MONTANA STATE UNIVERSITY

Monte Carlo simulation of some logging operations
by Frank Joseph Cesario

A thesis Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Industrial Engineering
Montana State University
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Abstract:
The first subsystem of the logging system, logmaking, was simulated by Monte Carlo methods on an IBM 1620 computer. Data gathered from a multitude of sources were used to construct a preliminary computer simulation "model" of the subsystem, by means of which all of the operations usually performed in the field could be done within the comforts of the office. By the use of this approach, hundreds of acres of timber could be "harvested" in just a few hours. An important advantage of the simulation technique is that the logger can make optimum decisions before . he begins logging, rather than making costly remedial ones in the field.

Although the model is by no means complete, the results obtained showed that this approach to logging analysis is very promising. A few of the many different alternatives were evaluated to give potential users an indication of how simulation can be used to advantage in logging analysis. The following is a list of the different simulation runs reported in this thesis: 1. Production rate was varied.

2. Different production policies were tested.

3. The amount of defective material was varied.

4. Various payment schemes were compared.

In these runs, cost, profit, and waste material were used as measures of effectiveness.

No statements were made concerning the reliability of either the model or the results, although some phases of the model did prove to be dependable. It was concluded that more data is needed if significant results are to be obtained. Also, the assumption of additivity of data would have to be investigated. The model given in this thesis can, at best, be used as a guide to the construction of a more elaborate model.

A section on logging processes and the production components of logmaking is given for readers unfamiliar with these operations.

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SYMBOLS USED AND THEIR MEANINGS

Two different symbolisms are encountered in this thesis. The first, algebraic symbols, are used in explaining some basic theoretical statistical notions. The second type of symbolism is the collection of FORTRAN II variable names used in the computer programs. Each symbolism is presented separately.

A. Algebraic Symbols.

Greek letters and English letters are listed separately according to their respective alphabets. Operation symbols are also listed separately.

a	(1) Constant term in a regression equation; (2) A positive parameter; (3) Distance in inches between bark and rotten core of a tree.
a_i	(1) Polynomial coefficient; (2) Number of logs of size i.
A	A random variable.
b	Diameter of rotten core.
b_i	(1) Regression coefficient for the "i.th" group; (2) Time to buck log i.
B_i	(1) Cross-sectional area of a log at point i; (2) Bucker number.
c	(1) A random variable; (2) A curve.
C_i	(1) Number of observations on feller i; (2) Length of log i containing limbs.
$C_{n,k}$	The number of combinations of n things taken k at a time.
d	(1) Diameter at a point; (2) Limbing time per foot; (3) Distance to next tree; (4) Diameter at small end of a log.
d_i	(1) Difference between an observed value and a value as estimated by a regression equation; (2) Polynomial coefficient.
D	(1) Stump diameter; (2) Dbh; (3) Diameter at big end of a log.
dbh	Diameter breast-high.
d	Degrees of freedom.
e	$\lim_{t \to \infty} (1 + \frac{1}{t})^t$, approximately equal to 2.7183.

f_i (1) Observed frequency in sample i; (2) Length of free stem of log i.

$f(x)$ Probability density function of x.

$f(X_1,X_2)$ Joint probability for X_1 and X_2.

$f(X_1/X_2)$ Conditional probability for X_1 given X_2.

F Ratio of two mean squares.

F_i (1) Expected frequency in sample i; (2) Feller number.

$F(x)$ Probability distribution function of X.

g On-ground-travel-time ratio.

h_{off} Time to disembark from the stem.

h_{on} Time to mount the stem.

h Distance up the vertical at which the diameter of rotten core equals half the diameter of the stem.

h' Distance along the stem at which the diameter of rotten core equals half the diameter of the stem.

H (1) Test statistic for comparing samples by means of the sum of ranks; (2) Total height.

Ho Null hypothesis.

i Variable subscript.

j Variable subscript.

k (1) Value of a random variable; (2) Number of units in a sample.

K A constant.

K_i Number of observations in group i.

l Limbing and bucking time.

L (1) A least-squares curve; (2) Length of a log; (3) Distance up the stem at which the rot tapers to a point.

L' Distance up the vertical at which rot tapers to a point.

m (1) Number of logs in the optimal set; (2) Time to make reference cut; (3) Length of butt log.

m_γ	Sum of γ uniform variables.
M	Population mean.
M_i	A variance factor.
MBF	Thousand board feet.
n	(1) Number of units in a sample; (2) Number of logs in a tree.
N	Number of units in a population.
Pi	Length of log i.
P	Production rate.
psi	Pounds per square inch.
P(A)	Probability of A.
P(AC)	Joint probability of A and C.
P(A/C)	Conditional probability for A given C.
r	(1) Correlation coefficient; (2) Time to retract tape; (3) equivalent to $(\not\geq -6)/4$.
r^2	Percentage of variation attributable to regression.
R	Uniform random variable.
R_i	Sum of ranks for group i.
S	Time to hook tape measure.
s(x)	Standard deviation of a sample.
$\bar{s}(x)$	Pooled standard deviation of two samples.
$s^2(x)$	Variance of a sample.
t	(1) Ratio of a variable with unit normal distribution to the sample estimate of the standard error of that variable; (2) Number of observations in a tied set.
T	(1) (t) (t + 1)(t - 1), where t is defined as in (2) above; (2) A Chebychev Polynomial.
u	Travel time.
V	Volume of a log.
V_h	Cubic-foot volume, Huber.
V_s	Cubic-foot volume, Smalian.

w	On-log walk time ratio.
w_i	An individual random variable.
x_i	(1) An individual sample measurement; (2) Length of log i; (3) Variable in a regression equation.
X	(1) Set of x's, $(x_1, \ldots, x_n)$; (2) Abcissa.
$\overline{X}$	Sample mean of variable x.
(X_i, Y_j)	An ordered pair.
X_i	The "i th" sample of X.
X_{ij}	The x value in the "i th" row and "j th" column.
y	Ordinate of a probability distribution.
y_i	The "i th" sample value of y.
$\bar{y}$	Sample mean of variable y.
Y	(1) Set of y's, $(y_1, \ldots, y_n)$; (2) Ordinate; (3) Random normal deviate.
Z	(1) A variable with unit normal distribution; (2) Abcissa of normal probability curve; (3) Sum of random numbers.
α	Level of significance.
χ^2	The Chi-squared test statistic.
Δ	An incremental length.
λ	Population parameter.
θ	Angle between vertical and tree stem.
π	Ratio of the circumference of a circle to its diameter, approximately equal to 3.1416.
γ	An integer.
σ	Population standard deviation.
σ^2	Population variance.
Σ	The sum of.
μ	Population mean.
∞	Infinity.

B. Fortran II Variables

The variables in the following list include all input and output variables, and a selected few defined within the program itself. Variables defined within the program strictly for computational advantage (e.g. transformation of a fixed-point variable to floating-point for use in a calculation) or for storage space conservation (e.g. subscript transformation or alteration of an integer set for use in a computed GO TO) are not given. Therefore, if a variable is not defined in the following list, its meaning is self-evident in the computer source program which appears in Appendix A.

A(I) The number of logs of size X(I) that can be cut from the merchantable stem, where I = 1, JBUCK.

ABFV The Scribner board-foot volume of a 16-foot log.

ACRES The number of acres in the timber stand to be cut.

ADBH Average dbh in the timber stand, in inches.

ADJ The length of a log, in relation to a 16-foot log, in percent.

ATHT Average total height in the timber stand, in feet.

AVE Average distance between trees, in feet.

B(K) The number of logs of size X(K) that can be cut from the difference between merchantable height and A(I) · X(I).

BAN The number of times the tape is hooked and retracted during the limbing of one tree.

BBFV The Scribner board-foot volume of the difference between volume of an actual log and the volume of a 16-foot log.

BBG Dummy variable.

BFV Actual simulated board-foot volume of a tree, Scribner rule.

BFVS1 Scribner board-foot volume of a log, no allowance for taper.

BFVS2 Scribner board-foot volume of a log, allowance for taper.

BFVWT Board-foot volume of a log, estimated by weight.

BNO(J) Length of log number J, in feet, where J = 1, MAX.

BSDBH	The summation of dbh in the stand, in inches.
BUCKT	An individual bucking time, in decimal minutes.
BUT	The total number of logs cut in one day.
C(L)	The number of logs of size X(L) that can be cut from the difference between merchantable height and the sum of A(I) · X(I) · and B(K) · X(K).
CAN	The number of 4-foot lengths in a log, floating point.
CFV	Actual simulated cubic-foot volume of a tree.
CFVL	Cubic-foot volume of a log.
CHOP	Time involved in making reference marks in limbing a tree, in decimal minutes.
COMBO	The portion of a log that contains live limbs, in feet.
COS	The cosine of the angle between the vertical and the tree stem.
CROWN	The portion of a tree containing live limbs, in feet.
CTIME	The time spent in making excess bucking cuts as a result of rot, in decimal hours.
CTREES	The number of merchantable trees per acre.
CUT	The number of trees cut per day, floating point.
D(M)	The number of logs of size X(M) that can be cut from the difference between merchantable height and the sum of A(I) · X(I), B(K) · X(K), and C(L) · X(L).
DIAM(J)	Diameter of the tree at bucking point J, in inches, where J = 1, MAX.
DIAMM	The diameter at some point between the ends of a log, in inches.
DBH(I)	The number of diameters in diameter class I, where I = 7,15.
DBHS	The number of merchantable trees in the stand remaining to be cut.
DBHSY	The total number of merchantable trees originally in the stand.

DBHX	An individual dbh, in inches.
DEAD	Proportion of dead trees in a stand of timber.
DEV	Random normal deviate.
DIFF1	The difference between merchantable height and A(I) · X(I), in feet.
DIFF2	The difference between merchantable height and the sum of A(I) · X(I) and B(K) · X(K), in feet.
DIFF3	The difference between merchantable height and the sum of A(I) · X(I), B(K) · X(K), and C(L) · X(L), in feet.
DIFF4	The difference between merchantable height and A(I) · X(I), B(K) · X(K), C(L) · X(L), and D(M) ·X(M), in feet.
DIFFA	Difference between expected and actual cubic-foot volume.
DIFFB	Difference between expected and actual Scribner board-foot volume.
DIGIT	An 8- digit odd random number, not divisible by 5.
DIST	A sample value of distance between trees, in feet.
DISTT	The length of the butt log of a tree plus DIST, in feet.
DS	The portion of a tree stem containing rot, in feet.
DSTY	The number of tree stems per acre.
EBFV	Estimated Scribner board-foot volume of a tree.
ECFV	Estimated cubic-foot volume of a tree.
ETC	The portion of a tree that was cut into logs, in feet.
EXTRA	The difference between merchantable height and a given combination of logs, in feet.
FACTOR	Cull factor, in percent.
FCOST	Fixed cost, in dollars.
FELLT	An individual felling time, in decimal minutes.
FREE	The portion of a tree containing no live limbs, in feet.
H	The distance up the vertical at which the radius of the rotten core exactly equals the radius of the bole, in feet.

HHT	Transformed total height, in feet.
HONOF	The combined time to mount and disembark a tree stem, in decimal minutes.
HP	The distance up the stem at which the radius of the rotten core exactly equals the radius of the bole, in feet.
HTS(II)	The number of heights in each dbh class II, where II = 7,15.
HTT(II,JZ)	The number of height observations in each height subclass JZ contained in diameter Class II, where JZ = JBEG(II), JEND(II).
HTU(II,JZ,K)	The number of height observations in each height sub-subclass K, where K = 1,10.
IBEG	The minimum diameter in the stand, in inches.
IEND	The maximum diameter in the stand, in inches.
JBEG(II)	The minimum height value in diameter class II, rounded to the nearest 10 feet, where II = 7,15.
JBUCK	The number of acceptable log lengths.
JEND(II)	The maximum height value in diameter class II, rounded to the nearest 10 feet, where II = 7,15.
JOP(M)	Equivalent to D(M), in fixed point.
KBEG(JJ)	The minimum crown/height ratio in each diameter class JJ rounded to the nearest 10 feet, where JJ = 7,15.
KEND(JJ)	The maximum crown/height ratio in each diameter class JJ, rounded to the nearest 10 feet, where JJ = 7,15.
KLOGS	The number of logs in any given log combination.
KOKE	The number of days to be simulated.
KOP(L)	Equivalent to C(L), in fixed point.
LBEG(KK)	The minimum on-log travel time ratio estimate in diameter class KK, in decimal hours, where KK = 7,15.
LBBJ	The number of trees cut in any simulated hour.
LBJ	The number of trees cut in any simulated day.

LDECC1	Scribner Decimal C volume of a log, no allowance for taper.
LEND(KK)	The maximum ratio of on-log travel time in diameter class KK, in decimal minutes where KK = 7,15.
LHR	The hour of the day.
LOGS1	The number of logs of largest size that can be cut from a tree.
LOGS2	The number of logs of next-to-largest size that can be cut from a tree.
LOGS3	The number of logs of the second lowest size that can be cut from a tree.
LOGS4	The number of logs of smallest size that can be cut from a tree.
LOP(K)	Equal to B(K), in fixed point.
MAN	Equal to BAN, in fixed point.
MAX	The number of logs contained in the optimum log combination.
MBEG(LL)	The minimum limbing time ratio estimate in diameter class LL, in decimal hours multiplied by 100, where LL = 7,15.
MEND(LL)	The maximum limbing time ratio estimate in diameter class LL, in decimal hours multiplied by 100, where LL = 7,15.
MIN	Equal to CAN, in fixed point.
MOP(I)	Equal to A(I), in fixed point.
N	The number of a tree in the cutting order.
NCULL	Cull type.
NCUT	Same as CUT, in fixed point.
NDBHS	Same as DBHS, in fixed point.
NDECC1	Scribner Decimal C volume of a tree, in fixed point.
NHRS	Number of working hours in a simulated day.
NPR	Production rate, in trees per hour.
NTRIAL	Number of the simulated day.

OI4	International ¼" volume of a fraction of a 4-foot section.
OIA(JJ,JZ)	The number of crown/height ratios in each crown/height class JZ contained in diameter class JJ, where JZ = KBEG(JJ), KEND(JJ).
OIB(JJ,JZ,K)	The number of crown/height ratios in each crown/height sub-subclass K, where K = 1,10.
OIO(JJ)	The number of crown/height ratios in each diameter class JJ, where JJ = 7,15.
OIOX	A sample value of the crown/height ratio of a tree, in percent.
ONTOP(KK)	The number of on-log travel ratios in each diameter class KK, where KK = 7,15.
ONTOPT	A sample value of an on-log travel time ratio, in decimal minutes.
ONTP(KK,JZ)	The number of on-log ratios contained in each ratio class JZ contained in each diameter class KK, where JZ = LBEG(KK), LEND(KK).
OTHER	Proportion of trees containing dockage.
PCT	Percentage of a tree, in terms of length, infected by cull.
PLEFT	Board-foot volume (Scribner) left in the woods.
PLIMB(LL)	The number of limbing time ratio estimates contained in each diameter class LL, where LL = 7,15.
PLMB(LL,JZ)	The number of limbing time ratio estimates in each ratio class JZ contained in each diameter class LL, where JZ = MBEG(LL), MEND(LL).
PRE	The portion of a tree from its base to a particular bucking point, in feet.
PROB	Probability that a tree contains cull.
PROD	Productive time, in hours.
PROFIT	Amount of profit, in dollars.
Q	The portion of a tree, in feet, containing rot.
QP	The distance up the vertical, in feet, to the uppermost penetration of rot.

QUAN	Equivalent to $DBH^2 \cdot HHT$.
R	Equivalent to (SUMJ - 6) /4.
RAN	A random number between 0 and 1.
RDIAM(J)	Small diameter of a log, in inches.
RDECC1	Scribner Decimal C volume of a tree, in floating point.
REFUEL	Refueling time, in hours.
REV	Total revenue in a day, in dollars, based on the Scribner rule.
REV1	Revenue based on volume by weight.
REV2	Revenue based on Scribner volume, no allowance for taper.
REV3	Revenue based on Scribner volume, allowance for taper.
REV4	Revenue based on Scribner Decimal C, no allowance for taper.
REV5	Revenue based on International ¼" log volumes.
RND	A random number between 0 and 1.
ROT	Proportion of rotted trees in the stand.
RUN	Equivalent to MIN, in floating point.
RVI4	Total International ¼" board-foot volume of a log.
SHARP	Time involved in sharpening the chain, in hours.
SMAX	Equivalent to MAX, in floating point.
STD	Standard deviation of distribution of distance-between-trees.
STHT	Summation of total heights in the stand.
STMP	Diameter of the stump (o.b.), in inches.
STUMP	Diameter of the stump (i.b.), in inches.
TBUCK	Total bucking time, in hours.
TCOST	Total cost, fixed plus variable in any given hour, in dollars.
TDIST	Total distance traveled, in feet.

TFELL	Total felling time, in hours.
THETA	The angle between the vertical and the tree stem, in radians.
THRS	The total number of working hours in a simulated day.
THT	Total height of a tree, in feet.
TLAB	Total limbing and bucking time, in hours.
TLIMB(J)	Limbing time of log J, in decimal minutes.
TLMB	Total limbing time, in hours.
TMHT	Merchantable height from base of a tree to a point on the stem 5.6" in diameter.
TOTAL	Total distance up the stem from base to bucking point, in feet.
TPFT	Sample value of travel time per foot.
TRAVT	Travel time between two trees.
TRE	Portion of a tree cut off by excess bucking cuts caused by rot, in feet.
TREES	Number of merchantable trees in the stand.
TTCOST	Total cost in a simulated day, in dollars.
TTRAV	Total travel time, in hours.
TUMP	Diameter of first log (o.b.), in inches.
TV	Travel time between excess bucking cuts caused by cull type II, in hours.
TVI4	Total International ¼" board-foot volume of a tree.
TVSCB	Total Scribner board-foot volume in a simulated day.
UNPROD	Total unproductive time in a simulated day, in hours.
VBF	Total estimated board-foot volume (Scribner) in the stand.
VCOST	Variable cost, in dollars.
VDIAM(J)	Small diameter of a 4-foot section, in inches.
VI4	International ¼" volume of a 4-foot section.

VOLWT	Board-foot volume of a tree estimated by weight.
VSCB1	Scribner board-foot volume of a tree, no allowance for taper.
VSCB2	Scribner board-foot volume of a tree, allowing for taper.
WAIT	Total time taken up by waiting for trees to fall, in hours.
WALK(J)	Time to traverse a free stem, in decimal minutes.
WORK	Total time taken up by felling, limbing, and bucking, in hours.
X(I)	The set of log lengths, I = 1, JBUCK, in feet.
XLIMBT	Sample ratio estimate of limbing time.
XM	Random normal deviate.
XMHT	An individual merchantable height, in feet.
XNO(II)	Length of log II, in feet, where II = 1, MAX.
Y(J)	The sum of J uniform random numbers, where J = 1,12.

ABSTRACT

The first subsystem of the logging system, logmaking, was simulated by Monte Carlo methods on an IBM 1620 computer. Data gathered from a multitude of sources were used to construct a preliminary computer simulation "model" of the subsystem, by means of which all of the operations usually performed in the field could be done within the comforts of the office. By the use of this approach, hundreds of acres of timber could be "harvested" in just a few hours. An important advantage of the simulation technique is that the logger can make optimum decisions before he begins logging, rather than making costly remedial ones in the field.

Although the model is by no means complete, the results obtained showed that this approach to logging analysis is very promising. A few of the many different alternatives were evaluated to give potential users an indication of how simulation can be used to advantage in logging analysis. The following is a list of the different simulation runs reported in this thesis:

1. Production rate was varied.
2. Different production policies were tested.
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INTRODUCTION

I.1 Historical Development of the Problem. Logging has and will always be a major American industrial activity; Wood is used to build our homes, to make our furniture, and to construct hundreds of other things that we use in our everyday lives. This wood must all be extracted from the forest, transported to the conversion center, and processed before it can be delivered to the consumer. The extraction and transportation phases of the above cycle constitute the logging industry.

Like any other manufactured product, the market price of wood reflects the costs incurred in transforming it from raw-material to final form. In general, the higher the purchase price of a product, the higher the costs incurred in producing it, and vice-versa. Also, the higher the market price of a product the more selective people become in its use. The cost of logging is so high in the lodgepole pine country of Montana (anywhere from 35% to 60% of the cost of finished product) that many consumers are discouraged by the resultant high cost of finished product. Often, when available, other less expensive, equally-useful materials are bought as a substitute. The high cost of logging lodgepole pine can, for the most part, be attributed to the nature of the timber itself-lodgepole pine is a small-stemmed species predominating on steep slopes. Current logging methods are not designed to efficiently cope with these two adversities. A barrier to remedying this condition has been the loggers' failure to support the efforts of the few who have endeavored to develop new cost-cutting logging methods. Many times these complacent loggers, suffering the effects of the "profit-squeeze," are forced out of business soon after a short run.

To help the logger in his plight, it has been proposed by the U.S. Forest Service that a long-range program of systematic engineering analysis geared to the development of new logging methods and/or the improvement of existing methods might yield significant results. The findings of such a program might be beneficial to both the logger (who seeks more profit) and the consumer (who yearns for lower market prices). Little work of this nature has been done in the past. This thesis presents one possible way to approach this undertaking.

I.2 Selection of a Problem Area. Out of many possible problem areas, the logmaking operations (i.e. felling, limbing, and bucking) were selected for study. Among the reasons for this choice are:

(1) Logmaking is the first subsystem in the logging system. It is always a good idea to start in at the beginning. By doing this, the entire logging system can be studied in progressive stages.

(2) Logmaking represents from 20-35 percent of the total cost of logging lodgepole pine. A cost reduction in this phase will result in a sizable reduction in total cost.

(3) Logmaking is amenable to the "tools" of industrial engineering. By the nature and simplicity of these operations, one is able to pursue either the "classical" or the more recent "systems" approach.

After a problem area had been chosen, a suitable plan of attack had to be selected.

I.3 Method of Attack. In the field of industrial engineering, the emphasis in recent years has been toward the use of operations research techniques for analyzing man-machine work systems. Operations researchers, in effect, attempt to describe real-world physical situations in terms of mathematical models. The independent variables in the model can be manipulated to coincide with changes in the real world for purposes of noting the effect this alteration has on certain other variables, called the dependent variables, of practical interest in the problem.

More often than not, a wide gap exists between many of the mathematical models of operations research and the real systems whose characteristics they are intended to portray. First, a basic model is constructed by formulating a simple abstraction of the real world and expressing this abstraction mathematically. Then, by introducing a certain amount of complexity into the model in progressive stages, the simple model is modified toward realism. However, the mathematical complexities usually tend to increase at a faster rate than that at which the model approaches reality. In many instances, the mathematical model becomes unsolvable, or must be restricted by such stringent simplifying assumptions as to render the results useless. It is here that sometimes the right answers are found to the wrong problems. The mathematical abstraction may be helpful in providing insight into the functional relationship between the properties of a system and the related variables, but some other analytical means are usually needed to bridge the gap between theory and useful design procedures.

In many cases simulation can be used to advantage. Simulation takes the real system and in some sense duplicates it. In other words, simulation is the inter-connection of system components in such a way that the inter-connected system simulates, or behaves, in the same manner as the system under study.

Simulation might be considered the last step before complete