Plant location : a quantified model for community and plant site selection

by Phillip Argene Brown

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Industrial and Management Engineering
Montana State University

Abstract:
The problem of determining where to locate new plants is usually-divided into two stages: (1) selecting the region and (2) selecting the community and plant site. The objective of this study is to reapply current location theory on the basis of least-total-cost-site by developing a general model using consistent quantification techniques for selecting the community and plant site.

A model, describing the location measure of a potential site for a hypothetical plant, is defined in terms of critical, objective, and subjective location factors. Its applicability is tested, with the aid of a computer, by calculating location measures of six potential sites in Montana.

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Date August 7, 1970

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ABSTRACT

The problem of determining where to locate new plants is usually divided into two stages: (1) selecting the region and (2) selecting the community and plant site. The objective of this study is to reapply current location theory on the basis of least-total-cost-site by developing a general model using consistent quantification techniques for selecting the community and plant site.

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Chapter I.

INTRODUCTION

Importance of Plant Location

Economic changes over the past 70 years have reduced the margin between unit selling price and unit cost for many industries. With the unit selling price usually fixed by competition, an increase in gross profit requires either an increase in sales or a reduction in unit cost. In either case, one limiting factor in increasing profit is the location of the plant. [6], [16], [18]

Another reason for new emphasis on plant location is that a mistake in plant location can be costly to correct. Once the location has been selected, the overall efficiency and effectiveness of the plant are limited to some extent and may preclude the possibility of a profitable operation. Increasing the profitability of the operation can depend on transferring the new facility to another site at tremendous cost.

History of Location Theory

According to Holmes [6], Reed [16], and Yaseen [26], existing plant location theories do not provide satisfactory solutions to the location problem. One reason for this lack of progress is the nature of the factors that should be considered in determining the best location. Many of these factors do not lend themselves to quantitative measurement. For this reason most plant location authors have confined their analyses to monetary factors. Non-monetary factors are usually defined but their application is left to the reader. Yaseen [26, p.1] recognized this void in plant location theory when he stated: "No scientific advances, unfortunately, can be claimed in the determination of where new plants should be built."

The analysis of the plant location problem has generally been based on one of the following concepts: least-production-cost-site, maximum-profit-site, least-distribution-cost-site, or least-total-cost-site.

Most of the early work in location theory was concerned with determining the least-production-cost-site. This concept includes only those factors which directly affect the cost of production. Outstanding contributors to this concept were two Germans, Johann von Thunen [5] and Alfred Weber [25] and an American, Edgar Hoover [8]. Von Thunen's theory assumes that the location is given, the land surface is homogeneous, and the type of industry is to be determined. The complete analysis is based on two concepts: the theory of rent [1] and the cost of transportation. The industry which has the minimum sum of rent and transportation costs is selected.

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[1]Weber [25] theorizes that land rents reflect advantages due to favorable locations of industry in relation to raw material deposits and market areas.

In contrast to von Thunen's work, Weber developed a theory based on the assumptions that the branch of industry is known and that the exact location is sought. In addition to the cost of transportation, Weber includes the cost of labor and agglomerating forces[2] as the principal factors in plant location. He divides the problem into two stages: (1) selection of the region, and (2) selection of the location. Transportation and labor costs are designated as regional factors, while agglomerating forces are considered to be local factors. Melvin Greenhut [5, p. 255] summarized Weber's theory as follows:

> The cost of transfer tends to draw industry to the site of least transfer expense, while the cost of labor may cause displacement of that site to a place where the savings in labor cost are larger than the additional transportation costs. The decentralizing tendencies of these two factors are counteracted or intensified by Weber's third consideration: the agglomerating force (marketing advantages, proximity to auxiliary industries, economics of scale) and its corollary, the deglomerating influence (land rent). These considerations draw industry closer together or disperse it, depending upon the respective strength of each force.

Edgar Hoover's location theory [8], which also considers cost factors, is characterized by the inclusion of demand factors. He divides the activities of a productive enterprise into three stages:

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[2]Reed [16, p. 5] defines agglomerating and deglomerating forces as: "Agglomerating forces are those forces which tend to cause industry to gather densely in limited areas: the factors or forces favoring urban location. Deglomerating forces are those forces which tend to cause industry to scatter or to seek locations away from other industry: the factors or forces favoring rural location."

1. Procurement: Purchasing and bringing the necessary materials and supplies to the site of processing.
2. Processing: Transforming the materials into more valuable forms (products).
3. Distribution: Selling and delivering the products.

Procurement and processing are cost factors and distribution is the demand factor.

Hoover states that he is not concerned with the analysis of the problem per se, but is interested instead in the formulation of principles. His major contributions to location theory are: (1) discussions of the influence of location factors, (2) recognition of the error in assuming transportation costs are proportional to distance, and (3) recognition of capitalistic influences on location.

In 1956 Melvin Greenhut [5] introduced a location theory based on the concept of the maximum-profit-site. He defines the maximum-profit-site as: "the site at which the spread between total receipts and total cost is the greatest." His mathematical description [5, pp. 286-287] of a general theory is:

$$L = f(R-C) \tag{1-1}$$

$$C = f(SR \times C_a) \tag{1-2}$$

$$R = f(SR \times m) \tag{1-3}$$

L stands for location, C for total cost, R for total revenues, and SR for sales radius.[3] C_a represents the average cost (per unit), exclusive of freight, and m stands for the profit maximizing net-mill price.[4] [Footnotes mine]

Greenhut proposes that if for each firm within an industry:

1. $m \neq C_a$, and $\Delta R = \Delta C$, a state of location unequilibrium exists, or
2. $m = C_a$, and $\Delta R = \Delta C$, there is location equilibrium.

He finalizes his proposal with a hypothesis that new locations or relocations will cause a movement toward equilibrium conditions.

Most of the recent work in location theory is founded on a concept of either least-distribution-cost-site or least-total-cost-site. A least-distribution-cost concept is usually based on "decision data related to delivered cost to customer" [16, p. 3], such as minimizing service times or distances to customers. Two characteristics of location theories utilizing a least production cost concept are:

1. They seldom include more than time and distance factors in their analysis, and
2. They primarily utilize analytical models such as linear programming.

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[3]According to Greenhut [5], sales radius is defined as the total volume of unit purchases which produce the greatest possible profit.

[4]According to Greenhut [5], profit maximizing net-mill price is defined as the average revenue (per unit) which, excluding freight, maximizes the total profit.

Reed [16, pp. 26-27] analyzed location theories founded on a criterion of least distribution cost as follows:

> Most of the pure quantitative work which has been done relative to location studies has used some form of the transportation or assignment methods of linear programming. These in general, however, have been solutions to the problem of assigning "n" plants to "n" locations. The problem in this case is not one of selecting a location. The decision as to which locations are to be considered in the model has been determined prior to the model formulation.

With regard to the use of analytical models in plant location in the future, Reed [16, p. 27] states:

> The procedure may offer possibilities for long-range planning where several locations have been purchased and the question of when to use individual locations and what to produce at that location must be decided.

Most of the literature on plant location theory is based on a concept of least-total-cost-site. [1], [12], [16], [26] In general, the criterion is an extension of the least-production-cost-site criterion. In addition to factors affecting production cost, factors affecting the overall operation are also included in the analysis. A distinguishing characteristic of this criterion is that decision data are related to all factors that might affect a plant's operation.

Reed, [16, p. 1] has defined the location problem in terms of least-total-cost-site as:

> ... the determination of that location which, when considering all factors, will provide minimum delivered to customer costs[5] of the product(s) to be manufactured. [Footnote mine]

The analysis of the location problem using a concept of least total cost is usually divided into two stages: (1) selection of the general territory or region, and (2) selection of the community and plant site. In selecting the region, information of a more general nature is required, whereas the selection of the community requires information regarding specific costs, attitudes, etc. A general list of the factors affecting regional and community selections is presented in Table 1-1. [12, p. 39]

Table 1-1. General location factors to be considered in a typical location problem.

Location Factor	Regional Selection	Selection of Site & Community
Market	*	
Raw Materials	*	
Transportation	*	*
Power	*	*
Climate & Fuel	*	
Labor & Wages	*	*
Laws & Taxation	*	*
Community Services & Attitude		*
Water & Waste		*

[5]Holmes [6] has defined delivered to customer cost as: "... the total of all costs which a commodity must bear from the time it is taken from nature until it is delivered to the manufacturer's customer."

A regional analysis is usually based on a cost evaluation of the regional factors shown in Table 1-1. That region which minimizes the sum of the costs of the regional factors is then selected.

The procedure[6] used in selecting the community and plant site is:

1. Assigning a weight to each factor
2. Ranking locations for each factor
3. Distributing the factor weights among their respective set of ranked locations
4. Summing the location weights for each location, and
5. Selecting that site with the maximum sum of location weights.

The four previously mentioned location concepts will not be applied because: (1) required input data is unavailable, (2) assumptions are not practical for real world problems, (3) quantitative measures for subjective location factors are inconsistent, or (4) objectives of management are variable. [12], [16], [26]

According to Reed [16], rather than apply existing location theory, an industry or firm normally: (1) designs or assumes a general approach to the problem, (2) establishes an incomplete set of factors affecting selection, (3) evaluates the factors, and (4) makes a decision based upon available data.

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[6]For a thorough discussion on the detailed procedure, see Reed. [16, pp. 19-21]

Chapter II.

DEVELOPMENT OF LOCATION MODEL

The strongest criticism of current location theory derived from the least-total-cost-site concept is that the quantitative methods used to evaluate community and site factors are inconsistent. As an example, personal whims are often allowed to overshadow important cost factors. [6], [16], [26].

The method employed in current theories for selecting the region, as defined in Chapter I, has received a more encouraging response. [16] A prime reason for this response is that all of the factors considered in selecting a region are monetary factors. [12] Therefore, with an acceptable method available for selecting the region, emphasis in this study will be placed on selecting the community and the plant site.

Objective of Study

The objective of this study is to reapply current location theory on the basis of least-total-cost-site by developing a general model using consistent quantification techniques for selecting the community and plant site. This objective is accomplished in four steps:

1. Classifying the location factors[7]

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[7]Community and plant site factors will be referred to as location factors from here on.

2. Defining a general model in terms of the classifications.

3. Quantifying the terms of the general model, and

4. Formulating the final model.

An example, illustrating the application of the model, is presented in Chapter III and a critique of the model is presented in Chapter IV.

Classification of Location Factors

There are numerous lists of factors available for use in plant location analyses. [1], [12], [16], [18] In addition, most authors present a list of subfactors for each main factor. For example, some of the subfactors for the main factor, "availability of transportation", are: (1) airline service, (2) trucking service, and (3) train service.

Development of the location model in this study is not based on the discovery of a new list of main factors or subfactors. It is however, characterized by the recognition of three classifications of location factors. Main factors and subfactors in all of the previously cited lists can be divided into: (1) critical factors, (2) objective factors, and (3) subjective factors.

A main factor is classified as critical if one or more of its subfactors precludes the location of a plant at a particular site regardless of other conditions that might exist at the site. For instance, an industry that uses a large amount of water would certainly not consider a site with possible shortages of water. In most cases critical factors cause a reduction in the number of sites to be considered in a location study. For this reason they should be considered first.

Objective factors are those factors which can be classified in terms of cost with little or no personal bias. They also contribute directly to operational costs and can be classified as critical factors. Examples of objective factors are labor, raw material, and transportation costs.

The third classification, subjective factors, is usually characterized by the impossibility for monetary measurement. For instance, community attitude is a subjective factor because it can seldom be given a dollar value. Some factors can be classified as both critical and subjective.

Table 2-1 represents a typical list of location factors by classification.

Table 2-1. Plant Location Factors

Critical Factors
Availability of Labor
Availability of Utilities
Community Attitude
Availability of Transportation

Table 2-1. Continued.

Objective Factors
Cost of Transporting Raw Materials
Cost of Transporting Finished Products
Cost of Utilities
Cost of Labor
Cost of Building
Cost of Taxes
Subjective Factors
Availability of Transportation
Industrial Sites - cost, size, location, etc.
Climate
Educational Facilities
Union Activity
Recreation Facilities and Opportunities
Housing - availability & condition of
Future Growth - opportunities for
Community Services
Employee Transportation Facilities
Cost of Living
Competition
Complementary Industries
Availability of Labor
Community Attitude

Definition of General Model

The general model is based directly upon the above classifications. That is, the location measure (or potential) for each site, LM, is a combination of three values: (1) critical factor measure, (2) objective factor measure, and (3) subjective factor measure.

The model which determines the location measure for site "i" (LM_i) is defined as follows:

$$LM_i = CFM_i \cdot [X \cdot OFM_i + (1-X) \cdot SFM_i] \qquad (2\text{-}1)$$

where:

1. CFM_i is the combined measure of the critical factors for site "i" (CFM_i = 0 or 1)
2. OFM_i is the combined measure of the objective factors for site "i" ($0 \le OFM_i \le 1$, and $\sum_i OFM_i = 1$)
3. SFM_i is the combined measure of the subjective factors for site "i" ($0 \le SFM_i \le 1$, and $\sum_i SFM_i = 1$), and
4. X is the objective factor decision weight ($0 \le X \le 1$).

That site which receives the largest location measure, as defined in Equation 2-1, is then selected.

Quantification of Location Measures

To determine the location measure of a site, there are four terms in Equation 2-1 that must be evaluated: (1) critical factor measure, (2) objective factor measure, (3) subjective factor measure, and (4) the value of the objective factor decision weight. This section will be devoted to quantifying these four measures.

Quantification of critical factor measure. The critical factor measure, in terms of the individual critical factors, is defined as:

$$CFM_i = \prod_j CFI_{ij}$$

where CFI_{ij} is defined as the critical factor index for site "i" and critical factor "j".

For each critical factor there is only one decision to be made for each site, "Does the site have the minimum requirement of the critical factor?" If the answer is yes, the value of the critical factor is one, indicating that on the basis of this critical factor the site does have a location measure. On the other hand if the answer is no, the critical factor index is zero indicating that the site should be excluded from further consideration since its location measure is zero. Therefore, if a site is to receive a location measure, LM, the critical factor indexes for that site must all equal one.

Quantification of objective factor measure. By definition, all objective factors can be measured in monetary units. However, in order to insure compatibility between objective and subjective factor measures, objective factor costs are converted to dimensionless indexes.

The objective factor measure for site "i", in terms of the objective factor costs, is defined as:

$$OFM_i = [OFC_i \cdot (\sum_i (1/OFC_i))]^{-1} \qquad (2\text{-}3)$$

where OFC_i is the total objective factor cost for site "i".

Development of Equation 2-3 is based on three restrictions:

1. The site with the minimum cost must have the maximum measure
2. The relationship of the total objective factor cost for each site as compared to all other sites must be preserved, and
3. The sum of the objective factor measures must equal one.

Restrictions 1 and 3 are imposed to insure that the objective factor measure will be compatible with the subjective factor measure.

The implication of restriction 2 is that a site with one-half the objective factor cost of another is assigned twice the objective factor measure of the other site.[8] Equation 2-4 defines this implication for any two sites "i" and "j":

$$OFM_i \cdot OFC_i = OFM_j \cdot OFC_j \qquad (2\text{-}4)$$

Based on the preceding three restrictions and assuming a total of "n" sites, the development of Equation 2-3 is as follows:

1. Imposing restrictions (1) and (2) and applying Equation 2-4 (for j = 1,2,...,i-1,i+1,...,n) results in the following set of n-1 equations:

$$OFM_1 \cdot OFC_1 = OFM_2 \cdot OFC_2 = \ldots\ldots = OFM_i \cdot OFC_i = \ldots$$
$$\ldots = OFM_n \cdot OFC_n \qquad (2\text{-}5)$$

---

[8]The inverse relationship is due to restriction 1.

2. Writing restriction (3) in its mathematical form results in:

$$OFM_1 + OFM_2 + \ldots + OFM_i + \ldots + OFM_n = 1 \qquad (2\text{-}6)$$

3. Combining the set of equations represented by Equation 2-5 and substituting in Equation 2-6 results in:

$$(OFC_i/OFC_1 + OFC_i/OFC_2 + \ldots + OFC_i/OFC_n) \cdot OFM_i = 1 \qquad (2\text{-}7)$$

4. Equation 2-7 can now be simplified to the form of Equation 2-3.

Quantification of subjective factor measure. The subjective factor measure for site "i" is influenced by two quantitites: (1) the relative weight of each subjective factor, and (2) the weight of site "i" relative to all sites for each of the subjective factors. Mathematically the subjective factor measure for site "i" is defined as:

$$SFM_i = \sum_i (SFW_k \cdot SW_{ik}) \qquad (2\text{-}8)$$

where:

1. SFW_k is the weight of subjective factor "k" relative to all subjective factors, and

2. SW_{ik} is the weight of site "i" relative to all potential sites for subjective factor "k".

Values of the subjective factor weight, SFW, and site weight, SW, are determined with the aid of a subjective quantification technique referred to as preference theory. [4]

Preference theory is a tool used to assign weights to subjective properties[9] in a consistent and systematic manner. For each decision (which is a comparison between two factors) there are three possible results: (1) the first property is preferred over (more important than) the second, (2) the second property is preferred over (more important than) the first, or (3) neither property is preferred, i.e., the decision maker is indifferent. The corresponding numerical values for each of the above results are: (1) "1" for the first property and "0" for the second, (2) "1" for the second property and "0" for the first, and (3) "0" for both properties. Each of the properties are then compared two at a time, recording the appropriate values beside the properties, until all possible combinations have been exhausted.[10]

For ease of comparison a preference matrix is usually constructed. Table 2-2 is an illustration of such a matrix for four properties. The first decision (Table 2-2) is to compare the importance of property 1 to that of property 2. P_1 and P_2 are preference values of 0 or 1,

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[9]Properties are referred to in preference theory as alternatives, elements, factors, components, etc.

[10]The total number of decisions to be made for a problem involving "n" properties is: $n!/[2!(n-2)!]$.

based on this decision. Next, properties 1 and 3 are compared, then 1 and 4, etc. until the final decision of comparing properties 3 and 4 is made.

Table 2-2. General preference matrix for four properties

	Decision					
Property	1	2	3	4	5	6
1	P_1	P_3	P_5			
2	P_2			P_7	P_9	
3		P_4		P_8		P_{11}
4			P_6		P_{10}	P_{12}

Once the preference values have been assigned, the next step is to calculate the property weight as illustrated in Table 2-3.

Table 2-3. General preference matrix with property weights for four factors

Property	Decision 1	2	3	4	5	6	Sum of Preference Values	Property Weight
1	P_1	P_3	P_5				$P_1+P_3+P_5$	$(P_1+P_3+P_5)/P$
2	P_2			P_7	P_9		$P_2+P_7+P_9$	$(P_2+P_7+P_9)/P$
3		P_4		P_8		P_{11}	$P_4+P_8+P_{11}$	$(P_4+P_8+P_{11})/P$
4			P_6		P_{10}	P_{12}	$P_6+P_{10}+P_{12}$	$(P_6+P_{10}+P_{12})/P$
Total							P	1

The property weight is the ratio of the number of times a factor was preferred to the total number of preferred decisions. The ratio is a measure of the relative importance of each property compared to all properties.

Values of the subjective factor weight and the site weight may now be determined with the direct application of preference theory. The procedures for calculating these values are:

1. Properties - Subjective Factors (constant for all sites)
   a. Define a list of subjective factors.
   b. Based on past experience or company policy, construct a preference matrix for the subjective factors.
   c. Calculate the subjective factor weight (SFW_k) from the preference matrix in part b.

2. Properties - Potential Sites

For each subjective factor "k":

a. Gather data concerning the subjective factor in question for each site.

b. Based on the research in part a., construct a preference matrix for the potential sites and the subjective factor in question.

c. Calculate the site weights (SW_{ik}) from the preference matrix in part b.

Quantification of objective factor decision weight. The objective factor decision weight, X, is defined as the measure of the relative importance of the objective factor classification of the entire location problem.[11] The value of the objective factor decision weight is based on a judgment decision by top level management of the locating plant. A decision of this nature is usually based on company policy or past experience, or both, which are subject to error. Therefore, the sensitivity of the location measures to changes in the objective factor decision weight should be investigated.

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[11]Defining the weight of the location problem to be based on objective factors automatically defines the subjective factor decision weight (1-X) since the range on X is: $0 \leq X \leq 1$.

Formulation of Final Model

The model which defines the location measure for site "i" (LM_i) can now be redefined in terms of the preceding factors as:

$$LM_i = [\prod_j CFI_{ij}] [(X \cdot OFC_i \cdot \sum_i 1/OFC_i)^{-1} + (1-X) \cdot \sum_k SFW_k \cdot SW_{ik}] \qquad (2-9)$$

where:

1. CFI_{ij} if the critical factor index for site "i" and critical factor
2. OFC_i is the total objective factor cost for site "i"
3. SFW_k is the weight of subjective factor "k" relative to all subjective factors
4. SW_{ik} is the weight of site "i" relative to all potential sites for subjective factor "k", and
5. X is the objective factor decision weight.

That site which receives the largest location measure, as defined by Equation 2-9 is then selected.

Chapter III.

APPLICATION OF PLANT LOCATION MODEL

Procedure for Selecting Best Location

The complete procedure for determining the best location for a plant can be divided into three phases:

1. Defining information necessary to compare potential sites
2. Collecting the information for each site defined in phase 1, and
3. Evaluating potential sites on the basis of information collected in phase 2 and the location model defined in Chapter II.

Phase 1 entails defining a list of location factors and subfactors by classification which can then be used to evaluate potential sites. As stated in Chapter II, there are many such lists available to aid a location analyst in this phase of location theory. Appendix A illustrates a list of factors and subfactors which can be used to evaluate potential sites.

Consulting government and local data collection agencies may reduce the time required in phase 2. However, past experience indicates that location data are frequently unavailable or inconsistent with respect to time, source, and description. In most cases the location analyst will be forced to gather data at each potential site. A typical set of data for a hypothetical plant and six cities (sites) in Montana is displayed in Appendix A.

Upon completion of phases 1 and 2, each potential site should be evaluated in terms of the critical factors defined in phase 1. All sites that meet the minimum requirements for these critical factors are then evaluated according to Equation 3-1, which is a reduction of Equation 2-9.[12]

$$LM_i = X \cdot OFM_i + (1-X) \cdot SFM_i \qquad (3\text{-}1)$$

Example Problem[13]

To illustrate the application of the location model presented in Chapter II, consider the problem of locating a plant in one of six cities. Appendix A displays the results of the first two phases of the procedure outlined above. Since the six cities defined in Appendix A have all met the minimum requirements of the critical factors, the remainder of the problem may be divided into five steps:

1. Evaluating objective factor measures for each site
2. Evaluating subjective factor measures for each site
3. Evaluating location measures for each site

[12]The notation for the critical factor measure (CFM_i) has been replaced with its numerical value, "1".

[13]The following example problem was solved with the aid of a computer. A detailed documentation and listing of the computer program is provided in Appendix D.

4. Investigating the sensitivity of location measures to changes in X, and

5. Investigating the sensitivity of location measures to changes in input data.

Evaluating objective factor measures for each site. The objective factor measure for any site "i", OFM_i, was defined in Chapter II, Equation 2-3 as:

$$OFM_i = [OFC_i \cdot (\sum_i 1/OFC_i)]^{-1} \quad (3\text{-}2)$$

where OFC_i is the total objective factor cost for site "i". Determination of the total objective factor cost for any site "i" in this example problem requires six individual costs:[14] (1) raw material, (2) finished product, (3) utilities, (4) labor, (5) building, and (6) taxes.

The basic cost of raw material is the same at all three sources (Appendix A, Table A-3) and only the cost of transporting the raw material to each site was considered. Furthermore, it was assumed that any one of the three sources could satisfy the raw material requirements for the locating plant. Therefore, the cost of raw materials for site "i" is:

$$CRM_i = \min_j [TRC_{ij}] \quad (3\text{-}3)$$

---

[14]All objective costs are evaluated on a monthly basis.

where:

1. CRM_i is the cost of raw materials at each site "i", and

2. TRC_{ij} is the raw material transportation cost from source "j" to site "i".

Transportation costs for each site "i" and source "j" are defined according to M. D. Ray's [15] transportation cost equation[15] as:

$$TRC_{ij} = aX_{ij}^{b}Y_{ij}^{c} \qquad (3\text{-}4)$$

where:

1. X_{ij} is the distance to "i" from "j"

2. Y_{ij} is the weight of raw materials shipped to "i" from "j", and

3. a, b, and c are constants to be determined by a multiple regression analysis.[11]

Values of X_{ij} and Y_{ij} are shown in Appendix A, Tables A-3 and A-4. The empirical data for the regression analysis is shown in Appendix A, Table A-5. Table 3-1 shows the resulting raw material costs for each site.

---

[15]Ray's [15] equation was selected on the basis of his extensive empirical study relating to transportation cost equations.

Table 3-1. Raw material sources and costs for each site

Site	Source of Raw Material	Cost of Raw Material
1	3	$ 1,079
2	2	$ 945
3	1	$ 490
4	1	$ 979
5	2	$ 925
6	3	$ 1,507

For example, the cost of raw materials at site 1 for each of the three sources of raw material is:

$$TRC_{11} = (.003567)(256)^{.509}(200{,}000)^{.8319} = \$\ 1{,}542$$

$$TRC_{12} = (.003567)(224)^{.509}(200{,}000)^{.8319} = \$\ 1{,}440$$

$$TRC_{13} = (.003567)(127)^{.509}(200{,}000)^{.8319} = \$\ 1{,}079$$

The raw material cost for site 1, as defined by Equation 3-3, is then $1,079.

Finished product costs for each site are based only on the cost of transporting the finished products to the markets according to M. D. Ray's [15] transportation cost equation:

$$MC_i = \sum_j aX_{ij}^b\ Y_{ij}^c \qquad (3\text{-}5)$$

where:

1. X_{ij} is the distance from site "i" to market "j",
2. Y_{ij} is the weight of finished product to be shipped from site "i" to market "j", and
3. a, b, and c are constants to be determined using a multiple regression analysis.

The values of X_{ij} and Y_{ij} and the empirical data for a regression analysis to determine the constants a, b, and c are listed in Appendix A, Tables A-6 through A-9. Table 3-2 shows the resulting finished product costs for each site. For example, the cost of transporting the finished products from site 1 to the 18 markets is:

1. $MC_1 = a(256)^b(334)^c + a(0)^b(44{,}516)^c + a(142)^b(4{,}056)^c + \ldots$
   $\ldots + a(308)^b(2{,}399)^c + a(280)^b(2{,}590)^c$
2. a = .003567, b = .509, and c = .8319
3. MC_1 = \$ 1,316

Table 3-2. Marketing (finished product) costs for each site

Site	Marketing Cost
1	\$ 1,316
2	\$ 1,485
3	\$ 1,467
4	\$ 1,600
5	\$ 1,263
6	\$ 1,950

All remaining costs (utilities, labor, building, and taxes) are calculated directly from the information in Appendix A, Tables A-10 through A-19. Table 3-3 shows these costs for each site.

Table 3-3. Cost of utilities, labor, building, and taxes for each site

	Cost ($)			
Site	Utilities	Labor	Building	Taxes
1	9,460	12,773	514	3,095
2	11,563	11,249	563	3,470
3	12,768	10,422	539	3,580
4	10,548	12,159	490	3,755
5	10,898	12,333	612	3,701
6	11,628	12,244	612	3,393

For example, the costs in Table 3-3 for site 1 are calculated as follows:

1. Utility Cost (i) = Gas Cost (i) + Water & Sewage Cost (i)

   a. Gas Cost (i) = $\sum_j A_{ij} B_j$

   where:

   i. A_{ij} is the gas rate at site "i" for gas category "j", and

   ii. B_j is the amount of gas in category "j".

b. Water & Sewage Cost (i) = $\sum_j C_{ij}D_j$

where:

i. C_{ij} is the water and sewage rate at site "i" for water and sewage category "j", and

ii. D_j is the amount of water and sewage in category "j".

c. Therefore, the Utility Cost for site 1 is:

Gas Cost (1) = (.33)(1)+(.33)(99)+(.33)(200)+ (.33)(700)+(.33)(19,000)

= $ 6,600.

Water & Sewage Cost (1) = (.33)(1,000)+(.19)(1,000)+ (.13)(18,000)

= $ 2,860.

Utility Cost (1) = $ 9,460.

2. Labor Cost (i) = $\sum_j E_j F_{ij} X + \sum_k G_k H_{ik}$

where:

a. E_j is the required number of type "j" hourly paid employees

b. F_{ij} is the hourly wage rate for type "j" employees at site "i"

c. X is the average monthly working hours

d. G_k is the required number of type "k" monthly paid employees, and

e. H_{ik} is the monthly salary for type "k" employees at site "i".

Therefore, at site 1

Labor Cost (1)=[(12)(1.75)+(12)(2.50)] [173] + (4)(750) + (1)(950)

= $ 12,773.

3. Building Cost (i) = CRF/12 $[\sum_j W_{ij} S_j]$

where:

a. CRF is the capital recovery factor for a period of 20 years and an interest rate of 10%

b. W_{ij} is the cost per square foot at site "i" for floor level "j", and

c. S_j is the total square footage of floor level "j".

Therefore, at site 1

Building Cost (1) = .1175/12 [(6)(2500)+(15)(2500)]

= $ 514.

4. Tax Cost (i) = (V) (M_i) (T)/12

where:

a. V is the value of the property

b. M_i is the mill levy at site "i", and

c. T is the percent of property value that is taxable.

Therefore, at site 1

$$\text{Tax Cost (1)} = (500,000)(.18569)(.40)/12$$
$$= \$\ 3,095.$$

The objective factor cost for each site may now be calculated by summing the costs in Tables 3-1 through 3-3 and applying Equation 3-2. Table 3-4 shows the resulting total objective factor costs along with the objective factor measures for each site.

Table 3-4. Total objective factor cost and measure for each site

Site	Total Objective Factor Cost (OFC)	Objective Factor Measure (OFM)
1	$ 28,237	.17433
2	$ 29,275	.16814
3	$ 29,266	.16819
4	$ 29,531	.16669
5	$ 29,732	.16556
6	$ 31,334	.15709

For example, the objective factor measure for site 1 in Table 3-4 is calculated as follows:

$$OFM_1 = [OFC_1 \cdot (\sum_i 1/OFC_i)]^{-1}$$
$$= [28,237(1/28,236 + 1/29,275 + 1/29,266 + 1/29,732 + 1/31,332)]^{-1}$$
$$= .17433$$

Evaluating subjective factor measures for each site. The subjective factor measure for any site "i" (SFM_i) is defined in Chapter II, Equation 2-8 as:

$$SFM_i = \sum_k (SFW_k \cdot SW_{ik}) \qquad (3-6)$$

where:

1. SFW_k is the weight of subjective factor "k" relative to all subjective factors, and
2. SW_{ik} is the weight of site "i" relative to all potential sites for subjective factor "k".

In order to determine the values of SFM_i for all "i", terms SFW_k and SW_{ik} are evaluated using the technique previously referred to as preference theory.

Consider first the subjective factor weights. The perference matrix[16] used to weight the subjective factors listed in Appendix A is shown in Table 3-5. Decision 1 (Table 3-5) implies that the decision maker considers factor "availability of transportation" more important than factor "industrial sites". To illustrate further, decisions 11 and 13 imply that the decision maker is indifferent towards factor "availability of transportation" and factors "competition" and "availability of labor".

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[16]The preference values shown in Table 3-5 are theoretical and would normally represent policies and practices of the management of the locating plant.

Table 3-5. Preference matrix for subjective factors

Decisions

Property	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
Availability of Transportation	1	1	1	1	1	1	1	1	1	1	0	1	0																																	
Industrial Sites	0													1	1	1	1	1	0	1	1	1	0	1	0																					
Climate		0												0												1	0	0	0	0	0	0	1	0	0	0										
Educational Facilities			0												0											0											0	0	0	0	0	0	0	0	0	0
Union Activity				0												0											1										1									
Recreation Facilities					0												0											1										1								
Housing						0												0											1										1							
Future Growth							0												1											1										1						
Community Services								0												0											1										1					
Employee Transportation Facilities									0												0											1										1				
Cost of Living										0												0											0										1			
Competition											0												1											1										1		
Complementary Industries												0												0											1										1	
Availability of Labor													0												1											1										1

Decisions

Property	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91
Availability of Transportation																																													
Industrial Sites																																													
Climate																																													
Educational Facilities																																													
Union Activity	1	1	0	0	1	1	0	0	0																																				
Recreation Facilities	0									1	0	0	1	1	0	0	0																												
Housing		0								0								0	0	0	1	0	0	0																					
Future Growth			1								1							1							1	1	1	0	1	0															
Community Services				1								1							1						0						1	1	0	0	0										
Employee Transportation Facilities					0								0							1						0					0					1	0	0	0						
Cost of Living						0								0							0						0					0				0				0	0	0			
Competition							1								1							1						1					1				1			1			1	0	
Complementary Industries								1								1							1						0					1				0			1		0		0
Availability of Labor									1								1							1						1					1				1			1		0	1

Subjective factor weights for each factor are then determined by summing the number of times each factor was preferred (considered more important) and dividing each sum by the total number of preferred decisions. A list of the subjective factor weights calculated from Table 3-5 is shown in Table 3-6. In Table 3-6 the weight of factor "availability of transportation" (.12644) is the quotient of 11 (number of times it was preferred) divided by 87 (total number of "preferred" decisions).

Table 3-6. Factor weights for subjective factors

Property	Sum of Preference Values*	Factor Weight
Availability of Transportation	11	.12644
Industrial Sites	9	.10345
Climate	2	.02299
Educational Facilities	0	.00000
Union Activity	6	.06897
Recreation Facilities	5	.05747
Housing	3	.03448
Future Growth	10	.11494
Community Services	7	.08046
Employee Transportation Facilities	4	.04598
Cost of Living	1	.01149
Competition	11	.12644
Complementary Industries	7	.08046
Availability of Labor	11	.12644
Total	87	1.00000

*Calculated from Table 3-5

Site weights are determined utilizing basically the same procedure used to evaluate subjective factor weights. The only difference in the two procedures is that site weights are evaluated on the basis of decision data at the sites while subjective factor weights are evaluated on the basis of company policies and practices. Appendix A, Tables A-20 through A-39, shows the information to be used in determining site weights for each subjective factor.

As an example, consider evaluating the site weights for subjective factor "availability of transportation". Table 3-7 shows the preference matrix with site weights for this factor. The matrix was arrived at by first subjectively examining the data in Appendix A, Tables A-20, A-21 and A-22. Then on the basis of this examination, the sites were compared two at a time. Decision 1 (Table 3-7) indicates that site 1 was preferred to site 2 for factor "availability of transportation". This decision was made because site 1 had more desirable transportation facilities than site 2.

Site weights for factor "availability of transportation" are then determined by dividing the number of preferred decisions by the total. For example, the site weight (.2000) for site 1 and factor "availability of transportation" is the quotient of 3 (number of times it was preferred) divided by 15 (total number of preferred decisions).

Appendix B displays the preference matrices used to calculate the site weights for each of the remaining subjective factors and Table 3-8 shows the resulting site weights calculated from these matrices.

Table 3-7. Preference matrix and site weights for factor "availability of transportation"

Property	Decision 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Sum of Preference Values	Site Weight
Site 1	1	0	0	1	1											3	.20000
Site 2	0					0	0	0	1							1	.06667
Site 3		1				1				1	1	1				5	.33333
Site 4			1				1			0			1	1		4	.26667
Site 5				0				1			0		0		1	2	.13333
Site 6					0				0			0		0	0	0	.00000
												Total				15	1.00000

Table 3-8. Site weights for each subjective factor

Subjective Factor	Site Number	Site Weight
Availability of Transportation	1	.20000
	2	.06667
	3	.33333
	4	.26667
	5	.13333
	6	.00000
Industrial Sites	1	.20000
	2	.06667
	3	.13333
	4	.33333
	5	.26667
	6	.00000
Climate	1	.25000
	2	.16667
	3	.08333
	4	.25000
	5	.25000
	6	.00000
Educational Facilities	1	.14286
	2	.35714
	3	.14286
	4	.28571
	5	.07143
	6	.00000
Union Activity	1	.33333
	2	.26667
	3	.06667
	4	.20000
	5	.13333
	6	.00000

Table 3-8. Continued

Subjective Factor	Site Number	Site Weight
Recreation Facilities	1	.07143
	2	.35714
	3	.21429
	4	.28571
	5	.07143
	6	.00000
Housing	1	.26667
	2	.20000
	3	.06667
	4	.33333
	5	.13333
	6	.00000
Future Growth	1	.21429
	2	.14286
	3	.07143
	4	.21429
	5	.35714
	6	.00000
Community Services	1	.28571
	2	.07143
	3	.14286
	4	.21429
	5	.28571
	6	.00000
Employee Transportation Facilities	1	.26667
	2	.13333
	3	.06667
	4	.20000
	5	.33333
	6	.00000

Table 3-8. Continued

Subjective Factor	Site Number	Site Weight
Cost of Living	1	.07692
	2	.23077
	3	.07692
	4	.38462
	5	.23077
	6	.00000
Competition	1	.20000
	2	.20000
	3	.20000
	4	.20000
	5	.20000
	6	.00000
Complementary Industries	1	.28571
	2	.21429
	3	.07143
	4	.07143
	5	.35714
	6	.00000
Availability of Labor	1	.20000
	2	.06667
	3	.13333
	4	.33333
	5	.26667
	6	.00000

Subjective factor measures for each site may now be calculated by applying Equation 3-6 to the data in Tables 3-6 and 3-8. Table 3-9 shows the resulting subjective factor measures for each site. For example, the subjective factor measure for site 1 in Table 3-9 is calculated as follows:

$$
\begin{aligned}
SFM_1 &= \sum_k (SFW_k \cdot SW_{1k}) \\
&= (.12644)(.20000) + (.10345)(.20000) + \ldots \\
&\quad \ldots + (.12644)(.20000) \\
&= .22234
\end{aligned}
$$

Table 3-9. Subjective factor measures for each site

Site	Subjective Factor Measure
1	.22234
2	.14688
3	.14861
4	.24432
5	.23785
6	.00000

Evaluating location measures for each site. The location measure for any site "i" is defined in Equation 3-1 as:

$$LM_i = X \cdot OFM_i + (1-X) \cdot SFM_i \qquad (3-7)$$

where:

1. OFM_i is the measure of the objective factors for site "i"
2. SFM_i is the measure of the subjective factors for site "i", and
3. X is the objective factor decision weight.

Assuming an arbitrary value of .8 for the objective factor decision weight (X) and applying Equation 3-7 to the data in Tables 3-4 and 3-9, the location measures for the various sites can be calculated. Table 3-10 shows the resulting location measures for the six cities under consideration. Since site 1 has received the largest location measure (Table 3-10) it would be selected as the new location on the basis of X equal to exactly .8.

Table 3-10. Location Measures for each site (X = .8)

Site	Objective Factor Measure	Subjective Factor Measure	Location Measure
1	.17433	.22234	.18393
2	.16814	.14688	.16389
3	.16819	.14861	.16428
4	.16669	.24432	.18222
5	.16556	.23785	.18001
6	.15709	.00000	.12567

For example, the location measure for site 1 in Table 3-10 is calculated as follows:

$$LM_1 = X \cdot OFM_1 + (1-X) \cdot SFM_1$$
$$= (.8)(.17433) + (.2)(.22234)$$
$$= .18393$$

Sensitivity to changes in decision weight. As stated in Chapter II, the value of the objective factor decision weight may be subject to error. As a safety precaution the sensitivity of the location measures to changes in the objective factor decision weight should be investigated before the best site is selected. Figure 3-1 illustrates the changes in location measures for each site and various values of X.

A change in the best location is detected when the objective factor decision weight is equal to approximately .75 (Figure 3-1). The final decision, that of selecting the best site, is now made with the knowledge that if site 1 is selected and the estimated value of X (.8) is overestimated by 6 percent, the wrong site will be chosen.

Sensitivity to input data. As observed in the previous section, changes in the objective factor decision weight affects the selection of the best site. The best site can also be affected by changing various input data. Changes in labor, utilities, and production requirements are good examples of input data that can affect the choice of locations.