Analysis for three-dimensional pipe structures by group relaxation
by Nicholas Bassar

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Givil Engineering at Montana State College
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Abstract:
The group relaxation procedure for determining moments and reactions at the fixed ends of a three-dimensional pipe structure subjected to expansion resulting from temperature change utilizes principles of the neutral point method and the moment distribution process. The given pipe structure is subdivided into groups or system such that each system has one terminal that is common to all systems and the other terminal a fixed end. This subdivision has the advantage of requiring only the control of S degrees of freedom of the common terminal during the solution. Before permitting expansion the common terminal is restrained against rotation in each plane and against translation in each direction. When expan-sion occurs, moments throughout the structure, end reactions and restraints at the common terminal are induced.

The restraints at joint B may be released by either of two methods as follows: (1) By first allowing rotational equilibrium to occur and then releasing the restraints against translation. (2) By releasing the restraints against translation first and then allowing rotational equilibrium to occur* but balancing shears each time the common terminal rotates.

In Appendix II the six neutral point equations are derived and are used to provide the following items needed to perform the moment distribution process and to eliminate the restraints against translation at the common terminal: (a) Distribution factors (b) Carry-over factors (c) Shear correction factors (d) Forces required to produce a unit translation of the common terminal For a pipe structure with from 1 to 4 degrees of freedom for translation it is probably more advantageous to use Method I. If there are more than 4 degrees of freedom for translation Method II appears to have merit*

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by

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ABSTRACT

The group relaxation procedure for determining moments and reactions at the fixed ends of a three-dimensional pipe structure subjected to expansion resulting from temperature change utilizes principles of the neutral point method and the moment distribution process. The given pipe structure is subdivided into groups or systems such that each system has one terminal that is common to all systems and the other terminal a fixed end. This subdivision has the advantage of requiring only the control of 3 degrees of freedom of the common terminal during the solution. Before permitting expansion, the common terminal is restrained against rotation in each plane and against translation in each direction. When expansion occurs, moments throughout the structure, end reactions and restraints at the common terminal are induced.

The restraints at joint B may be released by either of two methods as follows: (1) By first allowing rotational equilibrium to occur and then releasing the restraints against translation. (2) By releasing the restraints against translation first and then allowing rotational equilibrium to occur, but balancing shears each time the common terminal rotates.

In Appendix II the six neutral point equations are derived and are used to provide the following items needed to perform the moment distribution process and to eliminate the restraints against translation at the common terminal:

(a) Distribution factors
(b) Carry-over factors
(c) Shear correction factors
(d) Forces required to produce a unit translation of the common terminal

For a pipe structure with from 1 to 4 degrees of freedom for translation it is probably more advantageous to use Method I. If there are more than 4 degrees of freedom for translation Method II appears to have merit.

INTRODUCTION

Object

The primary objective of this thesis is to present a group relaxation procedure for determining moments and forces at the fixed ends of a three-dimensional pipe structure with 3 or more fixed ends subjected to expansion resulting from temperature change. A comparison of two methods by which the relaxation is executed is the secondary objective.

Previous Investigation

Two available methods of evaluating the moments and forces at the fixed ends of a three-dimensional pipe structure with three fixed ends require the solution of a minimum of six[1] and a maximum of twelve[2] simultaneous equations.

The method involving six simultaneous equations utilizes the moment area principles and the principles of the neutral point method.[3] If the structure has more than 3 fixed ends, the number of simultaneous equations to be solved increases.

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1. "Simplified Method of Analysis of Reactions Developed by Expansion in a Three-Anchor Piping System," by Boris Lochak, A.S.M.E. Trans., Vol. 66, 1944, pp. 311-318
2. "Design of Piping Systems," published by M. W. Kellogg Co., New York, N. Y., 1941
3. "Theory of Modern Steel Structures," Vol. 2, by L. E. Grinter, published by the Macmillan Co., N. Y. pp. 206-210

The equations of the general method of indeterminate structures[4] along with the virtual work principles[5] are applied in the method involving twelve simultaneous equations. This method requires that the structure be cut back to a statically determinate one; then knowing that the freed ends cannot translate or rotate, the displacement due to expansion is eliminated with the aid of virtual work principles.

A third method[6] makes use of the moment distribution[7] process. Each member of the structure is allowed to expand while restrained against rotation. Then each joint is allowed to rotate with translation prohibited. After a joint rotates, the restraint against translation is released. This procedure of alternately allowing a joint to rotate and then the entire structure to translate will finally result in a condition whereby there will be no further tendency for translation or rotation when the joints are released, indicating that the structure has reached equilibrium.

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4. Ibid pp. 74-75
5. Ibid pp. 35-41
6. Discussion of "Moment-Distribution Analysis for Three Dimensional Pipe Structures," by R. C. DeHart, A.S.M.E. Trans., Vol. 66, 1944, pp. A240-A244
7. "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan published by John Wiley and Sons, Inc., N. Y. pp. 81-125

Importance

The group relaxation procedure requires the solution of only 3 simultaneous equations at any one time and thereby reduces the tedious task of solving six or more simultaneous equations as presented by two of the available methods. A physical picture of the effect of joint rotation and joint translation on the moments and on the forces at the fixed ends is presented by applying the group relaxation procedure which is of assistance to the engineer analyzing the structure.

While the solution given is for square corners, the method can be applied to pipe structures with bends at the corners. When quarter bends are a part of the pipe structure, their lengths must be modified when computing the centroid of an orthogonal projection and when computing the moments of inertia and products of inertia to account for the added flexibility due to flattening of the curved section when subjected to bending. A three-dimensional pipe structure with two fixed ends containing circular quarter bends has been solved by the neutral point method by S. W. Spielvogel.[8] The use of quarter bends reduces stress concentrations at the corners.

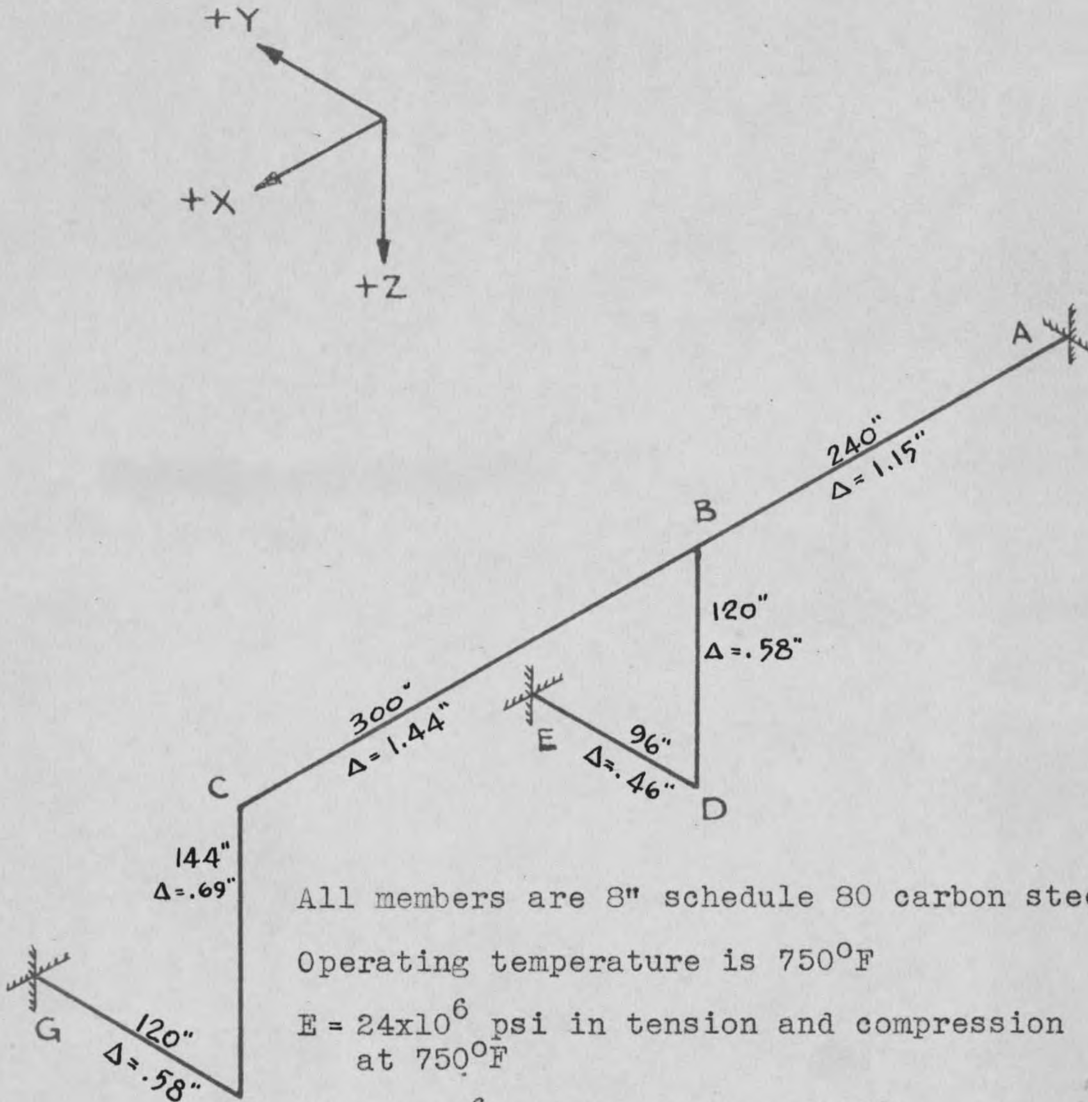
8. "Stress Calculation for High Temperature Piping-II," by S. W. Spielvogel, Power, February 1941, pp. 67-69

PROCEDURE

The pipe structure shown in Fig. 1[9] is subdivided into groups or systems with B being one terminal of each system and a fixed end the other terminal. These systems are AB, EDB, and GFCB. Joint B is restrained against rotation in each plane and against translation in the Y and Z directions before expansion is permitted. The effect of direct stress relative to elongation or contraction is to be neglected in this analysis; therefore AB must be allowed to expand. This permanently displaces joint B in the X direction by the amount of expansion of AB. After AB expands, it serves as a permanent restraint in the X direction.

For each orthogonal projection of the elastic areas, ds/EI, of each complex system as EDB and GFCB, the centroid or the neutral point is evaluated. For a member that appears as a point in an orthogonal projection, an equivalent length is used since in this plane the member acts in torsion rather than flexure. The equivalent length is equal to the product of the actual length of the member and a constant. This constant is equal to the ratio of EI to GJ. The coordinate axes pass through the neutral point. The moment

9. The pipe structure used is the same as the one given in closure to discussion of paper, "Moment Distribution Analysis for Three-Dimensional Pipe Structures," by R. C. DeHart, Journal of Applied Mechanics, A.S.M.E. Trans., Vol. 67, 1945, pp. A-188



All members are 8" schedule 80 carbon steel

Operating temperature is 750°F

E = 24×10^6 psi in tension and compression at 750°F

G = 9.6×10^6 psi in shear at 750°F

I = 105.7 $''^4$

J = 211.4 $''^4$

Fig. 1 Three-Dimensional Pipe Structure to be Analyzed of inertia and the product of inertia of the elastic areas about the axes through the neutral point are computed and tabulated in Tables I and II. Since EI is constant in this problem and since it is in the denominator, multiplying equations 3b through 8b shown in Appendix II by EI removes this denominator. The terms in Table I and II are of this form.

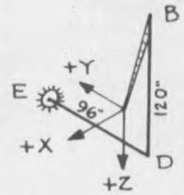
Sign Convention

The sign for translation is plus if the movement is in the positive direction of the axes as indicated in Tables I and II. The sign for a force is plus if it acts in the positive direction of the axes. Moment in a plane is positive if the action of the joint on the member is clockwise when the plane is viewed as indicated in Tables I and II.

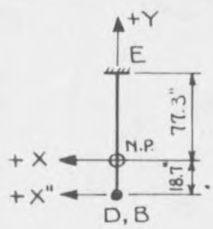
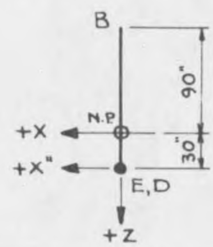
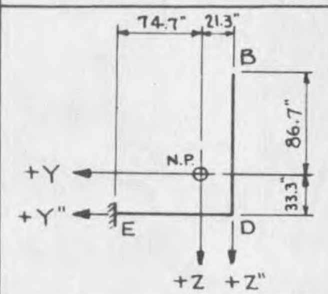
Unit Translation Effect

The translation of the neutral point in one of the coordinate directions implies that joint B also translates in the same direction and the same amount, but it is restrained against translation in the other two directions and restrained against rotation in each plane. The neutral point forces F_x, F_y and F_z corresponding to a positive unit translation in the X direction of the neutral point of system GFCB are computed from the neutral point equations 3b through 5b shown in Appendix II:

TABLE I: LOCATION OF NEUTRAL POINT AND ELASTIC PROPERTIES OF EDB SYSTEM



PROJECTION IN

X-Y PLANE	X-Z PLANE	Y-Z PLANE
		

CENTROID

X-Y Plane

	L	Y	LY	X"	LX"
ED	96	48	4608	0	0
DB	1.25×120	0	0	0	0
	246		4608		

$\frac{4608}{246} = 18.7"$

X-Z Plane

	L	Z	LZ	X"	LX"
ED	1.25×96	0	0	0	0
DB	120	-60	-7200	0	0
	240		-7200		

$\frac{-7200}{240} = -30"$

Y-Z Plane

	L	Z"	LZ"	Y"	LY"
ED	96	0	0	48	4608
DB	120	-60	-7200	0	0
	216		-7200		4608

$\frac{-7200}{216} = -33.3"$ $\frac{4608}{216} = 21.3"$

I_{XY} / I_{XZ} / I_{YZ}

	I_{XY}	I_{XZ}	I_{YZ}
ED	$96 \times 29.3 \times 0 = 0$	$1.25 \times 96 \times 30 \times 0 = 0$	$96 \times 33.3 \times 26.7 = .854 \times 10^5$
DB	$1.25 \times 120 \times (-18.7) \times 0 = 0$	$120 \times 30 \times 0 = 0$	$120 \times (-26.7) \times (-21.3) = .682 \times 10^5$
			1.536×10^5

I'_{XX} / I_{XX} / I_{YY}

	I'_{XX}	I_{XX}	I_{YY}
ED	$\frac{1}{12} \times 96^3 + 96 \times 29.3^2 = 1.561 \times 10^5$	$1.25 \times 96 \times 30^2 = 1.080 \times 10^5$	$96 \times 33.3^2 = 1.065 \times 10^5$
DB	$1.25 \times 120 \times 18.7^2 = .525 \times 10^5$	$\frac{1}{12} \times 120^3 + 120 \times 30^2 = 2.520 \times 10^5$	$\frac{1}{12} \times 120^3 + 120 \times 26.7^2 = 2.295 \times 10^5$
	2.086×10^5	3.600×10^5	3.360×10^5

I'_{YY} / I_{ZZ} / I'_{ZZ}

	I'_{YY}	I_{ZZ}	I'_{ZZ}
ED	NEGLECT	NEGLECT	$\frac{1}{12} \times 96^3 + 96 \times 26.7^2 = 1.421 \times 10^5$
DB	NEGLECT	NEGLECT	$120 \times 21.3^2 = .544 \times 10^5$
			1.965×10^5