A state-space approach to the design of pre- and postfilters in a sampled-data communication system

by Yern Yeh

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Electrical Engineering
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Abstract:
The basic problem considered in this thesis is how to discretize a random process observed at one end, and after transmission, to reconstruct the desired signal from the discretized data received at the other end. The discretizing process consists of sampling, quantization and a presampling operation. The system under consideration in this work can either be regarded as a data communication or a data processing system.

The main purposes of this work are: 1. To solve for the optimal realizable pre-transmission filter (prefilter) at one end, and the optimal realizable reconstruction filter (postfilter) at the other end using a state-space representation and the innovations approach.

2. To investigate the functions of the optimal realizable pre- and postfilters in comparison with that of the optimal unrealizable filters. 3. To investigate the smoothing problem as a function of allowable delay.

The major contributions of this work are: 1. The state-space and innovations approach has been successfully applied to derive the optimal realizable zero-lag pre-and postfilters. The initial step in the procedure is to find the optimal post (pre) filter with a given pre (post) filter. Joint optimization then is carried out for the special case in which the quantization noise and channel additive noise are absent. The optimal filters are specified as solutions to differential equations in the state variables. 2. In the absence of the optimal prefilter, the optimal finite-lag postfilter has been found. An error-reduction factor and a delay-efficiency factor have been defined in order to study the effect of introducing delay on reducing the error between the output of the system and the desired signal. 3. Numerical examples of the performances of various systems have been worked out, and interpretations are given.

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ABSTRACT

The basic problem considered in this thesis is how to discretize a random process observed at one end, and after transmission, to reconstruct the desired signal from the discretized data received at the other end. The discretizing process consists of sampling, quantization and a presampling operation. The system under consideration in this work can either be regarded as a data communication or a data processing system.

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The major contributions of this work are: 1. The state-space and innovations approach has been successfully applied to derive the optimal realizable zero-lag pre- and postfilters. The initial step in the procedure is to find the optimal post (pre) filter with a given pre (post) filter. Joint optimization then is carried out for the special case in which the quantization noise and channel additive noise are absent. The optimal filters are specified as solutions to differential equations in the state variables. 2. In the absence of the optimal prefilter, the optimal finite-lag postfilter has been found. An error-reduction factor and a delay-efficiency factor have been defined in order to study the effect of introducing delay on reducing the error between the output of the system and the desired signal. 3. Numerical examples of the performances of various systems have been worked out, and interpretations are given.

Chapter 1

INTRODUCTION

1.1 Problem Statement

In designing data communication or data processing systems, channel capacity is often one of the major designing factors. Whenever the amount of information exceeds the capacity of the channel being used for transmitting the information, it is advantageous, or even necessary, to compress the information before transmission. For example, one has to discretize an analog signal before transmitting it through a digital channel. After receiving the compressed information such as the discretized data at the other end, one then tries to reconstruct the original information from the compressed data according to a certain fidelity criterion.

Numerical analysis provide an example of the discretization and reconstruction process. For example, functions such as exp(x), cos(x), etc., are first tabulated and stored. One then approximates the value of exp(x) at an arbitrary point by some kind of interpolation process from a set of tabulated values.

The system under consideration in this thesis is depicted in Fig. 1.1. $n_1(t)$ in Fig. 1.1_a is the primary

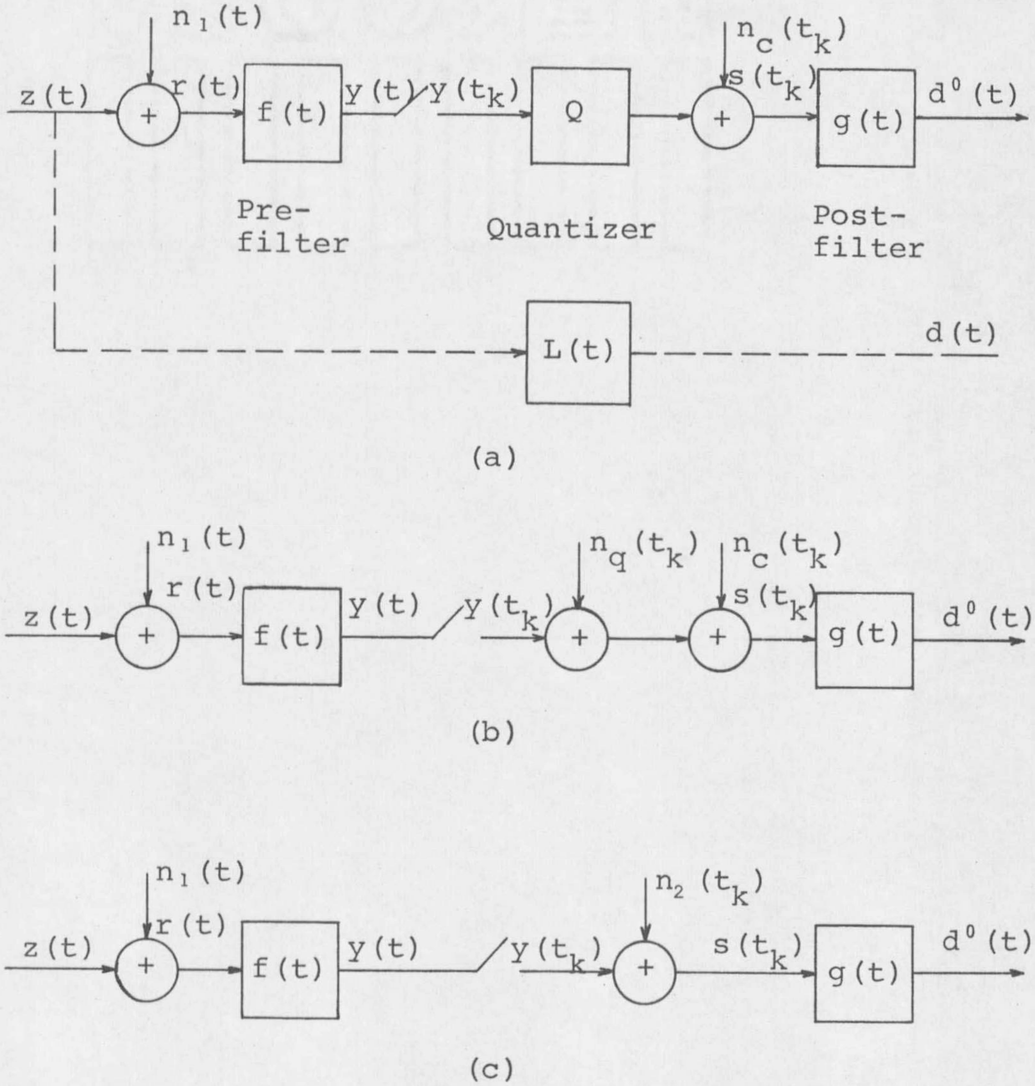

Figure 1.1 A sampled-data communication system.

observation noise. The observation process r(t) is discretized before transmission. The discretizing of r(t) could simply be accomplished by sampling and quantization, but in general it might be advantageous to operate on r(t) prior to sampling in such a way that the overall distortion is reduced. This is one of the basic reasons why a prefilter is inserted in the system. The functions of the prefilter will be investigated in more detail later. The function of the postfilter is obvious; to reconstruct the desired signal from the discretized and possibly contaminated data received at the other end. The functions of the quantizer are:

1. To combat the additive noises in the channel (55).
2. To reduce the amount of information to be transmitted (31).

In this work, the design of the quantizer is not undertaken; only its effects on the design of the optimal pre- and postfilters are considered. In Fig. 1.1_b, the input-output relationship of the quantizer is specified by an additive noise $n_q(t_k)$ called the quantization noise. If the sampling process is periodic, t_k=kT, otherwise t_k denotes the time the kth sampling takes place. In

Fig. 1.1_c, $n_q(t_k)$ and the channel additive noise $n_c(t_k)$ are combined to become $n_2(t_k)$. The characterization of the quantization noise will be discussed in Section 1.5.

Depending on whether n_1, n_c or/and n_q are present or not, the system shown in Fig. 1.1 can either be regarded as a PCM (Pulse Code Modulation) system, a PAM (Pulse Amplitude Modulation) system, or a data processing system.

The distortion between $d(t)$ and $d^0(t)$ is due to n_1, n_2, and the sampling process. The object is to apply the innovations approach to find the optimal realizable pre- and postfilters for reconstructing $d(t)$ according to a mean-square-error criterion, and to discuss their physical interpretations.

1.2 Historical Summary

Previous work in this area can be grouped into three categories: 1. The frequency domain analysis, where the optimal filters are specified by their transfer functions which are the frequency domain representation of the impulse response of the optimal filters. 2. The state-space approach, where the optimal filters are specified by differential or difference equations which relate the inputs to the outputs of the optimal filters. 3. Numerical methods; while both of the above two methods can be used in conjunction with numerical methods, the one that will be mentioned in this thesis is that of Kellog's (31). The three methods are discussed in more detail as follows.

1. Frequency domain analysis: Ever since Wiener's work in linear least-squares estimation of continuous random processes (the system is shown in Fig. 1.2_a), the result has been extended by various people. Costas (10) studied the system shown in Fig. 1.2_b, where an unrealizable pre-transmission filter with a power constraint is added. The error reduction resulting from the usage of this prefilter were shown to be moderate for a simple signal such as a first-order G.M. (Gauss-Markov) process with a white additive noise. Franklin (47), and Tretter and Steiglitz (57) considered the system shown in Fig. 1.2_c, where a sampler is included. They also considered fixed-lag postfilters but did not provide a detailed quantitative error analysis. Katzenelson (30), Steiglitz (54), and Ruchkin et al. (50) investigate the system shown in Fig. 1.2_d, where quantization is included. Since the effect of the quantization process can always be