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A method of studying solar prominence motion, wherein the outlines of an eruptive prominence are traced from photographic film at successive intervals of time by means of a specially designed projector, is described. The motion of the prominence in the x and y directions is then plotted against the time and the resulting curves analyzed by means of a curve fitting procedure due to Birge. Results indicate that, excepting large probable error, the acceleration of the prominence, in the eruption stage, is very nearly constant both in the x and y directions for the relatively short time interval during which measurements are taken. Main source of error is found to be the measurement of heights above the chromosphere, due to non-uniform centering of the occulting disk of the coronagraph with successive exposures.

ANALYSIS AND CRITIQUE OF THE ENVELOPE-TRACING METHOD FOR THE INVESTIGATION OF SOLAR PROMINENCES

by

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ABSTRACT

A method of studying solar prominence motion, wherein the outlines of an eruptive prominence are traced from photographic film at successive intervals of time by means of a specially designed projector, is described. The motion of the prominence in the x and y directions is then plotted against the time and the resulting curves analyzed by means of a curve fitting procedure due to Birge. Results indicate that, excepting large probable error, the acceleration of the prominence, in the eruption stage, is very nearly constant both in the x and y directions for the relatively short time interval during which measurements are taken. Main source of error is found to be the measurement of heights above the chromosphere, due to non-uniform centering of the occulting disk of the coronagraph with successive exposures.

INTRODUCTION

When, in 1931, the French astronomer, Bernard Lyot, first photographed the solar corona without the aid of a total eclipse, but by means of his newly developed coronagraph, it was apparent that astronomers had at last been given the instrument which would permit extensive research in the study of solar prominences and other interesting features which appeared within the corona itself. Only in recent years have some of these instruments been constructed in observatories at Climax, Colorado; Pic du Midi in the Pyrenees; Sacramento Peak, New Mexico; Arosa, Switzerland; Wendelstein, Bavaria; Kanzelhöhe, Austria; Mt. Norikura, Japan and a Russian station somewhere in the Caucasus.

Observational data concerning the solar corona, which has been supplied by these observatories, consist primarily of motion pictures of prominence activity. Perhaps more than 50,000 feet of film have been exposed at these various stations, with more than 20,000 feet from Climax alone.

To date, analysis of these films has consisted primarily of tracing the motion of well defined "knots" and "streamers". The method has been quite successful but has, nevertheless, had many shortcomings. One particular difficulty is that usually rather few "well defined" knots or streamers can be found which remain sharp and clear for an appreciable period of time. The disappearance and reappearance of prominence features is a familiar phenomenon,[1] although it usually occurs most noticeably in

1. Dodson and McMath, 1948 (11), have given evidence of a filament which disappeared and then reappeared. Others have claimed to have witnessed the same phenomenon.

the case of delicate streamers and knots which are quite tenuous to begin with.

The acquisition, by the Climax Observatory, of a projector which permits highly magnified and distortion-free pictures to be focused on graph paper with fine resolution, has suggested a new mode of study of the prominence motion. This new method is concerned with tracing the general outline or envelope of the prominence or certain prominence features, at successive intervals of time (usually equally spaced), and then proceeding to extract information from these curves. The idea is based on the supposition that the succeeding envelopes of the prominence motion should tend to become aligned with the equipotential surfaces of the force field; and that motion outward orthogonal to the envelope should indeed be along the lines of force. It is apparent then that, should this situation be realized, it is only a matter of measurement to obtain information on the force field.

This investigation proposes to examine one solar prominence by this method, and to give not only as much information about the forces and velocities as is measurable, but also to serve as a general critique of the method, specifying quantitatively, whenever possible, the general accuracy of the method and attempting to point out its shortcomings.

As might be expected, the envelopes traced (see figure 6) are rough and full of humps as the prominence itself appears; and hence it would seem that they are certainly not equipotential surfaces. Obviously, then, there is a large element of human judgment in assigning the directions of the lines of force. In this particular investigation, the measurements are confined to two areas, one in which the motion was entirely vertical and the other in which the motion was transverse.

DESCRIPTION OF THE PROMINENCE.

The solar prominence with which this investigation is concerned appeared over the southeastern limb of the sun on September 8, 1948. It was of the eruptive type, and was photographed with the coronagraph of the High Altitude Observatory of Harvard University and the University of Colorado at Climax, Colorado from 16:29 to 20:23 U.T. on September 8, 1948, at the rate of one picture per half minute. The measurements and data presented herein are concerned chiefly with the period $17^h45.5^m$ to 18^h15^m, a relatively brief episode in the life of the prominence, during which virtually the entire eruptive motion took place. The northern-most and southern-most extremities of the arch lay at approximately -23° and -44.5° heliographic latitude, respectively, and at approximately 330° heliograph longitude. A large bipolar sunspot (Mt. Wilson No. 9395) lay 1° south and roughly 10° west of the northern extremity. This was a relatively short-lived spot, since it was first seen September 8 and was classified 1βd at that time.[2]

A large bipolar group (Mt. Wilson No. 9400), classified 1βpl, was

2. Sunspots are classified as follows:
   β = bipolar members; approximately equal in size.
   βp = bipolar members; preceding member dominant.
   βf = bipolar members; following member dominant.
   γ = composite spots.
   γβ = composite spots with leading members bipolar.
   l = living.
   d = dead or dying; i.e., 1βd = living when first seen; dead or dying when last seen . . .
   See: Menzel, Donald H., OUR SUN, Blakiston's Sons and Company, Philadelphia, 1949.

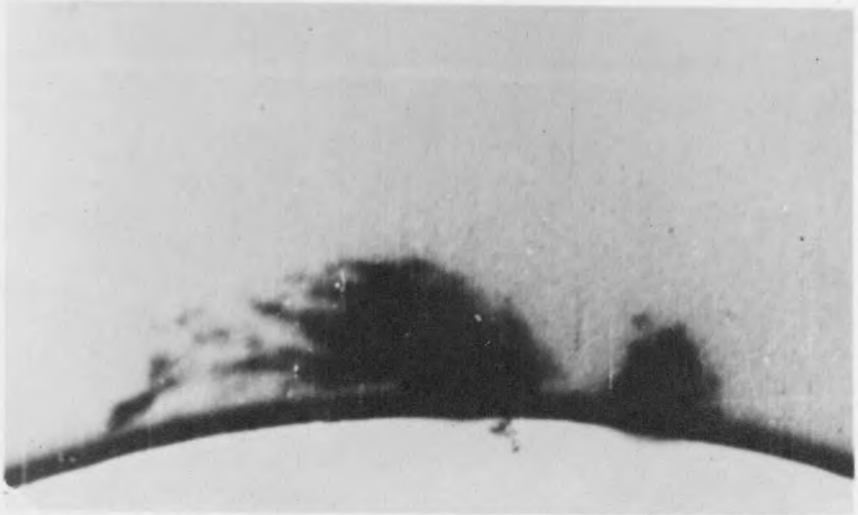

Figure 1. The prominence as it appeared when first observed (16^h30^m U.T.). Rapid upward motion or eruption has already commenced. The sunspots are at the extreme northern (left) end of the arch. At this time, matter seemed to be moving in that direction.

situated very near the northern tip of the arch, being roughly 3^{o} south of it. The prominence, as it appeared when first seen, is pictured in figure 1. At this time ($16^{h}30^{m}$), it was already in the process of slow expansion with acceleration. By $17^{h}40^{m}$, it had formed into a characteristic arch (figure 2) in which material was streaming into the photosphere along both ends. The prominence continued to rise and expand with increasing velocity, meanwhile developing into a well-defined loop as is illustrated in figures 3 and 4, and reaching a height of 400,000 kilometers (figure 4), at which time its upward velocity was in excess of 250 kms/sec. Thereafter, its upper portions became diffuse and disappeared from view. Matter continued to descend along the arches, particularly the southern-most, the northern arch having virtually disappeared by this time; so that by $19^{h}00^{m}$ there remained only two bright mounds of material at the extremities (figure 5).

The apparent[3] motion of material along the arches very strongly suggests that the extremities were indeed strong centers of attraction. This was true particularly of the southern-most, though there were no sunspots known to be near that end. On the contrary, a very large spot concentration was found near the northern extreme of the arch where matter visibly streaming downward was barely perceptible and fading. Now,

3. The word "apparent" is employed here to describe the motion because there has been no little conjecture as to whether material is actually moving or whether the effect is merely the result of moving excitation fields or some such phenomena. For example, consider the motion of a beam of light along a mass of clouds. Dodson and Weston (1950) have found significant evidence in favor of the hypothesis of moving material.

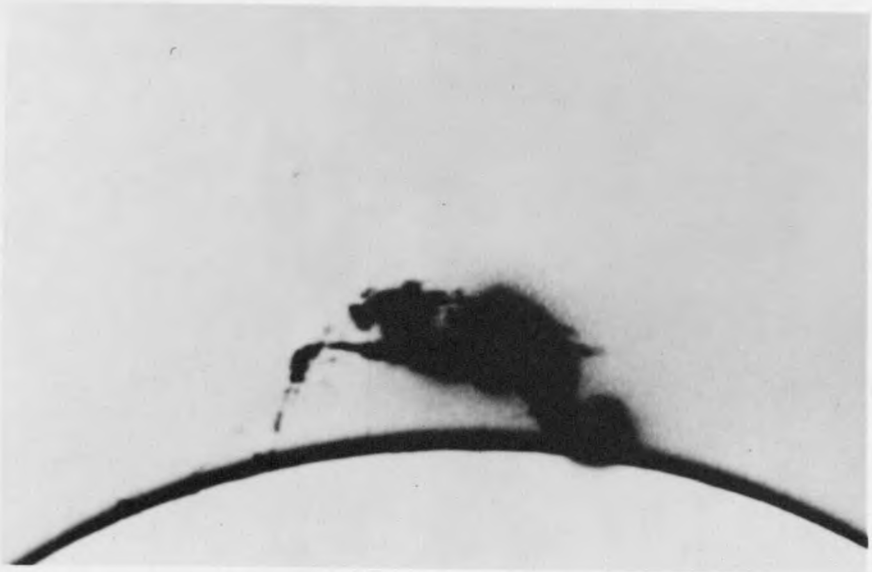

Figure 2. The prominence at 17^h40^m U.T. The region between the parallel lines is the region in which the motion of the envelope is measured and plotted (see figures 7 to 14). At this time matter appears to be streaming downward along the arches.

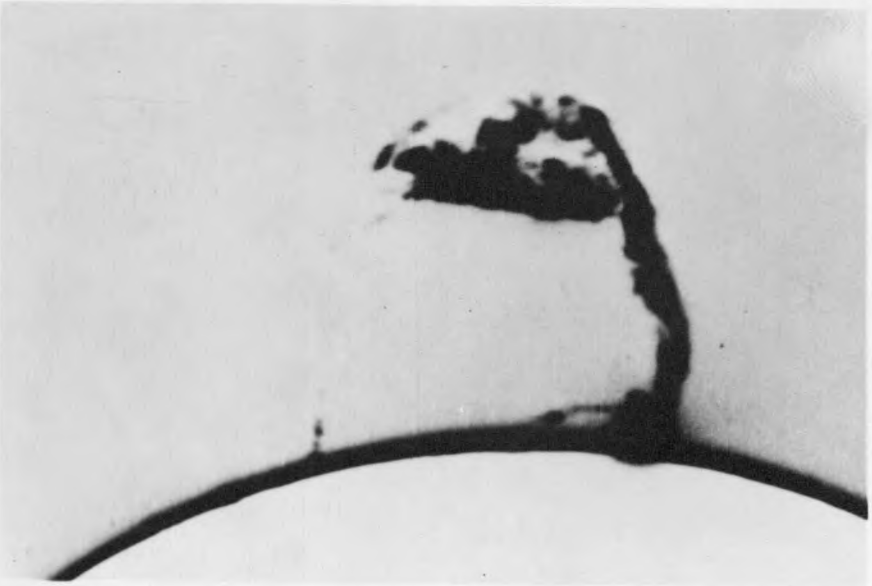

Figure 3. The prominence at 18^h00^m U.T. Note the gradual formation of the looplike arch.

streaming into the sun along this arch, there might very well have been large quantities of matter which was simply invisible. The prominences are known to be composed almost entirely of hydrogen and to be visible due to the fact that the gas is radiating. Some observers (among them, Dodson and McMath, 1948 - see footnote no. 1) have given evidence for prominence features which are at times visible and at other times not. The assumption that some matter in the prominence is invisible at times is a generally accepted one.

Figure 4. The prominence at 18^h15^m U.T. Unfortunately, reproduction of the photographs has caused the loss of much detail. On the original film, the southern (right) branch of the arch is very nearly intact although quite tenuous. Material has been rapidly dispersing. The vertical velocity at this time is approximately 250 kms/sec.

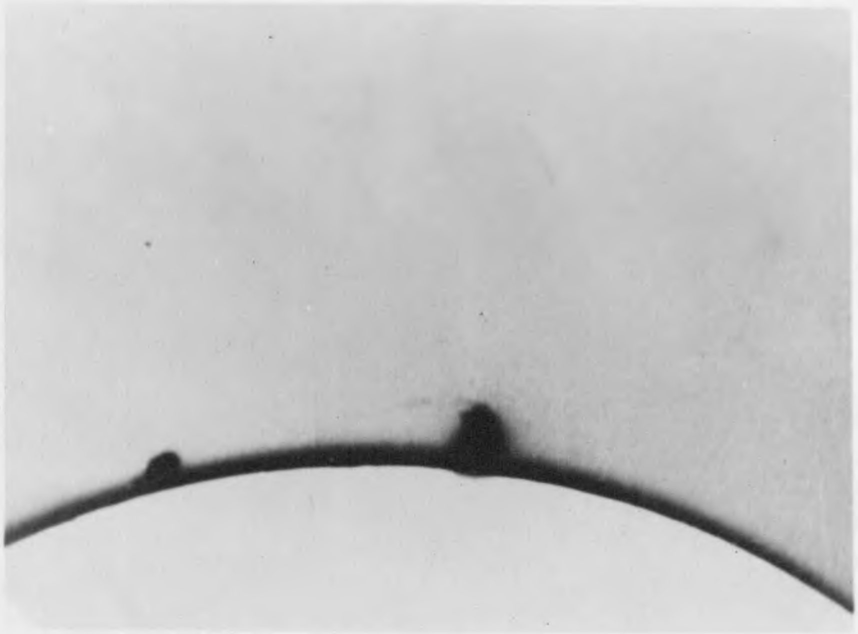

Figure 5. The prominence at 19^h00^m U.T. The material visible at the top of figure 4 has risen out of the scope of the camera. The two mounds of material shown here remained this way for more than an hour.

PLOTTING THE PROMINENCE MOTION

With the aid of the specially designed projector, selected frames from the film, chosen at equal intervals of time, have been traced on graph paper (figure 6). The pictures are "blown up" to a scale of 1 mm. to 1.39×10^3 kms., which corresponds to a solar radius of 50 cms. on the graph paper. In the rectangular system of coordinates, here employed, the origin is located on the solar limb at -37° heliographic latitude with the x-axis tangent to the limb at that point. We thus have a graph, scaled in distance and time, from which we can obtain measurements to study the motion of the prominence and the forces producing this motion. We shall not enter into any speculation as to the type of force field or fields involved, but shall attempt quantitative measurements.

Consider, for example, figure 7 and imagine a smooth curve drawn through the points so that for any instant of time t, the corresponding ordinate on the curve gives the y position (height) of the point on the envelope of the prominence whose x position is zero (in this particular case). In other words, the equation of the curve, call it $y(t)$, describes the motion of a point on the envelope of the prominence which has always the abscissa $x = 0$. The "best fit" curve through these points was obtained by the orthogonal polynomial least squares method of curve fitting as adapted by Birge[4] and Weinberg (4). The result was the

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4. For the sake of brevity, throughout the remainder of this paper the method shall be referred to as the Birge method.

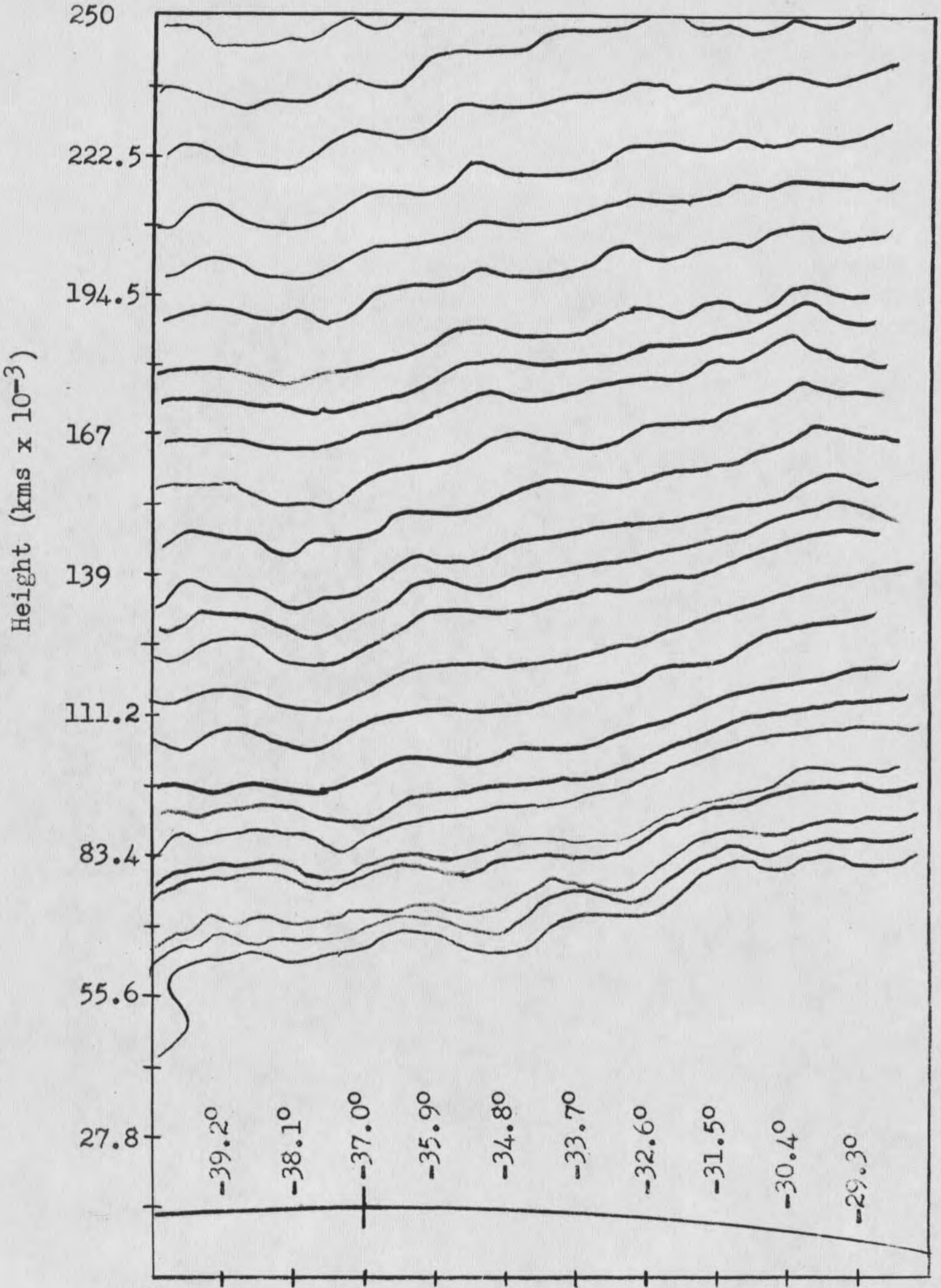

Figure 6. Showing the lower envelopes of the prominence as they were traced. The region shown is that lying between the parallel lines of figures 3 and 4. Time between any two curves is one minute.

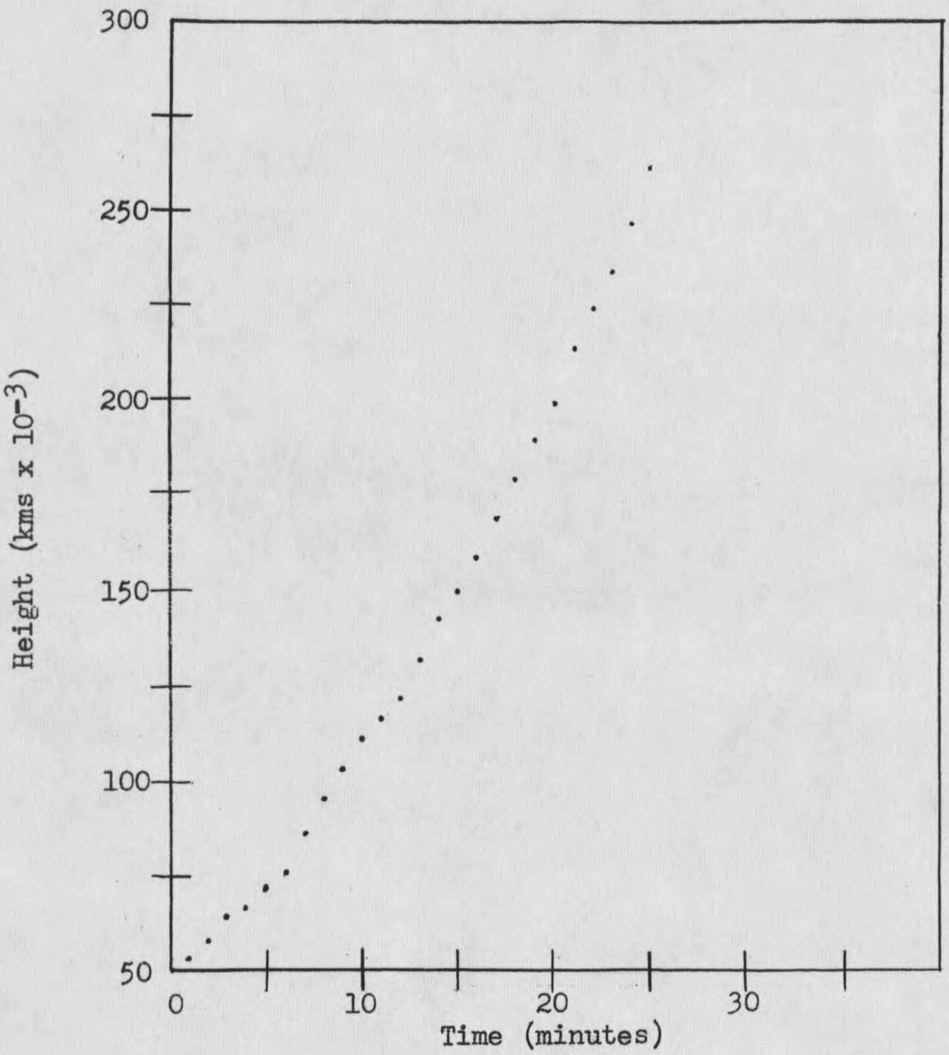

Figure 7. Plot of the vertical height (y) vs. the time along the ordinate $x = 0$ for the lower envelope of the prominence. Compare with figure 8.

parabola $y = 5.00 \times 10^4 + 3.89 \times 10^3 t + 182t^2$. Figure 8 illustrates how the parabola fits the observed points. See also figures 9 to 13 inclusive. Here the reader may very well object that a curve which passed through the points of figure 7 would have too many humps to be represented by a parabola. However, the method of Birge includes two interesting features which provide the physical scientist with criteria for best fit. First of all, the method predicts the probable deviation at any point between the calculated curve and the original or "true" curve. Secondly, the method employs a procedure whereby certain successive sums[5] are calculated, each sum corresponding to an additional degree in the resulting least squares polynomial and said successive sums being a measure of the increase in goodness of fit for that degree.

The two criteria of the Birge method in this case indicated that no better fit could be obtained with a third, fourth, or fifth degree curve than was obtained with the parabola. Furthermore, as many of the features of the prominence were diffuse and ephemeral, there was an element of error introduced with the tracing of the envelope on the graph

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5. The sums referred to here are the sums of the squares of the residuals of the several unweighted points, usually designated by Σv^2. The criterion of fit of any function to the experimental data is furnished by the magnitude of Σv^2. By the use of the methods of Birge, one can obtain, with comparative rapidity, the Σv^2 for a least squares polynomial of any degree, and in the process obtains, simultaneously, the value of Σv^2 for the least squares polynomials of all lower degrees. This convenience is a result of the fact that the Birge least squares polynomial solution is a sum of orthogonal polynomials. One, therefore, merely increases the degree until the value of Σv^2 has dropped to a roughly constant value. The size of this final value is a measure of the goodness of fit and the above mentioned probable deviation is calculated directly from it.

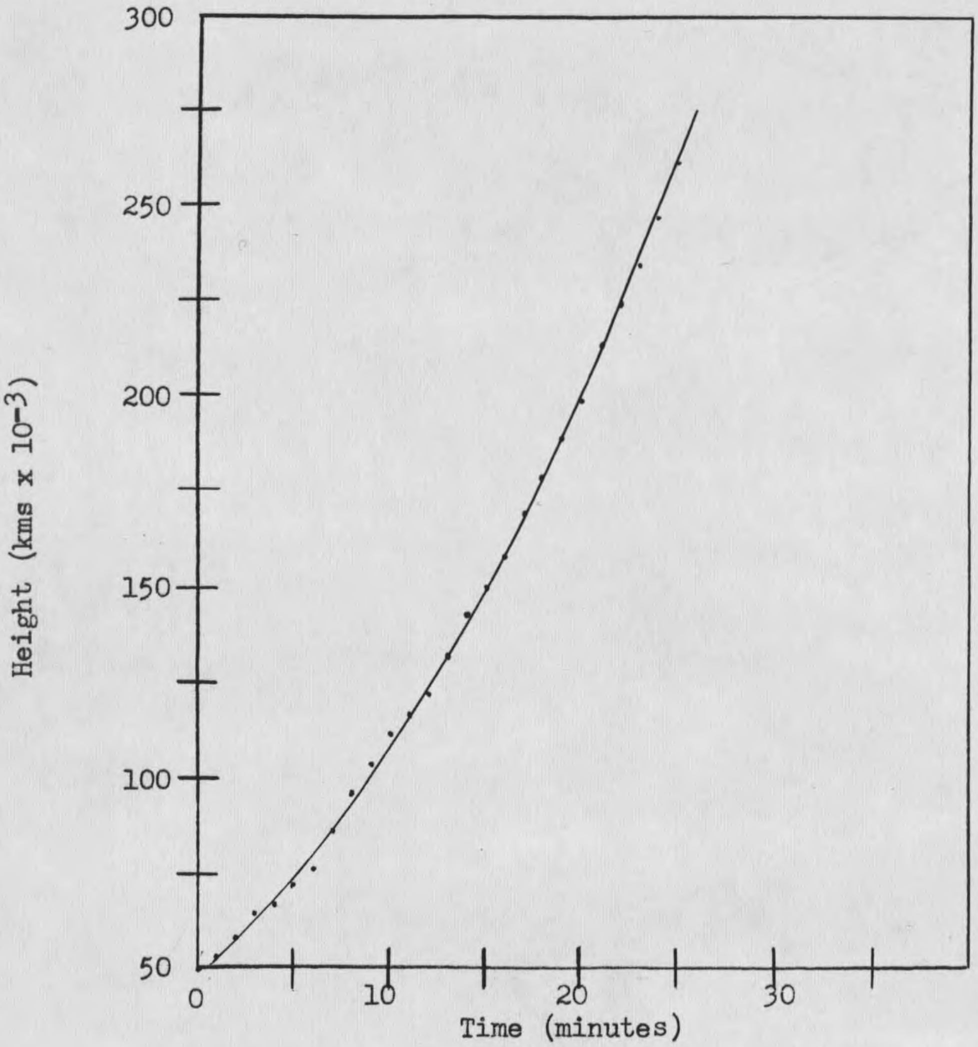

Figure 8. The smooth curve is a portion of the parabola $y = 5.00 \times 10^4 + 3.89 \times 10^3 t + 182t^2$ (kms and mins) which was obtained by applying the least squares curve fitting method of Birge. It is superimposed on the plot of the vertical height vs. the time for x = 0, i.e. the curve of figure 7.

paper and this error is coupled with the impossibility of reading a coordinate on the graph paper closer than approximately ± .2 mm. Thus, we see that the intrinsic uncertainty involved in determining the position of a feature in the prominence at successive intervals of time is of such magnitude as to preclude a more precise formulation of the equation of the curve.

Returning to the curves of figure 6, we note that, though the distance between them is erratic and non-uniformly increasing, they still exhibit a marked parallelism. Now, it is precisely this deviation from a uniform increase that gives rise to the humps in the curves plotted in figures 8 to 13. Concerning this situation, one might argue in the following fashion.

At any one value of x in the region, the y component of the force field might very well fluctuate in such a manner that the curve $y = y(t)$ for that value of x would be humped or in general be quite different from a parabola. Is it reasonable, then, to assume that the force field should fluctuate in precisely the same manner throughout the region of x concerned so as to produce precisely similar humps in all the $y = y(t)$ curves; that is, so that all the envelopes of the prominence in that region are markedly parallel . . . ? If so, one would be forced to admit that the force field throughout the region in question must not depend on phenomena occurring in that region; but, on the contrary, must be referred to some single source. By way of hypothetical analogy, consider the field due to a variable point charge. Were we to plot the motion of several particles moving in this field, starting simultaneously at approximately equal distances from the particle and journeying along different radii, we would find the curves to each have the same humps and dips. Now, the hypothesis of force fields generated through the interaction of the particles of the prominence, due to their motion, that is, of force fields depending on phenomena occurring within the prominence itself, is receiving considerable attention today. In fact, some writers, notably H. Alfvén (1), have gone so far as to explain certain phases of prominence motion on this hypothesis. If, then, we must escape the conclusion that the force field has a single source, how are we to explain the parallelism displayed by the prominence envelopes? The solution becomes apparent at once when we consider the tracing process itself.

In each photograph comprising the film there is a variation in the thickness of the coronal ring appearing around the occulting disk of the coronagraph. This variation results from the positioning of the occulting disk in the photography itself. The ring is the reference point by which the previously mentioned projector is centered with respect to the coordinate axis prior to the tracing of each subsequent picture. Since the variation in the thickness of the coronal ring produces a corresponding varying brightness of the solar limb and since the limb is not clearly sharp, but somewhat diffuse, an unavoidable, undetermined error is present with each centering of the projector. We ask: is this error, coupled with the error involved in tracing the similarly diffuse envelope of the prominence, of the same order of magnitude as the variation in the separations of the envelopes of figure 6? It is, of course, impossible to ascertain precisely the magnitude of the errors involved; but after due examination, it was decided to ascribe a value of ± 1 mm. to the uncertainty of reading a y coordinate on the graph.[6] This estimate includes the error associated with tracing and with subsequent measurement from the graph (due to the thickness of the pencil lines) and is felt to be conservative. We note carefully that this evidences only the shortcoming of the graphing procedure, and does not permit us to conclude anything about the force field. We have found, then, that from two considerations, one statistical (the Birge polynomials) and one physical (the last paragraph), that we can at best give a constant as a value for the force field by using this method.

6. 1 mm. on the graph corresponds to 1.39×10^3 kms. on the sun.

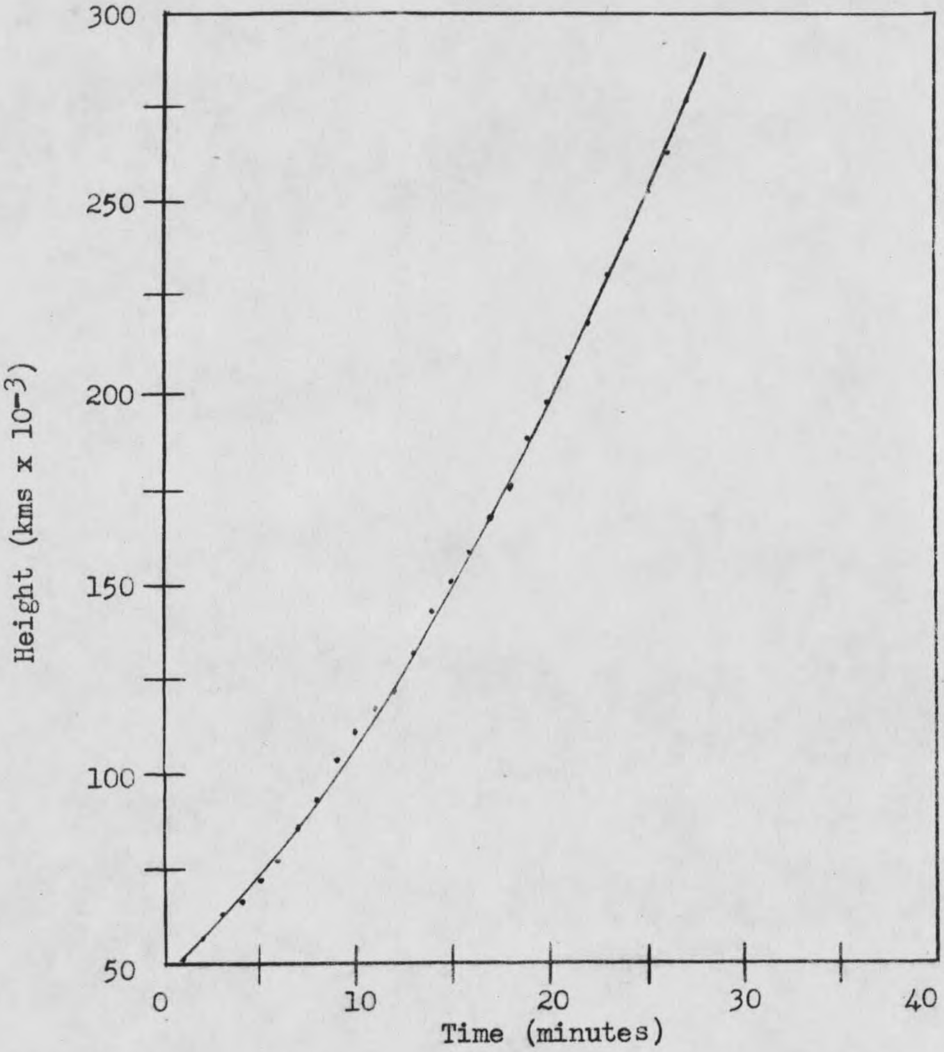

Figure 9. Illustrating the fit of the curve $y = 4.80 \times 10^4 + 3.62 \times 10^3 t + 129t^2$ to the observed points plotted from a measure of the height (y) vs. the time along the ordinate x = -20 for the lower envelope of the prominence.