Minimization methods for cellular logic arrays

by Harold Gregory Schmitz

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
Montana State University

Abstract:
This thesis is concerned with the development of minimization techniques for the efficient synthesis of cellular logic networks.

The main results can he summarized as follows: First, an algorithm is presented which obtains a realization for an arbitrary switching function using a minimal size two-dimensional cellular logic array in which each cell is capable of producing all two-input one-output unate functions and in which the interconnection structure is constrained to be "nearest neighbor." Also, the number of equivalence classes of unate cascade realizable functions is determined, and a necessary condition and improved test for determining unate cascade realizability is given. Second, a solution to a classical minimization problem involving the Reed-Muller canonic forms is provided and minimization techniques for two-dimensional cellular arrays based on the Reed Muller forms are developed. Third, an essential step in the cellular array minimization algorithms is formulated as a graph theoretic problem whose solution requires finding the cliques of a linear nondirected graph. For this, a new algorithm for detecting the cliques of a linear nondirected graph is developed.

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Approved:

Paul E. Uhlrich
Head, Major Department

A. Mukhopadhyay
Chairman, Examining Committee

H. Goering
Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

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ABSTRACT

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Chapter 1

INTRODUCTION AND REVIEW OF CELLULAR RESEARCH

1.1 Introduction

A cellular logic array is a one-, two-, or many-dimensional iterative arrangement of similar or identical combinational or sequential cells with a regular interconnection pattern on the cells. If the arrays are composed of simple cells (low complexity), they are referred to as microcellular. Arrays having more complex cells are called macrocellular.

Research in microcellular and macrocellular networks started about two decades ago. As early as 1952, studies in automata theory prompted research in the macrocellular area. Von Neumann(37) introduced the notion of a cellular space and discussed universal computing and constructing automata embedded in such a space.

A large part of the research in macrocellular networks has been motivated by the desire to build highly parallel computers. Holland's (18) machine was a two-dimensional array of iterative modules. He envisioned it as a universal computer capable of simultaneously executing an arbitrary number of subprograms. Modified Holland machines were subsequently considered by Comfort(5) and Gonzales.(12) At the same time Holland was envisioning his iterative circuit computer, Unger(36) independently proposed a special purpose modular computer oriented to solve problems in which data is arranged in a spatial form.

More recently, investigations by Slotnick, Borck, and McReynolds (35) led to the SOLOMON and ILLIAC IV parallel processing computers.

Concurrent with the development of macrocellular arrays, there were other areas of development that would now be classified as microcellular in nature. The first of these areas might be termed contact arrays. The crossbar switch of Scudder and Reynolds[33] exhibited cellularity. Each "cell" of the crossbar switch consisted of several contacts and a mechanical latching arrangement. By operating magnets in the desired row and column in the proper sequence, it was possible to activate and hold various cells in the crossbar switch.

Diode arrays also led to the development of modern microcellular techniques. Brown[3] appears to have been the first to suggest the diode array which had an interconnection structure that consisted of a set of horizontal and vertical busses.

Memory arrays using shift registers were proposed by Aiken in about 1948. Shortly after, Forrester[11] and Rajchman[31] proposed the magnetic-core storage which was also microcellular in nature.

However, the early areas of microcellular development received only limited attention because of what were then technological limitations. It is only with the advent of integrated circuit and batch-fabrication technologies that microcellular techniques have taken on a new importance. With the progress now being made in the field of large scale integration (LSI) technology, it is reasonably certain that the future generations of computers will employ cellular arrays in their completely integrated, highly parallel and sophisticated subsystems. The various advantages of cellular arrays in LSI are high packing density, high reliability, high manufacturing yield, flexibility in performance, and ease in error diagnosis.

To effectively utilize cellular arrays in LSI, logical designers must have efficient synthesis techniques at their disposal. The purpose of this research is the development of minimization algorithms for the efficient synthesis of cellular logic networks.

1.2 Review of Previous Work and Delimitation

Many cellular structures have been proposed for synthesizing combinational switching circuits. Maitra(21) has shown that cascaded networks composed of completely flexible two-input one-output logical cells may be used to realize a certain class of switching functions. He also gives a condition for realizing arbitrary switching functions by such networks.

Minnick's cutpoint(26) cellular array was a two-dimensional rectangular arrangement of cells, each of which had binary inputs on the top and left edges and outputs on the bottom and right edges. Each cell was connected with neighboring cells and specialized to produce a subset of the possible sixteen functions by a set of binary constants called "cutpoints". He showed that a cascade of cutpoint cells was capable of producing all the functions of the Maitra cascade. He also showed that a two-dimensional array of cutpoint cells could be arranged in such a way that an arbitrary function of n variables was produced. His synthesis technique relied on the ingenuity of the designer, however, and the realizations obtained did not necessarily have a minimum number of cells.

The cobweb array,[25] also introduced by Minnick, was a modification of the cutpoint array made by complicating the cell-interconnection structure. Improved realizations were possible with the cobweb array but the task of obtaining a realization for a particular function using a minimal number of cobweb cells was still up to the ingenuity of the designer.

Short[34] and Yoeli[38] studied two rail cascades. These cascades are functionally complete, i.e., any arbitrary function can be realized within the extent of a single cascade. However, the general two-output three variable cell of the two rail cascade is significantly more complex than the cell used in the arrays already described. Two rail realizations based on the Reed expansion may be obtained for an arbitrary function. However, this expansion does not always yield an economical realization. Indeed, only when the number of variables is one or two is the bound firm on the number of cells needed to realize a particular function.

All of the cellular arrays mentioned so far have at least one drawback. Given a particular switching function, there exist no algorithms to find an optimal design of the array to realize the function. Perhaps this limited success in the development of optimal cellular arrays lies in the problem of mathematically modeling these arrays. Several design parameters are involved which makes modeling difficult. Synthesis algorithms must recognize the trade-offs among such design parameters as the size of the array, the logical complexity of individual cells, the interconnection structure, and the speed of computation.

The cobweb array illustrates the fact that the total array size for realizing any arbitrary function can be reduced by increasing the logical complexity of the cells and by allowing interconnection among non-adjacent cells. Research on the (n,d,k) problem[7][8] has produced some analytical results concerning the interdependence among some of the purely topological parameters of cellular arrays such as the total number of connections incident on a cell, maximum and average intercell distance, and the size and planarity of the array. When both logical and topological parameters are considered, mathematical modeling becomes very vague.

Mathematical modeling of cellular arrays is a problem which needs further study. However, it should be realized that the urge to attain excellence in modeling should not be carried so far as to obscure the necessity of developing practical design algorithms. Therefore, in this thesis we will start with a particular two-dimensional array whose interconnection structure is constrained as in Figure 1.1.

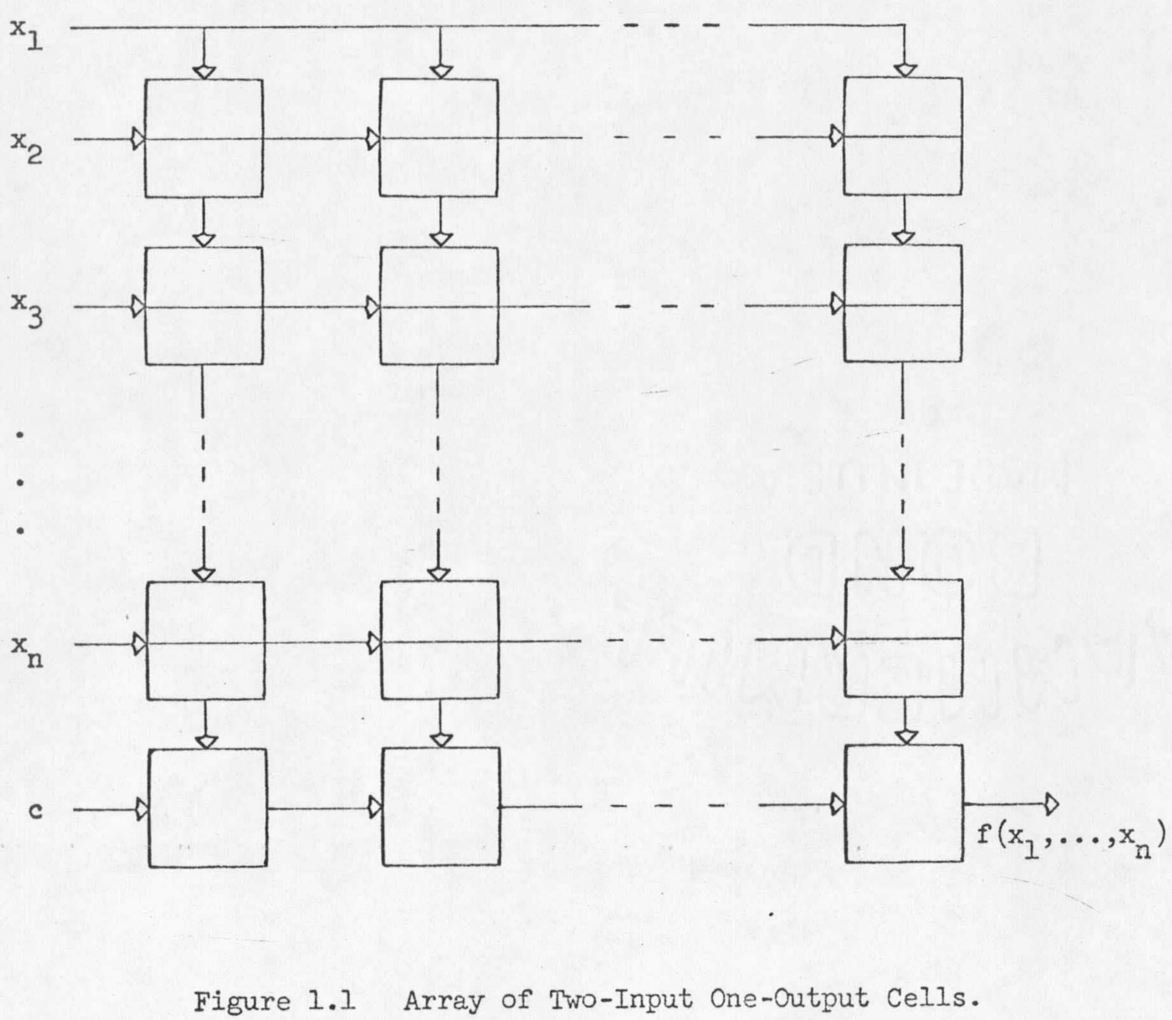

Figure 1.1 Array of Two-Input One-Output Cells.

Each cell produces an arbitrary two-input one-output combinational function. Input variables $x_1, x_2, \ldots, x_n$ are bussed horizontally and with the exception of the bottom row, vertical cascades of cells do not interact. The problem of synthesizing two-dimensional arrays in which the vertical cascades are allowed to interact is still open and seems to be very difficult. The bottom row of Figure 1.1 acts as a "collector" row and collects the functions produced in each cascade.

Cellular arrays whose general structure fits that of Figure 1.1 are considered in the following chapters. Minimization algorithms can usually be developed only after some constraints are placed on the allowable cell functions. These constraints are briefly mentioned in the next section and more fully discussed in Chapters 2 and 3.

1.3 Organization of Remaining Chapters

Chapter 2 develops minimization techniques for the case in which the cascade functions produced in Figure 1.1 are restricted to be unate. Also, the number of equivalence classes of unate cascade realizable functions is determined, a necessary condition for unate cascade realizability is established, and an improved test for determining unate cascade realizability is given.

In Chapter 3, the collector row of Figure 1.1 is set to produce the Exclusive Or or Logical Equivalence operation. Minimization techniques for arrays of this nature are developed and a solution to a classical minimization problem involving the Reed-Muller canonic forms is presented.

The algorithms developed in Chapters 2 and 3 require the determination of the cliques of a linear non-directed graph. Chapter 4 develops a new clique detection algorithm and compares it with other algorithms which have been programmed for this purpose.

The concluding Chapter 5 summarizes all the results obtained and discusses particular areas which merit further study.

Chapter 2

MINIMIZATION OF
UNATE CELLULAR ARRAYS

2.1 Introduction

The most general minimization problems as discussed in the introduction to this thesis are very complex. A more tractable problem is to consider particular array structures and develop minimization algorithms applicable to them. The particular array that will be considered in this chapter is a unate cellular array which is a one- or two-dimensional arrangement of two-input one-output combinational cells. Each cell is capable of being set to give any one of the fourteen unate functions of its two inputs; that is, excluding Exclusive-Or and Equivalence.

2.2 Unate Concepts

2.2.1 Definition

A switching function $f(x_1,x_2,\ldots,x_n)$ is said to be unate if and only if $f(x_1,x_2,\ldots,x_n)$ is representable as a disjunctive or conjuntive normal formula in which no literal x_i appears complemented and uncomplemented.

The following is a list of all unate functions of two variables, x and y: $0,1,x,x',y,y'$, AND(xy), NAND$(x'+y')$, OR$(x+y)$, NOR$(x'y')$, IMP$(x+y', x'+y)$, NIMP$(x'y,xy')$. The first six functions of this list are improper functions of two variables; that is, they are independent of some or all input variables.