A dynamic programming model for range management decisions with suggestions for further research
by William Albert Bussing

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Agricultural Economics
Montana State University

Abstract:
This study is an attempt to formulate a dynamic programming decision model of the management process of a range-livestock operation and to establish the empirical measurements required for the application of the model.

The dynamic programming formulation is used for the model because the range management problem may be accurately represented by a multistage decision process.

The analysis is performed on a per-acre basis on the level of the individual ranch. The model determines an optimal policy for range improvement, through fertilization and artificial seeding, and for grazing practice, through the adjustment of the stocking rate. The optimal policy is stated in terms of decision rules that are dynamic extensions of the static equimarginal principle.

An example is presented to demonstrate the derivation of the decision rules from a model with specified functional forms. The properties of the decision rules are examined for changes in the system parameters and relevant prices.

The data requirements for the application of the model are presented. It is necessary to specify the response of livestock output to changes in moisture and nutrient levels, the stocking rate, and the specie composition of the range area. The effects of each decision on the future production of the range area must also be measured.

The measurement problems of the model may be classified as: (1) problems that are independent of animal effects; (2) problems that are dependent on animal effects, but not on animal performance; and (3) problems that are dependent on animal performance. It is suggested that problems of the first and second type may be resolved in small-plot experiments; problems of the third type must be resolved in grazing trials.

The importance of this study is the presentation of the empirical measurements required for future research. The problem of range management must be approached in a unified research effort by physical scientists as well as economists. This study expresses the economic viewpoint on the relevant information and appropriate data for such research.

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Date July 21, 1970

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Any error or omissions in this study, of course, are the responsibility of the author.

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ABSTRACT

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CHAPTER I

INTRODUCTION*

In 1969, Montana ranchers produced $319.4 million worth of livestock and livestock products, an increase of 11 percent over the previous year. Cattle and calves produced $292.2 million of the total, an increase of $27 million or 10 percent over the previous year. In 1968, livestock accounted for 62.1 percent of the total cash receipts for all farm commodities and cattle and calves accounted for 51.3 percent.

The marketings of cattle declined between 1968 and 1969, so the increase in cash receipts resulted from an increase in the average price. In 1967, the average prices per hundredweight for cattle and calves were $22.00 and $28.30, respectively. In 1968, the average prices rose to $22.80 and $29.20, and in 1969, to $25.70 and $32.70.

About 55 percent of Montana's total land area is used for grazing, and Montana ranchers are interested in methods of improving their range areas. One method is fertilization, and the cost of nitrogen fertilizer has dropped from 13 or 14 cents to about 10 cents a pound. It is hoped that the lower costs of fertilization and the higher prices of cattle make range improvement by fertilization a practical and profitable alternative. Even at 10 cents a pound, however, nitrogen fertilizer represents a considerable investment in range improvement.

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*The figures reported are from the April 30, 1970 bulletin of the Montana Crop and Livestock Reporting Service.

Before making a decision on such a range improvement practice, the ranchers need to know the effect of the range improvement on their profits. This study is directed toward providing that information.

The Problem

The output of a range-livestock operation may be increased in several ways: by improving the forage yield of the range, by improving the livestock characteristics, and by improving the grazing management practices of the rancher. The forage yield of the range may be improved by a variety of methods. Those methods that have been used in Montana may be classified as mechanical and chemical treatments, artificial seeding, and fertilization.

In this paper, mechanical and chemical treatments of the range, and the characteristics of the livestock are considered to be determined by purely technical criteria. That is, the production function of the range-livestock operation is assumed to specify the maximum output, for a given factor mix, which results from the use of the best treatments and type of livestock which the existing state of technical knowledge provides. Then, the yield of the range-livestock operation, in pounds of marketable livestock products, is assumed to be determined by the nitrogen and moisture levels of the soil, the specie composition of the range, and the stocking rate of livestock. The site characteristics such as topography, available water, and native soil productivity are taken as given.

The nitrogen level, or the amount of available nitrogen per unit area of soil, is a measure of the controllable nutrient of the soil. It is expected that the range yield reacts positively, at least over some values, to an increase in the nitrogen level. Nitrogen fertilization also alters the specie composition of the range. It is expected that the change is toward more desirable specie with increased nitrogen levels.

The stocking rate, or the number of head of livestock grazed on a unit area or range, is a measure of range usage* The effect of an increased stocking rate on the range yield is complex.

> Grazing animals have harmful effects on plants because of over- and under-selective grazing; compaction of soils due to trampling, which retards water infiltration and root development; uneven fertility and its removal; and outright herbage wastage due to fouling, trampling, and selective grazing. [7:p.14]

Over-selective grazing is the case in which the animal is able to select the higher quality forage for its grazing. This case results from a stocking rate that is low relative to forage availability and produces more output per animal, generally at the expense of lower output per acre. Under-selective grazing is the case in which the animal is forced to graze low quality forage. This case results from a high

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*When considering different ages and classes of livestock, the stocking rate will have to be adjusted by animal unit equivalents.

stocking rate and produces lower output per animal, although the output per acre may increase. The effect of increased stocking rates on specie composition is expected to be negative.

This is the central problem of grazing management : the compromise between gain per animal and gain per acre. Generalized curves have been suggested by Harlan [3:p.140] and Mott [5:p.606], which clearly illustrate this compromise. Figure 1 shows the curves presented by Mott. The vertical axis represents the output per animal (solid line) and the output per acre (dashed line), though the scale is not the same. The horizontal axis represents the stocking rate, running from no animals to heavy stocking rates. It is readily seen that at the maximum output per acre the output per animal is falling off rapidly, which results in animals with lower market value.

The net revenue function of the rancher is a function of both the output per acre and the output per animal, which are both functions of the stocking rate. Such a revenue function may be written as

$$R(s) = \Phi[x(s),y(s)]$$

where s is the stocking rate, x(s) is the output per acre, and y(s) is the output per animal. Figure 1 shows that over some range of values, the output per acre increases with increased stocking rates. The output per animal decreases for any increase in the stocking rate. An increase in either output per acre or output per animal results in an increase in revenue. These relationships may be expressed as

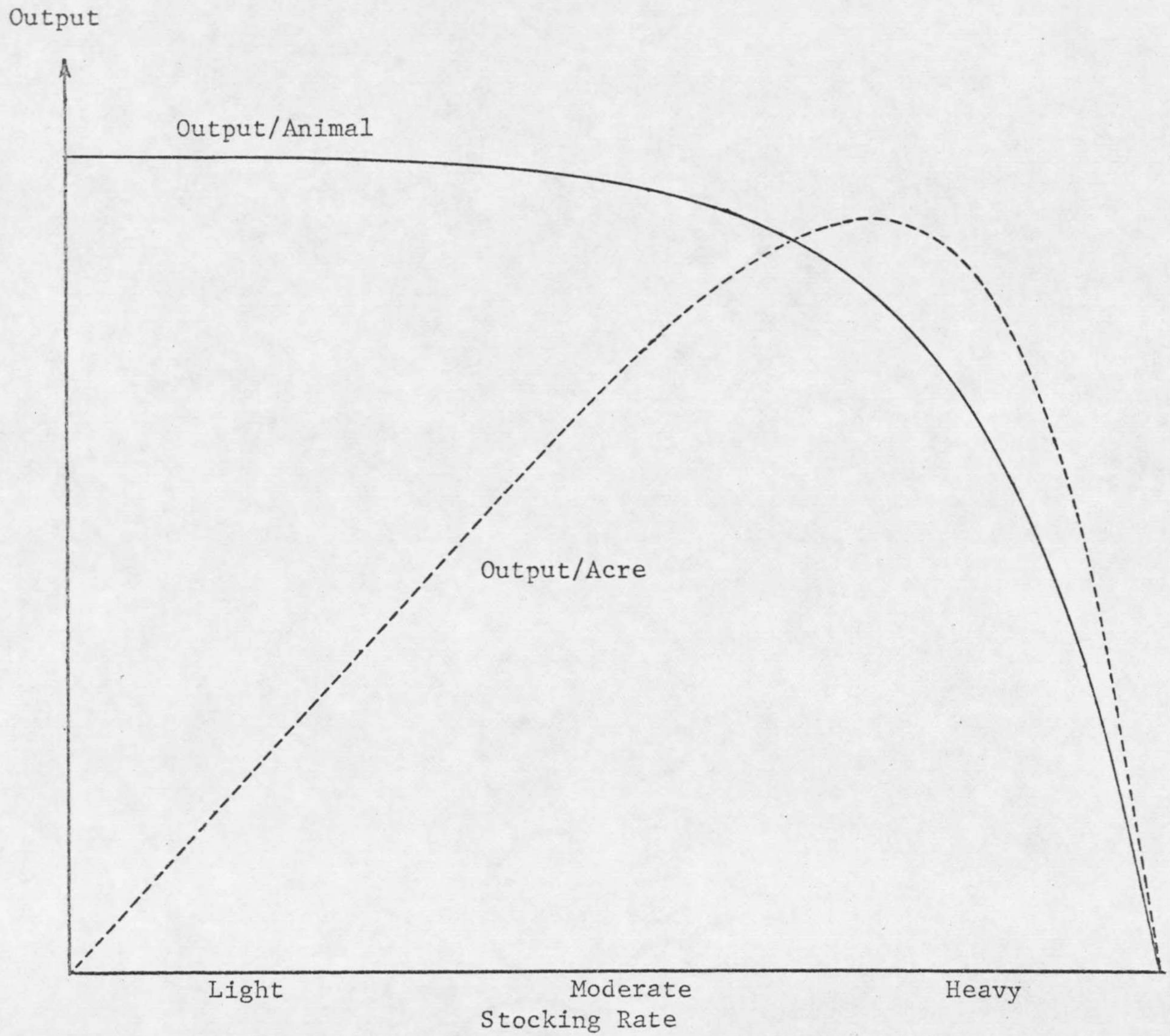

Figure 1. The influence of the stocking rate on output per animal and output per acre.

$$\frac{\partial y}{\partial s} < 0,\ \frac{\partial \Phi}{\partial x} > 0,\ \frac{\partial \Phi}{\partial y} > 0,$$

$$\frac{\partial^2 x}{\partial s^2} < 0 \text{ and } \frac{\partial x}{\partial s} > 0,\ s < s^*,$$

where s* is the stocking rate that results in the maximum output per acre ($\frac{\partial x}{\partial s} = 0$).

In a static framework, the rancher seeks to maximize the revenue function, R(s), which implies that $\Phi[x(s),y(s)]$ is a maximum. The necessary condition for the maximum is

$$\frac{\partial \Phi}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial \Phi}{\partial y} \cdot \frac{\partial y}{\partial s} = 0.$$

As the stocking rate increases toward s*, the output per acre increases at a decreasing rate and the output per animal falls at an increasing rate. The optimum stocking rate is that at which the incremental addition to revenue due to an incremental increase in output per acre is equal to the loss in revenue due to an incremental decrease in output per animal. The optimum stocking rate will not be s* unless an increase in the stocking rate does not result in a decrease in the output per animal or in revenue. The management problem is to determine the optimum stocking rate.

The rancher approaches this problem with his ability to adjust the stocking rate through the retention of young stock as yearlings.* In the

*The term <u>young stock</u> will be used to refer to the offspring of the mature livestock from birth to one year of age, such as calves. The term <u>yearling</u> will be used to refer to the offspring from age one to age two, such as yearling heifers and steers.

face of a good year, when moisture is expected to be high and forage yield larger than normal, the rancher may hold yearlings in order to take advantage of the higher yield. Before an expected bad year, the rancher may sell all of his young stock in order to reduce the grazing pressure on the expected low forage yield.

The specie composition of the range might be expressed as an index of the relative abundance of the various specie that comprise the forage. The index must reflect the desirability of the forage, including such things as energy, protein and fat content, plant characteristics such as root structure, and so on. It is then expected that the range yield reacts positively to an improvement in the index of specie composition. If a suitable index cannot be constructed, the specie composition may have to be expressed as a vector of forage quality measurements.

The moisture level is a measure of the available water in a unit area of the soil. It is expected that the range yield reacts positively to an increase in the moisture level. The moisture level, measured, for example, as soil moisture in the spring, is assumed to be determined by the precipitation during the year. The precipitation is assumed to be determined exogenously in a stochastic manner.

The yield of the range-livestock operation is to be expressed as the output of animal products per acre, which may be measured directly or based upon measurements of output per animal and the number of animals per acre, and is to be measured at the end of the production period. The revenue function is the market value of the output that is sold, measured on a per acre basis.

The assumption of a stochastic moisture level variable imposes a probability distribution on the return function*, but in order to achieve analytical results it is desirable to reduce the stochastic return function to a meaningful deterministic value. This is accomplished by assuming some specific behavior on the part of the rancher. For simplicity, it is assumed here that the rancher is concerned with his expected returns, the mean value of the return function over the probability distribution associated with it. The expected return function does not depend on the moisture level.

The managerial control of the rancher takes the form of alternatives through which he may influence his expected returns: he may adjust the nitrogen level of the soil by applying fertilizer, he may adjust the stocking rate by adding or removing livestock from the range area, and he may adjust the specie composition by reseeding the range.

The application of fertilizer entails some fixed cost per acre for application plus an increasing cost function of the amount of fertilizer applied per acre.

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*The fact that the return function itself is stochastic allows the imposition of subjective probabilities on the returns. For example, it might be desirable, on a behavioral basis, to weight higher moisture (and thereby, higher returns) by a greater probability than climatic data specifies. It is also possible to attach probabilities to the predicted future prices of livestock. The analysis remains the same.

The option of reseeding the range, if carried out, entails some fixed cost per acre for preparing and seeding the range.

It is assumed that the rancher has an initial breeding herd which may be represented by a specific stocking rate. Further, it is assumed that the rancher maintains the breeding herd at a constant level. In other words, the offspring of the breeding stock are fed for one production period and then sold, or fed for one additional production period, as yearlings, and then sold. Any young livestock held for longer than two production periods is considered to be held for replacement of older breeding stock, the older stock then being sold in place of the young. Thus, the adjustment in the stocking rate available to the rancher consists of the number of young livestock he holds over as yearlings, and entails a marketing cost for selling young stock and yearlings plus a wintering cost for holding young stock over to yearlings.*

The total cost function of the range-livestock operation is the sum of the three cost functions, and therefore depends on nitrogen application, changes in the stocking rate, and the option of reseeding.**

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*It is conceivable that the rancher may even wish to purchase additional stock to feed during an exceptionally good year. In that case, the cost of adjusting grazing intensity also entails the purchasing costs of the additional livestock.

**The cost function, like the return function, may be stochastic, in which case the rancher is assumed to consider his <u>expected</u> costs.

It is assumed that the rancher utilizes the alternatives available to him in such a way as to increase the output of his operation, so that the addition to his expected revenue exceeds his costs, and maximizes discounted expected profit over his planning horizon. The increased output may take the form of heavier animals or more animals or both. The objective of this paper is to establish a conceptual framework within which a rational policy of utilizing the alternatives may be found.

Dynamic Programming

The dynamic programming formulation was selected for the problem structure mainly because its characteristic multistage decision process most nearly approximates the decision process of the rancher.

> A multistage decision process is characterized by the task of finding a sequence of decisions which maximizes (or minimizes) an appropriately defined objective function. The stage is the interval into which the process is divided, a decision being made at each stage in the sequence of stages comprising the decision process. The state of the process, at a particular stage, describes the condition of the process, and is defined by the magnitudes of state variables and/or qualitative characteristics. Decision making at a given stage controls the state in which the process will be found in the following stage, the control being either deterministic (with certainty) or stochastic (with a probability distribution). [2:p.121]

These concepts will become clearer as the present model is formulated in the next chapter.

Several other characteristics of the dynamic programming formulation are also desirable. The dynamic nature of the formulation is important, as the continued operation of a range area is a dynamic problem. As noted above, the decision process may be stochastic, and the formulation serves to determine an optimum decision under uncertainty. The specific decision problem of any particular rancher is imbedded in a general formulation which specifically embodies any length of planning horizon and any initial state of the range operation, and thus contains two dimensions of sensitivity analysis.

The method of solution will be "value iteration," in which the maximum value of the objective function is computed at each stage. The principal advantage of this method is that the general solution displays, in its development, the optimal decision for each stage of the process.

The investigation will begin with the statement of the range management problem, the general assumptions and the relevant variables. The problem will then be stated in terms of a multistage decision process and formulated as a dynamic programming problem. The solution of the problem will be found, in the form of the necessary conditions for optimization. An example will be presented to illustrate the use of the model with specified functional forms. The properties of the solution of the example will be considered. Finally, the problems of empirical measurements will be outlined and suggestions for their resolution will be made.

CHAPTER II

THE MODEL

Formulation

In order to utilize the dynamic programming algorithm, it is necessary to formulate the problem stated above in terms of a multistage decision process.

Define a stage of the process to be one year, ending in late fall when the livestock is sold. The state of the process, at any given stage, is determined by the nitrogen level, the stocking rate, and the index of specie composition.

The functional form of the decision process for dynamic programming is restricted only by the Markov requirement:

> After any number of decisions, say k, the effect of the remaining N-k stages of the decision process upon the total return depends only upon the state of the system at the end of the k-th decision and subsequent decisions. [1:p.54]

In other words, at any time during the decision process, the rancher's future decisions depend only on the value of the state variables at the end of the last production period. It is assumed that the formulation of this model meets the Markov requirement, although problems with accomplishing the Markov requirement can usually be avoided by introducing additional state variables.

As a matter of notation, a superscript always refers to the time period or stage number. Exponents will always be written with parentheses.

The following variables are defined on a per acre basis to allow for any ranch size:

y^t = expected range yield at the end of period t, in animal products per acre,

x_1^t = nitrogen level of the range at the end of period t, in pounds per acre,

x_2^t = additional stocking rate in period t+1, due to yearlings held, in head per acre,

x_3^t = index of specie composition at the end of period t,

θ_1^t = nitrogen application at the beginning of period t, in pounds per acre,

θ_2^t = adjustment in stocking rate at the end of period t, due to the sale of young livestock, in head per acre,

θ_3^t = reseeding option, carried out at the beginning of period t.

To clarify the relationships of the state and decision variables to the stages, Figure 2 is presented. The horizontal line represents time, future time to the right. The occurrences of the state and decision variables are indicated by the variables' symbols.

A very important aspect of the multistage decision process is the effect of the decision variables at a given stage upon the state variables in the next stage. These effects, expressed as transformations of the state variables, provide the dynamic link between stages. The transformations are indicated in Figure 2 by the dashed arrows.