



Entrance level competencies needed for beginning mathematics courses at Montana State University
by John Dail Whitesitt

A thesis submitted in partial fulfillment of the requirements for the degree of DOCTOR OF
EDUCATION

Montana State University

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Abstract:

The problem of this study was: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Students enrolled in Math 100, an intermediate algebra course; Math 115, a trigonometry course; Math 117, a one quarter survey calculus course; and Math 121, a regular beginning calculus course were tested at the beginning of fall quarter to identify entrance level competencies. At the end of the quarter these students were classified as successful or not successful based on course grades. The independence of success and entrance level competencies in various areas was tested using a Chi Square Test of Independence. The effectiveness of the remedial courses was considered by estimating the proportion of successful students in Math 001, a beginning algebra course; Math 100; and Math 115 who demonstrated competence in the areas identified as important for success in subsequent courses. All hypotheses were tested at the .01 level of significance and the proportions were estimated using 95 percent confidence intervals.

Three of the nine competency areas tested in Math 100 were found to be significantly related to success. One of the ten areas tested in Math 115 and Math 117 was found to be significantly related to success, and seven of the thirteen areas tested in Math 121 were found to be significantly related to success. All of the remedial mathematics courses at Montana State University were ineffective in developing competency in most areas identified as important for success in subsequent courses.

More emphasis needs to be placed on developing competence in algebra for college bound students and less needs to be placed on trigonometry. Many competency areas in algebra were found to be significantly related to success in beginning mathematics courses, but only one trigonometry area was significantly related to success. All high school students should be encouraged to take at least two years of algebra before they graduate because of the ineffectiveness of college remedial courses.

ENTRANCE LEVEL COMPETENCIES NEEDED FOR
BEGINNING MATHEMATICS COURSES AT
MONTANA STATE UNIVERSITY

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A thesis submitted in partial fulfillment
of the requirements for the degree


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Approved:


Chairperson, Graduate Committee


Head, Major Department


Graduate Dean

MONTANA STATE UNIVERSITY
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ABSTRACT

The problem of this study was: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Students enrolled in Math 100, an intermediate algebra course; Math 115, a trigonometry course; Math 117, a one quarter survey calculus course; and Math 121, a regular beginning calculus course were tested at the beginning of fall quarter to identify entrance level competencies. At the end of the quarter these students were classified as successful or not successful based on course grades. The independence of success and entrance level competencies in various areas was tested using a Chi Square Test of Independence. The effectiveness of the remedial courses was considered by estimating the proportion of successful students in Math 001, a beginning algebra course; Math 100; and Math 115 who demonstrated competence in the areas identified as important for success in subsequent courses. All hypotheses were tested at the .01 level of significance and the proportions were estimated using 95 percent confidence intervals.

Three of the nine competency areas tested in Math 100 were found to be significantly related to success. One of the ten areas tested in Math 115 and Math 117 was found to be significantly related to success, and seven of the thirteen areas tested in Math 121 were found to be significantly related to success. All of the remedial mathematics courses at Montana State University were ineffective in developing competency in most areas identified as important for success in subsequent courses.

More emphasis needs to be placed on developing competence in algebra for college bound students and less needs to be placed on trigonometry. Many competency areas in algebra were found to be significantly related to success in beginning mathematics courses, but only one trigonometry area was significantly related to success. All high school students should be encouraged to take at least two years of algebra before they graduate because of the ineffectiveness of college remedial courses.

Chapter 1

INTRODUCTION

Mathematics education for the college bound student has undergone a rapid and often diverse reorganization during the past 20 years. The success of this reorganization has been questioned widely by mathematics educators such as Morris Kline (1973) and by parents and students as well. Declining achievement test scores during the past 15 years have focused public attention on the question and have led many people, including Kline, to conclude that the reorganization, commonly called "New Math," is the cause of the problem. "New Math" is a term that refers to all of the mathematics programs during a period of time spanning two decades. As such, it certainly is not a single entity and, therefore, is most likely not either entirely good or entirely bad.

As Ferguson (1976) and many others have pointed out, the nature of the college bound student has changed drastically during this time. The growing belief in free access to higher education and accompanying open admissions policies have had a definite effect on the variety and number of students attending college. The preparation of students for college level mathematics has, correspondingly, become much more complex than ever before. According to Lindberg (1974), Lindquist (1965), and Maxwell (1975), universities, colleges, and junior colleges have become increasingly involved in the process. Increased coordination of efforts has recently been called for in a joint publication of the Mathematical

Association of America and the National Council of Teachers of Mathematics (1978). It was within this rather complex setting that this study took place.

At Montana State University there are basically five courses that students can take as beginning mathematics courses. These courses are: (1) Math 001, a course in elementary algebra; (2) Math 100, a course in intermediate algebra; (3) Math 115, a course in trigonometry; (4) Math 117, a terminal one quarter calculus course for students who need only a broad overview of calculus; and (5) Math 121, the first quarter of the standard six quarter calculus sequence. A student and his advisor usually make the decision as to which course is an appropriate starting point for that student. It was on these five courses that this study centered.

Statement of the Problem

The problem of this study was two-fold: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

Need or Purpose of the Study

The first indication of a need for this study was provided by the low success rate of students in beginning mathematics courses at Montana State University. For the purpose of this study, a successful student was defined to be any student who received a course grade of A, B, or C. Data was available only for students who stayed in a course long enough to be listed on the registrar's final grade sheet. This gave success rates that appeared higher than they would have if all students who started the courses could have been considered. Even considering only those students, during fall quarter of 1977, 32 percent of the basic algebra students were unsuccessful; 26.5 percent of the intermediate algebra students were unsuccessful; 29 percent of the trigonometry students were unsuccessful; and 15 percent of the beginning calculus students were unsuccessful. There is no question that including students who dropped courses before final grade sheets were printed would have significantly increased these figures. This situation is fairly common at other institutions as well. Lindberg (1974), in a study of 450 universities, colleges, and junior colleges, found that 57 percent of the mathematics departments surveyed rated at least 25 percent of their beginning mathematics students as unsuccessful. Identification of the competencies needed for success in beginning mathematics courses was needed to help develop an understanding of the situation.

Another reason for doing this study arose from the fact that the tremendous variety of mathematics courses now being offered to college bound students at the secondary level may lead to poor preparation of these students. The variety is pointed out by the following examples. Bozeman Senior High School offered only four mathematics courses during the 1957-1958 school year: Algebra I, Plane Geometry, Algebra II, and Math IV. During the 1977-1978 school year they offered 13 mathematics courses: Basic Computer Applications, Data Processing, Algebra I-A, Algebra I-B, Algebra I, Geometry, Trigonometry, Calculus, Industrial Mathematics, Math Topics, Statistics, Algebra II, and Business Math. Bozeman Senior High School is not alone in this move. As another example, Helena Senior High School moved from four courses similar to those at Bozeman Senior High School in the 1957-1958 school year to a total of 13 courses in the 1978-1979 school year. This presents the college bound secondary student with a confusingly large number of alternatives. Which of these will provide the student with the best chance for success when he gets to college? Can the student wait until he gets to college to take algebra and still have a reasonable chance for success? Will computer programming or statistics be more valuable than a second year of algebra? The answers to these and many similar questions depend not only on the future plans of the student, but also on the identification of needed competencies and the evaluation of the remedial courses offered at the college level. The Carnegie Foundation

for the Advancement of Teaching believes that "colleges have an obligation to make clear to the high schools which skills they expect their students to acquire before they are admitted" (1978).

Effective placement of students once they get to college also depends on the identification of competencies needed for success in various possible beginning courses. No advanced placement testing program can be useful if the competencies being tested don't coincide with those needed by the student once he is placed. A recent study by Bers and Jaffe (1977) showed that there was no significant relationship between prerequisites for beginning chemistry courses at Oakton Community College and course grades. Prerequisites must provide the student with needed competencies in order to be meaningful.

Declining achievement test scores also point to a need for establishing the importance of prerequisite skills for success in college mathematics. There has been a significant decline in mean scores in both the American College Testing Program (ACT) test scores and the Scholastic Aptitude Test (SAT) scores during the past 15 years. Ferguson (1976) and Ferguson and Schmeiser (1978), discussing the ACT test score decline, and Braswell (1978) and Edson (1976), discussing the SAT score decline, list a number of probable causes, but point out that real evidence of a cause-effect relationship has not been found. Among the probable causes that they present are: (1) an increase in the number of elective courses being offered in secondary schools, (2) a

lowering of academic standards, (3) changes in our society, and (4) declining student motivation. In a survey of incoming freshmen at Berkeley, Maxwell (1975) found that although achievement test scores were declining, average grade point averages were increasing for college freshmen. She attributed this trend to grade inflation because of her belief that performance in college courses had also declined. Maxey and Others (1976) found a similar trend in high school grade point averages. Their findings were based on self-reported grades in four subject areas obtained from the ACT test results for 1970-1971 through 1974-1975.

Finally, there exists a controversy over whether the responsibility for remedial education belongs at the universities, the colleges, the junior colleges, or the secondary schools. Maxwell (1975), a professor at Berkeley, states:

I do not see the University's function to be one of offering basic elementary and junior high skills and subjects. It is too costly and too time consuming for both staff and students.

In a recent article to Montana mathematics teachers, Bill Stannard (1978), a member of the Board of Directors of the National Council of Teachers of Mathematics, said:

Clearly, the proper place to learn high school mathematics is in high school, not on a college campus. The pace in college for this kind of course is about four times that for the same material in high school.

If students are to be convinced to come to college better prepared than they do at present, it seemed critical to identify necessary entrance

level competencies as well as to provide some evidence that remedial mathematics courses at the college level are not as effective as they might be.

General Questions Answered

This study attempted to answer eight major questions. They were:

1. Are there certain mathematical competencies that a student should have before taking a beginning mathematics course at Montana State University if he or she expects to be successful?
2. Do the remedial mathematics courses at Montana State University effectively develop the competencies needed for success in subsequent courses?
3. What proportion of the students who register for and attend beginning mathematics courses at Montana State University are actually successful in that course?
4. What proportion of the students who register for and attend remedial mathematics courses at Montana State University have not already passed an equivalent course?
5. Do students who wait until they get to Montana State University to take their first algebra course have a reasonable chance for success in that course?
6. Does the age of the student have any effect on his or her chances for success in beginning mathematics courses at Montana State

University?

7. Does the sex of the student have any effect on his or her chances for success in beginning mathematics courses at Montana State University?

8. Does the method of instruction used for Math 100 have any effect on the success rate for students in that course?

Sex, age, and method of instruction were included in the study because they had been reported in other research as important factors and because they were of particular interest to either the researcher or the Mathematics Department at Montana State University.

General Procedures

Arrangements were made through the chairman of the Mathematics Department at Montana State University for the sampling and testing of mathematics students taking beginning mathematics courses during fall quarter of the 1978-1979 academic year.

Students were given a test of mathematical competencies at the beginning of fall quarter. The competencies of interest in this study were those competencies that a student could acquire from courses that precede the one for which he was currently registered. The competencies covered in the course for which the student was registered were not the subject of this investigation. Students who demonstrated a prior knowledge of the competencies covered by this course were eliminated from consideration for questions involving entrance level competencies.

This procedure was necessary because many students were required only to take a certain number of mathematics courses and often took the easiest course offered even though they already knew the material. Their success would not have been the result of needed competencies but of prior knowledge of the course itself.

In addition to the test, the students were asked to fill out a short personal data form which allowed the researcher to consider questions pertaining to the effectiveness of previous courses, the effect of age, the effect of sex, and the effect of instructional method.

At the end of the quarter, students were classified as either successful or unsuccessful according to the course grade that they received. Using the Chi Square Test of Independence, the researcher checked to see if the success of the students was related to competence as measured by the tests discussed earlier.

The researcher also checked to see what proportion of the successful students in each remedial course demonstrated competence in the areas identified as important for success in the next sequential course.

Limitations and/or Delimitations

The delimitations of this study were:

1. The study considered only the mathematics area and not any other areas subject to remediation at the college level.

2. The study considered only students who were taking mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year.

3. The study was supported by the resources available through the Montana State University library.

Definition of Terms

The following definitions were provided by the researcher or correspond to those used in other research as indicated by citation and are to be considered as operational definitions.

College level mathematics. Courses at or above the level of calculus or calculus and analytic geometry (Mathematical Association of America and the National Council of Teachers of Mathematics, 1978).

Remedial mathematics. Courses taught by colleges which cover one or more of arithmetic, elementary algebra, intermediate algebra, trigonometry, and high school geometry.

Successful student. Any student who received a course grade of A, B, or C (Lindberg, 1974). In pass-fail courses, students who passed were considered successful.

Unsuccessful student. Any student who registered for and attended class but failed to receive a course grade of A, B, or C with the following exception. Students who received an I (Incomplete) grade were eliminated from further consideration but students who withdrew, for any reason, were considered unsuccessful. In pass-fail courses,

students who failed or withdrew were considered unsuccessful and those who received an I were again eliminated.

Entrance level competency. A skill or an understanding based on material covered in previous courses that a student demonstrated at the beginning of a given course.

Exit level competency. A skill or an understanding that a student demonstrated at the end of a given course.

Younger student. Any student who was less than 25 years old.

Older student. Any student who was 25 years old or older.

Math 001. A course in elementary algebra offered at Montana State University for no credit.

Math 100. A course in intermediate algebra offered at Montana State University for five quarter credits.

Math 115. A course in trigonometry offered at Montana State University for five quarter credits.

Math 117. A one quarter, terminal calculus course offered at Montana State University for five quarter credits.

Math 121. A beginning calculus course offered at Montana State University for four quarter credits.

Competence. The ability to answer correctly the items on the tests in Appendix B as follows: answer both items correctly for areas covered by only two items, answer two out of three items correctly for areas covered by three items, and answer three out of four items

correctly for areas covered by four items.

Summary

Changing programs in mathematics at all levels during the past 20 years have created a confusing picture for college bound secondary students. This study attempted to clarify the situation by identifying those mathematical competencies that a student needs in order to be successful in beginning mathematics courses at Montana State University. The study also evaluated the remedial mathematics courses at Montana State University in an attempt to convince college bound students and their advisors of the need for such students to take certain mathematics courses in high school.

Chapter 2

REVIEW OF LITERATURE

For the purpose of this study, the related literature was considered under three main topics. These topics were: college preparatory high school programs, college remedial programs, and prediction of college success. National trends as well as the situation in Montana were considered within each of the first two topics. Prediction of college success was broken down in order to examine literature related to general college success as well as that related specifically to success in mathematics courses.

College Preparatory High School Programs

High school mathematics programs in the late 1950's consisted primarily of Algebra I for freshmen, Plane Geometry for sophomores, Algebra II for juniors, and some course for seniors that commonly included trigonometry and solid geometry (Fehr, 1957). Fehr predicted a drastic change in secondary mathematics in his 1957 article "Mathematics in Ragged Clothes." The Commission on Mathematics of the College Entrance Examination Board (CEEB), in its report Program for College Preparatory Mathematics (1959), made eight specific recommendations pertaining to the content of a college preparatory mathematics program. Their first recommendation indicated that college bound seniors should be prepared to take college mathematics courses beginning at the level

of calculus and analytic geometry. Another indicated a definite need for trigonometry at the junior level. The combination of plane and solid geometry was recommended along with the inclusion of inequalities in the consideration of equations. They recommended that seniors should study elementary functions and either probability or modern algebra from an abstract point of view. The other recommendations included stress on deductive reasoning in algebra as well as geometry and stress on the structure of mathematics and the use of sets, variables, functions, and relations as unifying concepts throughout the curriculum. This report represented the beginning of a major movement in mathematics curriculum reform that has been commonly called "New Math."

In 1974, following a period of rapid and varied change in the secondary mathematics curriculum, the Conference Board of the Mathematical Sciences appointed the National Advisory Committee on Mathematical Education (NACOME) to study the changes that had occurred in mathematics education during the preceding fifteen years. In the NACOME report, Overview and Analysis of School Mathematics Grades K-12 (1975), the changes that followed the CEEB recommendations are presented and discussed. According to the NACOME report, some of these recommendations were widely implemented. Plane and solid geometry have been combined, inequalities are considered with equations, and trigonometry is generally taught in college preparatory programs. Many other recommendations were less widely implemented and the result, as indicated by

this quotation from the NACOME report (1975), has been a wide diversity in college preparatory secondary mathematics programs.

As part of the NACOME effort to survey the status of mathematics education in the United States, we have collected whatever published mathematics objectives were available in each of the 50 states. The diversity of this collection defies detailed quantitative analysis.

Diversity of program offerings seems to be the present trend in secondary mathematics programs. The NACOME report also makes eight recommendations for mathematics curriculum change. It is important here to notice the similarities and differences between the NACOME recommendations and the CEEB recommendations presented fifteen years earlier. The NACOME report does not recommend that college bound seniors be prepared to take calculus and also fails to mention those items that it found to have been widely implemented. There is continued stress on deductive reasoning, structure, unifying concepts, and probability. Several recommendations in the NACOME report deal with mathematics for non college bound students, an area that was not covered in the CEEB report. Specific suggestions for college preparatory programs corresponding to those in the CEEB report are missing from the NACOME report, along with any indication of the desired level for college freshmen to start their mathematics work.

Recently, the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) jointly made some specific recommendations for college bound high school students in their publication, Recommendations for the Preparation of High School Students

for College Mathematics Courses (1978). This document again indicates that college mathematics begins at the calculus or calculus and analytic geometry level. The authors go on to say:

Any student who is unable to perform arithmetic calculations and algebraic operations with accuracy and reasonable speed, to understand which operations to use in a given problem, and to determine whether the results have meaning is severely handicapped in the study and applications of mathematics.

They further recommend a de-emphasis on trigonometry and a greater emphasis on algebra. A final recommendation of interest suggests more cooperation between secondary schools and colleges in the areas of remedial programs in schools and colleges and the preparation of students for college mathematics. This recommendation was foreshadowed by Rippey (1976) who claimed that confusing aims in secondary schools have contributed to poor preparation of high school students for college mathematics. In addition, the Carnegie Foundation for the Advancement of Teaching claims "colleges have an obligation to make clear to the high schools which skills they expect their students to acquire before they are admitted" (1978).

The college preparatory programs that have evolved during the past twenty years have not been totally successful. A study (Blai, 1976) of 272 freshmen (73 percent of the freshman class) at Harcum Junior College indicated that 61 percent of the entering freshmen felt that they were not adequately prepared for their college mathematics courses. In 1974, 41 percent of the incoming freshmen at Berkeley felt

that they needed remedial mathematics (Maxwell, 1975). Rosovsky (1976) reports that 35 percent of the freshmen entering Harvard in the fall of 1977 failed to achieve a score of 50 percent on the placement tests covering basic pre-calculus mathematics.

College Remedial Programs

The enrollment in remedial mathematics courses at the college level has increased dramatically in recent years. In a study of 450 colleges, junior colleges, and universities (Lindberg, 1974), 75 percent of the mathematics departments reported offering "preparatory mathematics programs." Lindberg defines preparatory mathematics as mathematics that is generally available to high school students including arithmetic, algebra, and geometry. The percentages varied from 65 percent in institutions with graduate programs to 96 percent in two year colleges. More than one third of the departments in this study reported that the enrollment in preparatory mathematics courses accounted for at least 30 percent of their total enrollment. Lindquist (1965), in a study of 855 post-secondary institutions, reported that 77 percent of them offered remedial mathematics courses. In a study of 194 junior colleges conducted by the American Mathematical Association of Two Year Colleges, Baldwin and Others (1975) found that 91 percent of the junior colleges actually offered "developmental mathematics" courses but 96 percent felt that there was a need for such courses. Developmental mathematics was defined to be arithmetic and basic algebra.

According to final enrollment figures kept by the Office of the Registrar at Montana State University, the enrollment in remedial mathematics courses at Montana State University has been rising faster than the total enrollment in mathematics courses in general. While the ratio held fairly constant during the early sixties, it has risen dramatically during the seventies. Table 2.1 shows these figures based on fall quarter ending enrollments for every third year beginning with 1961 and ending with 1979.

Table 2.1

Total Enrollment in Mathematics Compared to
Enrollment in Remedial Mathematics at
Montana State University

Year	Total Mathematics Enrollment	Remedial Mathematics Enrollment	Ratio Remedial/ Total
1961	1600	429	.268
1964	1956	526	.269
1967	2287	557	.244
1970	2415	922	.382
1973	2434	773	.318
1976	2321	1068	.460
1979	2769	1461	.528

There appears to be widespread concern about the effectiveness of college level remedial mathematics programs. Lindberg (1974), in the study previously discussed, reported that 67 percent of the departments responding to his questionnaire rated their preparatory mathematics programs as only "somewhat successful." He also reported that almost 57 percent of the departments rated fewer than 75 percent of their preparatory mathematics students as successful. Here, again, success was considered to be a course grade of A, B, or C. Baldwin and Others (1975), also previously discussed, found that only six percent of the junior colleges in their study reported more than an 80 percent success rate for developmental mathematics students in subsequent mathematics courses. Almost 50 percent of the institutions had never evaluated their remedial programs. Edmond (1976) reported that a majority of the remedial programs in junior colleges in the United States are unsuccessful in preparing students for subsequent non-remedial courses.

Maxwell (1975) of Berkeley shares the feeling of many mathematics educators that students who are admitted to Universities "should be able to write at an eighth grade level and have mastered fundamental arithmetic skills and basic algebra concepts."

The Carnegie Foundation for the Advancement of Teaching (1978) sums up the situation that college remedial programs are in as follows:

For the student who reaches college without such skills [elementary skills], even the best of remedial education may be too

little and too late. We therefore urge all states to take such measures as may be needed to ensure that elementary and high school students acquire proficiencies consistent with their abilities and grade levels in reading, computation, and written English.

Prediction of Success in College

Many authors have written about the prediction of success for college students. Armstrong (1976) reported that high school grade point averages (GPA), ACT test scores, and SAT test scores were all useful in predicting college success measured by college GPA. She used the entire freshman class of the University of Minnesota during the 1973-1974 academic year as her subjects. Mauger and Kolmodin (1975) showed that SAT test scores give valid prediction of success in college course work. Croft (1976) also reported high school GPA to be useful in predicting college success but did not find ACT test scores to be useful. Croft was, however, considering only multicultural students with low prior success in educational endeavors.

More than 20 years ago, Paul Horst (1956) recognized the need for predicting success in particular course areas in addition to predicting general success in college.

The prediction of general or over-all college success is certainly worthwhile, but the differential prediction of success for a wide variety of college course areas should be much more valuable to the student in helping him avoid the areas where he is weak and choosing those for which his chances for success are greater.

Achievement test scores, ranking in the high school graduating class, high school mathematics grades, and I.Q. scores have all been

shown to be useful predictors of success in college mathematics courses (Troutman, 1977). In a study of 142 women selected at random from a class of 706 freshmen at Longwood College, Gussett (1974) found all three of the Scholastic Aptitude Test scores to be useful predictors of mathematics course grades. The correlation coefficients between each test and student mathematics grades were .63 for SAT-T, .62 for SAT-M, and .48 for SAT-V. Troutman (1977) found only a .50 correlation between SAT-M scores and mathematics grades. His study used 123 students at York College. SAT-M was, however, the best predictor of mathematics grades that he found with high school rank, high school mathematics grades, and I.Q. scores also significant at the .01 level.

Having the necessary prerequisites would, by its very nature, seem to be important for the success of college mathematics students since all college mathematics courses require some background. Dahlke (1974) found that previous achievement in arithmetic was the best predictor of success in a college arithmetic review. However, in a study of 120 chemistry students at Oakton Community College, Bers and Jaffe (1977) examined the relationship between prerequisites and success in beginning chemistry courses and did not find it to be significant. In fact, the successful and unsuccessful groups each had 69 percent who had the necessary prerequisites and 31 percent who did not.

Summary

The college preparatory mathematics curriculum has undergone a

rapid and often diversified reorganization in recent years. The result has been a very non-uniform preparation of students for college mathematics. College mathematics programs have expanded their remedial course offerings in an attempt to meet the changing needs of their students. The efforts, however, have not been as successful as they could have been. In trying to properly advise college bound students, many methods have been developed for the prediction of success in college mathematics and achievement test scores seem to be the best single predictor.

Chapter 3

PROCEDURES

The problem of this study was two-fold: (1) to identify entrance level competencies that were important for success in beginning mathematics courses at Montana State University during fall quarter of the 1978-1979 academic year; and (2) to evaluate the effectiveness of the remedial mathematics courses at Montana State University in developing the competencies identified as important for success in subsequent courses.

This chapter begins with a description of the population and of the sampling procedure that was used. This is followed by an explanation of the categories that were compared and the methods that were used to account for contaminating variables. The next section of the chapter discusses the data collection procedure and the test instrument. Next, the statistical hypotheses that were tested are listed. Finally, the actual analysis of the data is explained along with the precautions taken to insure accuracy in the collection, analysis and presentation of the data.

Population Description and Sampling Procedure

This study took place on the campus of Montana State University, a land grant institution established in 1893 under the authorization of the Morrill Act of 1862. The University is located in Bozeman, Montana,

a community of approximately 20,000 people, and during fall quarter of 1978 had an undergraduate student population of approximately 9,400 with a graduate student population of approximately 520. The academic faculty numbers over 500 with an additional 200 professional staff members in the research and extension areas.

The Department of Mathematics at Montana State University consists of approximately 25 full time instructional faculty members and 30 part time graduate assistants. During fall quarter of 1978, there were 3,388 students enrolled in mathematics courses with 1,366, approximately 40 percent, enrolled in 39 sections of remedial algebra and trigonometry.

The population under consideration in this study consisted of the students enrolled in Math 001, Math 100, Math 115, Math 117, and Math 121 at Montana State University during fall quarter of the 1978-1979 academic year. There were 2,169 students enrolled in those courses at that time.

The statistics that were estimated in this study were to be used as estimates of parameters within each course and not as estimates of overall population parameters. For this reason, sample size calculations were made for each course separately. Since the sample size requirements for using the Chi Square Test of Independence are minimal, the sample sizes were determined by considering the accuracy desired for estimating the proportions of interest. The researcher wanted to

be able to estimate the proportion in question accurately to within five percent with 95 percent confidence, but was willing to settle for less accurate estimation due to reduction in sample size caused by elimination of students, etc. Because the researcher had reason to doubt the prior estimates of p , the proportion being estimated, and q , $1-p$, values of $p = q = .50$ were used for sample size calculations to insure that the sizes were large enough for the stated purpose. Cochran (1963) recommends the use of the following formulas for sample size calculations of this type:

$$(1) n_0 = (t^2)(p)(q)/(d^2)$$

$$(2) n = (n_0)/(1+[n_0/N])$$

The first formula gives an initial approximation of the sample size where t is the appropriate value of Student's t Statistic; p and q are both .50, and d is the allowable variation in the proportion to be estimated. The second formula corrects the initial estimate for finite populations of size N . The calculations of sample sizes for each course are summarized in Table 3.1.

The sample size selected in each case was the first multiple of 40 larger than n . The extra sample size was an allowance for students who would later have to be eliminated from the study and multiples of 40 were used to allow sampling by sections of 40 students each. These calculations indicated that samples of 200 for Math 001, 280 for Math 100, 200 for Math 115, 160 for Math 117, and 280 for Math 121 would be

sufficient.

Table 3.1
Sample Size Calculations

Course	Estimated Enrollment N	n_0	n	Projected Sample Size
Math 001	350	384	183	200
Math 100	850	384	265	280
Math 115	375	384	190	200
Math 117	225	384	142	160
Math 121	700	384	248	280

The Math 001 and Math 100 students presented a different sampling problem than the other students. Math 001 did not start until two weeks after the beginning of fall quarter to allow those Math 100 students who had difficulty to drop back to Math 001 without missing part of the course. All of the Math 001 students and part of the Math 100 students received individualized instruction and were not assigned to sections. A simple random sample of 200 Math 001 students was to be selected during the first week of the course. The computer fills sectioned courses to 40 students per section so four sections of Math 100 were chosen at random to get the first 160 students in that group and the remaining 120 were selected at random from the students who

received individualized instruction. Five, four, and seven sections of Math 115, Math 117, and Math 121 respectively, were chosen at random to produce the desired samples for those courses.

Categories Compared and Methods Used
to Account for Contaminating
Variables

Students in Math 100, Math 115, Math 117, and Math 121 were tested at the start of fall quarter and, based on the results of this pre-test, were divided into two groups relative to each competency being tested. Math 100 students took Test I, Math 115 and Math 117 students took Test II, and Math 121 students took Test III. The competency areas tested by each of these tests are listed in Table 3.3. One group, identified by this process, consisted of those students who demonstrated competence in the area and the other group consisted of those students who did not. For the purpose of this study, competence was defined in terms of the number of test items that the student answered correctly. Each area of competency was covered by from two to four test items as listed in Table 3.3. To be considered competent in a given area a student had to answer correctly both items for an area covered by only two items, two out of three items or better for an area covered by three items, and three out of four items or better for an area covered by four items. At the end of fall quarter the students involved in the study were classified as successful or unsuccessful based on their performance in the course. Students who received a course grade of A, B, or C were

considered successful while students who dropped the course or received a course grade of D or F were considered unsuccessful. Students who received an I (Incomplete) were eliminated from the study. In Math 001, which was offered only on a pass-fail basis, students who passed were considered successful and those who did not were considered unsuccessful. The same method applied to students who took one of the other courses on a pass-fail basis. The independence of student success and entrance level competence was then checked.

The students in Math 001, Math 100, and Math 115 were tested at the end of fall quarter to determine whether they had developed competence in the areas previously identified as important to success in subsequent courses. For this purpose Math 001 students took Test I, Math 100 students took Test II, and Math 115 students took Test III.

In addition to the comparisons mentioned in preceding paragraphs, sex, age, and method of instruction were compared to success. Finally, proportions were estimated to answer the remaining general questions in this study.

Student differences other than entrance level mathematics competencies were controlled by randomization. Students who took courses by an individualized method were actually selected at random. Students who took courses by the traditional lecture method were selected in groups by section. The sections were chosen at random and since the students were assigned to sections by the computer without consideration

being given to any variables that were considered contaminating, this process was also viewed as a random process in relation to contaminating variables.

The teacher effect was also controlled by randomization since the sections were selected at random. In some situations the teacher variable might well have deserved more careful control as a contaminating variable. The courses under consideration in this study, however, were all subjected to careful standardization because of their purpose. Since these are service courses, students go on from the numerous sections of these courses to subsequent courses that depend on the abilities developed in these prerequisite courses. To make possible the planning of the later courses, a great deal of effort is taken to minimize the effect of the teacher variable for these courses. Sequencing and scheduling of topics for these courses is done by the course supervisor and common final exams as well as common hour exams are used extensively.

One of the most important contaminating variables was the prior knowledge that a student had of the course for which he or she was registered. Many students are required to take a certain number of unspecified mathematics credits. For obvious reasons, some students take the easiest courses offered for credit regardless of their mathematics background. A student, for example, who did well in high school Algebra II might take Math 100 for an "easy A" and would most likely

wind up in the successful group. The success of this student would be strongly influenced by the fact that he had taken the course previously and would, consequently, shed very little light on the importance of the entrance level competencies to success in the course. Because of this problem, students were asked to indicate the courses that they had taken previously and those students who had already been exposed to the material covered in the course for which they were registered were eliminated from further consideration in relation to identification of needed entrance level competencies.

Data Collection Procedures and Test Instruments

All students were read the same cover letter and asked to fill out a short questionnaire at the beginning of fall quarter. This questionnaire provided the necessary background information for completing the analysis discussed later in this chapter. A copy of this questionnaire and the cover letter appear in Appendix A. Figure 3.1 shows the course sequence for the courses under consideration in this study. Any of these courses could be taken as a first course at Montana State University depending on a student's background. Each course in the sequence has the preceding course or its equivalent as a prerequisite. Three tests were used to measure mathematical competencies at various times and in various courses. Table 3.2 shows the data collection schedule used during fall quarter of 1978.

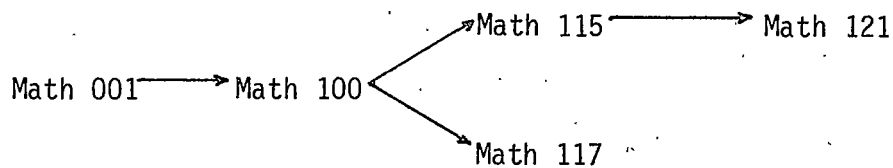


Figure 3.1
Course Sequence

Table 3.2
Data Collection Schedule

Course	Time	
	Beginning of Quarter	End of Quarter
Math 001	Questionnaire Only	Test I
Math 100	Test I & Questionnaire	Test II
Math 115	Test II & Questionnaire	Test III
Math 117	Test II & Questionnaire	
Math 121	Test III & Questionnaire	

The three tests were written by the researcher and appear in Appendix B. Content validity for these test instruments was established by the construction procedure. The competency areas to be tested and the number of items to be used for each appear in Table 3.3 and were

selected following examination of high school texts, examination of the texts used in Math 001, Math 100, and Math 115, and after consultation with the faculty supervisors for the college courses. From Table 3.3 the test items were written using tests from the texts and old hour exams and final exams from the college courses as models. The tests were then examined by the course supervisors and altered according to their recommendations. The predictive validity of the instruments was demonstrated by the results of the study and is discussed in a later chapter. The reliability of the test instruments was established through pre-testing of the instruments during summer quarter of 1978.

A test-retest technique was used and followed by the calculation of the standard correlation coefficient for the two sets of scores. Ferguson (1976) recommends the following formula:

$$r = \frac{N\sum XY - \sum X \sum Y}{\sqrt{[N\sum X^2 - (\sum X)^2][N\sum Y^2 - (\sum Y)^2]}}$$

In the formula, X is a first test score and Y is a second test score for the same person; N is the number of students who were tested. Students in education classes during summer quarter of 1978 were used as subjects to establish this reliability. The results of these calculations are summarized in Table 3.4. For Test I the correlation coefficient was .874, for Test II it was .802, and for Test III it was .805.

Table 3.3
Number of Test Items Per Competency

Competencies	Test I	Test II	Test III
Arithmetic	4		
Solution of Linear Equations	4	2	2
Graphing (Points and Lines)	4	2	2
Solution of Linear Inequalities	3	2	2
Rational Expressions	3	3	
Polynomials and Factoring	3	3	2
Exponents and Radicals	4	3	
Solution of Systems of Equations	3	3	
Solution of Quadratic Equations	2	4	2
Exponential and Logarithmic Functions		4	2
Conic Sections and Their Graphs		4	2
Trigonometric Functions (Definitions)			3
Trigonometric Identities (Basic)			3
Graphing (Trigonometric Functions)			3
Inverse Trigonometric Functions			2
Complex Numbers			3
Vectors			2
Total Number of Items	30	30	30

Table 3.4
Summary of Reliability Calculations for
Test Instruments

Test	Number of Students	Pearson Product - Moment Correlation Coefficient
I	21	.874
II	14	.802
III	12	.805

Statistical Hypotheses Tested

The following statistical hypotheses, in null hypotheses form, were tested in this study:

1. The success of a student in the courses considered is independent of entrance level competence in each competency area tested.
2. The success of a student in the courses considered is independent of the sex of the student.
3. The success of a student in the courses considered is independent of the age group to which the student belongs.
4. The success of a student in the courses considered is independent of the method of instruction that the student received.

All of the hypotheses were tested using a Chi Square Test of Independence. Snedecor and Cochran (1967) recommend the following

formula:

$$\chi^2 = \frac{N(|AD-BC|-N/2)^2}{(A+B)(C+D)(A+C)(B+D)}$$

In this formula, N is the total number of students in a given sample and A, B, C, and D are the numbers of students who wound up in various categories as determined by the two variables of interest. For example, for hypothesis number one there are two levels of success, successful and unsuccessful, and two levels of competence, competent and not competent. Table 3.5 shows the relationship of A, B, C, and D to these categories. In this case A, for example, is the number of students who were both competent and successful. The term N/2 is a continuity correction. The remaining general questions were answered by estimating the proportions of interest using interval estimation techniques and not by hypothesis testing.

Table 3.5
2x2 Contingency Table

	Successful	Not Successful	Total
Competent	A	B	A + B
Not Competent	C	D	C + D
Total	A + C	B + D	N

Analysis

The statistical hypotheses were tested at the $\alpha = .01$ level. The decision to use this level of significance was based on a consideration of the impact of the results of this study. With relatively large sample sizes used in order to get accurate estimates of the proportions only highly significant results can be considered meaningful. Those relationships that were found to be significant at the $\alpha = .01$ level formed the basis for the recommendations that accompany this study.

As mentioned in the discussion of sample size, the researcher wanted to be able to estimate proportions to within five percent with 95 percent confidence. This led to 95 percent confidence intervals for p whose width was approximately .10. A normal approximation to the binomial was used to set these intervals. Cochran (1963) suggests the following formula:

$$p \pm \{t\sqrt{1-f} \sqrt{pq/(n-1)} + 1/2n\}$$

In this formula p is the observed proportion, t is the appropriate value of Student's t Statistic, $f = n/N$ where n is the sample size and N is the population size, and $q = 1 - p$. The factor $\sqrt{1-f}$, is a finite population correction and the term $1/2n$ is a continuity correction. In addition, as Cochran points out, the $n - 1$ is usually replaced by n to simplify computations. The researcher used the simplified version with n replacing $n - 1$.

Summary

A sample of the students in beginning mathematics courses at Montana State University during fall quarter of 1978 was taken. These students were given pre-tests and post-tests of mathematical competency. By comparing the success rates for students who demonstrated competence and those who did not, those competencies that were important for success in beginning courses were identified. The exit level competencies of successful students in each course were measured to see if the remedial courses actually were effective in developing those competencies identified as important. Comparisons were also made of males and females, of younger and older students, and of students receiving individualized instruction and those receiving traditional lectures. Answers to other questions were gained by the use of descriptive statistics available in the data that was collected.

Chapter 4

RESULTS AND FINDINGS

This study produced data concerning mathematical competencies related to beginning mathematics classes at Montana State University. The results are presented under three major headings. First, the population and the samples that were drawn are discussed. Second, the results from the testing of the four statistical hypotheses are presented. Finally, the general questions not answered by hypothesis testing are each discussed.

Population and Samples

The population under consideration in this study was the students enrolled in Math 001, Math 100, Math 115, Math 117, and Math 121 at Montana State University during fall quarter of the 1978-1979 academic year. As mentioned in Chapter 3, the statistics that were calculated by this study were used as estimates of parameters within courses and not as estimates of overall population parameters.

Sample size calculations were based on estimated starting enrollments in each of the courses under consideration. Actual enrollments were not used because they were not available until after the collection of data had been started. Table 4.1 shows estimated starting enrollments, actual starting enrollments, calculated sample sizes, and actual sample sizes for each course that was considered. The actual sample sizes differ from the calculated sample sizes in Math 100, Math

