Natural convection heat transfer between a body and its spherical enclosure

by Norman Weber

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Aerospace and Mechanical Engineering
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Abstract:
Natural convection heat transfer between isothermal vertically eccentric spheres and centrally located vertical cylinders and their isothermal spherical enclosure was experimentally investigated. Pertinent dimensionless parameters and variables utilized in the investigation were Rayleigh number, based on gap width, N , Prandtl number, N PR, eccentricity to gap width ratio, e/(r - ri), gap width to inner radius ratio, L/r , and aspect ratio, H/2r, and covered the following ranges: 4.7< N PR < 4148, 4 x 102< N RA < 9 x 10 7, 0.09< L/ri< 1.18 and -0.75< e/(ro-ri - + 0.75 for the eccentric — spheres; 6< Npr< 14, 3.5 x 103< Nra< 2.2 x 108, 1.14< H/2ri< 2.00 ro-ri and 0.09 < —------ < 1.18 for the vertical cylinder.

— r. — i Examination of the experimental data showed that the eccentricity had a slight effect on the average heat transfer coefficient. The negative eccentricities were found to produce slightly higher average heat transfer coefficients than positive eccentricities. The largest eccentricities, either positive or negative, were also seen to produce heat transfer coefficients larger than the next lower eccentricity, i.e., the average heat transfer coefficient for e/(ro - ri) = ±0.75 > the average heat transfer coefficient for e/i(r-r) = ±0.50, etc. Gap thickness to inner radius ratio, L9ri was also found to have an effect on the average heat transfer coefficient. The larger values of L/r. were found to yield higher values of Nusselt number. An increase in the viscosity did tend to suppress this behavior, however, the same trend was still found to exist. Heat transfer correlations for each individual fluid, as well as an overall correlation, were obtained for the eccentric sphere configuration utilizing conformal mapping techniques to transform the eccentric sphere configuration. Temperature profiles at five angular locations (0 = 0°, 40°, 80°, 120°, and 160°) were taken and yielded some insight as to the flow behavior and heat transfer as a result of eccentricity. A negative eccentricity was seen to enhance the convective activity while a positive eccentricity seemed to suppress the convective activity. A multicellular flow field is postulated for the smallest diameter ratio configuration.

The experimental data for the cylinders were examined independently and showed that aspect ratio did have an effect on the average heat transfer coefficient. The smaller aspect ratios for each diameter ratio produced higher average heat transfer coefficients. Heat transfer correlations were obtained for each cylinder as well as an overall heat transfer correlation for all of the cylinders. Temperature profiles were taken and were enlightening as to the nature of the effect of aspect ratio and diameter ratio on the flow field. The larger aspect ratios for all diameter ratios seemed to curtail the convective activity while the smaller aspect ratios for all diameter ratios seemed to promote convective activity. A multicellular flow regime is postulated for the smallest diameter ratio cylinder investigated.

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ABSTRACT

Natural convection heat transfer between isothermal vertically eccentric spheres and centrally located vertical cylinders and their isothermal spherical enclosure was experimentally investigated. Pertinent dimensionless parameters and variables utilized in the investigation were Rayleigh number, based on gap width, N_{RA}, Prandtl number, N_{PR}, eccentricity to gap width ratio, $e/(r_o - r_i)$, gap width to inner radius ratio, L/r_i, and aspect ratio, $H/2r_i$, and covered the following ranges:

$4.7 < N_{PR} < 4148$, $4 \times 10^2 < N_{RA} < 9 \times 10^7$, $0.09 \leq L/r_i \leq 1.18$,

and $-0.75 \leq e/(r_o - r_i) \leq + 0.75$ for the eccentric spheres;

$6 < N_{PR} < 14$, $3.5 \times 10^3 < N_{RA} < 2.2 \times 10^8$, $1.14 \leq H/2r_i \leq 2.00$,

and $0.09 \leq \dfrac{r_o - r_i}{r_i} \leq 1.18$ for the vertical cylinder.

Examination of the experimental data showed that the eccentricity had a slight effect on the average heat transfer coefficient. The negative eccentricities were found to produce slightly higher average heat transfer coefficients than positive eccentricities. The largest eccentricities, either positive or negative, were also seen to produce heat transfer coefficients larger than the next lower eccentricity, i.e., the average heat transfer coefficient for $e/(r_o - r_i) = \pm 0.75$ > the average heat transfer coefficient for $e/(r_o - r_i) = \pm 0.50$, etc. Gap thickness to inner radius ratio, L/r_i, was also found to have an effect on the average heat transfer coefficient. The larger values of L/r_i were found to yield higher values of Nusselt number. An increase in the viscosity did tend to suppress this behavior, however, the same trend was still found to exist. Heat transfer correlations for each individual fluid, as well as an overall correlation, were obtained for the eccentric sphere configuration utilizing conformal mapping techniques to transform the eccentric sphere configuration. Temperature profiles at five angular locations ($\theta = 0°, 40°, 80°, 120°$, and $160°$) were taken and yielded some insight as to the flow behavior and heat transfer as a result of eccentricity. A negative eccentricity was seen to enhance the convective activity while a positive eccentricity seemed to suppress the convective activity. A multicellular flow field is postulated for the smallest diameter ratio configuration.

The experimental data for the cylinders were examined independently and showed that aspect ratio did have an effect on the average heat transfer coefficient. The smaller aspect ratios for each diameter ratio produced higher average heat transfer coefficients. Heat transfer correlations were obtained for each cylinder as well as an overall heat transfer correlation for all of the cylinders. Temperature profiles were taken and were enlightening as to the nature of the effect of aspect ratio and diameter ratio on the flow field. The larger aspect ratios for all diameter ratios seemed to curtail the convective activity while the smaller aspect ratios for all diameter ratios seemed to promote convective activity. A multicellular flow regime is postulated for the smallest diameter ratio cylinder investigated.

NOMENCLATURE

Symbol	Description
a, b, c, d	Characteristic physical dimensions; exponents in functional equations
A	Area
C	Constant
$C_{1...n}$	Constants in equation (4.9)
C_p	Fluid specific heat at constant pressure
D	Diameter; width of vertical enclosure
e	Base of natural logarithms (2.71828...); eccentricity
E	Voltage
f	Denotes function
F	Denotes function
g	Acceleration of gravity, 32.174 ft/sec^2
h	Length of straight cylindrical section
$\bar{h}$	Convective heat transfer coefficient (average, defined as $\bar{h} = q_{net}/A\Delta T$)
H	Height of vertical plate; total height of cylinder
I	Electrical current
ID	Inner body diameter
k	Fluid thermal conductivity
k_c	Equivalent thermal conductivity

Symbol	Description
k_{eff}	Effective thermal donductivity, $k_{eff}/k = qL/4\pi k(T_i - T_o)r_o r_i$
l	Overall cylindrical length, l = H
L	Distance between plates; gap thickness, defined as $L = (r_o - r_i)$; length of vertical enclosure
N_D	Characteristic physical dimension ratio, a/b
N_{GR_a}	Grashof number, $\rho^2 g\beta(T_i - T_o)a^3/\mu^2$, a is replaced by any desired characteristic dimension
N_{NU_a}	Nusselt number, $\bar{h}a/k$, a is replaced by any desired characteristic dimension
N_{NU*}	Modified Nusselt number, $N_{NU*} = N_{NU}/(1 + L/r_i)$
N_{PR}	Prandtl number, $C_p\mu/k$
N_{RA_a}	Rayleigh number, $\rho^2 g\beta(T_i - T_o)a^3 C_p/\mu k$
N_{RA*}	Modified Rayleigh number, $\rho^2 g\beta(T_i - T_o)L^4 C_p/\mu k r_i$
n	Exponent on functional notation equation
q	Heat transfer rate, total energy supplied to inner body
q_1	Heat transfer rate, losses
q_{net}	Heat transfer rate, net, $q_{net} = q - q_1$
r	Radius to a point between the inner and outer surfaces; dimensionless radius ratio, $r = (R - R_\theta)/(R_o - R_\theta)$
R	Radius to a point within the annulus

Symbol	Description
r_{avg}	Average radius, $r_{avg} = (r_i + r_o)/2$
r_e	Reading on dial caliper used in equation (3.6)
r_i	Radius of inner body
r_o	Radius of outer sphere
r_{ref}	Reference reading used in equation (3.6)
R_i	Mapped radius of inner sphere
R_θ	Distance from center of the enclosing sphere to the surface of the inner body along a θ = constant line
R_o	Mapped radius of outer sphere; radius of outer sphere as used in equation (3.7)
T_i	Temperature of inner body
T_o	Temperature of outer sphere
T	Temperature at some point between the inner body and the outer sphere
T_{am}	Arithmetic mean temperature
T_{vm}	Volumetric mean temperature
ΔT	Difference between inner body surface temperature and outer sphere surface temperature, $\Delta T = (T_i - T_o)$
u_3	Defined by equation (A2.7)
u_2	$u_2 = -u_3$
u_1	Defined by equation (A2.13)
u	Abscissa in w plane

Symbol	Description
v_3	Defined by equation (A2.7)
v_2	$v_2 = -v_3$
v_1	Defined by equation (A2.13)
V_{exp}	Value of experimental data
V_{eqn}	Value of equation
w_3	Defined by equation (A2.6)
w_2	Defined by equation (A2.11)
w_1	$w_1 = u_1 + iv_1$
w	Defined by equation (A2.15)
x_2	Defined by equation (A2.4)
x_1	Defined by equation (A2.3)
x	Defined by equation (A2.2)
x'	Defined by equation (A2.1)
X	Independent variable used in equation (2.29) and defined as $X = N_{RA}[1-D_i/D_o]^{6.5}$
$X_{1...n}$	Independent variable used in equation (4.9)
y	Dependent variable used in equation (4.9); defined by equation (A2.2)
y'	Defined by equation (A2.1)
z	Defined by equation (A2.9)
α	Thermal diffusivity, $k/\rho C_p$
β	Thermal expansion coefficient

Symbol	Description
Γ	Dimensionless temperature ratio, $\Gamma = (T - T_o)/(T_i - T_o)$
μ	Dynamic viscosity of the fluid
Φ	Functional notation
ρ	Fluid density
π	Ratio of circumference of circle to diameter, 3.1415927...
θ	Angular displacement, measured from the upward vertical axis
σ	Deviation
δ	Average percent deviation
η	Inverse relative gap width, $2r_i/(r_o - r_i)$

Symbol	Subscripts
am	Arithmetic mean
F	Freon-11
H	Based on height of plate
i	Inner body surface, at a point
l	Denotes losses
L	Based on distance between plates or gap thickness $(r_o - r_i)$
m	Mapped
o	Outer spherical surface
R	Inner radius
s	Stem
um	Unmapped
vm	Volume mean
θ	Based on angular position

CHAPTER I

INTRODUCTION

During the past fifty years, technology has advanced far beyond all its previous years. This occurrence has fostered many problems and yielded some developments almost too awesome and spectacular to comprehend. During the same fifty year period considerable attention has been given to the phenomenon of natural convection heat transfer from a body to an infinite atmosphere. However, many of the natural convection problems confronting modern technology are those of a body and its finite enclosure, and only a small amount of data exists for this case. Extrapolation of these existing data to new configurations not covered by the existing data has had to suffice for design criteria. Extrapolation is acceptable only when adequate data are not available, and then should be used with extreme caution.

Several papers dealing with natural convection to bodies within finite enclosures have been published in recent years in an effort to overcome the serious lack of data. These papers primarily deal with concentric cylindrical and spherical annuli, and the natural convection phenomena which occur within these geometries have been quite well defined for Prandtl number range 0.7 to 4148 and for a wide range of diameter ratios and Grashof numbers. Unfortunately, these data are not enough to characterize natural convection within the many geometry variations possible, even with these simple configurations. There exists a need to extend these concentric configurations to ones with the inner body in an eccentric location and, as a further extension, to completely change the internal body shape. Absolutely no knowledge of the resulting flow field, heat transfer, or temperature profiles which might be encountered as a result of these changes is available. The existing data might be extrapolated in some manner to yield a prediction of these phenomena, but experimental verification attesting to its accuracy and effectiveness would be non-existent. Extrapolation of data to a different case is quite dangerous and often produces disastrous results when used to establish design criteria.

It might be appropriate to consider the manners in which a heat transfer prediction may be accurately made for a selected geometry. The first attempt might be an analytical solution of the governing equations and associated boundary conditions. It is immediately evident that due to the non-linearity of the governing equations, any analytical solution will be extremely difficult and may, as in the case of Mack and Hardee [1], be severely limited in applicability as well as being quite difficult to employ. Numerical techniques can also be utilized to effect a solution, and attempts along these lines have been successful in accurately predicting occurrences and trends for simple geometries but are quite laborious and difficult to employ. Therefore, another method of dealing with the problem is considered -- an experimentally determined solution in the form of an empirical equation.

The purpose of the current study is to experimentally obtain information concerning natural convection heat transfer between an eccentrically located sphere and a vertical cylinder and their spherical enclosure; and to determine the temperature distributions that take place within these two geometric configurations. These new data should aid in extending the current data to include these two new configurations and perhaps yield the necessary insight to effect a correlation that will encompass more than one geometry. The temperature profiles to be taken will be helpful in describing the flow phenomena and the heat transfer that takes place within these two configurations.

The basis for the current study will be the recent work of Scanlan, Bishop and Powe [2] for concentric spheres. This work presents several heat transfer correlations -- one for each of the test fluids involved (air, water, Dow Corning 200 Fluid - 20 cs, and Dow Corning 200 Fluid - 350 cs) and a single equation for all four fluids. The Dow Corning 200 fluids are silicone base fluids, and the 20 cs and 350 cs designations refer to the kinematic viscosity in centistokes at 25°C. An attempt will be made in the current investigation to correlate the experimentally obtained data with the data for the concentric spherical case [2] whenever possible.

The overall goals of this study are thus:

(1) To obtain empirical relationships that will characterize the natural convection heat transfer taking place between the spherical enclosure and the inner body for a wide range of variation of the independent variables for the eccentric sphere configuration and the cylindrical inner body configuration. The eccentricity is defined as the vertical displacement of the horizontal centerline of the inner sphere relative to that of the enclosing sphere. It may be positive (upward) or negative but is confined to the vertical axis. The cylindrical inner body is defined

as a right circular cylinder with hemispherical ends. A concentric location of this inner body is defined as the centroids of the inner body and the enclosing sphere being coincident and the longitudinal axis of the cylinder being coincident with the vertical axis of the enclosing sphere. The heat transfer was measured for various eccentricities for a spherical body and for a concentrically located cylindrical body as a function of impressed temperature differential.

(2) To obtain temperature distributions within the gap as the temperature difference between the two bodies is varied for a wide range of the independent variables.

(3) To correlate the current data with those currently in existence [2] for the concentric spherical annuli.

The apparatus used to determine the heat transfer rates and the temperature profiles was constructed such that the independent variables included the temperatures of the two bodies; the size, shape, and location of the inner body; and the test fluid. The radial temperature profile can be determined at five angular locations starting with 0° (upward vertical axis) and spaced at 40° increments.

CHAPTER II

LITERATURE REVIEW

Considerable attention has been given to natural convection heat transfer over the last fifty years. The area which has received the most attention has been the case of convection from a body to its infinite surroundings. Other geometries that also received much attention were horizontal and vertical rectangular enclosures. The most recent contributions have been relative to convection from bodies to their finite enclosures with the geometries most often considered being concentric cylindrical and spherical annuli.

For clarity the discussion is presented in three parts. These are (1) horizontal and vertical rectangular enclosures (parallel flat plates), (2) concentric cylindrical annuli, and (3) concentric spherical annuli. The main topic of interest is concentric spherical annuli. However, the other geometries are presented for completeness and to demonstrate similarities between natural convection phenomena which occur in the various geometries.

The case of natural convection from a body to an infinite atmosphere will not be treated since this subject is quite adequately covered in standard textbook references. Jakob [3] and Gröber, Erk, and Grigull [4] present very complete and informative reviews on natural convection heat transfer from a body to an infinite atmosphere. They also treat very adequately the topic of convection within vertical and horizontal enclosed spaces. The reader is referred to these sources for the case of a body to an infinite atmosphere and for a complete list of pertinent references for this situation.

Natural convection heat transfer within enclosures will be the general topic covered, and specific attention will be given to natural convection from bodies to a finite enclosure. In order to accomplish this, the introduction of some terminology at this point might be helpful. The study of the natural convection process involves both fluid mechanics and heat transfer considerations. This fact dictates that any characteristic parameters which describe the phenomena will be derived from a combination of these disciplines. The literature on natural convection generally agrees that the following dimensionless groups characterize the phenomena for finite enclosures:

$$N_{GR_a} = \frac{\rho^2 \beta \Delta T a^3 g}{\mu^2} , \qquad (2.1)$$

$$N_{PR} = \frac{C_p \mu}{k} , \qquad (2.2)$$

$$N_D = \frac{a}{b} \qquad (2.3)$$

where "a" and "b" are characteristic dimensions, ΔT is a suitably defined temperature difference, N_{GR_a} is called the Grashof number based on "a", N_{PR} is the Prandtl number, and N_D is a dimensionless ratio of the characteristic dimensions. An additional dimensionless group, the Rayleigh number, N_{RA}, is often used in place of the Grashof number and is the product of the Prandtl and Grashof numbers:

$$N_{RA_a} = N_{PR} \cdot N_{GR_a} = \frac{g \beta a^3 \Delta T}{\alpha \mu} . \qquad (2.4)$$

When considering natural convection heat transfer within enclosures, the literature, in general, states that the heat transfer taking place can be determined in functional notation as

$$N_{NU_a} = \Phi \left[N_{GR_a}, N_{PR}, N_D \right] , \qquad (2.5)$$

where

$$N_{NU_a} = \bar{h}a/k \quad . \tag{2.6}$$

This type of functional relationship can easily be verified from an examination of the governing equations and the appropriate boundary conditions.

Jakob [3] and Gröber, Erk, and Grigull [4] indicate that this functional relationship can take the final form

$$N_{NU} = C\ [N_{GR}\ N_{PR}]^{n} \tag{2.7}$$

for natural convection from a body to an infinite atmosphere and for either laminar or turbulent flow. In this expression and all subsequent expressions, the Nusselt number has the same characteristic dimension as the Grashof or Rayleigh number appearing in the expression. Therefore, the subscript on Nusselt number will be deleted in subsequent expressions. (2.7) is indicative of no explicit dependence of the correlation on the aspect ratio. Here, aspect ratio would be defined as the ratio of characteristic dimensions. Further, Jakob [3] and Gröber, Erk, and Grigull [4] define a pseudothermal conductivity, k_{eff}, first postulated by Beckman [3], which when combined in a ratio with the thermal conductivity of the fluid for pure conduction, describes the relative increase of heat transfer due to convection. The k_{eff} term may be described physically as the thermal conductivity necessary to conduct the same amount of heat as actually transferred by convection and conduction combined. Therefore, it is immediately seen that k_{eff}/k has a lower limit of 1. The same dimensionless variables and the same functional relationships associated with Nusselt number hold for k_{eff}/k and take the functional form

$$\frac{k_{eff}}{k} = \Phi\ [N_{GR},\ N_{PR},\ N_D]\ . \qquad (2.8)$$

HORIZONTAL-VERTICAL-RECTANGULAR ENCLOSURES

Now consider natural convection heat transfer between horizontal and vertical enclosures. Jakob [3] gives expressions for heat transfer based on the data of Mull and Reiher which for horizontal air layers reduced from the form of the equation (2.6) to

$$\frac{k_{eff}}{k} = 0.195\ N_{GR_L}^{\ 1/4} \text{ for } 10^4 < N_{GR_L} < 4 \times 10^5\ , \qquad (2.9)$$

and

$$\frac{k_{eff}}{k} = 0.068\ N_{GR_L}^{1/3} \quad \text{for } N_{GR_L} > 4 \times 10^5 \quad . \tag{2.10}$$

For air contained between vertical plates, Jakob [3] gives the following equations:

$$\frac{k_{eff}}{k} = 0.18\ N_{GR_L}^{1/4} \left(\frac{H}{L}\right)^{-1/9} \quad \text{for } 2 \times 10^4 \leq N_{GR_L} \leq 2 \times 10^5 \ ,$$

$$\text{and } 10.6 \leq \frac{H}{L} \leq 42.2 \quad ; \tag{2.11}$$

$$\frac{k_{eff}}{k} = 0.065\ N_{GR_L}^{1/3} \left(\frac{H}{L}\right)^{-1/9} \quad \text{for } 2 \times 10^5 \leq N_{GR_L} \leq 11 \times 10^6 \ ,$$

$$\text{and } 10.6 \leq \frac{H}{L} \leq 42.2 \quad . \tag{2.12}$$

Here, H is the height of the plates and L is the distance between them. These expressions for both horizontal and vertical enclosures were much simpler than those of Mull and Reiher.

Globe and Dropkin [5] used mercury, water, and silicone fluids in studying heat transfer in liquids confined between horizontal plates heated from below. The work covered large ranges of Prandtl number, 0.02 to 8750, and Rayleigh number, $1.51(10)^5$ to $6.76(10)^8$. They arrived at the following correlation:

$$N_{NU} = 0.069\ (N_{RA})^{1/3}\ (N_{PR})^{0.074} \quad . \qquad (2.13)$$

Here all properties are evaluated at the arithmetic mean temperature of the two plates, and Nusselt and Rayleigh number are based on the distance between the plates. An extension of this work was carried out by Dropkin and Somerscales [6]. They studied the heat transfer between parallel plates at various angles of inclination with the horizontal for the same fluids as Globe and Dropkin. Their findings were in agreement with those of Globe and Dropkin [5]. They had the same form of equation including exponents on the Rayleigh and Prandtl numbers. The constant in front of the equation did, however, vary as a function of the angle of inclination from 0.069 for $\theta = 0°$ to 0.049 for $\theta = 90°$. The Prandtl number range was 0.02 to 11,560 and the Rayleigh number range was 5×10^4 to 7.17×10^8, which represents somewhat of an extension to the previous investigation [5]. Landis [6], in commenting on the investigation of Dropkin and Sommerscales [6], stated that their results indicate the correctness of the conclusion that, for natural convection flows in enclosures, Nusselt number correlations are particularly insensitive to geometry, boundary conditions, and flow regimes. Eckert and

Carlson [7]utilized a Mach-Zender interferometer to investigate the flow and temperature profiles in a long vertical rectangular duct for air. The heat transfer coefficients were then evaluated on the basis of the temperatures obtained. For the boundary layer region, they propose the following expression for an average Nusselt number based on the height of the plate:

$$N_{NU} = 0.119 \, (N_{GR})^{0.3} \tag{2.14}$$

They further proposed another correlation based on the distance between the plates. This expression is

$$N_{NU} = 0.119 \, (N_{GR})^{0.3} \left(\frac{L}{H}\right)^{0.1} \, . \tag{2.15}$$

Batchelor [8] carried out an analytical solution for the problem of vertical walls. He was able to predict temperature and velocity profiles and, from these data, evaluate a Nusselt number. This work has been qualitatively verified by Eckert and Carlson [7] and produced the following expression for the heat transfer for small to moderate Rayleigh number, based on distance between plates:

$$N_{NU} = \frac{L}{D} + \left(\frac{2\gamma - 1}{720}\right) N_{RA} \quad \text{for } N_{RA} < 3 \times 10^4 \, , \tag{2.16}$$

where γ is a constant usually taken to be 1. For large values of Rayleigh number,

$$N_{NU} = 0.43\, N_{RA_L}^{1/4} \left(\frac{L}{D}\right)^{3/4} . \qquad (2.17)$$

Batchelor also set up the mathematical mechanism by which subsequent analytical attempts to solve the governing equations and associated boundary conditions for other geometries were made.

Poots [9] applied numerical techniques to the problem of vertical walls with conducting connecting strips. His findings were in good agreement with both those of Jakob [3] and Batchelor [8]. He found the heat transfer to be determined by

$$N_{NU} = 1 + 5.08 \times 10^{-8}\, N_{RA_L}^{2} \text{ for } N_{RA_L} < 10^3 , \qquad (2.18)$$

and

$$N_{NU} = 0.16\, N_{GR_L}^{1/4} \text{ for } N_{RA_L} \sim 10^4 . \qquad (2.19)$$

He also hypothesized that the expression for $N_{RA_L} \sim 10^4$ would probably hold for larger values of Rayleigh number. This was demonstrated by Jakob [3] who gives

$$N_{NU} = 0.18\, N_{GR_L}^{1/4} \left(\frac{H}{L}\right)^{-8/9} \text{ for } 2 \times 10^4 \leq N_{GR_L} \leq 2 \times 10^5 , \qquad (2.20)$$

which is essentially the same expression except for the aspect ratio term.

Wilkes and Churchill [10] applied numerical techniques to a long rectangular channel with air as the enclosed fluid. Their results agreed well with those of Poots [9] but there was a discrepancy between the results and the equation reported by Jakob [3] and given previously. Their data [10] varied from 0 to 70% in excess of the values predicted by equation (2.20). The variation was attributed to the range of (H/L) used in determining equation (2.20) being considerably beyond the range of (H/L) used for the numerical solution.

Another numerical solution to the governing equations for closed vertical walls is that of Newell and Schmidt [11]. They found their results to be in good agreement with existing experimental data and gave the following relationships for heat transfer:

$$N_{NU} = 0.0547\, N_{GR_L}^{\;0.397} \quad \text{for} \left(\frac{H}{L}\right) = 1 , \qquad (2.21)$$

$$N_{NU} = 0.155\, N_{GR_L}^{\;0.315} \left(\frac{H}{L}\right)^{-.265} \quad \text{for } 2.5 \leq \left(\frac{H}{L}\right) \leq 20 . \qquad (2.22)$$

Their range of Grashof number was 4×10^3 to 1.4×10^5.

Elder [12, 13] performed both experimental and numerical investigations of this problem. He did not, as did

Eckert and Carlson, determine any Nusselt number relationships from his data. He did, however, check his numerical solution against both his experimental data and the data of Eckert and Carlson [7] for validity and found the numerical predictions to be comparable to the experimentally determined values. On the basis of his experimental study, he described various types of flow and temperature fields which are generally in agreement with those of other investigators.

CONCENTRIC CYLINDRICAL ANNULI

Natural convection heat transfer in concentric cylindrical annuli has received considerable attention in recent years. This geometry has been investigated for a wide range of fluids, Grashof numbers, and diameter ratios. It has been investigated analytically, numerically, and experimentally.

Beckman [3] investigated this configuration in 1931 for air, hydrogen, and carbon dioxide as the enclosed fluids for diameter ratios (D_o/D_i) of 1.2 to 8.1. His Grashof number range was 640 to 1.5×10^7. He postulated an effective thermal conductivity as a correlating implement and, utilizing dimensional analysis in conjunction with physical arguments, arrived at the functional relationship

$$\frac{k_{eff}}{k} = \Phi[N_{GR}, N_{PR}, \frac{D_o}{D_i}] \quad . \tag{2.23}$$

Kraussold [3] in 1934 extended Beckman's data utilizing water and two kinds of oil. His experiments were carried out for diameter ratios, (D_o/D_i), of 1.2 to 3.0 and Prandtl numbers from 7 to 4000. He used the gap thickness, $L = (D_o - D_i)/2$, as the characteristic length and arrived at the following expressions for heat transfer:

$$k_{eff} = 1 \text{ for } N_{RA} < 10^3 \quad , \tag{2.24}$$

$$k_{eff} = 0.11\, N_{RA_L}^{0.29} \text{ for } 6.4 \times 10^3 < N_{RA_L} < 10^6 \quad , \tag{2.25}$$

$$k_{eff} = 0.40\, N_{RA_L}^{0.20} \text{ for } 10^6 < N_{RA_L} < 10^8 \quad . \tag{2.26}$$

Liu, Mueller, and Landis [14] performed an experimental investigation with five sets of concentric tubes containing air, water, or one of two silicone fluids to obtain a Prandtl number range of 0.7 to 3672. It was again found that the heat transfer could be correlated in terms of k_{eff}/k. The resulting expressions were

$$\frac{k_{eff}}{k} = 0.135 \left[\frac{N_{PR}{}^{2} N_{GR_L}}{1.36+N_{PR}}\right]^{0.278} , \qquad (2.27)$$

where

$$3.5 \leq \log \left[\frac{N_{PR}{}^{2} N_{GR_L}}{1.36+N_{PR}}\right] < 8.0 \text{ for } 0.25 \leq \frac{L}{D_i} \leq 3.25$$

and

$$\frac{k_{eff}}{k} = 1.0 \text{ for } \log \left[\frac{N_{PR}{}^{2} N_{GR_L}}{1.36+N_{PR}}\right] < 3.0 . \qquad (2.28)$$

They also compared their data with those of Beckman and Kraussold. Beckman's data were considerably higher than values predicted by their correlation. This was attributed to Beckman's failure to end insulate his apparatus. Kraussold's data, however, followed the general trend of the correlation but, on the average, was about 10% below the predicted values.

An experimental investigation relative to simple and obstructed annuli was performed by Lis [15]. His work covered a Rayleigh number range of 4×10^4 to 4.7×10^{10} for diameter ratios from 2.0 to 4.0, with Prandtl number varying from 0.645 to 1.32. He found the relation given on the following page to correlate his data within ±12%. The correlating expression for simple annuli is

$$\log\left(\frac{k_{eff}}{k}\right) = 0.0794 + 0.625 \log X + 0.0154 (\log X)^2 \quad , \tag{2.29}$$

where $X = N_{RA} [1 - \frac{D_i}{D_o}]^{6.5}$ and N_{RA} is based on the inner diameter. His data showed considerable discrepancy from the predictions of Kraussold. This was attributed to the possibility that the use of the gap thickness in evaluating Rayleigh number did not entirely compensate for all the geometrical effects as suggested by Kraussold.

Another study, carried out at essentially the same time, was that of Grigull and Hauf [16]. They used a Mach-Zender interferometer to make their measurements. Their experimental data covered a Grashof number, N_{GR_L}, range from 320 to 716,000 at a Prandtl number of approximately 0.7 (air). The following equation was presented for the mean Nusselt number:

$$N_{NU_L} = [0.2 + 0.145 (L/D_i)] N_{GR_L}^{0.25} e^{-0.02(L/D_i)}$$

$$\text{for } 30{,}000 \leq N_{GR_L} \leq 716{,}000 \quad ,$$

$$\text{and } 0.55 \leq L/D_i \leq 2.65 \quad . \tag{2.30}$$

Their data were approximately 20% higher than the correlation of Liu, Mueller and Landis [14], but did agree with those of Beckman and the predictions of Crawford and Lemlich [19].

Bishop [17], in a critique of the paper by Grigull and Hauf, utilized k_{eff}/k to obtain a simple correlating expression,

$$\frac{k_{eff}}{k} = 0.163 \, (N_{GR_L})^{0.2578} \text{ for } 5 \times 10^3 \leq N_{GR_L} \leq 7.16 \times 10^5 ,$$

$$\text{and } 0.32 \leq L/D_i \leq 2.65 \quad , \qquad (2.31)$$

which correlated their data to -7.42% to +10.2%. This represented an extension in the range of applicable Grashof number and gap thickness to inner diameter ratio with the additional advantage of its being a simpler expression.

An analytical investigation for concentric cylinders was carried out by Mack and Bishop [18]. They utilized the technique set forth by Batchelor [8] and expanded the stream function and temperature in infinite series of powers of the Rayleigh number. They obtained the first three terms of the series for both the stream function and temperature. The solution, however, involves a large number of constants which must be determined sequentially. Utilizing the temperature and stream function solutions, they determined local and overall heat transfer in terms of appropriate Nusselt numbers. Due to the fact that only the first three terms for both stream function and the temperature were obtained, the solution is necessarily limited to low values of Rayleigh number.

Numerical methods have also been applied to this problem by three investigators, [19], [20], and [21]. Crawford and Lemlich [19] utilized finite difference techniques in conjunction with a Gauss-Seidel iterative procedure to effect a solution to the concentric cylindrical case. They calculated both local and overall heat transfer effects through evaluation of the local and overall Nusselt number. Comparison of their data was made with those of Liu, Mueller, and Landis [14] and those of Beckman [3]. The agreement was fair for the data of Liu, et al, and within the scatter of the experimental data on which their correlations are based. The comparison with Beckman is much better. Therefore, quantitative as well as qualitative credibility has been established for this numerical model.

Abbott [20] also carried out a numerical solution to this problem utilizing iterative techniques requiring matrix inversions and multiplication, and his results compared quite well with the work of Crawford and Lemlich [19]. The matrix method was found to converge faster than the finite difference technique for N_{GR_L} $(L/r_i)^3$ less than 6 x 10^5 to 2 x 10^6. No definite value of the limit was established.

The latest numerical solution to this problem is that of Powe, Carley, and Carruth [21]. Their work is for air (N_{PR} ∿ 0.7) and utilizes finite difference techniques to obtain a solution. The main concern of the effort was to numerically predict values of velocity, temperature, stream function, and Nusselt number near Rayleigh numbers where the flow will go from a steady condition into an unsteady condition for a wide range of inverse relative gap width η, (2.8 - 12.5). The existence of an unsteady condition was quite well documented, [14], [16], [17], [22], and [23]. The existing experimental data had been correlated to provide some insight to conditions for onset of the unsteady condition, but the data were sparse enough so that a well defined curve was not possible.

Powe, Carley, and Carruth [21] compared their results with the existing data and were reasonably successful in their attempts. They observed that the onset or existence of an unsteady condition did not noticeably affect the Nusselt number for any of the inverse relative gap widths considered. Temperature and velocity profiles were affected in accordance with the inverse relative gap thickness,