The formulation of a technique for finding an optimal skidding road layout
by Michael Richard Carter

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE in Industrial and Management Engineering
Montana State University
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Abstract:
The objective of this thesis is the formulation of a technique for the determination of the minimum cost
layout of logging work (skid) roads. A study of the factors affecting layout costs showed that the most
important factor in layout is spacing. The spacing factor was also found to be most important in the
layout for landings.

A total cost equation is set up which contains all the factors found to affect cost. The relationships
among these factors are derived to permit their combination. The form of the total cost equation
indicates that, by using the methods of calculus, an optimal spacing can be determined.

The calculus approach to optimization proved a successful technique for determining the optimal
layout and therefore the minimum cost of log removal. The results, however, are dependent on the
accuracy of the input data and the exactness of simplifying assumptions made in the derivations. Any
application of this technique would require knowledge of these factors.

The conclusion is that this technique of determining optimal layout is a feasible approach to the
problem of road and landing spacing. In addition, it is felt that it can be used to give a deeper
understanding of the various factors affecting log removal costs.
THE FORMULATION OF A TECHNIQUE FOR FINDING AN OPTIMAL "SKIDDING" ROAD LAYOUT

by

MICHAEL RICHARD CARTER

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Industrial and Management Engineering

Approved:

[Signatures]

MONTANA STATE UNIVERSITY
Bozeman, Montana

August, 1968
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To Dr. Vernon E. McBryde, chairman of my graduate committee, and to the other members, Dr. R. J. McConnen and Dr. Donald W. Boyd, I offer my thanks for the many helpful ideas and attitudes presented in their instruction this past year.

A special thanks also goes to Mr. David B. Brown, a former Graduate Research Assistant, for his help in familiarizing me with the research program, "Engineering Analysis of Logging Operations", being carried on by the Forest Service Experiment Station, Bozeman, Montana.

Finally, to my wife goes my sincerest thanks for her encouragement throughout my graduate work.
TABLE OF CONTENTS

CHAPTER I INTRODUCTION 1
- Historical Background of the Problem 1
- Statement of the Problem 3
- Summary of Past Work 5

CHAPTER II DEVELOPMENT OF A TOTAL COST EQUATION 7
- Introduction 7
- Identification of Costs and Cost Factors 7
- Development of a Total Cost Equation 12
- Summary 24

CHAPTER III OPTIMIZATION OF TOTAL COST EQUATION 30
- Method 30
- Solution 31
- Summary 37

CHAPTER IV RESEARCH RESULTS 39
- Introduction 39
- Sample Problem 40
- Summary 51

APPENDIX I DEVELOPMENT OF AN EXPRESSION FOR PRODUCTION RATE OF CONSTRUCTION EQUIPMENT 55
- Introduction 55
- Development 58
- Cost Equation for Landing Construction 68
- Cost Expression for Switchback Construction 70
- Summary 74

APPENDIX II COMPUTER FLOW DIAGRAM 75
- Introduction 75
- Flow Diagram 75

APPENDIX III DEVELOPMENT OF AVERAGE SKIDDING DISTANCES 79

LITERATURE CITED 86
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table I</td>
<td>Relationship of System Variables to Costs</td>
<td>13</td>
</tr>
<tr>
<td>Table II</td>
<td>Summary of Optimum Values for Each Combination</td>
<td>43</td>
</tr>
<tr>
<td>Table III</td>
<td>Common Excavation as a Function of Slope and Road Width</td>
<td>56</td>
</tr>
<tr>
<td>Table IV</td>
<td>Rock Excavation as a Function of Slope and Road Width</td>
<td>56</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Figure 1</td>
<td>Diagram of the Iterative Solution</td>
<td>36</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Removal Cost as a Function of Percent of Optimal Road Spacing</td>
<td>44</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Removal Cost as a Function of the Percent of Optimal Road and Landing Spacings</td>
<td>45</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Removal Cost as a Function of Slope, Percent Rock, and Volume of Timber per Acre, No Landings</td>
<td>46</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Optimum Horizontal Road Spacing as a Function of Slope, Percent Rock, and Volume of Timber per Acre, No Landings</td>
<td>47</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Removal Cost as a Function of Slope, Percent Rock, and Volume of Timber per Acre, with Landings</td>
<td>48</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Optimum Horizontal Road Spacing as a Function of Slope, Percent Rock, and Volume of Timber per Acre, with Landings</td>
<td>49</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Optimum Landing Spacing as a Function of Slope, Percent Rock, and Volume of Timber per Acre</td>
<td>50</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Values from Table II and Table III</td>
<td>57</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Slope versus Excavation Ratio of 14 ft Road Width to 20 ft Road Width</td>
<td>65</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Quantity of Common Excavation for a 14 ft Road Width as a Function of Sidehill Slope</td>
<td>67</td>
</tr>
</tbody>
</table>
ABSTRACT

The objective of this thesis is the formulation of a technique for the determination of the minimum cost layout of logging work (skid) roads. A study of the factors affecting layout costs showed that the most important factor in layout is spacing. The spacing factor was also found to be most important in the layout for landings.

A total cost equation is set up which contains all the factors found to affect cost. The relationships among these factors are derived to permit their combination. The form of the total cost equation indicates that, by using the methods of calculus, an optimal spacing can be determined.

The calculus approach to optimization proved a successful technique for determining the optimal layout and therefore the minimum cost of log removal. The results, however, are dependent on the accuracy of the input data and the exactness of simplifying assumptions made in the derivations. Any application of this technique would require knowledge of these factors.

The conclusion is that this technique of determining optimal layout is a feasible approach to the problem of road and landing spacing. In addition, it is felt that it can be used to give a deeper understanding of the various factors affecting log removal costs.
HISTORICAL BACKGROUND OF THE PROBLEM

The forest industry has always been an important segment of our national economy. A study by the U. S. Forest Service shows that nearly five percent of our national income is obtained from wood related industries. The continuous use of this limited resource requires that our forest reserves be used in a prudent and economical manner.

The managers of our forests have maintained a high and steady level of production by using scientific forestry methods. This type of management dictates the amount of wood which can be removed annually. The logging operators, who actually remove the wood, are therefore limited in the amount they can remove. Their problem is to move this amount of wood from the forest to the mill at minimum cost.

The process of moving wood from the forest to the mill can be broken down into several steps. The first is the conversion of standing trees into logs. This basically involves falling the tree and cutting it into measured log lengths. The second step is the moving of the logs from the forest to the road where they can be loaded for movement to the mill. This step has two major parts: the construction of a system of work (skid) roads and landings to receive the
logs and the actual skidding of the logs to the road system. The final step in getting the wood to the mill is the loading of the logs on the truck or other means of transportation and then hauling them to the mill where they are unloaded.

For most practical purposes, the degree of independence among the above steps makes it possible to minimize the total cost of wood removal by minimizing the cost of each step. Work has been done on each of the steps with the hope of finding the most economical way to perform that function. There has not, however, been much innovation in either the methods or machinery used in performing the steps. Probably the greatest amount of study has been concentrated on skidding logs to the road. This concentration of effort is due primarily to the fact that skidding costs vary widely because of variations in terrain and stand characteristics. The cost of falling depends primarily on the number of trees, the cost of loading depends basically on the number of logs, and the cost of hauling depends mainly on the distances hauled. Due to these stable factors, these costs are not as variable as the cost of skidding.

Research on skidding costs has been done by different groups, but this author found that the most extensive work has been done by the U. S. Forest Service, Intermountain Forest and Range Experiment Station at Bozeman, Montana, in
cooperation with the Industrial and Management Engineering Department of Montana State University. This work, consisting of several master's theses and supplementary research work, has progressed to the stage where D. B. Brown has developed reliable techniques for determining the effect of terrain and stand variables on the cost of skidding. At this point in the research there is a need for a method of determining how these terrain and stand variables affect the cost of the road system used in skidding. With knowledge of how skidding costs and road costs are affected by the terrain and stand variables, it will be possible to determine which skidding method and corresponding road network would provide removal of the logs at a minimum cost. This component of cost, for one of the three steps involved in moving the wood from where it stands in the forest to the mill, is the primary concern of this paper.

The research presented here was undertaken for the purpose of formulating a model which would relate terrain and stand variables to the total removal cost. By applying optimization techniques to this model, the road layout required for minimum removal cost could then be found.

STATEMENT OF THE PROBLEM

To achieve the minimum cost of moving logs from stump
to road, one must determine the best combination of skidding method and road layout which results in the lowest cost. The total cost involved can be broken into the cost of skidding and the cost of the road system. The cost of skidding can generally be reduced by shortening the distance the logs must be moved. This would imply that the roads should be very close together in order for a minimum cost to be achieved. The road cost, however, increases as the length of road increases. This implies that the roads should be built farther apart, which would reduce the length required and therefore the cost. It is easy to see that these are two opposing costs and cannot therefore be minimized at the same time.

Since the two costs oppose each other, the nature of their relationship to some common variable must be found if the total cost is to be minimized. Once these relationships are determined, a mathematical optimization technique can possibly be used to find the combination of the two costs which produces a minimum total cost. The problem is then to determine the total cost equation as a function of skidding and road costs and to optimize it with respect to the common variable. This would then allow one to determine the best combination of skidding method and road layout for a given logging area.
SUMMARY OF PAST WORK

Past research on this subject has been scattered and mostly unrelated to the actual problem set forth here. A group of Japanese researchers in the fields of logging, forest engineering, and forest management have published several studies on forest road systems. A number of these studies describe work performed by a group headed by Dr. Seihei Kato. This group was concerned with the density of roads within an entire forest area. The roads they studied were permanent access or primary roads, which makes them different from skidding roads, since the latter are built in a logging area for temporary use in the removal of logs from the forest. In addition, Kato's macro approach dictated that he consider only certain classes of forests and the general type of equipment best suited for each class. This approach does not lend itself to the problem of determining the combination of skidding method and road layout that yields the minimum cost for a single logging area.

A Canadian forest management researcher, L. J. Lussier, has developed several simplified models for dealing with the problem of finding the road and landing spacing which give the minimum log removal costs. His method of optimization is similar to the method used by the Japanese in their macro outlook. As it turns out, the same method is
used by this author in this work, which could be termed a micro look at log removal costs. The biggest drawback with Lussier's work is that it does not consider enough of the variables affecting cost. His work is more of a general example showing how to minimize a total cost equation when one already knows the variables and their relationships.

The work of D. B. Brown was not directly related to the study of optimal road layout. It concerned the determination of the effect of various variables on the cost of skidding. This, however, has set the stage for the development of a total cost equation which can be optimized to obtain the minimum total removal cost and the optimal road layout.
CHAPTER II

DEVELOPMENT OF A TOTAL COST EQUATION

INTRODUCTION

The development of a total cost equation requires the identification of all the costs involved and the variables that affect these costs. After the various costs and variables are determined, they must be combined in such a way that the resulting equation represents the total cost of the phenomenon under study. Once the equation is established, it can then be optimized with respect to the variable(s) of interest. In this study the desired result is to obtain the minimum total cost of log removal as a function of controllable system variables.

IDENTIFICATION OF COSTS AND COST FACTORS

Each of the phases of log removal consists of several costs. To build the system of roads and landings one must
1. Move in construction equipment
2. Plan and lay out the road and landing system
3. Construct roads
4. Construct switchbacks
5. Construct landings.

Each of these activities incurs a cost, and while they may not all be required on a particular logging site, in general all five of these functions must be accomplished. The costs of the last three activities are similar since roads, switch-
backs, and landings are normally constructed with the same machinery. But, as will be shown later, it is better to treat these three costs separately, in order to more accurately describe the system.

For the process of skidding the logs to the road or landing, the following functions are required:

1. Move in skidding equipment
2. Set up equipment for each section
3. Skid logs.

The set-up operation is different from the move-in operation, in that it may have to be repeated periodically while skidding the total area. The set-up operation is required for high-lead and jammer skidding, two of the most popular methods of line skidding.

All costs associated with the various activities in road building and skidding are functions of several variables. Each cost and the associated variables are listed below. Values shown in parentheses are used throughout this paper to refer to the corresponding costs.

I. Cost to move in construction

   equipment \((C_1)\) = function of

   1. distance moved

   2. type of equipment moved

   3. moving method used.
II. Cost to plan and layout roads ($C_2$) = function of
1. planning and layout method
2. cost of men and materials
3. productivity of the men
4. length of road required
   = function of
   a. road spacing
   b. total area.

III. Cost of road construction ($C_3$) = function of
1. cost of equipment (owning and operating, including men)
2. production rate of equipment
   = function of
   a. slope of sidehill
   b. percent rock in excavation
3. amount of road required
   = function of
   a. road spacing
   b. road width

IV. Cost of switchback construction ($C_4$) = function of
1. cost of equipment (owning and operating, including men)
2. production rate of equipment
   = function of
10

a. slope of sidehill
b. percent rock in excavation

3. swichback size = function of
   a. radius of swichback
   b. road width
   c. backslope of cut

4. number of swichbacks
   = function of
   a. road spacing
   b. distance between swichbacks

V. Cost of landing = function of

   construction (C_5) = function of
   1. cost of equipment (owning and operating, including men)
   2. production rate of equipment
      = function of
      a. slope of sidehill
      b. percent rock in excavation
   3. landing size = function of
      a. timber volume / unit area
      b. landing spacing
      c. road spacing
   4. number of landings
      = function of
      a. landing size
1. Cost to move in skidding equipment ($C_6$) = function of
   1. distance moved
   2. type of equipment moved
   3. moving method used.

VII. Cost to set up skidding equipment ($C_7$) = function of
    1. skidding method
    2. set-up time
    3. number of set-ups
       = function of
       a. road spacing
       b. distance between set-ups.

VIII. Cost of skidding ($C_8$) = function of

   1. cost of skidding equipment (owning and operating)
   2. volume of timber / unit area
   3. productivity of equipment
      = function of
      a. slope of sidehill
      b. number of logs per cycle
      c. size of the logs.
The variables listed above are summarized in Table I on page 13. This table shows the functional relationship between each variable and the various costs. It assigns to each variable a symbol which will be used to identify it throughout the remainder of this paper. Also, the units used for each variable are listed.

DEVELOPMENT OF A TOTAL COST EQUATION

The total cost of moving the logs from the forest to the road is equal to the summation of the component costs (I to VIII) listed in the last section. To make the costs comparable and the summation possible, all costs are converted to the common units of dollars per one thousand board feet of wood removed. The symbol, $/MBF, will be used to denote these units.

The following discussion sets forth the logic used in setting up the equations for component costs. All equations are represented by symbols beginning with capital letters. Other factors are represented by lower-case letters and are defined as they are introduced.

In the development of the following cost equations, the measure of timber volume per unit area is given the units of thousand board feet per horizontal acre of land or MBF/acre where acre is defined to be a measure of horizontal
### Table I: Relationship of System Variables to Costs

<table>
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<tr>
<th>Symbol</th>
<th>Variable Description</th>
<th>Units</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
<th>$g_4$</th>
<th>$g_5$</th>
<th>$g_6$</th>
<th>$g_7$</th>
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<td>A</td>
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<tr>
<td>B</td>
<td>backslope of road cut</td>
<td>decimal %</td>
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<td>x</td>
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<tr>
<td>Dc</td>
<td>construction method cost</td>
<td>$/hr</td>
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<td>F</td>
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<td>x</td>
<td></td>
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<td>ft/hr</td>
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<td>yd/landing</td>
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<td>x</td>
<td></td>
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<td>Iv</td>
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</tbody>
</table>
area. This method of density measurement is used because in practice the total volume of timber for an area is divided by the horizontal area covered by the timber. Horizontal area is used because it is easily obtained from maps. Actual area of the rolling terrain would be very difficult to measure. This density is higher than would be the density measured with respect to the actual surface area covered by the trees.

The timber density over horizontal area is used to establish the unit of \$/MBF for the component costs. The areas used in the development of these cost equations must therefore be horizontal. Horizontal areas are therefore used to develop the cost equations. In this paper all measurements of road spacing and skidding distance are horizontal distances.

The first cost involved is for the move in of construction equipment. This cost, in terms of \$/MBF, would be equal to the total move-in cost divided by the product of the total logging area in acres and the volume of timber per acre. It should be noted that the total move-in cost is independent of variables at the logging site and is therefore a constant for a given site. The expression for cost to more in construction equipment is therefore

\[ C_1 = \frac{N_c}{AV} \quad (\$/MBF) \]

where
\[ Ne = \text{total cost to move in construction equipment (\$)} \]
\[ A = \text{total area to be logged (horizontal acres)} \]
\[ V = \text{volume of timber per horizontal acre (MBF/acre)} \]

The cost to plan and lay out a road system depends on the cost and productivity of the method used as well as the length of road. The length of road required to remove one MBF is developed as follows. Given an area \( y \) (ft) by \( X \) (ft) where \( y \) is the length of road along one side of the area, as shown in the sketch,

\[
\begin{align*}
\text{Road} & \quad \text{Road} \\
\text{Road} & \quad \text{Road} \\
y \ (\text{ft}) & \quad X \ (\text{ft}) \\
\end{align*}
\]

the amount of road per unit area would be

\[
\frac{y \ (\text{ft of road})}{yX \ (\text{sq ft})} = \frac{1}{X} \ (\text{ft of road}) \ (\text{sq ft})
\]

Converted to length of road per acre this would be

\[
\frac{1}{X} \ (\text{ft of road}) \times \frac{43560 \ (\text{sq ft})}{1 \ (\text{acre})} = \frac{43560 \ (\text{ft of road})}{X \ (\text{acre})}
\]

Now the volume of timber removed per foot of road is obtained

\[
\frac{43560 \ (\text{ft of road})}{X \ (\text{acre})} \times \frac{1 \ (\text{acre})}{V \ (\text{MBF})} = \frac{43560 \ (\text{ft of road})}{X \ V \ (\text{MBF})}
\]
The expression for the cost to plan and lay out the roads would then be

\[ C_2 = \frac{D_m}{P_m F_m} \left( \frac{\$}{hr} \right) \times \frac{43560 \text{ (ft of road)}}{X V \text{ (MBF)}} = \frac{43560 D_m}{P_m F_m V X \text{ (MBF)}} \]  

(2)

where  

- \( D_m \) = cost per hour of layout method (\$/hr)  
- \( P_m \) = productivity coefficient for layout (no units)  
- \( F_m \) = productivity of layout method (ft/hr).

The cost of building the road, like laying it out, depends on the cost and productivity of the method used and the amount of road required. The value, \( \frac{43560 \text{ (ft of road)}}{V X \text{ (MBF)}} \), was obtained earlier for the latter. The cost and productivity of a particular construction method present somewhat of a problem, since the productivity of road building equipment depends on a number of factors. The most important factors are the slope of the sidehill, the percentage of rock in the excavation, and the width of the road. These are the factors that have been found by a past study to cause significant variation in the productivity of road building equipment. However, no one has developed relationships among these factors. Appendix I presents the development of a relationship among the above factors. This development uses graphical methods. Other methods, such as regression analysis, could be used in developing...
these relationships, but sufficient data for this approach was not available to the author and its collection was not a part of the plan for this work.

At this point we denote equipment productivity by the symbol, \( F \), with units of (cu yd/hr) where \( F \) is a function of the slope and percent rock. Also, the variable, \( H_r \) (cu yd/ft of road), is introduced and will be a function of the road width and slope. The resulting expression for road cost is then

\[
C_3 = \frac{D_e}{P_c F} \left( \frac{\text{\$}}{\text{cu yd}} \right) \times H_r \left( \frac{\text{cu yd}}{\text{1 ft of road}} \right) \times \frac{43560}{X V} \left( \text{ft of road} \right) \times \frac{1}{\text{MBF}} =
\]

\[
= \frac{D_e H_r}{P_c F X V} \left( \frac{\text{\$}}{\text{MBF}} \right)
\]

where

- \( D_e \) = hourly cost of construction method in dollars
- \( P_c \) = productivity coefficient of construction equipment in dimensionless units
- \( F \) = productivity of construction equipment in cu yd per hr.

The cost of constructing switchbacks is found in much the same manner as for roads. The productivity of the construction equipment will again be denoted by \( F \). \( H_s \) (cu yd/switchback), which denotes the excavation required for each switchback, will be written as a function of the
sidehill slope, the road width, the backslope of the road, and the radius of curvature of the switchback. In road construction we developed an expression for the feet of road required per MBF of timber; similarly, in switchback construction we want an expression for the number of switchbacks required per MBF. Such an expression is now developed in terms of the system variables listed in Table I.

Let:  
\[ h = \text{the difference in elevation between road levels in feet} \]
\[ d = \text{horizontal distance in feet separating switchback and level section of road} \]
\[ g = \text{horizontal distance in feet as shown on the sketch} \]
\[ S = \text{slope of hillside in decimal \%} \]
\[ Z_b = \text{length of level road in feet as shown on the accompanying diagrams} \]
Assuming the road climbs from one level to the other at a constant rate, \( Mx \), then

\[
h = 2 \, Mx \, d \, (ft) \quad \text{and} \quad h = X \, S \, (ft)
\]

therefore

\[
2 \, Mx \, d = X \, S
\]

\[
d = X \, S / 2 \, Mx
\]

and also

\[
g^2 = d^2 - (\frac{1}{2}X)^2 = (X^2S^2/4Mx^2) - (X^2/4) =
\]

\[
= (X^2/4)((S^2/Mx^2) - 1)
\]

or

\[
g = (X/2)((S^2/Mx^2) - 1)^{\frac{1}{2}}
\]

The area per switchback, assuming a constant value for \( Zb \), is as shown by the dashed lines in the following diagram and may be expressed as

\[
\text{Area (horizontal)} = X(Zb+2g) = X(Zb+2(X/2)((S^2/Mx^2) - 1)^{\frac{1}{2}}) =
\]

\[
= X(Zb+X((S^2/Mx^2) - 1)^{\frac{1}{2}}) \quad (\text{sq ft})
\]
The number of switchbacks required per MBF is obtained as follows:

\[
\frac{1}{X(Zb+X((S^2/Mx^2)-1))^{\frac{3}{2}}} \times \frac{43560}{(\text{sq ft})} \times \frac{1}{(\text{acre})} V (\text{MBF})
\]

\[
= \frac{43560}{VX(Zb+X((S^2/Mx^2)-1))^{\frac{3}{2}}} \quad \text{(switchbacks)} \quad \text{(MBF)}
\]

Now the cost equation for switchback construction can be set up as

\[
C_4 = \frac{43560}{VX(Zb+X((S^2/Mx^2)-1))^{\frac{3}{2}}} \quad \text{(switchbacks)} \times \frac{Hs}{\text{(yd)}} \times \frac{1}{\text{(switchback)}}
\]

\[
\times \frac{Dc \ (\$)}{Pc \ (hr)} \times \frac{1 \ (hr)}{F \ (yd)} = \frac{43560 \ Hs \ Dc}{PcFVX(Zb+X((S^2/Mx^2)-1))^{\frac{3}{2}}} \quad (\$) \quad (4)
\]

The last cost component for construction of the road network is the cost to build landings. The rate of
production of the construction equipment will once again be represented by $F$. $H_l$ (cu yd / landing) will be used to represent the excavation required per landing as a function of slope, road width, and landing size. The number of landings per MBF is obtained as shown below.

\[
\text{Horizontal Projection of Area per Landing}
\]

\[
\begin{array}{c}
\text{Road} \\
\hline
a & b \\
\hline
d & X \\
\hline
\text{Landing} & \text{Landing} \\
\hline
k & Y \\
\end{array}
\]

where $Y =$ level distance in feet between landings.

The area $(a,b,c,d)$ is served by one landing if we assume that skidding will be downhill to the nearest landing. The size of this area is $XY$ (sq ft). Therefore the landings per MBF are

\[
\frac{1}{XY} \text{ (landing)} \times \frac{43560}{1} \text{ (sq ft)} \times \frac{1}{V} \text{ (acre)} = \frac{43560}{XYV} \text{ (landings) (MBF)}
\]

and the cost of landings is

\[
C_5 = \frac{43560}{XYV} \text{ (landings) (MBF)} \times \frac{Dc}{Pc} \text{ ($) (hr)} \times \frac{H_l}{F} \text{ (yd)} = \frac{1}{1} \text{ (hr)}
\]
The cost to move in the skidding equipment is found in the same manner as the move-in cost for construction equipment was found. Letting $N_s$ ($) represent the total cost to move in the equipment, we have:

$$C_6 = \frac{N_s (\$)}{A \text{ (acre)}} \frac{1 \text{ (acre)}}{V \text{ (MBF)}} = \frac{N_s (\$)}{AV \text{ (MBF)}}.$$  

(6)

The cost per MBF to set up the skidding equipment is found in the following manner.

Assuming that the logs are skidded to the nearest set-up area, the area covered by one set-up is equal to $X Z_s$ (sqft). The set-ups per MBF would be
and the cost of set-ups is

\[
C_I = \frac{43560 \text{(set-ups)}}{ZsXV \text{(MBF)}} \times \frac{D_s \text{ (\$)}}{1 \text{(hr)}} \times \frac{Sp \text{ (hr)}}{1 \text{(set-up)}}
\]

\[
= \frac{43560 D_s Sp \text{ (\$)}}{Zs X V \text{(MBF)}} \tag{7}
\]

where

\( D_s = \) hourly cost of skidding equipment in dollars

\( Sp = \) time in hours to set up the skidding equipment

\( Zs = \) distance in feet between set-up locations.

The final cost component is the one for skidding the logs to the road or landing. The regression equation developed by D. B. Brown is used to relate the productivity of the skidding equipment to the system variables. This equation is of the form

\[
\text{time per cycle in hr} = a_1 + a_2 \text{(slope in decimal \%)} + a_3 \text{(logs per cycle)} + a_4 \text{(one-way skidding distance in ft)} \tag{8}
\]

where \( a_1, a_2, a_3, \) and \( a_4 \) are regression coefficients.
Using the symbols in Table I, it is

\[
time \text{ per cycle in hours} = a_1 + a_2 S + a_3 VcLv + a_4 \left( d_{\text{average}} \right) \quad (9)
\]

where

- \( Vc \) = volume in MBF per skidding cycle
- \( Lv \) = number of logs per MBF of timber
- \( d_{\text{average}} \) = average skidding distance in feet, see Appendix III for the development of expressions for this value.

The skidding cost equation is

\[
C_8 = \frac{D_5 \left( \$/hr \right) \times \left( a_1 + a_2 S + a_3 VcLv + a_4 \left( d_{\text{average}} \right) \right)}{Ps \left( \$/hr \right)} \times \frac{1 \text{ (cycle)}}{1 \text{ (cycle) } Vc \text{ (MBF)}} \times \frac{1 \text{ (cycle)}}{1 \text{ (cycle) } Vc \text{ (MBF)}} \quad (10)
\]

where \( Ps \) = productivity coefficient for the skidding equipment in unitless dimensions.

**SUMMARY**

Expressions for the eight component costs have now been developed, as well as expressions for the average skidding
distance in Appendix III and expressions for construction costs in Appendix I. The expressions for component costs, as developed throughout this chapter, are listed for easy reference. All costs are in terms of \$/MBF.

Move-in, construction: \( C_1 = \frac{N_c}{AV} \)

Road planning & layout: \( C_2 = 43560 \frac{D_m}{P_cF_{mXV}} \)

Road construction: \( C_3 = 43560 \frac{DcH_r}{P_cF_{VX}} \)

Switchback construction: \( C_4 = \frac{43560 \frac{Dc}{H_s}}{P_cF_{VX}(Z_bX((S^2/N_x^2)-1)^{0.5})} \)

Landing construction: \( C_5 = 43560 \frac{DcH_l}{P_cV_{XY}} \)

Move-in, skidding: \( C_6 = \frac{N_s}{AV} \)

Set-up, skidding: \( C_7 = 43560 \frac{DcS_p}{Z_vX} \)

Skidding: \( C_8 = (Dc/P_{Vc})(a_1+a_2S+a_3V_cL_v+a_4(d_{average})) \)

From Appendix I are obtained the following expressions. They were developed from data found in a past study.

The cost per linear foot of road construction is

\[
\frac{DcH_r}{F} = \frac{(0.079+0.617R_o)(1-R_o/3)P_o}{(14/N)^{1.7}} - 0.17S \quad \text{(\$/ft of road)}
\]
and $W = $ road width in feet

$Po = $ excavation in cu yd for each foot of road where the road is 14 ft wide and the excavation has zero percent rock.

In terms of the system variables

$Po = 7.4S$ where $10 \leq S \leq 0.54$

or $Po = 4 + 13.8(S-0.54)$ where $0.54 < S$.

The cost of constructing one switchback is

$$\frac{DcHs}{F} = (0.079+0.617Ra)\left(\left(\frac{Ra^2-(SW)^2}{2B-2S}\right)^{\frac{1}{3}}\frac{(2Po(1-Ro/3))}{(14/W)-0.17S}\right) +$$

$$+ \frac{1}{3}\left(\frac{Ra -(\frac{Ra^2-(SW)^2}{2B-2S})}{2B-2S}\right)^{\frac{1}{3}}\left(\frac{2Po(1-Ro/3)}{(14/W)-0.17S}\right) + \frac{Po(1-Ro/3)}{6(W^2-2Ra)^{\frac{1}{2}}-0.17S}\right) +$$

$$+ \frac{W}{3}\left(\frac{Po(1-Ro/3)}{(7/(W^2+2RaW)^{\frac{1}{2}}-0.17S)}\right)$$

(switchback) (12)

where $B =$ backslope of the cut in decimal %

$Ra =$ interior radius of the curve in feet

and $Po$ is as defined above.

The cost of excavation per landing is
\[
\frac{DcHL}{F} = Le(0.079+0.617Ro)\left(\frac{Po(1-Ro/3)}{(14/(W+WL))}-0.17S\right)
\]

\[
= \frac{Po(1-Ro/3)}{(14/W)-0.17S}\frac{(#)\text{ (landing)}}{\text{($\$/$\text{landing}$)}}
\]

where \(Le\) = length of landing in feet
\(Wl\) = landings width in feet
and \(Po\) is as previously defined.

From Appendix III the following expressions for the average skidding distance are obtained.

The average horizontal skidding distance for the case without landings is

\[
d_{\text{average}} = Zb(\frac{1}{2}X)+(2X^2/3)((S^2/Mx^2)-1)^{\frac{1}{3}}
\]

\[
\frac{Zb + X((S^2/Mx^2)-1)^{\frac{1}{3}}}{Zb + X((S^2/Mx^2)-1)^{\frac{1}{3}}} \text{ (ft)}.
\]

The average horizontal skidding distance for the case with landings is

\[
d_{\text{average}} = (X/3) + (X^2 + Y^2)^{\frac{1}{2}}/6 \text{ (ft)}.
\]

Combining the eight component costs, one obtains the following total cost equation,
\[
C_t = \frac{Ne}{AV} + \frac{43560 \text{ Dm}}{\text{PmFmVX}} + \frac{43560 \text{ DcHr}}{\text{PcFVX}} + \frac{43560 \text{ Dc Hs}}{\text{PcFVX(Zb+X((S^2/Mx^2)-1)^{0.3})}} \\
+ \frac{43560 \text{ DcHl}}{\text{PcFVXY}} + \frac{43560 \text{ Ds Sp}}{\text{Zs VX}} + \frac{Ns}{AV} \\
+ \frac{Ds(a_1 + a_2 S + a_3 VcLv + a_4 (d_{average}))}{Ps Vc} \quad (16)
\]

The appropriate expressions for \(Po\) and \(d_{average}\) must be placed in this equation before it can be solved. The expressions for \(DcHr/F\), \(DcHs/F\), and \(DcHl/F\) can also be substituted into the total cost equation. However, they are not because in the next chapter equation (16) is simplified and the above three expressions would just have to be placed back into the total cost equation again.

Equation (16) represents the average cost in dollars to remove one thousand board feet of timber from an area. This cost reflects the skidding and construction methods used, as well as the terrain and timber stand characteristics. The objective of the logger is to pick the construction and skidding methods which give the minimum value of total cost, \(C_t\), for his logging site. As previously stated, the minimum value of \(C_t\) for a given combination of methods occurs at some unique value of road spacing or some unique
combination of values for road and landing spacing. The
first step in finding the optimum combination of methods is
to find the layout or road spacing which yields the minimum
cost for each of the possible combinations. When the
optimum spacings have been determined for each of these
combinations, the task then is to identify the combination
which gives the minimum total cost.
CHAPTER III
OPTIMIZATION OF TOTAL COST EQUATION

METHOD

The total cost equation contains many controllable variables but of these only road and landing spacing appear in both the numerator and denominator. Any variable with a positive value which does not have this property cannot cause the function to have a minimum point, except at a boundary condition. The above conclusion agrees with the idea stated earlier that as the spacing is increased, the skidding cost increases and the construction cost decreases. This implies that a spacing exists between the boundaries, zero and infinity, where the total cost is a minimum. This agrees with the way spacing values appear in the total cost equation.

The method of calculus as used to find maximum and minimum points of a continuous function is found to be appropriate for use here. The partial derivatives of the two cases are therefore taken with respect to road spacing, X, and landing spacing, Y. This allows the calculation of the X and Y values which give the minimum total cost for a given combination of methods and logging site.

The total cost equation has many values which are fixed for a given logging site. They therefore act as constants in the optimization process. They are combined into groups and
set equal to a new variable. This makes the partial derivatives simpler because they do not contain as many symbols. The new variables and their equivalents are

\[ K_1 = \frac{43560}{P_m F_m V} \]
\[ K_2 = \frac{43560}{P_c F V} \]
\[ K_3 = \frac{43560}{D_s S / Z_s V} \]
\[ K_4 = \frac{(D_s / P_s V_c)(a_1 + a_2 S + a_3 V_c L_v)}{} \]
\[ K_5 = \frac{D_s a_4}{P_s V_c} \]
\[ K_6 = \left(\frac{S^2}{M_x^2} - 1\right)^{1/2} \]

Substituting these values into the total cost equation, we have

\[ C_t = \frac{N_c + K_1 + K_2 H_r + K_2 H_s}{AV X} + \frac{K_2 H_1}{XY} + \frac{N_s + K_3}{AV X} \]
\[ + K_4 + K_5 (d_{\text{average}}) \]

**SOLUTION**

The partial derivatives of the above equation are taken for each of the cases, landings and no landings. This is first done for the case where there are no landings or where

\[ d_{\text{average}} = \frac{Z_b (X/2) + K_6 (2X^2/3)}{} \]
and $K_2H_1/XY = 0$, because there are no landings.

Therefore

$$\frac{dC_t}{dx} = -\frac{K_1}{x^2} - \frac{K_2Hr}{x^2} - \frac{K_2Hs(Zb+2XK_6)}{(ZbX+X^2K_6)^2} - \frac{K_3}{x^2} +$$

$$+ \frac{(K_6)((Zb^2/2)+(4/3)(XZbK_6)+(2/3)(x^2K_6^2))}{Zb^2+(2ZbXK_6)+(x^2K_6^2)} = 0 \quad (18)$$

$$\frac{d^2C_t}{dx^2} = +\frac{K_1}{x^3} + \frac{2K_2Hr}{x^3} + \frac{2K_2Hs(Zb^2+3XK_6Zb+3X^2K_6^2)}{(ZbX+X^2K_6)^3} +$$

$$+ \frac{2K_3}{x^2} + \frac{(Zb^2K_6/3)(Zb+XK_6)}{(Zb^2+2ZbXK_6+X^2K_6^2)^2} = 0 \quad (19)$$

Equation 19 indicates the cost is a minimum rather than a maximum value. Therefore solving equation 18 for the value of $X$ gives the value of road spacing required to obtain a minimum cost for the set of conditions used in this solution of the total cost equation. The $X$ value cannot be solved for directly so iteration is used. This involves picking a value of $X$, solving the equation, and then checking the sign of the answer. This procedure is repeated until the sign of the answer changes. The change of sign means the desired value of $X$ is between the last two values tried.
In this manner the interval containing the solution value is obtained and by the choice of X values is closed down to any degree accuracy. The solution value of X is therefore said to be approximated by the midpoint of this interval.

The situation where landings are used is solved in the same manner as above, only now there are two variables X and Y. The equation is

\[ C_t = \frac{Nc}{AV} + K_1 + \frac{K_2}{X} + \frac{K_3}{X(Zb+XK_6)} + \frac{K_4}{XY} + \frac{K_s}{AV} + \frac{K_5}{X} + \frac{K_6}{X(Zb+XK_6)} + \frac{K_7}{X^2} + \frac{K_8}{X^3} + \frac{K_9}{X^4} \]

So the partial derivatives are

\[ \frac{d C_t}{d X} = - \frac{K_1}{X^2} - \frac{K_2 H_r}{X^2} + \frac{K_3}{X(Zb+XK_6)} - \frac{K_4 H_s}{X^2 Y} - \frac{K_5}{X^2} \]

\[ + \left( \frac{K_6}{X(Zb+XK_6)} + \frac{K_7}{X^2} + \frac{K_8}{X^3} + \frac{K_9}{X^4} \right) = 0 \]  

\[ \frac{d C_t}{d Y} = - \frac{K_4}{XY^2} + \frac{K_5 Y}{6(X^2+Y^2)^{\frac{3}{2}}} = 0 \]

\[ \frac{d^2 C_t}{d X^2} = \frac{2K_2}{X^3} + \frac{2K_4 H_r}{X^3} + \frac{2K_6 H_s}{X(Zb+XK_6)^3} + \frac{2K_8}{X^3} \]
Equations 23 and 24 are sufficient conditions for guaranteeing that the simultaneous solution of equations 21 and 22 for \( X \) and \( Y \) will yield a minimum cost. Generally equations 21 and 22 are solved by using one equation to find one variable as a function of the other. This relationship is then substituted into the other equation which is solved by iteration as previously outlined. In this situation, however, the above procedure does not work. The reason is that one of the terms in the final equation contains the square root of a value which is negative for certain ranges of several variables, and therefore has no meaning.

The two equations, 21 and 22, are solved by iteration as follows. Set initial values for \( X \) and \( Y \) and solve the two equations. Next change one unknown, \( X \), by a small increment and recalculate the equations. Proceed in this manner until the sign of one equation changes. Now increase the other unknown, \( Y \), by a small increment and set \( X \) back to its initial value. Keep repeating the above operation until a
change in X of one increment causes both equations to change sign. This results in an interval for each unknown which contains the values required for minimum total cost. Just as with the case of one unknown, these intervals can be made as small as desired.

What this method does is shown by Figure 1 on page 36. Setting the initial values of X and Y at zero, the value of X is increased by e and the point on the surface given by these values of X and Y is found. The slope of the surface in both the X and Y direction is checked in this interval to see if it changes sign. This is repeated until we come to the interval between X and X+e where the slope of the function in the X direction changes sign. However, since Y does not change sign, we must increase Y by e and start the process over with X set equal to its initial value. As Figure 1 shows, when the area, Y to Y+e by X to X+e, is reached the slope changes sign in both directions. This change in sign of the slope is the same as the change in sign of the partial derivatives in the mathematical solution. The minimum value of $C_t$ is contained in this area, so one knows the values of X and Y within $\pm e/2$ that are required for the minimum cost solution.

The solution of this problem by iteration is much too long to be attempted by hand calculation. One iteration
The minimum value of $C_t$ is contained in the shaded area on the surface. The values of $X$ and $Y$ which yield this minimum are contained in their respective intervals, $X$ to $X + e$ and $Y$ to $Y + e$.

Figure 1: Diagram of the Iterative Solution
takes over an hour to complete using a slide rule. Therefore the problem must be solved by a computer where the entire solution is a matter of seconds. A computer flow diagram for the solution of this problem is given in Appendix II.

SUMMARY

The method used to optimize the total cost equation has now been presented. In summarizing the procedure, we should recall the major steps. First, the system variables were determined and combined into a total cost equation. Study of this equation indicated that the only variables which are of interest are road and landing spacing. This indicated that the equation should be optimized with respect to these variables, X and Y. Using the calculus method of partial derivatives and iteration to solve the resulting partial equations, it was possible to find the road and landing spacings which gave the minimum total cost of log removal.

Throughout this paper the road spacing is used as a measure of the horizontal distance between roads. This method of measurement is used because it is easiest to handle. Application of results will be more realistic if the spacing distance along the slope is known. The slope distance is a function of the slope, S, and the horizontal road spacing, X, and is obtained by the following expression.
\[ Q = \frac{X}{\cos (\tan^{-1} S)} \]  

where \( Q \) = road spacing in feet along the slope.

It is therefore very easy to convert the optimal road spacing from the horizontal plane to a surface measurement if desired.

It should be remembered that the results are based on certain assumptions which are used to approximate the relationships between certain variables. A new solution must be worked out for each set of variables since a change in any one variable will yield a different minimum removal cost. Finally, one should keep in mind that the solution technique is not practical without the use of an electronic computer.
CHAPTER IV
RESEARCH RESULTS

INTRODUCTION

The objective of this research was to find a method of solution to the logger's problem of determining the best combination of construction and skidding methods for a particular logging site. To determine the most economical combination of methods one must first know the particular road and landing spacing which give minimum cost for each possible combination. The problem therefore is one of finding the optimum spacings.

In the preceding chapters relationships among selected system variables were developed. When used with the optimization technique demonstrated in Chapter III, the relationships allow the loggers to determine optimum road and landing spacings for each combination of construction and skidding methods. This can be done for any logging area.

The model presented in this paper, as with any mathematical model, gives output only as good as the input. As mentioned earlier, the research presented in this paper was undertaken to develop a method of solution to the problem of road and landing layout. The research was not to determine the exact values of the system variables or equation coefficients. With this in mind, it is possible to proceed with a demonstration of the type of results obtained
from this method. This is done with a sample problem where a value for each variable is randomly picked from its possible range.

SAMPLE PROBLEM

The following values are used for the terrain and stand characteristics of a hypothetical area. See Table I for additional information on each variable.

Ro = 0.2 = percent rock in excavation in decimal %
S = 0.5 = sidehill slope in decimal %
V = 20 = timber volume density in MBF/acre
Zb = 2000 = level road in ft per switchback
Lv = 10 = size of logs in logs per MBF
A = 100 = size in acres of total logging site
Nc = 400 = cost in $ to move in construction equipment
Ns = 400 = cost in $ to move in skidding equipment

Next it is arbitrarily decided that only a large crawler tractor or a medium crawler tractor are to be used to skid the hypothetical area. A large crawler tractor equipped with a blade and ripper tooth is to be used for construction. The equipment and layout for the area have the following characteristics.
Layout

\[ D_m = 10 = \text{cost of two-man crew in } \$/\text{hr} \]
\[ P_m = 0.75 = \text{coefficient of production of layout crew} \]
\[ F_m = 500 = \text{productivity of layout crew in ft/hr} \]
\[ W = 14 = \text{road width in ft for case without landings} \]
\[ W = 10 = \text{road width in ft for case with landings} \]
\[ R_a = 30 = \text{interior curve radius in ft} \]
\[ M_x = 0.1 = \text{maximum road grade in decimal } \% \]
\[ L_e = 100 = \text{length of landing in ft} \]
\[ B = 1 = \text{slope of cut in decimal } \% \]

Construction equipment

\[ P_c = 0.8 = \text{productivity coefficient of equipment} \]
\[ D_c/F = 0.079 + 0.617 \text{ Ro} = \text{average cost of excavation} \]
\[ \text{in } \$/\text{cu yd, from Appendix I} \]

Skidding equipment

\[ D_s = 14 = \text{cost in } \$/\text{hr to operate medium crawler} \]
\[ D_s = 16 = \text{cost in } \$/\text{hr to operate large crawler} \]
\[ P_s = 0.75 = \text{productivity coefficient for both methods} \]
\[ S_p = 0 = \text{time in hr to set up the equipment} \]
\[ Z_s = 0 = \text{distance in ft between set-ups} \]
\[ V_c = 0.7 = \text{volume in MBF per cycle of the medium crawler} \]
\[ V_c = 1 = \text{volume in MBF per cycle of large crawler} \]
Regression coefficients for skidding equipment are

Medium crawler

\[
\begin{align*}
a_1 &= 0.10518 \\
a_2 &= -0.00201 \\
a_3 &= 0.01257 \\
a_4 &= 0.00027
\end{align*}
\]

Large crawler

\[
\begin{align*}
a_1 &= 0.10736 \\
a_2 &= 0.00133 \\
a_3 &= 0.01502 \\
a_4 &= 0.00048
\end{align*}
\]

these values were obtained from the work of D. B. Brown.

The preceding data values are used in a computer program which was developed to solve the partial derivative equations on pages 32, 33, and 34. The program uses the iterative solution technique described on pages 35 and 36. A flow diagram of this computer program is given in Appendix II. The computer output for this sample set of data is presented in Table II on the following page and is plotted in Figures 2 and 3 on pages 44 and 45.

The solution technique set forth in this paper is also used to make a set of graphs. The graphs show how removal cost and optimal spacing are affected by changes in certain variables. These graphs enable the logger to determine values of spacing that are near the optimal value.
<table>
<thead>
<tr>
<th>Construction Method</th>
<th>Skidding Method</th>
<th>No Landings</th>
<th>With Landings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cost $/MBF</td>
<td>Horiz rd space ft</td>
</tr>
<tr>
<td>Large Crawler</td>
<td>Medium Crawler</td>
<td>12.83</td>
<td>645</td>
</tr>
<tr>
<td>&quot; &quot;</td>
<td>Large Crawler</td>
<td>13.23</td>
<td>585</td>
</tr>
</tbody>
</table>

The optimum value of spacing is obtained only if the values of the particular logging site exactly match those values used to obtain the graphs.

On the graphs of Figures 4 to 8 on pages 46 to 50 are shown the results of the sample problem's solution for various combinations of sidehill slope, percent rock in the excavation, and volume of timber per acre. These results are obtained using the data listed on pages 40, 41, and 42 and the computer program represented by the flow diagram in Appendix II.

From the form of the output obtained for the sample problem, one can see the possibility of developing handbooks to help the logger determine optimal spacing policy. This booklet could contain various types of graphs and tables that show the effect of various variables on optimum spacing. Also, the handbook could contain graphs that show how sensitive the removal cost is to the spacing. Therefore
Removal Cost
($/MBF)

Decimal % of Optimum Horizontal Road Spacing

where:
- Optimum = 645 (ft) for medium crawler
- Optimum = 585 (ft) for large crawler

Figure 2: Removal Cost as a Function of Percent of Optimum Road Spacing
Decimals % of Optimum Landing Spacing
Where Optimum = 725 (ft), medium crawler
Optimum = 675 (ft), large crawler

Decimals % of Optimum Horizontal Road Spacing
where Optimum = 735 (ft) for medium crawler
Optimum = 665 (ft) for large crawler

Figure 3: Removal Cost as a Function of the Percents of Optimum Road and Landing Spacings
Figure 4: Removal Cost as a Function of Slope, Percent Rock, and Volume of Timber per Acre
Figure 5: Optimum Horizontal Road Spacing as a Function of Slope, Percent Rock, and Timber Volume per Acre
Removal Cost ($/MBF)

Construction: large crawler
Skidding: medium crawler
Case: with landings

MBF/acre
10 (solid lines)
20 (dashed lines)

% rock
(shown on line)

Figure 6: Removal Cost as a Function of Slope, Percent Rock, and Volume of Timber per Acre
Figure 7: Optimum Horizontal Road Spacing as a Function of
Slope, Percent Rock, and Volume of Timber per Acre
Construction: large crawler  
Skidding: medium crawler  
Case: with landings

MBF/acre
10 (solid lines)  
20 (dashed lines)

% rock  
(shown on lines)

Figure 8: Optimum Landing Spacing as a Function of Slope, Percent Rock, and Volume of Timber per Acre
the method developed in this paper could not only be used to obtain results for a given area, but to make a handbook that covers most combinations of values for the terrain and stand characteristics. The handbook would not give exact answers, however, it would give a close approximation which would be a useful guide to the logger.

SUMMARY

The problem of determining at what distance logging work roads should be spaced was investigated in this research. To find the optimal spacing, a criterion for optimal had to be chosen. Minimum total removal cost was chosen as the criterion for optimum road and landing spacings. Next all the variables or factors affecting removal cost were identified and combined in a total cost equation. The minimization of this equation with respect to spacing enabled one to determine the optimum values of spacing.

The most important result of this research is its demonstration of the fact that a total cost equation can be set up and optimized to find the optimal road spacing for minimum log removal cost. The method developed here makes it possible to find the optimal spacing for any set of stand and terrain characteristics. The fact that a mathematical model now represents the removal system is important. This means we can determine how each system
variable affects the total removal cost and optimal spacing.

The method presented in this paper depends on two things for successful application. One is the collection of accurate values for the system variables and the other is the availability of expressions that accurately relate the variables. The first requirement is easily met because none of the variables are difficult to measure. The difficulty lies in the second requirement where accurate relationships must be obtained. There are methods available for determining these relationships but they all require large amounts of data. The data requirement is great due to the large number of variables and the difficulty in finding the required combinations of value levels for the variables. These factors greatly increase the cost of data collection. An example of the data collection problem is the work of D. B. Brown on a productivity expression for skidding. The data collection for that study required over 1500 man-hours. Even so, data were not obtained for enough combinations of variable levels. The regression equations are therefore not accurate for all possible ranges of the variable. The reason data takes so long to gather is because the characteristics of a logging site are fairly uniform. Therefore if data is collected all day it may still give information on only several combinations of
values for the system variables.

A suggestion for future study on this problem is to continue work on the development of better expressions which would more adequately relate the variables. This should only be done with experiments designed to require small amounts of data. These studies should only strive for a fair degree of accuracy in relating the variables. It is felt the cost of too much refinement would be more than the savings due to improved accuracy. The system a logger operates in is natural and attempts to relate the infinite variety of combinations in nature may never be completely successful. Therefore it is suggested that future work be carried only to the point where reasonable relationships exist. When this is accomplished research should be directed toward finding usable methods of presenting the results of this solution technique. The ultimate use of this method will likely be in helping loggers to understand how various factors affect the minimum cost layout of work roads. This can probably best be done by using some type of graphical presentation.
APPENDIXES
INTRODUCTION

A study by the Bureau of Land Management, U. S. Department of the Interior, on various sizes of construction equipment gave results in the form of average cost per yard of excavation. The study found that these costs have no significant relationship to the percent slope of the sidehill. The actual excavation costs found by the study are $0.079 per cubic yard of common excavation and $0.696 per cubic yard of rock excavation. Other data from the study are given in the tables on the following page.

In the tables the usable widths are listed as ten and twelve feet. This assumes that roads are ditched and surfaced which is generally not the case for logging work roads. Work roads are built, used, and abandoned all in one season. The subgrade width is therefore a better estimate of the usable road width.

Figure 9 on page 57 shows the data from the following tables in graphical form. This data will be used to develop an expression for the productivity of construction equipment. The expression will be a function of the slope, percent rock, and road width. This means the data plotted in Figure 9 and the excavation costs on page 55 will be
Table III: Common Excavation as a Function of Slope and Road Width\(^1\)

<table>
<thead>
<tr>
<th>% Sidehill Slope</th>
<th>Cubic Yards per Station (100 ft)</th>
<th>14 ft subgrade (10 ft usable)</th>
<th>20 ft subgrade (12 ft usable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>93</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>93</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>147</td>
<td>309</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>220</td>
<td>346</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>321</td>
<td>462</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>370</td>
<td>617</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>485</td>
<td>768</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>622</td>
<td>1088</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>763</td>
<td>1333</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>907</td>
<td>1636</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1133</td>
<td>2045</td>
<td></td>
</tr>
</tbody>
</table>

Table IV: Rock Excavation as a Function of Slope and Road Width\(^1\)

<table>
<thead>
<tr>
<th>% Sidehill Slope</th>
<th>Cubic Yards per Station (100 ft)</th>
<th>14 ft subgrade (10 ft usable)</th>
<th>20 ft subgrade (12 ft usable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>64</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>64</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>86</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>96</td>
<td>206</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>194</td>
<td>276</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>263</td>
<td>509</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>393</td>
<td>597</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>473</td>
<td>861</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>569</td>
<td>990</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>638</td>
<td>1180</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>735</td>
<td>1335</td>
<td></td>
</tr>
</tbody>
</table>
Excavation per Station (100 ft) in Cubic Yards

Common & 20' subgrade
Rock & 14' subgrade
Common & 20' subgrade
Rock & 14' subgrade

Figure 9: Values from Table III and Table IV Plotted
contained in one equation. This expression will then be used to solve for road costs, and with some modification to solve for landing and switchback costs.

**DEVELOPMENT**

The expression is developed by the following steps. First, the cost of excavation in soil containing both common and rock material must be found. The assumption is made that the cost per cubic yard remains the same for each type of excavation regardless of the percentage combination of them. In truth the above is probably not true, however, lack of knowledge requires that some assumption be made. The assumption of constant costs leads to the following expression.

Letting \( R_0 = \text{decimal \% of rock in excavation} \)

\[ (1-R_0) = \text{decimal \% of common in excavation} \]

and with \( \$0.079 = \text{cost per cu yd in common excavation} \)

\( \$0.696 = \text{cost per cu yd in rock excavation} \)

the equation is

Average Cost of Excavation = \( 0.079(1-R_0)+0.696R_0 \)

\[ = 0.079+0.617R_0 \ (\$/\text{cu yd}). \]

Next one must determine how the percentage of rock
affects the volume of excavation required. If the volume of excavation in rock for a cross section of given slope and road width is divided by the volume of common for a cross section with that slope and road width the following ratios are obtained.

<table>
<thead>
<tr>
<th>Slope in %</th>
<th>14' road width</th>
<th>20' road width</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.69</td>
<td>0.57</td>
</tr>
<tr>
<td>20</td>
<td>0.58</td>
<td>0.39</td>
</tr>
<tr>
<td>30</td>
<td>0.44</td>
<td>0.60</td>
</tr>
<tr>
<td>40</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>50</td>
<td>0.71</td>
<td>0.82</td>
</tr>
<tr>
<td>60</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>70</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>80</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>90</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>100</td>
<td>0.65</td>
<td>0.65</td>
</tr>
</tbody>
</table>

$x_{14} = 0.669$  
$x_{20} = 0.666$

$x_{\text{average}} = (0.669 + 0.666) / 2 = 0.6675$

The author does not know the exact procedure used in data collection for this study. His past experience indicates that the data were probably gathered at random on various logging sites, so this will be assumed. Also, it will be assumed the data are from populations of excavation rates which are normal and independently distributed with
equal variances. With these admittedly loose assumptions, one can test the excavation ratio to see if it is independent of slope and road width. Following is a two-way analysis of variance calculation for the randomized, complete block design which the data will fit under the above assumptions.

**Analysis of Variance Calculation**

Data values are coded by \((\text{true value} - 0.67)100 = \text{table value}\) and \(n = 2, k = 10, N = 20\)

<table>
<thead>
<tr>
<th>Road Width (ft)</th>
<th>Slope in %</th>
<th>(T_{1.})</th>
<th>(T_{1.}^2/k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14</td>
<td>-1</td>
<td>0.1</td>
</tr>
<tr>
<td>20</td>
<td>-10</td>
<td>-4</td>
<td>1.6</td>
</tr>
</tbody>
</table>

\[ T_{.j} = -5 = T_{..} \]

\[ T_{.j}^2/n = 2133 = \sum_{j=1}^{10} T_{.j}^2/n \]

\[ \sum_{i=1}^{2} x_{ij}^2 = 2575 = \sum_{j=1}^{10} \sum_{i=1}^{2} x_{ij}^2 \]

\[ 1.7 = \sum_{i=1}^{2} T_{1.}^2/k \]

\[ SS_{\text{slope}} = \frac{10}{j=1} T_{.j}^2/n - T_{..}^2/N = 2131.75 \]

\[ SS_{\text{road width}} = \sum_{i=1}^{2} T_{1.}/k - T_{..}^2/N = 0.45 \]
\[ SS_{\text{total}} = \sum_{j=1}^{10} \sum_{i=1}^{2} x_{ij}^2 - T^2 / N = 2573.75 \]

\[ SS_{\text{error}} = SS_{\text{total}} - SS_{\text{slope}} - SS_{\text{road width}} = 441.55 \]

Table of Values for the Analysis of Variance Calculation:

<table>
<thead>
<tr>
<th>Source of Error</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>( F_{\text{test}} )</th>
<th>( F_{\text{critical}} (\alpha = 0.01) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>9</td>
<td>2131.75</td>
<td>239.0</td>
<td>4.87</td>
<td>5.35</td>
</tr>
<tr>
<td>Road Width</td>
<td>1</td>
<td>0.45</td>
<td>0.01</td>
<td>0.61</td>
<td>10.6</td>
</tr>
<tr>
<td>Error</td>
<td>9</td>
<td>441.55</td>
<td>49.2</td>
<td>( F_{\text{test}} )</td>
<td>( F_{\text{critical}} )</td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>2573.75</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the test of the slope, \( F_{\text{critical}} > F_{\text{test}} \), therefore it can be said that at the 0.01 level of significance there is no significant difference in the means of the ratio of rock excavation to common excavation at various values of slope. At the 0.05 level we cannot make the above statement, so the test results are not strongly conclusive. The results of the test on road widths yield, \( F_{\text{critical}} > F_{\text{test}} \), so no significant difference exists in the excavation ratio for different road widths. The test on road widths is very conclusive in showing that width has no effect on the ratio. The results of these tests indicate that the excavation ratio can be used in the development in a manner independent
of slope and road width. Results of the preceding tests are only true if the assumptions preceding the tests are true, and it must be remembered that the author had no knowledge of the means of data collection and no empirical backing for his assumptions.

The availability of cost data for only two percentages of rock, zero and one hundred, means that only a straight line can be plotted for the cost of excavation as a function of percent rock, so a straight line relationship is assumed. Cost of excavation as a function of percent rock can be shown by the following expression which adjusts the yardage of excavation required in common material for the percentage of rock present. The excavation ratio of 0.6675 or 2/3 relates excavation required in one hundred percent rock cross sections to the excavation required in one hundred percent common cross sections. This ratio is used in the following expression to adjust the excavation yardage required for the percentage of rock in the excavation. The expression adjusting excavation for the percentage of rock present is

\[(1-Ro) + \frac{2Ro}{3} = \frac{1-Ro}{3}\]

Percent of Common Exc. + Percent of rock Exc. = Equivalent Exc. reduced to Common Exc. in Percent Common
\[
\text{Excavation for a given slope, road width, and percent rock} = (1-\frac{Ro}{3}) \text{Excavation for a given slope, road width, and zero percent rock}.
\]

The required amount of excavation has now been adjusted for the percentage of rock present. The next requirement is a relationship between the amount of excavation required and the road width. This relationship is obtained by dividing the cubic yards of excavation required for a 14 ft width by the cubic yards required for a 20 ft width. The results of this division are

<table>
<thead>
<tr>
<th>% Slope</th>
<th>100% Common</th>
<th>100% Rock</th>
<th>Average Value of 1 and 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.72</td>
<td>0.86</td>
<td>0.790</td>
</tr>
<tr>
<td>20</td>
<td>0.48</td>
<td>0.72</td>
<td>0.600</td>
</tr>
<tr>
<td>30</td>
<td>0.64</td>
<td>0.47</td>
<td>0.555</td>
</tr>
<tr>
<td>40</td>
<td>0.70</td>
<td>0.70</td>
<td>0.700</td>
</tr>
<tr>
<td>50</td>
<td>0.60</td>
<td>0.52</td>
<td>0.560</td>
</tr>
<tr>
<td>60</td>
<td>0.63</td>
<td>0.66</td>
<td>0.645</td>
</tr>
<tr>
<td>70</td>
<td>0.57</td>
<td>0.55</td>
<td>0.560</td>
</tr>
<tr>
<td>80</td>
<td>0.57</td>
<td>0.57</td>
<td>0.570</td>
</tr>
<tr>
<td>90</td>
<td>0.55</td>
<td>0.54</td>
<td>0.545</td>
</tr>
<tr>
<td>100</td>
<td>0.55</td>
<td>0.55</td>
<td>0.550</td>
</tr>
</tbody>
</table>

The results for common and rock are averaged in the
list on the preceding page because the previous statistical test showed that changes in excavation ratios for various road widths are independent of the percent rock in the excavation. Independence means the ratio of excavation required for cross sections of two different widths does not change as the percent of rock changes. The average values of the ratios are seen to decrease as values of slope increase. This is shown graphically in Figure 10 on page 65.

The line plotted on the graph in Figure 10 is arrived at by minimizing the sum of the distances the data points deviate from the line. The line has a secondary constraint in that it must pass through the point, 0.7, on the ratio axis. This is required because at zero slope the excavation ratio is exactly equal to the road width ratio or (14'/20'= 0.7).

The expression representing the relationship shown by Figure 10 is

\[
\frac{\text{Ratio of Excavation for 14' width}}{\text{Excavation for 20' width}} = 0.7 - 0.17 S
\]

Then, if 14 ft is chosen as the base measurement of road width the above expression generalizes to

\[
\frac{\text{Ratio of Excavation for 14' width}}{\text{Excavation for W' width}} = \frac{14}{W} - 0.17 S
\]
Figure 10: Slope versus Excavation Ratio of 14 ft Road Width to 20 ft Road Width
where \( W = \) road width in feet.

The above expression is also based on the assumption that the relationship between road widths and the amount of excavation is linear over all values. This assumption cannot be empirically tested because data is available for only two road widths.

The previous expression is rewritten as

\[
\text{(Excavation for width } W) = \frac{(\text{Excavation for width } 14 \text{ ft}) \cdot (14/W) - 0.17 \cdot S}{(14/W) - 0.17 \cdot S}
\]

An adjustment for the percentage of rock in excavation is added to this expression by using the relationship developed on page 62. The equation then becomes

\[
\frac{(\text{Excavation for width } W)}{(\text{and percent rock } R_o)} = \frac{(\text{Excavation for width } 14') \cdot (1 - R_o)}{(14/W) - 0.17 \cdot S}
\]

Now a relationship must be obtained for the slope and the excavation required for 14 ft width and zero percent rock. The data from Table III for this combination of conditions is plotted in Figure 11 on page 67. It is seen that the data is approximated by two straight line segments. The equation for each line segment and range of slope values it is valid for are obtained from Figure 11 and are
Figure 11: Quantity of Common Excavation for a 14 ft Road Width as a Function of Sidehill Slope
Excavation in cu yd = 7.4 S, where 0 ≤ S ≤ 0.54 per ft of road

Excavation in cu yd = 4 + 13.8(S-0.54), where 0.54 < S.

The combination of the equations on pages 58 and 66 gives an expression for the cost of road construction per linear foot of road. This equation is

\[ \text{Cost per ft} = \frac{DcHr}{F} (\$) = \frac{(0.079+0.617Ro)(1-Ro/3)Po}{(14/W)} - 0.17 S \]

where Dc, Hr, and F represents the cost in \$/ft in terms of the system variables listed in Table I, and

\[ Po = \begin{cases} \text{Excavation for each} \\ \text{ft of road width 14 ft} \\ \text{and percent rock zero} \end{cases} \]

or

\[ Po = 7.4 S, \text{ where } 0 \leq S \leq 0.54 \]

\[ Po = 4 + 13.8(S-0.54), \text{ where } 0.54 < S. \]

COST EQUATION FOR LANDING CONSTRUCTION

The preceding expression for road construction cost can be applied to landing cost very easily. On the following page are shown a road cross section and a landing cross
section which includes the road.

Road only

Road and Landing

where $W_l = $ landing width in feet

$E_1 = $ area of excavation for width $W$

and $E_2 = $ area of excavation for width $W + W_l$.

From geometry $E_2 = e_1 + e_2$ and $E_1 = e_1$

therefore $e_2 = E_2 - E_1$

where $e_2 = $ additional area of cross section due to landing.

The above result allows use of the road cost expression to determine the construction cost of landings. The following expression gives this cost by finding the additional area of excavation required and multiplying it times the landing length in feet and the construction cost per linear foot. This expression is
Excavation cost per landing = \( DcHl/F \) ($/landing) =

\[
= Le(0.079+0.617) \left( \frac{Po(1-Ro/3)}{(14/(W+Wl)-0.17S)} - \frac{Po(1-Ro/3)}{(14/W)-0.17S} \right)
\]

where \( Dc, Hl, \) and \( F \) are system variables from Table I that represent the cost of landing construction and \( Po = 7.4S \), where \( 0 \leq S \leq 0.54 \)

\[ Po = 4 + 13.8(S-0.54) \], where \( 0.54 < S \).

COST EXPRESSION FOR SWITCHBACK CONSTRUCTION

The cost expression for switchbacks uses the cost expression for roads in a manner similar to the way it was used for landings. The switchback is divided into three sections with the volumes being calculated for each section by taking the average cross sectional area of its end sections times its length. On the following page is a diagram of a switchback. It is followed by sketches of various cross sections. The variables shown by capital letters are defined in Table I on page 13.
Switchback Layout - Plane View

Cross Section (A' - A'')
Cross Section ( $B' - B''$ )

\[ (\text{Ra}^2 - m^2)^{\frac{1}{2}} \]

Cross Section ( $C' - C''$ )

\[ 2(W^2 + 2\text{Ra}W)^{\frac{1}{2}} \]

Cross Section ( $D' - D''$ )

There is no cut because the outer edge of the switchback coincides with the ground surface at only one point for this cross section.
Before the volumes of the sections are calculated the value of \( m \) must be found in terms of the system variables. This is done with the equations obtained from the geometry of cross section \( (B' - B'') \).

If \( h = 2B(Ra^2 - m^2)^{\frac{1}{3}} \) and \( h = S(W + 2(Ra^2 - m^2)^{\frac{1}{3}}) \)

then \( 2B(Ra^2 - m^2)^{\frac{1}{3}} = SW + 2S(Ra^2 - m^2)^{\frac{1}{3}} \)

\[ (2B-2S)(Ra^2 - m^2)^{\frac{1}{3}} = SW \]

\[ Ra^2 - m^2 = \frac{S^2W^2}{(2B - 2S)^2} \]

\[ m^2 = Ra^2 - \frac{S^2W^2}{(2B - 2S)^2} \]

therefore \( m = (Ra^2 - \frac{SW}{(2B - 2S)})^{\frac{1}{3}} \).

Now the volume of excavation is found by multiplying the section lengths times their respective average end areas. The results are

Section I:

Volume (cu yd) = \( V_1 = \left( Ra^2 - \frac{(SW)^2}{(2B-2S)^2} \right)^{\frac{1}{3}} \times \left( \frac{Po(1-Ro/3)}{(14/W)-0.17S} \right) \)

Section II:

Volume (cu yd) = \( V_2 = \frac{1}{3} \left( Ra^2 - \left( \frac{(SW)^2}{(2B-2S)^2} \right)^{\frac{1}{3}} \right) \times \)
The expression for the total cost of constructing a switchback then becomes

$$\text{Excavation cost per switchback} = \frac{DcHs}{F} \left( \$ \right) = (0.079 + 0.617 \text{ Ro})(V_1 + V_2 + V_3)$$

where \( Dc, Hs, \) and \( F \) are system variables representing the cost of excavation per switchback.

SUMMARY

Expressions have now been developed for the cost of constructing roads, landings, and switchbacks. When these expressions are combined with the eight component cost expressions developed in the main text, the total cost expression is complete. Now data values for any area can be collected and used with this total cost equation to determine the minimum cost layout for any combination of construction and skidding methods.
INTRODUCTION

As stated in the main text of this thesis, the method of solution for optimal spacing problems requires an electronic computer. Starting below and continuing for several pages, is a flow diagram for the solution of this problem by a computer. It was felt by the author that a flow diagram should be presented rather than an actual program. This is due to the fact that many different computer codes are currently in use, and a flow diagram would be more useful than a program in only one of the codes.

The symbols used in the flow diagram are those in standard usage. The small letters at the top and bottom of each page label the lines of control that extend from one page to another.

FLOW DIAGRAM:

Start

Read: Terrain and Stand Characteristics
Read: Layout Design Criteria and N

Let: M = 0
Let: M = M + 1

M > N

Check Value of M

M ≤ N

M = N

End
Print out for each method its minimum costs and optimal road and landing spacings

End

Read: (N) Skidding methods and their characteristics

Read: Construction method and its characteristics

Let: \( T = 0 \)

Calculate constants for the partial equations

Check slope value

\[ S \leq 0.54 \]

Calculate partial equations that depend on road width

Initialize road spacing at zero

Increase spacing by an increment

Calculate value of the partial derivative

Check value of sign

\[ (-) \]

\[ (+) \]

Calculate production rate of construction equipment on mild terrain

Check value of \( T \)

\[ T = 0 \]

Calculate partial equations that depend on road width for case without landings

Calculate road width

Calculate partial equations that depend on road width
Calculate average of last two spacing values
Calculate the total cost at this spacing
Record this cost and road spacing

\[ T = T + 1 \]

If Neu is <

Increase landing spacing by an increment
Initialize landing spacing

Increase road spacing by an increment
Initialize road spacing

Calculate the values of the partial equations with respect to road and landing spacing

Check sign of partial with respect to road spacing

(-)

Check sign of partial with respect to landing spacing

(+)

Calculate total cost for case with landings

Compare with previous value of total cost

If Neu is >

If Neu is <
Let road spacing equal its present value minus the increment it was increased by.

Average this value of road spacing with the one preceding it.

Calculate the total cost with this value of road spacing and the last value of landing spacing.

Record the total cost and the road and landing spacing.
APPENDIX III

DEVELOPMENT OF AVERAGE SKIDDING DISTANCES

The average skidding distance, $d_{\text{average}}$, will be determined for two situations. The first is where all logs are skidded straight to the road which serves as a continuous landing. The second is the case where all logs are skidded to landings. These landings will be assumed equally spaced at an interval of $Y$ (ft). The following derivations for these cases assume that the trees are randomly distributed and because of the large area involved this distribution can be approximated by a uniform distribution. Also, the skidding path will be assumed to follow the straight line between the stump and the road or landing. This assumption can be modified to more closely approximate the situation for methods with curved skidding routes by multiplying the distance by a factor greater than one.

The first case to be considered will be that where the road serves as a continuous landing. The diagram below shows this situation.

Horizontal View of Layout for Skidding to Roads
Letting \( d_i = \) skidding distance (horizontal) in feet

then

\[
\overline{d} = \frac{1}{n} \sum_{i=1}^{n} d_i.
\]

With the previous assumption that a uniform distribution of trees exists, one can find the average skidding distance by finding the shortest distance from the centroid of the area to the road. The following diagram shows the area covered by one switchback section. Assuming that each section will be constructed to the same dimensions, the average skidding distance for this section will be the average for the total logging area.

To find the centroid of the switchback section would be difficult. The area has therefore been divided into sub-areas whose centroids are easily found. The average skidding distance for the area is then found by calculating the average of the distances from the sub-area centroids straight downhill to the nearest road. This average is weighted with respect to the area skidded over that distance. Following is the derivation of the average skidding distance equation for the case of no landings.

Let \( a_1, \ldots, a_5 \) denote sub-areas and \( d_1, \ldots, d_5 \) denote the horizontal distance in feet,
where \( g = \frac{X}{2} \left( \frac{S^2}{N^2} - 1 \right)^{\frac{1}{2}} \), see derivation on page 19.

The areas in sq ft of the sub-areas are

- \( a_1 = \frac{gX}{4} \)
- \( a_2 = gX \)
- \( a_3 = \frac{gX}{4} \)
- \( a_4 = XZb \)
- \( a_5 = \frac{gX}{2} \)

and the total area is

\[
\left(\frac{gX}{4}\right) + \left(gX\right) + \left(\frac{gX}{4}\right) + (XZb) + \left(\frac{gX}{2}\right) = X(Zb + 2g) \quad \text{(sq ft)}
\]

Distances from the centroids to the road, in feet, are

- \( d_1 = \frac{1}{3} \left(\frac{X}{2}\right) + (X) + \frac{2}{3} \left(\frac{X}{2}\right) = \frac{3X}{2} \)
- \( d_2 = \left(\frac{X}{2}\right) + \frac{1}{2} \left(\frac{X}{2}\right) = \frac{3X}{4} \)
- \( d_3 = \frac{1}{3} \left(\frac{X}{2}\right) = \frac{X}{6} \)
\[ d_4 = \frac{x}{2} \]
\[ d_5 = (2/3)(x/3) = x/3 . \]

The average skidding distance for the case with no landings is then

\[ d_{\text{average}} = \sum_{i=1}^{5} \frac{d_i^2}{a_i} = \frac{X^2((Zb/2)+(4/3)g)}{X(Zb+2g)} = \frac{(Zb(X/2))+(4X(g/3))}{Zb+2g} . \]

Substituting for the value of \( g \) its expression given on the last page we have

\[ d_{\text{average}} = \frac{Zb(X/2)+(2X^2/3)((S^2/Nx^2)-1)^{1/2}}{Zb+X((S^2/Nx^2)-1)^{1/2}} \text{ (ft)} . \]

In the second case, where distinct landings are used, the average skidding distance is developed in a manner similar to the above. In this case skidding is over the shortest route to the landing. The spacing of landings, however presents a problem. When the best landing spacing is determined, in all probability, it will not be an even division of the level distance between switchbacks. This is illustrated in the sketch on the following page.
As can be seen, the last landing to the right does not serve the same area as the others and would have to be treated separately. Another possibility is to force the distance between landings to be such that it gives an integer number of sections. This, however, would not give the optimum spacing, but only an approximation to it. This approximation would increase in accuracy as Zb is increased. However, as Zb is increased, the accuracy of the first method would also increase. This follows because the fraction of the landing section not in the switchback area would be a smaller percent of the total area. The two methods therefore appear similar in their degrees of accuracy.

It is felt that the first method has more merit and should therefore be used. Once the optimum spacing is found, the field designer can then make adjustments so that an integer number of landings results. In this manner the designer can make the final decision as to what landing
spacing should be used, and he will know how far that spacing deviates from the optimal spacing.

The following derivation will assume that the total road system serving the area can be approximated by parallel roads. Once again it is assumed that the trees are uniformly distributed. Using the method explained above and the assumptions made in the preceding sentences, the derivation of the equation for average skidding distance in the case with landings is set forth in the following paragraph.

The area skidded to each landing is divided into sub-areas for ease of calculation. The distances from the centroids to the landings are calculated and their weighted average with respect to area is calculated. These distances and areas are shown on the following sketch.

\[
\text{Horizontal Projection}
\]
\[
\text{Area Skidded to Each Landing}
\]

where \( a_1, a_2, a_3 \) are sub-areas
and \( d_1, d_2, d_3 \) are horizontal distances in feet to landing.
The areas are

\[ a_1 = a_3 = \frac{1}{2}X(Y) = \frac{XY}{4} \text{ (sq ft)} \]

\[ a_2 = \frac{1}{2}(XY) = \frac{XY}{2} \text{ (sq ft)} \]

The distances in feet are

\[ d_1 = d_3 = \left( \frac{X}{3} \right)^2 + \left( \frac{Y}{6} \right)^2 = \left( \frac{x^2 + y^2}{3} \right) \]

\[ d_2 = \frac{2X}{3} \]

The average skidding distance for the case with landings is

\[ d_{\text{average}} = \frac{\sum_{i=1}^{3} d_i a_i}{\sum_{i=1}^{3} a_i} = \left( \frac{X}{3} \right) + \left( \frac{x^2 + y^2}{3} \right)/6 \text{ (ft)} \]

The two expressions developed in this appendix are used in the main text of this paper for the average skidding distance for their respective cases, with and without landings.
LITERATURE CITED


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<thead>
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