Bridged-T networks
by Nirmal Kumar Dhar Choudhury

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University

Abstract:
Bridged- T networks, though not unknown in the field of radio engineering, have received little attention. Only the resonant conditions of simple symmetrical bridged-T networks have been discussed in literature. This paper deals with bridged-T networks in general and discusses two typical circuits in detail.

Using matrices and Kirchhoff's laws a general mathematical theory of bridged-T networks is developed from consideration of a perfectly general theory applicable to any complex network. From this theory are derived expression for transmission, i.e. the ratio of output voltage to input Voltage, under no-load conditions, and the condition of null transmission of general bridged-! networks. Equations are also derived for transforming these networks (as a matter of fact any complex network) to their equivalent Pi-circuits, These equations are applied to determine the input and output impedances of the network. Two typical bridged-T circuits are then selected for investigation, and the general expressions and equations derived are applied to these particular cases to determine their resonant conditions,, transmission, phase angle, input loading and output loading. These circuits are quite general since both symmetrical and nonsymmetrical circuits are considered.

The circuit characteristics, as determined from the theory are then checked by experimental investigation. Choosing proper circuit elements, transmission and phase-shift characteristics of both circuits are determined for symmetrical and unsynmetrical cases. The experimental results are then compared with the theoretical predictions and are found to be in good agreement.

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Approved:

R. C. Seibel

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Bozeman, Montana
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Abstract.

Bridged-T networks, though not unknown in the field of radio engineering, have received little attention. Only the resonant conditions of simple symmetrical bridged-T networks have been discussed in literature. This paper deals with bridged-T networks in general and discusses two typical circuits in detail.

Using matrices and Kirchhoff's laws, a general mathematical theory of bridged-T networks is developed from consideration of a perfectly general theory applicable to any complex network. From this theory are derived expression for transmission, i.e. the ratio of output voltage to input voltage, under no-load conditions, and the condition of null transmission of general bridged-T networks. Equations are also derived for transforming these networks (as a matter of fact any complex network) to their equivalent Pi-circuits. These equations are applied to determine the input and output impedances of the network. Two typical bridged-T circuits are then selected for investigation, and the general expressions and equations derived are applied to these particular cases to determine their resonant conditions, transmission, phase angle, input loading and output loading. These circuits are quite general since both symmetrical and nonsymmetrical circuits are considered.

The circuit characteristics, as determined from the theory are then checked by experimental investigation. Choosing proper circuit elements, transmission and phase-shift characteristics of both circuits are determined for symmetrical and unsymmetrical cases. The experimental results are then compared with the theoretical predictions and are found to be in good agreement.

Introduction.

Frequency selective networks containing inductance and capacitance have been extensively discussed in literature and widely used in practice. Bridged-T networks of simple types have been in use as wave traps since perfect suppression of a single frequency can be easily obtained, though considerable dissipation is present in the components of the circuit. Rejection characteristics of antiresonant circuits or their equivalents can be improved by the addition of resistances in such a manner as to form a bridged-T network.

In bridged-T networks the component values can be so chosen as to produce perfect null at a desired frequency in either the audio or radio frequency range. Since these circuits are four terminal networks, this property of perfect balance at resonance has been utilized to a limited extent in alternating-current bridge measuring instruments for measuring inductance, capacitance, resistance, quality factor, etc. But in such use of these circuits no attention is paid to their frequency response or phase-shift characteristics; the only requirement being a good balance at the desired frequency.

The other property of bridged-T networks is their selective response over a band of frequencies. The selectivity and phase-shift in these cases can be easily controlled by variation of circuit parameters, this variation being much more flexible than in the case of antiresonant circuits. In view of this advantage bridged-T networks may be used as wave filters having the desired frequency response characteristic over the desired band of frequencies, and also as feed-back circuits in vacuum tube amplifiers.

In operation, bridged-T networks are very simple, the generator and the load being directly connected across the input and output, respectively, without requiring a coupling transformer. Moreover, no balance-to-ground operation is required, the ground terminal being common to both the generator and the load. Hence they require no Wagner earth connection. Their disadvantage is due to insertion loss since they absorb power from the input.

To the author's knowledge very little work of any investigational nature has been done on bridged-T networks. Only the resonant conditions of some simple bridged-T networks have been discussed by Tuttle.[1] In view of the possible wide application of bridged-T networks and in view of some of their inherent merits, a systematic extensive investigation on these circuits was taken up. This paper deals with:

1. A general mathematical theory of bridged-T networks.
2. Transmission and phase-shift characteristics of two typical circuits for different values of circuit Q.
3. Transmission and phase-shift characteristics of two typical circuits having circuit parameters bearing no simple relationship with each other.
4. Dependence of circuit-selectivity and phase-shift on circuit Q and degree of dissymmetry of circuit parameters.
5. Comparison of the experimental results with the theoretical work.

1. W. N. Tuttle, BRIDGED-T AND PARALLEL-T CIRCUITS FOR MEASUREMENTS AT RADIO FREQUENCIES, Proc. I.R.E, page 23, January, 1940.

The Mathematical Theory.

The theory of bridged-T networks will be developed from consideration of a very general mathematical theory,[1] which may, with requisite changes, be applied to any complex four terminal network.

Network transmission and resonance:-

The network transmission T will be defined[2] as the vector ratio of the output voltage E_2 to the input voltage E_1, under condition of no load. This condition is closely realized in practice if the load connected across the output terminals has a high impedance. For the no load condition the expression for network transmission, as derived in appendix 1a., is

$$T = \frac{E_2}{E_1} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3 + Z_3 Z_4}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3 + Z_3 Z_4 + Z_1 Z_4} \quad \ldots\ldots\ldots \quad 4.$$

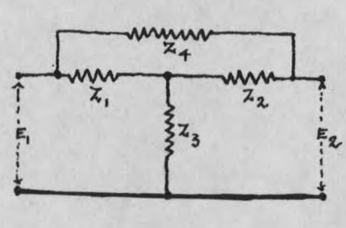

Fig. 1.

1. M. B. Reed, GENERAL FORMULAS FOR T AND Π-NETWORKS EQUIVALENTS, Proc. I.R.E., pp 897, December, 1945.

2. L. Stanton, THEORY AND APPLICATION OF PARALLEL-T RESISTANCE-CAPACITANCE FREQUENCY SELECTIVE NETWORKS, Proc. I.R.E., pp 447, July, 1946.

It may be noted that the network transmission T will be zero when the numerator of the expression for T is zero. Under this condition the whole circuit appears as an infinite impedance to the source. This condition is called resonance, since the behaviour is similar to the resonance of a parallel L-C circuit. Hence the condition of null transmission or resonance of the general bridged-T network is given by the equation

$$Z_1Z_2 + Z_2Z_3 + Z_1Z_3 + Z_3Z_4 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 5.$$

where the disposition of the component impedances Z_1, etc. is shown in Fig. 1.

Equivalent Pi-circuit.

In order to facilitate the study of some of the special characteristics of bridged-T networks, the given circuit is transformed to its equivalent Pi-circuit shown by Fig. 2. The relationship between the component impedances of the equivalent Pi-circuit and those of the original circuit, as derived in appendix 1b, is given by the following equations.

$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_2} \quad \ldots\ldots\ldots \quad 9a.$$

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_1} \quad \ldots\ldots\ldots \quad 9b.$$

$$Z_C = \frac{Z_4(Z_1Z_2 + Z_2Z_3 - Z_1Z_3)}{Z_1Z_2 + Z_2Z_3 + Z_1Z_3 + Z_3Z_4} \quad \ldots \quad 9c.$$

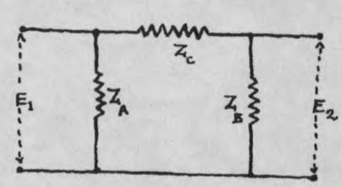

Fig. 2.

From the equivalent Pi-circuit it may also be found, as given in appendix 1c, that

$$T = \frac{E_2}{E_1} = \frac{1}{1 + Z_C/Z_B} \quad \text{.................... 11.}$$

If in this equation the values of Z_B and Z_C are substituted from Eq. 9, the expression for T is identical with that of Eq. 4, as it should be. The Eq. 11 clearly indicates that transmission T is zero when the ratio Z_C/Z_B is infinite. Since Z_B can never be zero owing to its finite component impedances, Z_C/Z_B is infinite only when the denominator of Z_C is zero, that is

$$Z_1Z_2 + Z_2Z_3 + Z_1Z_3 + Z_3Z_4 = 0, \quad \text{.................... 5.}$$

which is the same condition for null transmission as derived earlier.

Input and output loading.

Bridged-T networks place a load across the source as well as the detector. These loads can be determined from the equivalent Pi-circuit. Both the input and output loading vary with frequency in a complicated manner and are extremely difficult to determine theoretically. However, it is relatively easy to find, theoretically, the loading at resonance, since the impedance Z_C is then infinite. The input and output impedances of the circuit are Z_A and Z_B, respectively, where Z_A and Z_B are given by Eq. 9. As the frequency deviates from resonance, Z_C is no longer infinite and hence both the input and output impedances vary in a manner which could be determined experimentally.

Thus,

Input impedance at resonance = Z_A .... 12.

Output impedance at resonance = Z_B .... 13.

These are the impedances at resonance which the bridged-T circuit puts in parallel with the generator and the detector or load.

Bridged-T Network Type 1.

The first bridged-T network that was investigated is shown in Fig. 3. In this circuit the bridging arm consists of an inductance in series with a resistance, which in some cases may be the coil resistance alone.

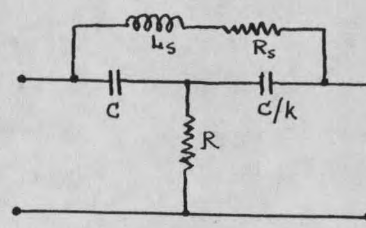

Fig. 3.

The arbitrary number k gives the degree of dissymmetry of the circuit. If k = 1, the circuit is symmetrical.[1] The two equations that the circuit must satisfy for null transmission, or resonance, are developed in appendix 2a. They are

$$R.R_s = k/(\omega C)^2 \qquad \text{.....................15.}$$

and

$$\omega L_s = \frac{1 + k}{\omega C} \; . \qquad \text{.....................16.}$$

For a symmetrical circuit the conditions reduce to

$$R.R_s = 1/(\omega C)^2$$

$$\omega L_s = 2/\omega C,$$

which are exactly Tuttle's conditions of resonance. From Eq. 16, it may

1. W.N.Tuttle, BRIDGED-T AND PARALLEL-T CIRCUITS FOR MEASUREMENT AT RADIO FREQUENCIES, Proc. I.R.E., pp 23, January, 1940.

F.E.Terman, RADIO ENGINEER'S HANDBOOK, First edition, pp 918.

be observed that the resonant frequency is the frequency at which the two capacities in series are in parallel resonance with L_s. Thus at resonance the circuit behaves as if an antiresonant circuit were placed in series with the line. Hence Eq. 16 is the frequency determining equation which fixes the resonant frequency of the circuit. If Eq. 15 is satisfied the transmission will be zero at resonance; otherwise it will simply pass through a minimum at the resonant frequency.

Circuit transmission.

The expression for circuit transmission under no load condition, as derived in appendix 2b, is

$$T = \frac{1}{1 + j\,\frac{\gamma}{1-\gamma^2}\cdot\frac{1+k}{k}\cdot\frac{1}{Q_o}} \qquad \ldots\ldots\ldots\ldots 17.$$

Hence the magnitude of T is

$$|T| = \frac{1}{\sqrt{\left[1 + \left(\frac{\gamma}{1-\gamma^2}\cdot\frac{1+k}{k}\cdot\frac{1}{Q_o}\right)^2\right]}} \qquad \ldots\ldots\ldots\ldots 18.$$

and the phase angle θ of T, i.e., the angle by which E_x leads or lags E_1 is

$$\theta = -\tan^{-1}\frac{\gamma}{1-\gamma^2}\cdot\frac{1+k}{k}\cdot\frac{1}{Q_o}, \qquad \ldots\ldots\ldots\ldots 19.$$

where

$$\gamma = \frac{f}{f_o} = \frac{\omega}{\omega_o} = \frac{\text{actual frequency}}{\text{resonant frequency}}$$

and $\qquad Q_o = \omega L_S/R_S$ = quality factor of the coil including the series resistance, if any.

These simple expressions for T and Q indicate that at resonance, when $\gamma = 1$, the transmission is zero and phase angle is $\pm 90^\circ$. For high circuit selectivity off-resonance transmission must be high, which occurs when both Q_o and k are large. It may be noted here that for a given value of C and of f_o higher values of k will automatically necessitate higher values of L_S and consequently higher Q_o. The effect is therefore cumulative towards higher selectivity. Moreover, larger values of Q_o and k will produce less phase-shift at frequencies off resonance. Hence maximum transmission at a given frequency will also correspond to minimum phase-shift. The phase-shift characteristic is such that the phase angle decreases on both sides of the resonant frequency.

There are some limitations against the use of higher values of Q_o and k. One limitation against the use of high values of k is that it makes the capacity C/k of Fig. 3 small for a given value of C. Under such circumstances the effect of stray capacity will be noticeable. The effect of stray capacity is negligible only when it is small compared with the effective capacity C/(1 + k) of the circuit.

Another limitation is that higher values of k and Q_o will increase the input and output loading as is evident from Eqs. 23 and 24. Higher input loading means shunting the generator by the network. Hence for smaller input loading both k and Q_o should be small. For low output loading Q_o should not be very large, but k should be large.

Input and output loading.

The expression for input impedance at resonance, as derived in appendix 2c, is

$$|Z_{io}| = \frac{1+k}{k} \cdot R\sqrt{1+1/Q_o^2} \qquad \ldots\ldots\ldots\ldots 23.$$

For small loading of the source $|Z_{io}|$ must be large. Hence R must essentially be large. Moreover, for low loading both k and Q_o should be small as has already been stated. Thus the values of k and Q_o should be so selected as to make a compromise between high selectivity and low input-loading.

The output impedance of the circuit at resonance, as found in appendix 2c, is

$$|(Z_{out})_o| = (1+k)\, R\sqrt{1+1/Q_o^2} \qquad \ldots\ldots\ldots\ldots 24.$$

For low output loading also R must be large. Small values of Q_o and large values of k are desirable.

It may therefore be mentioned that the values of k and Q_o should be carefully chosen so as to make a reasonable compromise between network selectivity and input and output loading.

Effect of stray capacity.

It should be noted that the capacity of the junction point O, Fig. 4, with respect to ground is not without effect, since the impedance level of the point O at resonance is high. The capacity of the points A and B with respect to ground are across the

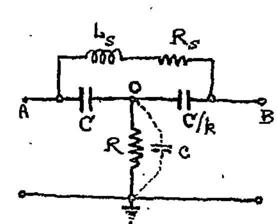

Fig. 4

source and the detector, respectively, and hence do not affect the circuit behaviour. If the effect of c is considered, the conditions of resonance are altered as shown in appendix 4a. They are

$$R_s R_S = k/(\omega C)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 15.$$

and

$$\omega L_S = \frac{1+k}{\omega C}\left[1 + \frac{kc}{(1+k)C}\right]. \qquad \ldots\ldots\ldots\ldots\ldots 40.$$

These equations show that while the first condition remains unaltered the resonant frequency given by the second condition is slightly increased by the term $kc/(1+k)C$. In order that the effect of c will not disturb the circuit behaviour, c must be very small compared with C. The effective capacity in presence of c is given by

$$C' = \frac{C^2}{(1+k)\,C + kc}, \qquad \ldots\ldots\ldots\ldots\ldots 41.$$

where kc is the correction term, and this should be small compared with $(1+k)C$.