\dot{e}$
4. Longitudinal steel content, $\rho$
5. Effective prestress, $\varepsilon_{se}$
6. Stress-strain curve of concrete
7. Stress-strain curve of steel

A theoretical analysis requires a stress-strain curve for concrete confined by transverse reinforcement and a stress-strain curve for steel which includes strain hardening. Ideally, the effects of cyclic loading should be considered, but the increase in expense and complexity of analysis reduces its usefulness. The complexity of the analysis would increase for prestressed concrete column because of the scarcity of experimental results.
Park and Sampson [39] have been advocating the use of monotonic moment-curvature analysis to assess the expected ductility of columns subjected to earthquake loading. In order to indicate the acceptability of this practice, the behavior of concrete and steel materials under cyclic loading must be considered.

Sinha, Gerstle, and Tulin [53] were the first investigators to indicate that the monotonic stress-strain curve for unconfined concrete is an envelope curve for the cyclic stress-strain characteristics of unconfined concrete. Figure 5 shows this result. The investigators [53] showed that this was also true for idealized behavior of the reinforcing (see Figure 6). Karsan and Jirsa [25] confirmed the results of Sirha, Gerstle, and Tulin for unconfined concrete.

Experimental results for an unconfined concrete section, concrete and steel acting together, indicated that the monotonic moment-curvature is an envelope curve for cyclic moment-curvature curves [35,37].

Park and Sampson [39] were the first to assume that the behavior of confined concrete would follow the same general characteristics of unconfined concrete. They assumed that the monotonic moment-curvature curve was an envelope curve for cyclic moment-curvature curves for confined concrete. Just recently, investigators from the University of New Zealand have proven this behavior, for both square-confined and spirally-confined reinforced concrete columns [38,43].

Some work [50] has been performed to indicate that the monotonic stress-strain curve of confined concrete at high strain rates will also be an envelope curve for high strain rate cyclic loading. These results have yet to be totally confirmed.

There appears to be enough evidence [8,35] to suggest that the monotonic moment-curvature curve for prestressed concrete will be an envelope curve for cycling moment-curvature curves of prestressed concrete columns. Therefore, the assessment of maximum
Figure 5. Envelope curve for concrete [53].

Figure 6. Envelope curve for steel [53].
curvature ductility for prestressed concrete column sections subjected to earthquake will
be based on the same practice as reinforced concrete [37] monotonic loading.

**Derivation of Moment-Curvature Curves**

**Assumptions**

For the analysis of moment-curvature characteristics of reinforced and prestressed
concrete column sections under monotonic loading, the following assumptions will be
made:

1. Plane sections before bending remain plane after bending.
2. The stress-strain curve for nonprestressed and prestressed reinforcing is known,
3. The tensile strength of concrete is ignored.
4. The stress-strain curve for biaxially and triaxially confined concrete in compres­
sion is known.
5. The stress-strain curve for unconfined concrete cover follows the stress-strain
curve for confined concrete at strains less than 0.004. For concrete strains greater
than 0.004, all the concrete cover is assumed to have spalled.
6. Buckling of longitudinal steel in compression is assumed to be prevented by the
   close spacing of transverse reinforcement.
7. Short-term loading only is considered.
8. The prestressing steel is initially bonded to the concrete.

**Analysis of Sections**

Theoretical moment-curvature curves for reinforced and prestressed concrete sections
under flexure and axial load can be derived on the basis of the assumptions presented
earlier. The derivation here was originally presented by Park and Paulay [37]. The curva­
tures associated with bending moment and axial load are determined using the presented
assumptions and requirements of strain compatibility and equilibrium of forces. In this analysis only bending about the principal axes will be considered. For a given concrete strain in the extreme compression fiber, \( e_{cm} \), and neutral axis depth, \( c \), the steel strains \( e_{s1}, e_{s2}, e_{s3}, \ldots e_{sn} \) can be determined from the strain diagram.

The steel stresses \( f_{s1}, f_{s2}, f_{s3}, \ldots, f_{sn} \) corresponding to strains \( e_{s1}, e_{s2}, e_{s3}, \ldots e_{sn} \) can be found from the stress-strain curve for steel. The effective strain in the steel due to prestressing is added to the strains imposed by the loading. Then the steel forces \( S_1, S_2, S_3, \ldots, S_n \) can be determined from the steel stress and reinforcing areas.

The distribution of compressive concrete stress on a section can be determined from the strain diagram and stress-strain curve for concrete. For any given concrete strain \( e_{cm} \), the concrete compressive force \( C_c \) and its position can be defined by parameters \( \alpha \) and \( \gamma \), where

\[
C_c = \alpha f'c bc
\]

(6)

acting at a distance \( \gamma c \) from the extreme compression fiber. The mean stress factor \( \alpha \) and the centroid factor \( \gamma \) (see Figure 7b) for any strain \( e_{cm} \) can be determined for rectangular sections from stress-strain relationships:

\[
\varphi = \frac{\int_0^{e_{cm}} f'c fcede \, \varepsilon_c}{f'c e_{cm}}
\]

(7)

\[
\gamma = 1 - \frac{\int_0^{e_{cm}} e_{cm} fcede \, \varepsilon_c}{e_{cm} \int_0^{e_{cm}} fcede \, \varepsilon_c}
\]

(8)

The force equilibrium equations can be written as (see Figure 7c)

\[
P = \alpha f'c bc + \sum_{i=1}^{n} f_{si} A_{si}
\]

(9)
Figure 7. Theoretical moment-curvature determination. (a) Steel in tension and compression. (b) Concrete in compression. (c) Section with strain, stress, and force distribution [58].
\[ M = \alpha f'c \cdot bc \left( \frac{h}{2} - \gamma c \right) + \sum_{i=1}^{n} f_{si} \cdot A_{si} \left( \frac{h}{2} - d_{i} \right) \]  

where

- \( \alpha = \) mean stress factor
- \( \gamma = \) centroid factor
- \( f'c = \) concrete stress
- \( b = \) width of concrete section
- \( c = \) neutral axis depth
- \( f_{si} = \) steel stress of bar \( i \)
- \( A_{si} = \) steel area of bar \( i \)
- \( h = \) depth of concrete section
- \( d_{i} = \) depth to centroid of reinforcing \( i \) from extreme fiber in compression

The curvature can then be found as

\[ \phi = \frac{\epsilon_{cm}}{c} \]

The theoretical moment curvature relationship for a given axial load may be determined by incrementing the concrete strain, \( \epsilon_{cm} \). For each value of \( \epsilon_{cm} \), the neutral axis depth, \( c \), that satisfies force equilibrium is found by iteration using the secant method. The internal forces and neutral axis depth which satisfy equilibrium are then used to determine the moment, \( M \), and curvature, \( \phi \). By carrying out the calculation for a range of \( \epsilon_{cm} \) values, the moment-curvature can be plotted.

A computer program was developed to calculate the moment-curvature response of square rectangular reinforced and prestressed concrete sections under monotonic loading. The program is described in Appendix B.
Models

Stress-Strain Curve for Confined Concrete. Because transverse reinforcement provides passive confinement it would seem reasonable to expect confined and unconfined stress-strain curves to be approximately the same until the transverse reinforcement becomes effective.

Chan [16], Blume et al. [9], Baker and Amarakone [3], Roy and Sozen [48], Soliman and Yu [54], Sargin et al. [49], and Kent and Park [26] have proposed stress-strain relationships for concrete confined by rectangular hoops. On the basis of existing experimental evidence, Kent and Park [26] proposed the stress-strain curve in Figure 8 for concrete confined by rectangular hoops. At the present state of the art, Kent and Park's stress-strain curve is the most acceptable.

A discussion of the assumptions made in the derivation of Kent and Park's curve will follow the analytical description of the curve. As shown in the figure, there are three regions:

Region AB: \( e_c \leq 0.002 \)

\[
f_c = f'_c \left[ \frac{2e_c}{0.002} - \left( \frac{e_c}{0.002} \right)^2 \right]
\]

where

- \( f_c \) = concrete stress, psi
- \( f'_c \) = concrete cylinder strength, psi
- \( e_c \) = concrete strain, in/in

Region BC: \( 0.002 \leq e_c \leq e_{20c} \)

\[
f_c = f'_c \left[ 1 - Z \left( e_c - 0.002 \right) \right]
\]

where

\[
Z = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - 0.002}
\]
Figure 8. Stress-strain relations for concrete [58].

\[ Z = \frac{\tan \theta}{f'_c} \]
\[ \varepsilon_{50u} = \frac{3 + 0.002 f'_c}{f'_c - 1000} \]  
(15)

\[ \varepsilon_{50h} = \frac{3}{4} \frac{\rho_s}{S_h} b'' \]  
(16)

\[ \rho_s = \text{ratio of volume of transverse reinforcement to volume of concrete core measured to outside of hoops} \]

\[ b'' = \text{width of confined core measured to outside of hoops} \]

\[ S_h = \text{spacing of hoops} \]

Region CD: \[ \varepsilon_{20c} \leq \varepsilon_c \]  
(17)

where \[ \varepsilon_{20c} = \text{concrete strain at 0.2 of maximum stress on descending branch of stress-strain curve} \]

Region AB

The expression for the ascending part of the curve follows the generally accepted second degree parabola for unconfined concrete and assumes that the transverse reinforcement has no effect on the shape of this part of the curve or the strain at maximum stress. Investigations [6,52] indicate that transverse reinforcement may increase the maximum concrete compressive stress beyond \( f'_c \). However, this increase may be small, and in Roy and Sozen's tests [48], no increase in strength was found. Therefore, it is conservatively assumed that the maximum stress reached by the confined concrete is the cylinder strength, \( f'_c \).

Region BC

The falling branch is assumed to be linear and its slope is specified by the strain when the concrete stress has fallen to 50% of the maximum stresses as suggested by Roy and
Sozen [46]. The parameter Z defines the slope of the assumed linear falling branch. Equation for $\varepsilon_{50u}$ takes into account the effect of concrete strength on the slope of the falling branch of unconfined concrete, high strength concrete being more brittle than low-strength concrete. Equation for $\varepsilon_{50h}$ gives the additional ductility due to transverse reinforcement and was derived from the experimental results of three investigations [7,48,54].

Region CD

It assumed that concrete can sustain a stress of 0.2 $f'_c$ from $\varepsilon_{20c}$ to infinity. Yamashiro and Siess [63] had previously used the same assumption and Barnard [4] has shown that concrete can sustain some stress at indefinitely large strains. Roy and Sozen [48] found that with rectangular hoops as confinement, load deformation curves extended well beyond concrete strains of 5%.

**Stress-Strain Curve for Unconfined Concrete.** For simplicity the stress-strain curve for unconfined concrete in compression is assumed to follow the curve for confined concrete at strains less than 0.004. This assumption is important when considering concrete covering transverse reinforcement. At large strains it is likely that the unconfined concrete outside the hoops (the concrete cover) will spall away. Because the spalling process occurs gradually, it is difficult to determine the strain at which spalling of the concrete cover commences. This assumption of the ineffectiveness of the concrete cover has also been made by Baker and Amarakone [3], at strains greater than 0.0035, and by Blume [9] at strains greater than 0.004. Some investigators have ignored spalling of concrete cover at high strains. Kent and Park [26] assumed that at strains greater than 0.004, the concrete cover carries no stress.

In this analysis, once the extreme compression fiber reaches a strain of 0.004, it will be assumed that all the unconfined concrete (concrete cover) spalls, even the concrete cover near the neutral axis. This assumption was based on:
(a) test results [22,23,33,38] indicating that the presence of high quantities of steel hoppes precipitate concrete cover spalling.

(b) that concrete cover will most likely become ineffective under several reversals of high intensity loading such as would occur if subjected to earthquake loading.

**Stress-Strain Relationship for Nonprestressed Steel.** The stress-strain characteristics of longitudinal reinforcing steel including the effects of strain hardening are required for the determination of the moment-curvature relationship of reinforced concrete members. The stress-strain curves for longitudinal steel in tension and compression are assumed to be identical. This is considered to be a reasonable assumption although strain hardening may occur at a lower strain in compression steel.

While the stress-strain curve for the longitudinal steel is important, the same information for the confining steel does not appear to be very useful. Kent and Park [26] could find no experimental evidence to indicate that the strength of the transverse reinforcement had an effect on the stress-strain curve of the concrete. This is reflected in the parameter $Z$, where the strength of the transverse reinforcement is neglected in Equation 14. Therefore, the stress-strain characteristics of the transverse reinforcement are not required.

Figure 9 shows the stress-strain curve for steel subjected to monotonic loading which will be used in this investigation. It is assumed that the curve is composed of a straight line up to the yield strain, a straight line representing the yield plateau to commencement of strain hardening, and a curved line beyond strain hardening. The curved portion from commencement at strain hardening to ultimate strain is similar to an expression proposed by Burns and Siess [15] but modified by Kent and Park [26].

The stress-strain curve is in three parts as follows:

Region AB: $\epsilon_s \ll \epsilon_s \ll \epsilon_y$

$$f_s = \epsilon_s E_s$$  \hspace{1cm} (18)
Figure 9. Stress-strain relations for non-prestressed steel [58].
where \( f_s \) = steel stress
\( \varepsilon_s \) = steel strain
\( \varepsilon_y \) = yield strain
\( E_s \) = modulus of elasticity

Region BC: \( \varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh} \)
\[ f_s = f_y \] (19)

where \( f_y \) = yield stress
\( \varepsilon_{sh} \) = steel strain at beginning of strain hardening

Region CD: \( \varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_{su} \)
\[ f_s = \frac{m (\varepsilon_s - \varepsilon_{sh}) + 2}{60 (\varepsilon_s - \varepsilon_{sh}) + 2} + \frac{(\varepsilon_s - \varepsilon_{sh}) (60 - m)}{2 (30r + 1)^2} \] (20)

where \( m = \frac{(f_{su}/f_y)}{15r^2} (30r + 1)^2 - 60r - 1 \) (21)

\[ r = \varepsilon_{su} - \varepsilon_{sh} \] (22)

\( \varepsilon_{su} \) = steel strain at ultimate stress
\( f_{su} \) = ultimate steel stress

The parameters for the stress-strain curve were chosen based on information provided by Burns and Siess [15], Kent and Park [26], and Mirza and MacGregor [32]. The parameters for Grade 60 steel are:

\( E_s = 29,200 \text{ ksi} \)
\( \varepsilon_{su} = 0.15 \)
\( \varepsilon_{sh} = 0.016 \)
\( f_{su} = 110.8 \text{ ksi} \)
\( f_y = 60 \text{ ksi} \)
Stress-Strain Relationship for Prestressing Steel. The stress-strain relation for prestressing steel in tension was derived by Blakeley and Park [8] and illustrated in Figure 10. The stress-strain relation has been used by various investigators [8,58]. The relation comprises three regions which are defined by the following equations:

Region AB: \( \epsilon_p \leq \epsilon_{pb} \)
\[
f_p = E_p \epsilon_p
\]
in which \( \epsilon_p \) = steel strain
\( \epsilon_{pb} \) = steel strain at Point B (proportional limit)
\( f_p \) = steel stress
\( E_p \) = modulus of elasticity of prestressing steel

Region BC: \( \epsilon_{pb} \leq \epsilon_p \leq \epsilon_{pc} \)
\[
f_p = \frac{f_{pc} \epsilon_{pc} - f_{pb} \epsilon_{pb}}{\epsilon_{pc} - \epsilon_{pb}} + \frac{\epsilon_{pb} \epsilon_{pc} (f_{pb} - f_{pc})}{\epsilon_p (\epsilon_{pc} - \epsilon_{pb})}
\]
in which \( \epsilon_{pc} \) = steel strain at Point C
\( f_{pb} \) = steel stress at Point B
\( f_{pc} \) = steel stress at Point C

Region CD: \( \epsilon_{pc} < \epsilon_p \leq \epsilon_{pu} \)
\[
f_p = f_{pc} + \left[ \frac{\epsilon_p - \epsilon_{pc}}{\epsilon_{pu} \epsilon_{pc}} \right] (f_{pu} - f_{pc})
\]
in which \( \epsilon_{pu} \) = ultimate steel strain
\( f_{pu} \) = ultimate steel stress

Numerical values for the stresses and strains at Points B, C, and D should be obtained from experimentally measured stress-strain curves for the prestressing steel used.

The parameters used for Grad 270 in this study follows those determined experimentally by Thompson and Park [58]. They are:
Figure 10. Stress-strain relations for prestressing steel \(^\text{[58]}\).
\[ e_{pb} = 0.007 \]
\[ e_{pc} = 0.010 \]
\[ e_{pu} = 0.050 \]
\[ f_{pb} = 189 \text{ ksi} \]
\[ f_{pc} = 240 \text{ ksi} \]
\[ f_{pu} = 281 \text{ ksi} \]
\[ E_{ps} = 27,000 \text{ ksi} \]
CHAPTER II

ANALYTICAL STUDY

Introduction

For this analytical study, square reinforced and prestressed concrete columns were analyzed. Figure 11 shows the columns properties.

An important consideration in the analytical study is the determination of the maximum usable concrete strain. Since for the development of moment-curvature curves the concrete strain is incremented by 0.001, its choice would dictate somewhat the final values of curvature ductility. Limited experimental results have suggested that the maximum concrete strain is unlimited. However, this assumption is based on a “confined” concrete. If the hoop steel fractures, the core is no longer confined. Therefore, the question of permissible maximum concrete strain is complicated by the characteristics of the hoop steel.

For this study, the maximum concrete strain is assumed to be 0.052 in/in. The basis for this choice was (a) limited full scale experimental results measured concrete strains approaching 0.05 in/in and (b) the strain was chosen large enough to indicate any variation of the moment-curvature curves at values of high concrete strains.

To follow the concrete strains on all moment-curvature plots, a symbol was placed at every 5th increment of strain beginning at \( e_c = 0.003 \). Therefore, a symbol will appear on the plots for concrete strains of 0.003, 0.008, 0.013, 0.018, 0.023, etc. This should prove useful in the interpretation of the curves.

Values of 3000 psi and 5000 psi are used for the strength of the concrete in reinforced and prestressed columns, respectively. These values were used to approximate usual
Prestressed

- $f_{pu} = 270$ ksi
- $f_{yh} = 40$ ksi
- $f'_c = 5$ ksi

16" X 16"

3 layers of steel

Figure 11. Column properties.

Reinforced

- $f_y = 60$ ksi
- $f_{yh} = 40$ ksi
- $f'_c = 3$ ksi

16" X 16"

2 layers of steel
building practices. For the reinforcing steel, the assumptions are as follows: Grade 60 for non-prestressed longitudinal steel, Grade 40 for non-prestressed hoop steel and Grade 270 for prestressing strand. These also were considered practical choices.

The moment-curvature curves are plotted in non-dimensional form. The column size for both reinforced and prestressed columns is 16 in. x 16 in. This is a common building size. The choice of the amount of steel, 3% for the reinforced concrete column, was chosen for two reasons. First, it satisfied ACI 318-77 and ACI 318-83 steel requirements and second, it provided a strength capacity approximately equal to a prestressed concrete column with 8-½" Grade 270 strands. This should enable certain comparisons to be made between the two columns.

The balanced load for each column is approximately 0.35 $f'_c$ Ag. The axial loads considered in the analyses are 0.10 $f'_c$ Ag, 0.20 $f'_c$ Ag, and 0.30 $f'_c$ Ag. This represents the tension control region of the moment axial load interaction diagram. The value of 0.10 $f'_c$ Ag corresponds to 0.4 $P_b$, the value ACI 318-77 and ACI 318-83 indicates as the separation between beam and column behavior.

The following variables were considered for the reinforced concrete column analysis: confinement, strain hardening of non-prestressed longitudinal steel, longitudinal steel content, and concrete cover. For prestressed concrete columns variable investigated were: confinement, axial load, bond compatibility factor, concrete cover, and percentage of non-prestressed steel.

The analysis for the columns is presented in terms of curvature ductilities. The selection of $\phi_y$, the curvature when the steel yields, is dependent on the choice of $\varepsilon_y$ for the prestressing steel. Unlike non-prestressed steel, prestressing steel has no well defined yield point. ASTM suggests a yield point for prestressing steel. In this study, the ASTM recommendation was used, i.e., $\phi_y = 0.01$, but in no case was the yield point of the steel considered to have occurred at a concrete strain greater than 0.003. This was done to conform
to recommended practice [39] and also to facilitate comparison of reinforced and pre-
stressed moment-curvature curves.

For the prestressed concrete columns, the initial strain in the prestressing strand is
considered to be the result of an initial prestress force of $0.70 f_{pu} \times A_{ps}$. The concrete
strain, $e_{ce}$, is considered to be the resulting prestress force divided by the gross concrete
column area. However, after the concrete cover spalls, the concrete strain due to prestress
is increased due to the resulting smaller concrete core.

For each figure showing the analysis results, all parameters are indicated in the cap-
tions. These computer generated plots (Figures 14 through 32) are located in Appendix A.

Confinement

The effect of confining hoop steel on the moment and curvature ductility is shown in
Figure 14. The confinement, $Z$, is shown in Table 1 below with the hoop steel correspond-
ing to each $Z$.

<table>
<thead>
<tr>
<th>$Z$</th>
<th>Hoop Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td># 4 @ 5½&quot;, # 3 @ 4&quot;</td>
</tr>
<tr>
<td>12</td>
<td># 4 @ 9&quot;, # 3 @ 6&quot;</td>
</tr>
<tr>
<td>18</td>
<td># 4 @ 12&quot;, # 3 @ 8½&quot;</td>
</tr>
<tr>
<td>24</td>
<td># 3 @ 10½&quot;</td>
</tr>
<tr>
<td>30</td>
<td># 3 @ 13&quot;</td>
</tr>
<tr>
<td>36</td>
<td># 3 @ 15&quot;</td>
</tr>
</tbody>
</table>

The axial load considered is $0.20 f'_c A_g$ and the longitudinal steel content is 3% (dis-
tributed equally to the top and bottom). The concrete strength is 3000 psi.

At a concrete strain of 0.052, the curvature ductility for $Z = 6$ is 32.6 and for $Z = 36$
is 25.7. These curvature ductilities would usually be considered adequate for seismic load-
ing, according to Park and Paulay [37] and Park and Sampson [39].
The loss of moment capacity is disturbing. Just after the concrete spalls, the section loses approximately 13% of its moment capacity, regardless of the confining steel. Investigators [35,37,39] have proposed that a useful curvature ductility criteria be defined at a moment capacity of not less than 0.85 of the maximum moment capacity. If this criteria were adopted, an examination of Figure 14 indicates that the confinement represented by a Z of 6 and 12 would be acceptable. Other levels of confinement do not adequately maintain moment capacity at high ratios of curvature ductility.

ACI 318-77, Appendix A, requires columns to be confined at a Z level equal to 12. Figure 14 indicates this to be a prudent requirement for the proposed moment capacity reduction criteria.

Figure 15 represents 3 M - \( \phi \) curves of Figure 14. It shows moment-curvature ductility curves for Z values of 6, 12, and 18. Figures 16 and 17 show curves for Z values of 6, 12, and 18 for axial loads of 0.10 \( f'_c \) Ag and 0.30 \( f'_c \) Ag, respectively.

The effect of steel strain hardening is shown in Figures 18, 19, and 20. A point C represents the start of strain hardening of the compression steel and a point T represents the start of the tension steel strain hardening.

Figure 16 indicates the inadequacy of the ACI 318-77 provisions. For low axial load levels, the required confinement could be reduced. The curves of Figure 16 indicate a Z of 6, 12, and 18 to be satisfactory in maintaining moment capacity. The initial reduction in moment, due to concrete spalling, is approximately 8%. The moment capacity continues to increase after the first initial moment reduction. Moment-curvature ductility curves shown in Figure 17 represent an axial load of 0.30 \( f'_c \) Ag and Z levels of 6, 12 and 18. After concrete spalling occurs, there is approximately a 23% reduction in moment capacity. Regardless of Z, the moment reduction continues until the onset of steel strain hardening.
While the curvature ductility appears to be adequate, the large reduction in moment capacity due to concrete spalling could be disastrous during an earthquake. It indicates an anomaly with the ACI 318-77 provisions. At large axial loads, the confinement level has no effect on satisfying the minimum moment capacity criteria.

For the column section analyzed, an axial load of 0.30 $f'_c$ Ag represents approximately the balanced load ($P_b = 0.35 f'_c$ Ag) and an axial load of 0.10 $f'_c$ Ag represents approximately 0.4 $f'_c$. ACI uses 0.4 $f'_c$ as the dividing line between beam and column behavior. The results shown by Figures 14, 15, 16, and 17 indicate the ACI provisions to be adequate for columns with axial loads below 0.4 $f'_c$. However, above this level of axial load, the ACI provisions would be considered inadequate based on current criteria. The results indicated by this study would require the level of confinement to be increased as the axial load increases. The large reduction in moment capacity due to concrete spalling, at axial loads approaching $P_b$, is of great concern. The effect of confinement is of no value in counteracting this reduction. The model used for the concrete of course assumes all the concrete to spall when the extreme concrete fiber strain reaches 0.004. A limited amount of work [38, 43, 58] indicates that the initial reduction in moment capacity will occur more gradually than indicated by the curves. However, the final moment reduction will be close to the predicted moment capacity, regardless of the gradual spalling.

Non-Prestressed Steel Strain Hardening

From previous results of moment vs. curvature ductility curves, it is obvious that steel strain hardening increases the moment capacity of the column sections. It was felt necessary to investigate the beneficial effect of steel strain hardening since ASTM standards have no requirements for the initiation of steel strain hardening.

Strain hardening of reinforcing steel may begin at any plastic strain, $\varepsilon_{sh}$. For this study, strain hardening was assumed to begin at $\varepsilon_{sh} \approx 16 \varepsilon_y$. Figures 18, 19, and 20
represent an $\epsilon_{sh}$ of $\infty$ (no strain hardening), 16 $\epsilon_y$, and 8 $\epsilon_y$ for axial load levels of 0.10 $f'_c$ Ag, 0.20 $f'_c$ Ag, and 0.30 $f'_c$ Ag. The concrete strength is 3000 psi and $Z = 12$. Figure 18 indicates that early onset of strain hardening ($\epsilon_{sh} = 8 \epsilon_y$) may help in reducing the initial moment capacity drop. It also indicates the largest available moment capacity of the section. An $\epsilon_{sh} = 16 \epsilon_y$ represents the middle curve on all three figures. For an $\epsilon_{sh} = \infty$, it provides the lowest possible moment capacity. Figure 18 indicates that at low axial load levels (0.10 $f'_c$ Ag) the effect of steel strain hardening is not required in order to meet proposed moment capacity criteria.

However, Figure 19 shows that if steel strain hardening is not included in the analysis, the moment capacity will continue to decrease after concrete spalling due to the continued yielding of the steel. At higher axial loads, the occurrence of early steel strain hardening is shown to produce a rapid moment increase. However, if strain hardening is neglected, the moment capacity rapidly decreases.

Because ASTM does not standardize $\epsilon_{sh}$ or the required shape of the strain hardening region, it would appear wise and also conservative to neglect strain hardening when developing general design requirements. Neglecting strain hardening will place a greater emphasis on confining steel to achieve stable moment capacity at high curvature ductility ratios.

**Longitudinal Steel Content**

The steel content considered is 1%, 3%, and 5%. The values of 1% and 5% represent the minimum and maximum steel ratios permitted for earthquake design by the ACI Code. The axial load levels are 0.10 $f'_c$ Ag, 0.20 $f'_c$ Ag and 0.30 $f'_c$ Ag. The concrete strength is 3000 psi and $Z = 12$.

Figure 21 indicates that the effect of an axial load decreases the curvature ductility as the longitudinal steel content decreases. Table 2 gives the results of maximum moment reduction and curvature ductility obtained for different axial load and steel content.
Table 2. Moment Reduction and Curvature Ductility.

<table>
<thead>
<tr>
<th>Steel Content</th>
<th>1%</th>
<th>3%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Load</td>
<td>Max Moment Reduction</td>
<td>Curvature Ductility</td>
<td>Max Moment Reduction</td>
</tr>
<tr>
<td>0.10 $f'_c$ Ag</td>
<td>19</td>
<td>59</td>
<td>7</td>
</tr>
<tr>
<td>0.20 $f'_c$ Ag</td>
<td>36</td>
<td>32</td>
<td>15</td>
</tr>
<tr>
<td>0.30 $f'_c$ Ag</td>
<td>76</td>
<td>24</td>
<td>31</td>
</tr>
</tbody>
</table>

The results of Table 2 indicate the following:

1. As axial load increases, curvature ductility decreases. However, the higher the longitudinal steel content, the smaller the ductility decrease.

2. As axial load increases, the maximum reduction in moment capacity increases. The increase in moment reduction is less for higher steel ratios.

3. For a given axial load, the lower the steel ratio, the higher the curvature ductility. However, this behavior is only really evident for columns with an axial load less than 0.4 $f'_p$ (beam behavior).

Confinement of longitudinal reinforcing for a constant axial load has the same approximate beneficial result regardless of longitudinal steel content. Figure 22 shows this to be true. Some investigators [52, 54] have found that higher steel contents tend to assist the hoop steel in confining the inner concrete core. Due to the limited tests on the variety of positions the reinforcing may be placed, this refinement appears to be unwarranted.

Concrete Cover

The effect of concrete cover ($cc = 1.5, 2.5, 3.5$) is considered for axial load levels of 0.10 $f'_c$ Ag, 0.20 $f'_c$ Ag, 0.30 $f'_c$ Ag. It was no surprise that as the concrete cover increased, the amount of initial moment reduction increased. It is interesting to note that Figures 23,
24, and 25 show that as the axial load increases, the moment reduction increases for a constant value of concrete cover. This would indicate that statistical variations in concrete cover (workmanship) would be more important for heavily loaded columns.

Prestressed Concrete

Moment-curvature curves were developed for prestressed concrete columns for the following variables: confinement, axial load, bond compatibility factor, concrete cover, and non-prestressed reinforcing.

Confinement

For prestressed concrete columns, the effect of confinement is illustrated in Figure 26. The column was analyzed for an axial load of 0.20 $f_c' A_g$, and $f_c' = 5000$ psi and a concrete cover of 2.5 inches. It is obvious that an increasing confinement (increasing $Z$) provides a more stable $M - \phi$ curve at higher curvatures. For an ACI specified confinement $Z = 12$, the moment capacity remains relatively constant from curvature ductilities of 8 to 20.

In all curves, the moment capacity is increasing after the concrete cover spalls. This is due to the shape of the stress strain curve for the prestressing steel. For the prestressing steel used, the stress continues to increase for any increasing values of strain.

At an extreme compression fiber strain of approximately 0.018, for all curves, the moment capacity begins to decrease. After investigating the changing steel strains, neutral axis locations and compression zone characteristics ($\alpha$ and $\gamma$), the cause of the decrease is due basically to the compression prestressing strand reaching its yield point. When it reaches its yield point, it increases in stress, but at a very slow rate. The total compressive force ($C_c + C_s$) drops in magnitude due to the decreasing mean stress factor ($\alpha$) and little increasing compression steel stress. In order for the total tension force ($T_1 + T_2$) to be
equal to the total compression force, the distance from the extreme fiber in compression to the neutral axis must increase. This process effectively diminishes the internal level arm, and therefore decreases the moment capacity since $C_s$ and $T$ remain relatively unchanged.

Perhaps the most obvious and disturbing feature of Figure 26 is the large drop in moment capacity after the concrete spalls (at $e_c = 0.004$). Generally speaking, for any level of confinement, the reduction in moment capacity after spalling is approximately 50%. This must be considered very poor behavior of a column subjected to earthquake loading. The confinement is of little concern, since after spalling just about any level of confinement produces an increasing moment capacity.

**Axial Load**

The effect of axial load on the $M - \phi$ curve is interesting. As expected, since the axial loads considered are less than the balanced load, an increase in axial load should increase the moment capacity. This is indeed true as shown in Figure 27. Table 3 shows some important information obtained from Figure 27 and additional computer printouts.

<table>
<thead>
<tr>
<th>Axial Load</th>
<th>Reduction in Moment Capacity, %</th>
<th>Curvature Ductility</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10 $f'_c$ Ag</td>
<td>39</td>
<td>12</td>
</tr>
<tr>
<td>0.20 $f'_c$ Ag</td>
<td>51</td>
<td>22</td>
</tr>
<tr>
<td>0.30 $f'_c$ Ag</td>
<td>65</td>
<td>23</td>
</tr>
</tbody>
</table>

For the lowest axial load, 0.10 $f'_c$ Ag, the prestressing strand fractured at a concrete strain of 0.023. The curvature ductility was therefore very low. It is interesting to note that an increase in axial load increases the curvature ductility. This is somewhat different than the results of curvature ductility of reinforced concrete columns. The increase in axial
load permits a lower tension strain at each concrete strain considered. This is due to the lowering of the neutral axis to achieve internal force equilibrium.

The percent of initial moment capacity reduction increases as axial load increases. In all three cases, the moment reduction is still greater than current acceptable criteria.

**Bond Compatibility Factor**

It is well known that the outer shell of concrete columns will spall at a concrete strain of approximately 0.004. For a prestressed member, local spalling may leave the prestressing strand exposed. The bond or lack of bond between the strand and the surrounding concrete will invalidate the use of a strain diagram. If the strand is considered to have no bond at various spots or locations along the column, it can be treated in the same manner as an unbonded prestressed member. A bond compatibility factor (F) was used to relate concrete strains to steel strains, \( \epsilon_s = F \epsilon_c \). The bond compatibility factor was varied to indicate any problems associated with loss of bond due to concrete shell spalling.

Figure 28 indicates \( M - \phi \) curves for bond compatibility factors of 1.0, 0.9, 0.8, 0.7, 0.6, and 0.5. The maximum curvature ductility was approximately the same for all curves. The only major difference was for “perfect bond” where the strand fractured.

Between a curvature ductility of 8 to 16, the moment capacity begins to decrease. This again, as explained in the previous section, is due to the prestressing steel reaching its compression yield strain. As the bond compatibility factor decreases, the steel strain increases but at a decreasing rate.

The bond compatibility factor was introduced analytically into the problem as soon as the concrete spalled. The major difficulty is still the 50% reduction in moment capacity upon onset of concrete spalling.
Concrete Cover

The effect of the amount of concrete cover specified for a column was studied. This effect was primarily studied after other figures indicated the large reduction in moment capacity after the outer concrete shell has spalled.

The following amount of concrete cover was considered: 0.5, 1.5, 2.0, and 2.5 inches. This is shown in Figure 29. It is easily observed that a decrease in concrete cover, i.e., a decrease in the amount of spalled concrete area, will decrease the reduction in moment capacity at \( \varepsilon_c = 0.004 \). Table 4 indicates the decrease in moment capacity at 0.004 for each value of concrete cover.

<table>
<thead>
<tr>
<th>Concrete Cover inches</th>
<th>Reduction in Moment Capacity, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>19</td>
</tr>
<tr>
<td>1.5</td>
<td>36</td>
</tr>
<tr>
<td>2.0</td>
<td>43</td>
</tr>
<tr>
<td>2.5</td>
<td>51</td>
</tr>
</tbody>
</table>

ACI 318-77 and 318-83 recommend a clear cover value for precast prestressed concrete of 1.5 inches. However, even if a cover of 1.5 inches is used, the moment capacity reduction would be more than considered acceptable.

Note that for the concrete cover cases studied, as the cover increased, the curvature ductility increased.

Improving Curvature Ductility of Prestressed Columns

It has always been interesting to encounter technical literature arguing the pros and cons of improving the curvature ductility of a member by either adding non-prestressed steel or confining hoop steel. Both definitely would improve the ductility but to what degree was uncertain. Figure 30 shows \( M - \phi \) curves in which hoop steel has been increased or non-prestressed reinforcing was added.
The basic case for this example was considered to be a prestressed concrete column with light hoop reinforcing, i.e., that satisfying Chapter 7 of the ACI Code (Z = 76). The decision is made to upgrade the ductility of the column for earthquake loading. The two choices are to

(a) increase the hoop steel to conform to Appendix A of the ACI Code [10] (i.e., \(Z = 12\)) or

(b) to add non-prestressed steel to improve the ductility.

Figure 31 indicates that the obvious choice is to add more confining steel rather than non-prestressed steel. This is chosen assuming column strength to be satisfactory. The moment capacity continues to decrease for all columns that had nominal hoop reinforcing and bar reinforcing. The moment capacity remains fairly constant for all columns in which extra hoop steel was added. In addition, the curvature ductility is greater for the column with the extra hoop steel.

An increase of both extra hoops and bar reinforcing will improve the amount of moment capacity reduction after concrete spalls. Table 5 shows the possible beneficial effort of increasing both the hoop steel and bar reinforcing.

Adding non-prestressed steel provides no benefit in improving curvature ductility and very little in improving the initial reduction in moment capacity.

Table 5. Column Characteristics.

<table>
<thead>
<tr>
<th>Confinement (Z)</th>
<th>Non-prestressed Steel</th>
<th>Curvature Ductility</th>
<th>Reduction in Moment Capacity, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2 - #4</td>
<td>20</td>
<td>44</td>
</tr>
<tr>
<td>12</td>
<td>4 - #4</td>
<td>20</td>
<td>42</td>
</tr>
<tr>
<td>12</td>
<td>6 - #4</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>38</td>
</tr>
</tbody>
</table>
CHAPTER III

ENGINEERING EXAMPLES

To indicate the usefulness of the computer program developed, two practical design examples were investigated. The first is a reinforced concrete column in the Life Sciences Building (Johnson Hall) on the campus of Montana State University. The second, a standard PCI prestressed concrete pile is investigated using AASHTO guidelines for confining steel.

The Life Sciences Building is a ten story reinforced concrete structure designed for Seismic Zone 3. The column chosen for investigation, D-6, is a 20" x 20" interior column. The column was symmetrically reinforced with 16-#11 bars and confined with 3 overlapping #4 bars at 4" o.c. for the first 28" from the beam and then thereon with 3 overlapping #3 bars at 18" o.c. The engineer has provided greater confinement at expected locations of plastic hinging. The longitudinal reinforcing is Grade 60 and confining steel Grade 40. The required concrete strength, $f'_c$, is 3750 psi. The dead and live load levels from each floor are available from the drawings. The column dead load (first floor) is estimated to be 400 kips. A portion of the live load should be considered for the column. It was deemed appropriate to consider 20% of the live load. The column load considering total dead load and 20% live load is 500 kips. Using the above mentioned information, a plot of moment versus curvature ductility was generated. Figure 31 shows the curve.

The results of the analysis indicate that the difference in axial loads, 400 kips versus 500 kips, only slightly affects the $M - \phi$ curve for the confinement of 3 - #4's at 4" o.c. For a light confined column, the axial load affects the $M - \phi$ curve to a more significant extent.
The $M - \phi$ for the column section confined with 3 - #4's at 4" o.c. ($Z = 24$) exhibits acceptable behavior. That is, low initial moment reduction and stable or increasing moment capacity at high levels of curvatures ductility. For the column section confined with 3 - #3's at 18" o.c. ($Z = 130$), the initial moment reduction would be considered unacceptable.

The design of column D-6 appears to be appropriate. The key to the design is the assumption of plastic hinge length. As long as plastic hinging occurs in the heavily confined region, the column behaves satisfactorily. If, however, plastic hinging can occur in the lightly confined region, the column would be considered inadequate. This example illustrates the necessity of an accurate concrete section will add little to the overall displacement ductility of the structure if hinging occurs at other locations.

The PCI Design Handbook [40] was used as a reference for a typical prestressed pile. A 12" x 12" prestressed concrete pile with 8 - #" Grade 270 strands is considered. The required concrete strength is 5000 psi. The pile is considered to be subjected to axial load levels of $0.10 f'_c A_g$ and $0.30 f'_c A_g$. Hoop reinforcement was that suggested by AASHTO [20,40]. It was assumed that for a bridge foundation, the standard PCI piles are used, but would be required to satisfy the confining hoop requirements of AASHTO. Two levels of confinement are considered. A $Z = 44$ satisfying AASHTO, Standard Specifications for Highway Bridges, 12th Ed., 1977 and a $Z = 3$ satisfying Guide Specifications for Seismic Design of Highway Bridges, 1983 (AASHTO), are considered for the $M - \phi$ analysis of the pile.

The $M - \phi$ curves (Figure 32) for the pile indicate typical results. A high level of confinement provides for a smaller decrease in initial moment capacity and a greater moment at high curvature ductilities. The most evident response showed by the $M - \phi$ curves is the
almost complete loss of moment capacity of the lightly confined pile. This moment loss has to be considered disastrous. A pile will be very difficult to repair or replace. Prestressed concrete piles should be considered with extreme caution, if they are likely to be subjected to dynamic loads during their service life.
CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

From the results of the analysis, certain general and specific conclusions can be drawn regarding the curvature ductility of reinforced and prestressed concrete columns.

In general, certain shape characteristics of the moment-curvature curves were evident. Figure 12 represents the general shape of a moment curvature curve for reinforced concrete column and Figure 13 for a prestressed concrete column.

The most obvious characteristics of the \( M - \phi \) curve for reinforced concrete are the initial reduction in moment capacity at the onset of concrete spalling, the first increase in moment capacity due to strain hardening of the tension steel, and then later from the compression steel, and the final curvature ductility values controlled by the choice of maximum concrete strain.

For the \( M - \phi \) curve illustrated in Figure 13, the observable characteristics include a large initial reduction in moment capacity, the increase in moment capacity because of the steel performing in the elastic range, the decreasing moment capacity when the steel strains reach yield, and the final curvature ductility value being dependent on the fracture strain of the prestressing steel.

The confinement level tends to affect the increase or decrease in moment capacity while the choice of maximum concrete strain or steel fracture strain control the final value of curvature ductility.

Confining steel proves to be more beneficial for reinforced concrete columns than prestressed concrete columns. This benefit was because of the much greater reduction in moment capacity due to spalling of prestressed concrete columns. The confinement level

Figure 12.


Figure 13.
specified by ACI 318-77 [10], \( Z = 12 \), is adequate for low levels of axial load but not for higher levels of axial load. This conclusion applies to reinforced concrete columns.

The beginning of steel strain hardening is beneficial in increasing the moment capacity of reinforced concrete columns. The beginning of strand yielding is the start of a decreasing moment capacity for prestressed concrete columns.

The effect of axial load in changing the initial moment capacity is the same for both reinforced and prestressed concrete columns. However, its effect on curvature ductility is different. For reinforced concrete columns, a lower level of axial load increases curvature ductility, while a low level of axial load decreases the curvature ductility (strand fractures) for prestressed concrete columns. This effect on prestressed concrete columns could be because of the assumed value of strand fracture.

Concrete cover affects the amount of initial moment capacity reduction in both reinforced and prestressed concrete columns.

Local loss of bond—concrete to strand—appeared to have little effect on performance of a prestressed concrete column. A complete bond loss, especially near end anchorages, will cause a severe reduction in capacity and most likely column failure.

The most effective way to improve stable moment capacity at high curvatures and greater final curvature ductilities for prestressed concrete columns is by increasing the level of confinement.

For the proposed criteria of acceptable curvature ductility, i.e., permissible moment capacity reduction of approximately 15% and a curvature ductility greater than 16, a reinforced concrete column could meet the criteria, a prestressed concrete column could not. Generally speaking, the prestressed concrete columns could not satisfy the level of permissible moment capacity reduction. Also, depending on the stress-strain curve of the prestressing strand, curvature ductilities greater than 16 may not be reached.

Based on this analytical study, the following recommendations are presented.
Reinforced Concrete Columns

1. The required level of confinement varies with axial load.

2. Strain hardening not be considered in analysis which is used to develop criteria for building codes. This recommendation could change if ASTM were to consider manufacturing requirements which include the beginning of strain hardening.

3. Proposed criteria for confinement include statistically permitted variations in concrete cover.

For Prestressed Concrete Columns

1. The use of prestressed concrete columns be considered unacceptable for regions associated with earthquakes, until experimental tests verify otherwise.

2. That engineers (or ASTM) consider requiring a specified value of steel strain fracture for prestressing steel used in prestressed concrete buildings subjected to earthquakes.

3. For ductility, the level of hoop steel should be increased rather than adding non-prestressed steel.
REFERENCES


10. "Building Code Requirements for Reinforced Concrete (ACI 318-77)," American Concrete Institute, Detroit, Michigan, 1977.

11. "Building Code Requirements for Reinforced Concrete (ACI 318-83)," American Concrete Institute, Detroit, Michigan, 1977.


47. Richart, F. E., Brandtzaeg, A., and Brown, R. L., "The Failure of Plain and Spirally Reinforced Concrete in Compression," Bulletin No. 190, Engineering Experiment Station, University of Illinois, April 1929.


APPENDICES
APPENDIX A

COMPUTER GENERATED M– φ CURVES
Figure 14. Effect of confining hoop steel.
Figure 15. Effect of confining hoop steel; $P = 0.20 f'_c$ Ag.
Figure 16. Effect of confining hoop steel; $P = 0.10 f'_c$ Ag.
Figure 17. Effect of confining hoop steel; $P = 0.30 f'_c$, Ag.
Figure 18. Effect of steel strain hardening; $P = 0.10 f'_c$, $Z = 12$. 
Figure 19. Effect of steel strain hardening; $P = 0.20 f'_c$, $Ag, Z = 12$. 
Figure 20. Effect of steel strain hardening; $P = 0.30 f'_c \, Ag, Z = 12$. 
Figure 21. Effect of longitudinal steel content; \( Z = 12 \).
Figure 22. Effect of confinement and varying steel percentages; $P = 0.20 f'_c$ Ag.
Figure 23. Effect of concrete cover; $P = 0.10 f'_c$ Ag.
Figure 24. Effect of concrete cover; $P = 0.20 f'_c$ Ag.
Figure 25. Effect of concrete cover; $P = 0.30 f'_c A_g.$
Figure 26. Effect of confinement; $P = 0.20 f'_c$ Ag.
Figure 27. Effect of axial load; Z = 12.
Figure 28. Effect of bond compatibility factor; $P = 0.20 f'_c$, $A_g$, $Z = 12$. 
Figure 29. Effect of concrete cover; $P = 0.20 f'_c$ Ag, $Z = 12$. 
Figure 30. Effect of non-prestressed steel and hoop steel; $P = 0.20 f'_c$ Ag.
Figure 31. Ductility of concrete column Life Sciences Building, Bozeman, MT.
Figure 32. Ductility of prestressed bridge piles.
APPENDIX B

PROGRAM DOCUMENTATION AND LISTING
APPENDIX B

PROGRAM DOCUMENTATION AND LISTING

Documentation for Program Column

A. Problem Identification Cards:

The information input on the first ten cards is printed. There must be ten cards, either blank or containing relevant information concerning the problem. The information on the first card is printed as a heading on a moment-curvature plot, if a plot is requested.

10 Cards:

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-80</td>
<td>A</td>
<td>Problem information to be output with results.</td>
</tr>
</tbody>
</table>

B. Problem Iteration Control Card:

This card identifies the number of problems to be run at one time.

1 Card:

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>I2</td>
<td>IK</td>
</tr>
</tbody>
</table>

If IK > 1, then the input cards for Sections C through H are required for each IK case.

C. Plotting and Printing Control Card:

This card provides the user with the opportunity to select the appropriate plots and output for a problem. It is possible to “Print All” which includes the neutral axis, concrete compression force, moment, and curvature for each increment of concrete strain and steel areas, locations, strains, stresses, and forces in the steel for each increment of concrete
strain. A summary of concrete strains, moments, and curvatures is then printed. A “Print Summary” option provides a means of reducing the amount of output.

The plotting option provides for a plot of moment versus curvature (ft. - kips vs. 1/in.), moment versus curvature (nondimensional), and to calculate nondimensional coordinates, but to save them for the combined plotting option. If a plot of all the moment curvature curves is required, the combined plotting option is used. More than one moment curvature curve is generated by using IK on the Problem Control Iteration Card.

II Print Option
= 0 Print All
= 1 Print Summary

KK Plotting Option
= 0 No Plot
= 1 Plot – Moment vs. Curvature
= 2 Plot – Moment vs. Curvature (Non-dimensional)
= 3 No Plot – Calculates Non-dimensional Coordinates for Combined Plotting
= 4 Plot – Non-dimensional Moment vs. Curvature Ductility
= 5 No Plot – Calculates Non-dimensional Moment vs. Curvature Ductility for Combined Plotting

KKK Combined Plotting Option
= 0 No Plot
= 1 Plot – Combined Non-dimensional Moment vs. Curvature Curves
= 2 Plot – Combined Non-dimensional Moment vs. Curvature Ductility
KI Plotting Symbol Option

= 1 Symbol Every Data Point

= 2 Symbol Every Other Data Point

= 5 Symbol Every Fifth Data Point

etc.

1 Card:

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>F10.1</td>
<td>AXIAL</td>
</tr>
<tr>
<td>3-4</td>
<td>F10.3</td>
<td>B</td>
</tr>
<tr>
<td>5-6</td>
<td>F10.3</td>
<td>H</td>
</tr>
<tr>
<td>7-8</td>
<td>F10.3</td>
<td>C0</td>
</tr>
</tbody>
</table>

D. Column Information:

The program uses the secant method to determine the neutral axis location for a concrete section by iterating the force equilibrium equation. The secant method requires two initial starting values, C0 and C1. The width B, depth H, and axial force level AXIAL, are also inputted on this card.

1 Card:

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>F10.1</td>
<td>AXIAL</td>
</tr>
<tr>
<td>11-20</td>
<td>F10.3</td>
<td>B</td>
</tr>
<tr>
<td>21-30</td>
<td>F10.3</td>
<td>H</td>
</tr>
<tr>
<td>31-40</td>
<td>F10.3</td>
<td>C0</td>
</tr>
<tr>
<td>41-50</td>
<td>F10.3</td>
<td>C1</td>
</tr>
</tbody>
</table>
E. Reinforcement Properties:

E1. Prestressed Steel

Properties of prestressed steel and the number of layers of prestressed steel, NP, are inputted on this card. The stress-strain curve for the steel consists of three segments. The modulus of elasticity, EP, is also inputted. This card is required to be blank if the column is not prestressed.

1 Card:

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>F10.6</td>
<td>EPB</td>
</tr>
<tr>
<td>11-20</td>
<td>F10.6</td>
<td>EPC</td>
</tr>
<tr>
<td>21-30</td>
<td>F10.6</td>
<td>EPU</td>
</tr>
<tr>
<td>31-40</td>
<td>F10.3</td>
<td>FPB</td>
</tr>
<tr>
<td>41-50</td>
<td>F10.3</td>
<td>FPC</td>
</tr>
<tr>
<td>51-60</td>
<td>F10.3</td>
<td>FPU</td>
</tr>
<tr>
<td>61-70</td>
<td>F10.1</td>
<td>EP</td>
</tr>
<tr>
<td>71-72</td>
<td>I2</td>
<td>NP</td>
</tr>
</tbody>
</table>

E2. Non-prestressed Reinforcement

Properties of non-prestressed steel and the number of layers of non-prestressed steel, NN, are inputted on the card. The stress-strain curve for non-prestressed steel is assumed to be elastic-plastic with a strain-hardening region. The modulus of elasticity, MOES, is also inputted. This card requires to be blank if column is only prestressed.

1 Card:
F. Prestressing Information:

The information input on this card is the effective strain in the prestressed steel due to prestressing, EPE, the effective strain in the concrete due to prestress, ECE, and a bond compatibility factor (CF), between the concrete and steel. (ECE, can also be modified to account for long term effects, i.e., creep.) If NP = 0, see section E1, this card is not required. EPY, is the yield point of the prestressing steel.

1 Card (optional):

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>F10.6</td>
<td>EY</td>
</tr>
<tr>
<td>11-20</td>
<td>F10.6</td>
<td>ESH</td>
</tr>
<tr>
<td>21-30</td>
<td>F10.6</td>
<td>ESU</td>
</tr>
<tr>
<td>31-40</td>
<td>F10.3</td>
<td>FY</td>
</tr>
<tr>
<td>41-50</td>
<td>F10.3</td>
<td>FSU</td>
</tr>
<tr>
<td>51-70</td>
<td>F20.3</td>
<td>MOES</td>
</tr>
<tr>
<td>71-72</td>
<td>I2</td>
<td>NN</td>
</tr>
</tbody>
</table>

G. Reinforcing Areas and Effective Depths:

G1. Prestressed Reinforcement

The program is currently dimensioned to handle 10 layers or 10 different locations of prestressed reinforcement. The area of each layer PAS, or each reinforcing bar and the distance from the extreme fiber in compression to the centroid of the area, PD, is input on this card.
The maximum number of cards which can be read is 10. The number of cards required is equal to NP, input in section E1. If NP = 0, this card must not be input.

0 – 10 Cards (optional):

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>F10.3</td>
<td>PAS</td>
</tr>
<tr>
<td>11-20</td>
<td>F10.3</td>
<td>PD</td>
</tr>
</tbody>
</table>

G2. Non-prestressed Reinforcement

The program is currently dimensioned to handle 10 layers or 10 different locations of non-prestressed reinforcement. The area of each layer, AS, or each reinforcing bar and the distance from the extreme fiber in compression to the centroid, D, is input on this card. The maximum number of cards which can be read is 10. The number of cards required is equal to NN, input in section E2. If NN = 0, no cards are required.

0 – 10 Cards:

<table>
<thead>
<tr>
<th>Columns</th>
<th>Format</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>F10.3</td>
<td>AS</td>
</tr>
<tr>
<td>11-20</td>
<td>F10.3</td>
<td>D</td>
</tr>
</tbody>
</table>

H. Column Confinement Information:

Information required to calculate the confined stress-strain curve for concrete is input on this card. The concrete strength, FCI, volume of hoop reinforcement, PS, width of the confined core, B2, transverse spacing of hoops, SH, and concrete cover, CC, are all input via this card. A Z value may be input, bypassing the calculation of Z. The yield strength of the transverse reinforcement is inputted as FYH.

The user can choose from three different stress-strain curves. The original stress-strain curve for concrete proposed by Park and Paulary, a second which is a modified version of the original to account for strength increase due to confining steel, and a third for a high strain rate version of the modified curve.
NM Stress-Strain Curve Option

- 1 Original Curve
- 0 Modified for Strength Increase
- 1 High Strain Rate Including Strength Increase

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Program Listing

09:11 MAY 15 '84 COLUMN.UEMBS

****
***** PROGRAM NAME: COLUMN DATE PROGRAMMED: APRIL, 82
***** COLUMN CALCULATES THE MOMENTS AND CURVATURES OF AN
***** AXIALLY LOADED CONFINED PRESTRESSED, REINFORCED,
***** OR PARTIALLY PRESTRESSED CONCRETE COLUMN SECTION.
***** THE CONFINED STRESS-STRAIN CURVE FOR CONCRETE CAN
***** BE THE ORIGINAL, MODIFIED, OR HIGH STRAIN CURVES
***** PROPOSED BY PARK & KENT OR PARK & OTHERS.
***** THE PROGRAM USES THE SECANT METHOD TO DETERMINE
***** THE NEUTRAL AXIS LOCATION BY ITERATING THE FORCE
***** EQUILIBRIUM EQUATION.
***** ASSUMPTIONS:
***** 1. BERNOULLI'S PRINCIPLE.
***** 2. TENSILE STRENGTH OF CONCRETE IS IGNORED.
***** 3. BUCKLING OF LONGITUDINAL STEEL IN COMPRESSION IS
***** PREVENTED BY TRANSVERSE REINFORCEMENT.
***** 4. THE STRESS-STRAIN CURVE FOR UNCONFINED CONCRETE
***** (CONCRETE COVER) FOLLOWS THE STRESS-STRAIN CURVE
***** FOR CONFINED CONCRETE AT STRAINS LESS THAN 0.004.
***** FOR STRAINS GREATER THAN 0.004, THE CONCRETE
***** COVER IS ASSUMED TO HAVE SPALLED AND THEREFORE
***** HAS ZERO STRENGTH.
***** 5. SHORT-TERM LOADING IS CONSIDERED.
***** 6. PERFECT BOND EXISTS BETWEEN THE CONCRETE AND
***** NONPRESTRESSED OR PRESTRESSED REINFORCEMENT.
***** PROGRAM RESTRICTIONS:
***** THE PROGRAM IS CURRENTLY DIMENSIONED TO HANDLE
***** 10 LAYERS OF PRESTRESSED REINFORCEMENT AND 10
***** LAYERS OF NONPRESTRESSED REINFORCEMENT.
***** THE PROGRAM CALCULATES THE MOMENT AND CURVATURE
***** FOR CONCRETE STRAINS .003 THRU .052, INCREMENTED BY.
***** .001.
***** ALL UNITS ARE ASSUMED TO BE IN INCHES AND POUNDS.
***** AXIAL LOAD IF COMPRESSION MUST BE INPUT AS POSITIVE
***** IF TENSION AS NEGATIVE.
***** PROGRAM WILL HANDLE ONLY UNIFORM CONCRETE STRAIN
***** DUE TO PRESTRESSING I.E., COLUMNS SYMMETRICAL
***** REINFORCEMENT WITH SAME PRESTRESS FORCE.
***** THIS IS THE MAIN PROGRAM IN COLUMN.
***** STEEL STRAINS, STRESSES, AND FORCES ARE IN COMPRESSION
***** WHEN THEY ARE NEGATIVE.
***** THE AXIAL LOAD IS COMPRESSION WHEN POSITIVE AND TENSION
SUBROUTINE YIELD CALCULATES THE YIELD CURVATURE AND MOMENT FOR A PRESTRESSED, NONPRESTRESSED, OR PARTIALLY PRESTRESSED COLUMN.

CALL YIELD (EPB, EPC, EPU, FPB, FPC, FPU, EPC, EPE, EC, CO, C1, PD, PAS, TPE, S$, S$, PSS, PFS, NP, CF, NM, EY, ESH, ESU, FSU, FY, MOES, AS, ES, SS, FS, NN, ALPHA, FC1, S$, AK, B, AXIAL, C, GAMMA, CURVY, TOTMY, H, CC, II, FYH, FS, B2, SH, Z, EPY)

DO 150 I = 1, 50
EC = 0.001*1 + .002

THE NEXT LINE DENOTES WHEN THE CONCRETE STRAIN IS GREATER THAN 0.004 THE COLUMN DIMENSIONS ARE TO BE CHANGED.

IF (EC - 0.004) 525, 525, 220

EC = ECE1

IF (NP .EQ. 0.0) GO TO 515

DO 919 J = 1, NP
PD(J) = PD1(J)
919 CONTINUE

IF (NN .EQ. 0.0) GO TO 525

DO 929 J = 1, NN
D(J) = DL(J)
929 CONTINUE

SUBROUTINE CONFINED CALCULATES THE CENTROID AND MEAN STRESS FACTOR FOR THREE DIFFERENT TYPES OF STRESS-CURVES.

CALL CONFINED (EC, FC1, FYH, PS, B2, SH, Z, NM, ALPHA, GAMMA, II, AK)

SUBROUTINE SOLVE USES THE SECANT METHOD TO CALCULATE THE NEUTRAL AXIS LOCATION.

CALL SOLVE (EPB, EPC, EPU, FPB, FPC, FPU, EPC, EPE, EC, CO, C1, PD, PAS, TPE, S$, S$, PSS, PFS, NP, CF, NM, EY, ESH, ESU, FSU, FY, MOES, AS, ES, SS, FS, NN, ALPHA, FC1, S$, AK, B, AXIAL, C, GAMMA, CURVY, TOTMY, H, CC, II, FYH, FS, B2, SH, Z, EPY)

COMP = -(ALPHA*FC1*AK*B*C)

IF (II .EQ. 1) GO TO 617

SUBROUTINE OUTPUT WRITES THE STRAINS, STRESSES, AND FORCES FOR EACH INCREMENT OF CONCRETE STRAIN.

CALL OUTPUT (NP, TPE, S$, PSS, PFS, C, COMP, AXIAL, PAS, PD, ES, SS, FS, AS, D, NN, S$)

SUBROUTINE MOMENT CALCULATES THE MOMENT OF THE COLUMN.

CALL MOMENT (NP, PFS, PD, ALPHA, GAMMA, B, C, H, EC, I, TOTM, CURV, FC1, CC, FS, S$, D, NN, II, AK)

SUBROUTINE SUMMARY PRINTS A SUMMARY OF THE CONCRETE STRAIN.
C WHEN NEGATIVE.
C
C IMPLICIT REAL (M)
DIMENSION PSS(IO),PFS(IO),PAS(IO),PD(IO)
DIMENSION TPES(IO),TOTM(50),CURV(50)
DIMENSION AS(IO),ES(IO),SS(IO),FS(IO),D1(IO),D(IO)
DIMENSION PDI(IO),PXCURV(52),PYTOTM(52)
DIMENSION XARRAY(502),YARRAY(502)
DIMENSION L(IO),L(0:10),PXCURV(10:50),PYXTOTM(10:50)
CHARACTER*80 HEADING(IO)
C
C THESE LINES READ AND WRITE ANY INFORMATION OR HEADING
C THAT THE USER WANTS OUTPUTTED WITH HIS RESULTS.
C
READ (105,222) (HEADING(I), I=1,10)
222 FORMAT (A80/A80/A80/A80/A80/A80/A80/A80/A80/A80)
WRITE (108,444) HEADING(I), I=1,10
444 FORMAT (1',A80/A80/A80/A80/A80/A80/A80/A80/A80/A80)
READ (105,226) .IK
226 FORMAT (12)
DO 921 NL=I,10
921 L(NL)=0.0
DO 923 NI = I,11
923 LI(NI)=0.0
DO 939 JK=I,IK
C SUBROUTINE INPUT READS IN THE VARIABLES TO THE MAIN PROGRAM.
C
CALL INPUT (AXIAL,B,H,CO,Cl,5 PB,E PC,EPU,F B,FPC,FPU,EP,NP,FC1,PS,B
$2,SH,CC,PAS,PD,PEP,KEE,FM,EM,ESH,ESU,FSU,MOES,NN,AS,D,I,I,
$K,CF,KK,EPY,KI)
C
C THE NEXT LINE CALCULATES THE INCREASE IN CONCRETE STRAIN
C DUE TO PRESTRESSING AFTER THE OUTER CORE HAS SPALLED.
C
ECE1=ECE*(AG/AN)
IF (NP ,EQ. 0) ECE=0.0
IF (NP ,EQ. 0) EPE=0.0
IF (NP ,EQ. 0) ECE=0.0
C DO 500 ND=1,10
500 D1(D)=D(D)-CC
C THE NEXT LINE CALCULATES THE INCREASE IN CONCRETE STRAIN
C DUE TO PRESTRESSING AFTER THE OUTER CORE HAS SPALLED.
C
ECE1=ECE*(AG/AN)
IF (NP ,EQ. 0) GO TO 414
DO 979 J=1,NP
979 PD1(J)=PD(J)-CC
414 CONTINUE
IF (NN ,EQ. 0) GO TO 230
DO 121 J=1,NN
121 D1(J)=D(J)-CC
230 CONTINUE
CALL SUMMARY (TOTM, CURV, EPE, ECE, NM, CF, FC, BG, HG, CURVY)
L(JK) = I-1

SUBROUTINE CALCOMP PLOTS MOMENT VERUS CURVATURE FOR
ONLY ONE ANALYSIS.

CALL CALCOMP (TOTM/CURV/HEADING, KK, HG, BG, FC, PXCURV, PYTOTM, L, NPTS, 
SJ, CURVY, KI)
GO TO 909

WRITE (108, 505)
505 FORMAT (///, TS, 'CONVERGENCE IS NOT ACHIEVED')
GO TO 949
909 CONTINUE

THE FOLLOWING LINES REARRANGE INFORMATION SO THAT
MORE THAN ONE PLOT CAN BE PUT ON A PAGE, I.E., IT
IS PLACING ALL THE INFORMATION IN ONE FILE SO THAT
IT CAN SCALE THAT FILE FOR THE PLOT PARAMETERS.

DO 35 KJ = 1, L(JK)
PXYCURV(JK, KJ) = PXCURV(KJ)
35 PYXTOTM(JK, KJ) = PYTOTM(KJ)
939 CONTINUE

IF (KKK .EQ. 0) GO TO 949

DO 343 JK = 1, IK
DO 434 KJ = 1, L(JK)
L(I0) = 0
LI(JK) = L(JK) + IF(JK - I)
I = LI(JK) - L(JK) + KJ
NCPTS = I
NCPTS1 = I + 1
NCPTS2 = I + 2
XARRAY(I) = PXYCURV(JK, KJ)
434 YARRAY(I) = PYXTOTM(JK, KJ)
343 CONTINUE

SUBROUTINE CALSUM PLOTS ALLCOMBINED INFORMATION ON ONE
GRAPH.

CALL CALSUM (XARRAY, YARRAY, IK, HEADING, L, PXYCUMV, PYXTOTM, NCPTS, NCPT, 
S1, NCPTS2, KKK, KI)
949 CONTINUE

END

SUBROUTINE SOLVE (EPC, EPE, EPU, FPB, FPC, FPU, EPE, EPE, EC, CO, C1, PD, P, 
S1, TPS, FSS, NP, CF, NM, EY, ES, ESU, FSU, FY, MD, AS, ES, SS, FS, NN, ALP, 
SHA, FC, A, B, AXIAL, C, D, ***)

THIS SUBROUTINE ITERATES THE FORCE EQUILIBRIUM EQUATION
BY USING THE SECANT METHOD. THE EXACT NEUTRAL AXIS LOCATION
IS DETERMINED.

IMPLICIT REAL (M)
DIMENSION SUMP0(0:10), SUMP1(0:10), SUMFS0(0:10), SUMFS1(0:10)
DIMENSION PSS(10), PFS(10), PAS(10), PD(10), TPS(10)
DIMENSION ES(10), SS(10), FS(10), AS(10), D(10)
JI = 0.0
CONTINUE
IF (NP .EQ. 0.0) GO TO 878
CALL PRESTEEL (EPB, EPC, EPB, FPB, FPC, FPU, EPC, EPE, EC, CO, PD, PAS, TPE, 
$S$, TSS, PFS, SSMPFO, NP, CF, NM, *301)
CALL PRESTEEL (EPB, EPC, EPB, FPB, FPC, FPU, EPC, EPE, EC, C1, PD, PAS, TPE, 
$S$, TSS, PFS, SSMPFI, NP, CF, NM, *301)
878 CONTINUE
IF (NN .EQ. 0.0) GO TO 787
CALL STEEL (EY, ESH, ESU, FSU, FY, MOES, EC, CO, D, ES, SS, FS, SSMPFO, NN, 
$S$, TSS, PFS, SSMPFI, NN, *301)
CALL STEEL (EY, ESH, ESU, FSU, FY, MOES, EC, C1, PD, ES, SS, FS, SSMPFI, NN, 
$S$, TSS, PFS, SSMPFO, NN, *301)
CONTINUE
IF (NP .EQ. 0.0) SSMPFO(O) = 0.0
IF (NP .EQ. 0.0) SSMPFI(O) = 0.0
IF (NN .EQ. 0.0) SSMPFO(O) = 0.0
IF (NN .EQ. 0.0) SSMPFI(O) = 0.0
ERRORO = (ALPHA*FC1*AK*B*CO + SSMPFO(NP) + AXIAL + SSMPFI(NN))
ERROR1 = (ALPHA*FC1*AK*B*C1) + SSMPFI(NP) + AXIAL + SSMPFI(NN)
C2 = C1 - ((ERROR1*(C1-C0)) / (ERROR1-ERRORO))
IF (ABS(C2-C1) .LT. 0.01) GO TO 160
C0 = C1
C1 = C2
JI = 1.0 + JI
IF (JI .GT. 1500) RETURN 1
GO TO 260
160 CONTINUE
C = C2
GO TO 171
301 WRITE (108, 404)
404 FORMAT (///,15,'STRAND FRACTURES')
RETURN 2
401 WRITE (108, 405)
405 FORMAT (///,15,'BAR FRACTURES')
RETURN 2
171 CONTINUE
RETURN END
SUBROUTINE YIELD (EPB, EPC, EPB, FPB, FPC, FPU, EPC, EPE, EC, CO, C1, PD, 
PAS, TPE, S, TSS, PFS, SSMPFO, NP, CF, NM, EY, ESH, ESU, FSU, FY, MOES, AS, ES, 
SS, FS, SSMPFO, NN, ALPHA, FC1, AK, B, AXIAL, GAMMA, CURVY, TOTMY, CC, II, FYH, 
PS, B2, SH, Z, EPY $)
C
C THIS SUBROUTINE CALCULATES THE NEUTRAL AXIS LOCATION WHEN THE
C THE BOTTOM STEEL (PRESTRESSED OR NONPRESTRESSED) IS AT ITS
C YIELD POINT. IT THEN CALCULATES THE YIELD MOMENT AND YIELD
C CURVATURE AND PRINTS BOTH. THE STRAIN IN THE BOTTOM STEEL
C IS PRINTED OUT FOR EACH ITERATION (AS A CHECK).
C
IMPLICIT REAL (M)
DIMENSION ES(10), TPE(10), TOTM(50), CURV(50)
DO 20 I = 1, 40
EC = 0.3010 + 0.00005*1
CALL CONFINED (EC, FC1, FY, PS, B2, SH, Z, NM, ALPHA, GAMMA, II, AK)
CALL SOLVE (EPB, EPC, EPB, FPB, FPC, FPU, EPC, EPE, EC, CO, C1, PD, PAS, TPE, 
S, TSS, PFS, SSMPFO, NP, CF, NM, EY, ESH, ESU, FSU, FY, MOES, AS, ES, SS, FS, 
NN, ALPHA, FC1, AK, B, AXIAL, CO, *155, *932)
C
IF (NP .GT. 0.0) GO TO 18
WRITE (108,345) ES(NN)
345 FORMAT (/T08, ' ES(NN) IS', 3X, F10.7)
IF (ABS(ES(NN)-EY) .LT. .00005) GO TO 21
GO TO 20
WRITE (108,456) TPES(NP)
456 FORMAT (/T08, ' TPES(NP) IS', 3X, F10.7)
IF ((TPES(NP) - EPY) .LT. .0005) GO TO 21
CONTINUE
GO TO 21
WRITE (108,43) CONVERGENCE IS NOT ACHIEVED'
43 FORMAT (/T08, ' CONVERGENCE IS NOT ACHIEVED'
WRITE (108,44) STEEL FRACTURES'
44 FORMAT (/T08, ' STEEL FRACTURES'
RETURN
END
SUBROUTINE CONFINED (EC, FC1, FYH, PS, B2, SH, Z, NM, ALPHA, GAMMA, II, AK)
C THIS SUBROUTINE DETERMINES THE MEAN STRESS AND CENTROID OF
C A CONFINED STRESS-STRAIN CURVE.
C NM=-1; STRESS-STRAIN CURVE OF PARK AND KENT
C NM=0; MODIFIED STRESS-STRAIN CURVE OF PARK AND KENT
C NM=1; MODIFIED STRESS-STRAIN CURVE AT HIGH STRAIN RATES
C
IF (NM .EQ. -1) RATE = 1.0
IF (NM .EQ. 0) RATE = 1.0
IF (NM .EQ. 1) RATE = 1.25
IF (NM .EQ. 1) AK = 1.25*(1.0 + (PS*FYH)/FC1)
2 IF (NM .EQ. 1) AK = 10.0*(PS*FYH)/FC1
10 AREABC = (FYH*EC**2.0)/(0.002*AK)*(1.0 - EC/(0.006*AK))
ALPHA = AREABC/(EC*FC1*AK)
FMAB = ((2.0*FC1*EC**3.0)/0.006)*(1.0 - 3.0*EC/(0.016*AK))
GAMMA = 1.0 - FMAB/(EC*AREABC)
GO TO 5
15 AREAAB = (FC1*EC+2.0)/0.002*(1.0+EC/(0.006*AK))
ALPHA = AREAAB/(EC*FC1*AK)
FMAB = ((2.0*FC1*EC**3.0)/0.006)*(1.0 - 3.0*EC/(0.016*AK))
GAMMA = 1.0 - FMAB/(EC*AREABC)
GO TO 5
20 IF (FC1 < EC) 30,30,40
30 AREABC = AK*FC1*Z/AK + AK*FC1*(EC - AK*Z/2 + EC - AK*Z) + 0.002*AK*FC1*Z*(EC - 0.002*AK)
ALPHA = AREABC/(EC*AK*FC1)
FMABC = 5.0/12.0*FC1*.002*.002*AK**3.0 + AK*FC1*0.5*(EC**2.0 - (.002*AK)**2.0)

GAMMA = 1.0 - FMABC/(EC*AREABC)

GO TO 50

40 AREACD = AK**2.0*FC1*2.0*.002/3.0 + AK*FC1*(E2OC -EC02*AK) - AK*F

$C1*Z*0.5*(E2OC**2.0 - (.002*AK)**2.0) + AK**2.0*FC1*2.0*.002*(E2OC -

S*EC02*AK) + 0.2*AK*FC1*Z/3.0*(E2OC**3.0 - (.002*AK)**3.0) + AK**

$2.0*FC1*Z*0.002*0.5*(E2OC**2.0 - (.002*AK)**2.0) + 0.2*AK*FC1*0.5(*

SEC**2.0 - E2OC**2.0)

GAMMA = 1.0 - FMACD/(EC*AREACD)

50 CONTINUE

IF (I .EQ. 1) GO TO 777

WRITE (108,60) EC,ALPHA,GAMMA

60 FORMAT ('CONCRETE STRAIN IS ',1X,F8.6,3X,'MEAN STRESS FACTOR IS ',1X,F8.6,3X,'CENTROID IS ',1X,F8.6)

WRITE (108,70) FC1/2

70 FORMAT ('28 DAY COMP. STRENGTH IS ',1X,F10.3,'PARAMETER IS ',1X,F10.6)

IF (EC-.004) 777,777,666

666 WRITE (108,888)

888 FORMAT ('CONCRETE COVER HAS SPALLED')

777 CONTINUE

RETURN

END

SUBROUTINE PRESTEEL (EPB#EPC#EPp,FPB,FPC#FPU,EP»ECE#EPE»EO,C,PD,PA

C

THIS SUBROUTINE CALCULATES THE STRAIN; STRESS, AND FORCE IN EACH

LAYER OF PRESTRESSING STEEL. TENSION IS POSITIVE,

COMPRESSION IS NEGATIVE.

C

DIMENSION PES(I),PSS(I),PFS(I),SUMPF$,PAS(10),PD(10),TPES

$S(10),TPEZ(10)

IF (NM .EQ. 1) FPB=1.25*FPB

IF (NM .EQ. 1) FPC=1.25*FPC

IF (NM .EQ. 1) FPU=1.25*FPU

DO 171 J=1,NP

PES(J) = 0.0

PSS(J) = 0.0

PFS(J) = 0.0

TPES(J)=0.0

TPEZ(J)=0.0

171 SUMPF$(J) = 0.0

SUMPF$(0) = 0.0

DO 101 J=1,NP

PES(J) = ((EC*(PD(J)-C))/C)*CF

TPES(J) = EPE + PES(J) + ECE

TPEZ(J)=ABS(TPES(J))

IF (TPEZ(J) .GT. EPU) RETURN 1

IF (TPEZ(J) .LT. EPB) GO TO 410

IF (TPEZ(J) .LT. EPC) GO TO 420

PSS(J) = FPC + ((TPEZ(J)-EPC)/(EPU-EPC))*(FPU-FPC)

GO TO 320
92

\[ PSS(J) = TPEZ(J) \times EP \]

GO TO 320

\[ \text{PART1} = \left( \frac{(FPC \times EPC - FPB \times EPB)}{(EPC - EPB)} \right) \]
\[ \text{PART2} = \left( \frac{(EPB \times EPC - FPB \times FPC)}{(TPEZ(J) \times (EPC - EPB))} \right) \]

\[ PSS(J) = \text{PART1} + \text{PART2} \]

320 CONTINUE

IF (TPES(J) \LT 0.0) \text{CORR} = -1.0
IF (TPES(J) \GT 0.0) \text{CORR} = 1.0

\[ PFS(J) = PSS(J) \times PAS(J) \times \text{CORR} \]

\[ \text{SUMPFS(J)} = PFS(J) + \text{SUHPFS(J-1)} \]

101 CONTINUE

RETURN

END

SUBROUTINE STEEL (EY, ESH, ESU, FSU, FY, MOES, EC, D, AS, ES, SS, FS, SUMFS, $NN, NM, *)

\( \text{DIMENSION ES}(10), SS(10), FS(10), \text{SUMFS}(0:10), AS(10), D(10), EZ(10) \)

IF (NM .EQ. 1) FY=1.25*FSU
IF (NM .EQ. 1) FSU=1.25*FSU

DO 717 J=1, NN
ES(J)=0.0
SS(J)=0.0
FS(J)=0.0
SUMFS(J)=0.0
717 EZ(J)=0.0

\( \text{M} = \frac{(FSU/FY) \times \left( (30.0 \times R + 1.0) \times 2.0 - (60.0 \times R) - 1.0 \right) \times (15.0 \times (R^2.0))}{(60.0 \times R + 1.0)} \)

DO 202 J=1, NN
ES(J)=(EC*(D(J)*C))/C
IF (ABS(ES(J)) \LE. EY) GO TO 200
IF (ABS(ES(J)) \LE. ESU) GO TO 210
IF (ABS(ES(J)) \GT. ESU) RETURN 1
IF (ES(J) \LT 0.0) \text{CORR} = -1.0
IF (ES(J) \GT 0.0) \text{CORR} = 1.0

\( \text{EZ(J)} = \text{ABS}(ES(J)) \)

\[ \text{PART1} = \left( \frac{M \times (EZ(J) - ESH) + 2.0}{(60.0 \times (EZ(J) - ESH) + 2.0)} \right) \]
\[ \text{PART2} = \left( \frac{(EZ(J) - ESH) \times (60.0 - M)}{(2.0 \times (30.0 \times R + 1.0) \times 2.0)} \right) \]

\[ \text{SS(J)} = FY \times (\text{PART1} + \text{PART2}) \times \text{CORR} \]

GO TO 230

200 SS(J)=ES(J) \times MOES
GO TO 230

210 IF (ES(J) \LT 0.0) SS(J)=FY
IF (ES(J) \GT 0.0) SS(J)=FY

230 SUMFS(Q)=0.0
FS(J)=SS(J) \times AS(J)
SUMFS(J)=FS(J) + SUMFS(J-1)

202 CONTINUE

RETURN

END

SUBROUTINE MOMENT (NP, PFS, PD, ALPHA, GAMMA, B, C, EC, I, TOTM, CURV, FC1, SCC, FS, D, NN, II, AK)

\( \text{DIMENSION NP}(10), PFS(10), PD(10), ALPHA(10), GAMMA(10), B(10), C(10), EC(10), TOTM(10), CURV(10), FC1(10), SCC(10), FS(10), D(10), NN(10), II(10), AK(10) \)

\( \text{DIMENSION NP}(10), PFS(10), PD(10), ALPHA(10), GAMMA(10), B(10), C(10), EC(10), TOTM(10), CURV(10), FC1(10), SCC(10), FS(10), D(10), NN(10), II(10), AK(10) \)

\( \text{DIMENSION NP}(10), PFS(10), PD(10), ALPHA(10), GAMMA(10), B(10), C(10), EC(10), TOTM(10), CURV(10), FC1(10), SCC(10), FS(10), D(10), NN(10), II(10), AK(10) \)

\( \text{DIMENSION NP}(10), PFS(10), PD(10), ALPHA(10), GAMMA(10), B(10), C(10), EC(10), TOTM(10), CURV(10), FC1(10), SCC(10), FS(10), D(10), NN(10), II(10), AK(10) \)

\( \text{DIMENSION NP}(10), PFS(10), PD(10), ALPHA(10), GAMMA(10), B(10), C(10), EC(10), TOTM(10), CURV(10), FC1(10), SCC(10), FS(10), D(10), NN(10), II(10), AK(10) \)
C THIS SUBROUTINE CALCULATES AND PRINTS THE MOMENT AND CURVATURE
C FOR A PRESTRESSED CONCRETE COLUMN WITH A GIVEN AXIAL LOAD.
C
IMPLICIT REAL (M)
DIMENSION FS(10),D(10),MPS(10),SUMMPS(0:10),TOTM(50),CURV(50)
DIMENSION FS(10),D(10),MS(10),SUMMS(0:10)
DO 161 L = 1,50
TOTM(L) = 0.0
DO 161 L = 1,50
SUMMPS(O) = 0.0
SUMMS(O) = 0.0
IF (NP .EQ. 0.0) GO TO 413
DO 401 J = 1,50
MPS(J) = FS(J) * (D(J) - H/2.0)
SUMMPS(J) = MPS(J) + SUMMPS(J-1)
401 CONTINUE
IF (NN .EQ. 0.0) GO TO 409
DO 41 I = 1,50
MS(I) = FSU * (D(I) - H/2.0)
SUMMS(I) = MS(I) + SUMMS(I-1)
41 CONTINUE
MC = (ALPHA*FC1*AK*B*C) * (H/2.0 - GAMMA)
TOTM(I) = MC + SUMMPS(NP) + SUMMS(NN)
CURV(I) = EC/C
IF (I .EQ. 1) GO TO 599
WRITE (108,500) TOTM(I)
500 FORMAT (//,5X,9HMOMENT IS,3X,F15.3)
WRITE (108,510) CURV(I)
510 FORMAT (//,5X,12HCURVATURE IS,3X,F10.8)
599 CONTINUE
RETURN
END

SUBROUTINE INPUT (AXIAL,B,H,C0,C1,EPB,EPC,EPU,FPC,FPU,EP,NP,FC
$1,PS,B2,SH,CC,PAS,PD,EPE,ECE,Z,FYH,NM,EGH,ESU,FY,FSU,MOES,NN,AS
$1,II,KK,CF,KKK,Epy,KI)
DIMENSION PAS(10), PD(10),AS(10),D(10)

C THIS SUBROUTINE READS THE INPUTTED VARIABLES
C
IMPLICIT REAL (M)
READ (105,111) II,KK,KKK,KI
111 FORMAT (4I2)
READ (105,100) AXIAL,B,H,C0,C1
100 FORMAT (4F10.3)
READ (105,101) EPB,EPC,EPU,FPC,FPU,EP,NP
101 FORMAT (3F10.6,3F10.3,F10.1,E2)
READ (105,107) EY,ESH,ESU,FY,FSU,MOES,NN
107 FORMAT (3F10.6,2F10.3,F20.3,I2)
READ (105,104) EPE,ECE,CF,Epy
104 FORMAT (4F10.6)
DO 802 I=1,NP
READ (105,102) PAS(I),PD(I)
802 CONTINUE
RETURN
END
94

DO 809 I=1,NN
READ (105,109) AS(I),D(I)
109 CONTINUE
809 CONTINUE
READ (105,103) FC1,PS,B2,SH,CC,Z,FYH,NM
103 FORMAT (F10.3,3F10.5,3F10.5,I2)
RETURN
END

SUBROUTINE OUTPUT (NP,TPES,PSS,PFS,COMP,AXIAL,PAS,PD,E,S,SS,FS,A
                     SS,D,NN)
DIMENSION TPES(IO), PSS(IO), PFS(IO), PAS(IO), PD(IO)
DIMENSION ES(IO),SS(10),FS(10),AS(IO),D(10)
C
C  THIS SUBROUTINE PRINTS OUT INFORMATION
C
IF (NP .EQ. 0.0) GO TO 622
WRITE (108,601)
601 FORMAT (///,T30, 'PRESTRESSED STEEL')
WRITE (108,600)
600 FORMAT (///,T10, 'STEEL LOCATION',T30, 'STRAIN',T45, 'STRESS',T60, 'FORCE',T75, 'AREA',T85, 'E')
DO 620 J=1,NP
       WRITE (108,610) J,TPES(J),PSS(J),PFS(J),PAS(J),PD(J)
610 FORMAT (10X,I5,10X,F10.5,5X,F10.3,3X,F15.3,1X,F10.3)
620 CONTINUE
622 CONTINUE
IF (NN .EQ. 0.0) GO TO 626
WRITE (108,602)
602 FORMAT (///,T30, 'NONPRESTRESSED STEEL')
WRITE (108,600)
DO 625 J=1,NN
       WRITE (108,603) J,ES(J),SS(J),FS(J),AS(J),D(J)
603 FORMAT (10X,I5,10X,F10.5,5X,F10.3,3X,F15.3,1X,F10.3)
625 CONTINUE
626 CONTINUE
WRITE (108,630) C
630 FORMAT (///,T20, 'NEUTRAL AXIS IS',3X,F7.4)
WRITE (108,640) COMP
640 FORMAT (///,T20, 'CONCRETE COMPRESSION FORCE IS',3X,F12.3)
WRITE (108,650) AXIAL
650 FORMAT (///,T20, 'AXIAL FORCE IS',3X,F15.3)
RETURN
END

SUBROUTINE SUMMARY (TOTM,CURV,EPE,ECE,NM,CF,FC1,BG,HG,CURVY)
DIMENSION TOTM(SO), CURV(SO), CTOTM(SO), CURVZ(SO)
DIMENSION CNTOTM(SO), CNCURV(SO)
C
C  THIS SUBROUTINE PRINTS A SUMMARY OF STRAINS, MOMENTS,
C  AND CURVATURES
C
WRITE (108,700)
700 FORMAT ('SUMMARY')
IF (NM) 393,394,395
393 WRITE (108,933)
933 FORMAT ('ORIGINAL STRESS-STRAIN CURVE - KENT & PARK')
GO TO 945
394 WRITE (108,943)
393 FORMAT (///110,'STRESS-STRAIN CURVE WITH STRENGTH INCREASE*)
395 GO TO 945
399 WRITE (108,953)
393 FORMAT (///110,'HIGH STRAIN RATE STRESS-STRAIN CURVE WITH STRENGTH
$^H$ INCREASE')
945 CONTINUE
IF (EPE .EQ. 0.0) GO TO 701
WRITE (108,705) EPE
705 FORMAT (///T5,'EFFECTIVE STEEL STRAIN IS',3X,F10.6)
WRITE (108,706) ECE
706 FORMAT (///T5,'PRESTRESSED CONCRETE STRAIN IS',3X,F10.6)
WRITE (108,704) CF
704 FORMAT (///T10,'STRAIN COMPATIBILITY FACTOR IS',2X,F10.6)
701 CONTINUE
WRITE (108,710)
710 FORMAT (///T10,'CONCRETE STRAIN*/T30,'MOMENT*/T60,'CURVATURE*/T90,'MOMENT*/T70,'MOMENT*/T80,'CURVATURE*/T95,'CURVATURE')
WRITE (108,714)
714 FORMAT (10X/Fl0.5/5X/.F15.3/3X/F10.8/2X#F10.3/2X/F-5.8/5X/F10.8/3X/F-8.3)
800 CONTINUE
RETURN
END
SUBROUTINE CALCOMP (TOTM,CURV,HEADING,KK,HT,BG,FC1,PXCURV,PYTOTM,L
$^S$/NPTS,JK,CURVY,KJ)
DIMENSION TOTM(50),CURV(50),PXCURV(52),PYTOTM(52),HEADING(10)
DIMENSION L(10)
C
C THIS SUBROUTINE PLOTS THE MOMENT CURVATURE CURVE
C
IF (KK .LE. 0) GO TO 464
DO 747 KL=1,52
PXCURV(KL)=0.0
747 PYTOTM(KL)=0.0
KJ=0.0
DO 474 KJ=1,L(JK)
PXCURV(KJ)=CURV(KJ)
474 PYTOTM(KJ)=ABS(TOTM(KJ)/12000.0)
IF(KK .LE. 1) GO TO 512
DO 475 KJ=1,L(JK)
PXCURV(KJ)=PXCURV(KJ)+HT
475 PYTOTM(KJ)=ABS(TOTM(KJ)/(FC1*BG*(HT**2.0)))
IF (KK .LE. 2) GO TO 512
IF (KK .LE. 3) GO TO 464
DO 213 KJ=1,L(JK)
213 PXCURV(KJ)=CURV(KJ)/CURVY
```fortran
IF (KK .EQ. 5) GO TO 464

512 CONTINUE
CALL PLOTS (0.0, 8)
CALL NEUPEN (2)
CALL FACTOR (2.54)
NPTS = L(JK)
NPTS1 = L(JK) + 1
NPTS2 = L(JK) + 2
CALL SCALE (PXCUR, 7.0, NPTS, 1)
CALL SCALE (PYTOTM, 5.0, NPTS, 1)
IF (KK .EQ. 2) GO TO 596
CALL AXIS (0.0, 0.0, 'CURVATURE', -16, 7.0, 0.0, PXCURV(NPTS1), PX
CURV(NPTS2))
CALL AXIS (0.0, 0.0, 'MOMENT', -15, 5.0, 90.0, PYTOTM(NPTS1), PYT
OTM(NPTS2))
GO TO 595

596 CONTINUE
CALL FLINE (PXCURV, PYTOTM, -NPTS, 1, KI, 4)
CALL SYMBOL (2.0, 0.0, 4, HEADING(I), 0.0, 80)
CALL QLOT (12.0, 0.0, 999)

464 CONTINUE
RETURN

END

SUBROUTINE CALSUM (XARRAY, YARRAY, IK, HEADING, L, PXCURV, PYXTOTM, NCPTS
$1, NCPTS2, KKK, KI)

C THIS SUBROUTINE PLOTS COMBINED NONDIMENSIONALIZE
C MOMENT - CURVATURE CURVES.
C
DIMENSION XARRAY(502), YARRAY(502), PXCURV(5%), PYTOTM(5)
DIMENSION PXCURV(10, 50), PYXTOTM(10, 50), L(10), INTE(10)
DIMENSION HEADING(10)
INTE(1) = 0
INTE(2) = 1
INTE(3) = 2
INTE(4) = 3
INTE(5) = 4
INTE(6) = 5
INTE(7) = 6
INTE(8) = 7
INTE(9) = 8
INTE(10) = 9

CALL PLOTS (0.0, 8)
CALL FACTOR (2.54)
CALL SCALE (XARRAY, 7.0, NCPTS, 1)
CALL SCALE (YARRAY, 5.0, NCPTS, 1)
IF (KKK .EQ. 2) GO TO 313
CALL AXIS (0.0, 0.0, 'CURVATURE', -16, 7.0, 0.0, XARRAY(NCPTS1), XA
RRAY(NCPTS2))
GO TO 314

313 CALL AXIS (0.0, 0.0, 'CURVATURE DUCTILITY', -19, 7.0, 0.0, XARRAY(NCPTS1
$), XARRAY(NCPTS2))

314 CONTINUE
```
CALL AXIS (0.0, 0.0, 'MOMENT - M/F = FC + H/4.195.0, 90.0, YARRAY(NCPTS1))
YARRAY(NCPTS2))
DO 43 J = 1, K
DO 46 KJ = 1, L(J)
NT1 = L(J) + 1
NT2 = L(J) + 2
PXCURV(KJ) = PXYCURV(J, KJ)
PXCURV(NT1) = XARRAY(NCPTS1)
PXCURV(NT2) = XARRAY(NCPTS2)
PYTOTM(KJ) = PYXTOTM(J, KJ)
PYTOTM(NT1) = YARRAY(NCPTS1)
PYTOTM(NT2) = YARRAY(NCPTS2)
46 PYTOTM(NT2) = YARRAY(NCPTS2)
CALL FLINE (PXCURV, PYTOTM, L(J), KJ, INTE(J))
43 CONTINUE
CALL SYMBOL (2.0, 4.5, 14.0, HEADING(1), 0.0, 80)
CALL PLOT (12.0, 0.0, 999)
RETURN
END
Suprenant, B. A.
cop.2 Curvature ductility of reinforced and prestressed...