Cooling by night radiation from water with evaporation and convection

by Gregory Nixon Cunniff

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University

Abstract:
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An experimental test of the proposed equations showed agreement between the predicted and experimentally measured heat transfer.

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Signature Gregory A. Cunniff

Date October 28, 1970

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NOMENCLATURE

Symbol	Description
a	empirical constant
b	empirical constant
c	specific heat at constant pressure, Btu/(lb_m °F)
C	cloud cover in tenths
d	mass diffusion coefficient, ft^2/sec
$\dot{E}''$	evaporative mass flux, lb_m/(sec ft^2)
E_b	black body emmisive power, Btu/(sec ft^2)
g	acceleration of gravity, ft/sec^2
G	irradiation, Btu/(sec ft^2)
Gr	Grashof number, $Gr = L^3 g \beta (t_o - t_\infty)/\nu^2$
h	cloud height in thousands of feet
i	enthalpy, Btu/lb_m
i_{fg}	heat of vaporization, Btu/lb_m
I	intensity of radiation, Btu/(sec ft^2 solid angle)
J	radiosity, Btu/(sec ft^2)
k	thermal conductivity, Btu/(sec ft^2 °F/ft)
k_λ	monochromatic absorption coefficient
L	length of pond, ft
Le	Lewis number, $Le = d/\alpha$
$\dot{m}$	vapor concentration, lb_m/ft^3

Symbol	Description
p	pressure, lb_f/ft^2
p_v	vapor pressure, millibars
Pr	Prandtl number, $Pr = \nu/\alpha$
$\dot{q}''$	heat flux, Btu/(sec ft^2)
$\vec{\dot{q}}''$	heat flux vector, Btu/(sec ft^2)
$\dot{q}''_{o_{net}}$	net loss of radiant energy, Btu/(sec ft^2)
$\vec{\dot{q}}''_R$	radiant heat flux vector, Btu/(sec ft^2)
$\dot{q}''_{o_{Total}}$	total heat flux, Btu/(sec ft^2)
Re_L	Reynolds number, $Re = u_\infty L/\nu$
Sc	Schmidt number, $Sc = \nu/d$
t	temperature, °F
T	absolute temperature, °R
u	velocity in x-direction, ft/sec
v	velocity in y-direction, ft/sec
w	velocity in z-direction, ft/sec
x	distance from leading edge, ft
y	vertical distance from water surface, ft
z	distance parallel to leading edge

Greek Symbols	Description
α	thermal diffusivity, $\alpha = k/\rho c$, ft^2/sec
β	thermal expansion coefficient, 1/°R
β_λ	monochromatic extinction coefficient

Greek Symbols	Description
$\frac{D}{D\theta}$	substantial derivative, $\frac{D}{D\theta} = \frac{\partial}{\partial\theta} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$
∇	del operator, $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$
ε	emmisivity
ε_M	eddy diffusivity for momentum, ft^2/sec
ε_H	eddy diffusivity for heat, ft^2/sec
ε_D	eddy diffusivity for mass, ft^2/sec
γ_λ	monochromatic absorption coefficient
μ	viscosity, $lb_m/(sec\ ft)$
ν	kinematic viscosity, ft^2/sec
ϕ	dissipation function (10)
ρ	fluid density, lb_m/ft, (eq. 1)
ρ	reflectivity (eq. 17)
σ	Stefan-Boltzman constant, Btu/(sec ft^2 $°R^4$)
θ	time, sec

Subscripts	Description
∞	refers to conditions in the free stream
j	refers to component j of the mixture
L	based upon the length of pond
λ	refers to monochromatic conditions

Subscripts	Description
o	refers to conditions at the water-air interface
x	based upon distance from the leading edge

ABSTRACT

A mathematical expression of the heat transfer involved in the cooling of a pond of water exposed to the night sky under varying wind velocity, air temperature, vapor concentration, and cloud cover and cloud height was obtained from the turbulent boundary layer equations, and an empirical equation was used to predict the radiant energy exchange between the pond and the night sky.

An experimental test of the proposed equations showed agreement between the predicted and experimentally measured heat transfer.

INTRODUCTION

Maintaining a habitable environment is one of the prime considerations in the design and use of any building or fallout shelter.

In the analysis of a fallout shelter, an acceptable environment with respect to gas concentrations and effective temperature can sometimes be maintained by ventilation with outside air only (18). However, in a large number of shelters and in most structures not under emergency conditions another means of conditioning the space must be found.

At the present time there are only a few practical and economical systems that can be used to condition a structure. Among these are (18):

1. Mechanical air conditioning;
2. Well water cooling;
3. Evaporative cooling;
4. Dehumidification by means of a chemical dessicant;
5. Cooling by night radiation from water with evaporation and convection (sometimes referred to as natural air conditioning).

The first system is the most reliable, but its cost for use in a shelter that one hopes is never used is difficult to justify. This system is also expensive in any commercial application. Well water cooling can also be expensive due to the cost of drilling. In addition, chemical treatment of the water is frequently required. Also, maintaining a consistent flow rate is often not possible. Evaporative cooling is not very effective in humid regions and requires considerable maintenance. Dehumidification lowers the humidity, but if the resulting latent heat that is converted to sensible heat is not removed the effective temperature can actually be increased rather than decreased.

The last system, cooling by night radiation from water with evaporation and convection, can be utilized by exposing a pond of water to the night sky and using the cooled water to condition a building or shelter throughout the day. The ponds could be roof ponds as proposed by Hay and Yellot (8) or for an underground shelter, water could be placed in an above-ground pond. Thus, if the heat transfer between a pond and the night sky could be quantitatively predicted this system would be very economical in comparison to standard air conditioning systems.

The purpose of this thesis is then to develop a means to quantitatively predict the heat transfer from a pond of water exposed to the night sky.

LITERATURE REVIEW

Several investigators have attempted to use night radiation to air condition a residence. Yanagimachi (26) has built several houses in Japan that use solar energy for heating and night radiation for cooling. Bliss (2) used these same principles in a building in Tucscon, Arizona. Thomason (21) also constructed two solar houses in Washington, D. C. He noted the decreased heat transfer rates under humid and cloudy conditions. The other investigators also noted some difficulties in radiation cooling.

Recently, Hay and Yellot (6, 7, 8, 25) reported success with a system employing roof ponds and movable insulation. Their system is also very economical in comparison to the standard methods of air conditioning. Their experimental work was conducted in Phoenix, Arizona. In addition, Yellot (24) demonstrated the effectiveness of roof cooling with intermittent water sprays.

In these studies the defficiency in performance of the systems is a result of not being able to quantitatively predict the heat transfer rates under existing meterological conditions.

There have been several studies of atmospheric radiation in recent years. An analysis of the atmospheric heat balance was conducted by London (11). In the Lake Hefner Studies (22), an empirical equation was developed that related long wavelength atmospheric radiation to local water vapor pressure, air temperature, and cloud cover and height. The report indicates that the emissivity of water is independent of water temperature and composition and is given as 0.970 $\pm$ 0.005.

Evaporation studies have been carried out by a number of investigators. Deardorff (5) and Sverdrup (20) attempted to correlate evaporation rates as a linear relationship of wind velocities. Millar (13), Hickox (9), Powell (15), and others found a nonlinear relationship between evaporation and wind velocity. Powell and Hickox also noted that evaporation rates are almost unaffected by composition of the water or the type of surface at the water air interface as long as the surface is saturated. Roll (16), Pasquill (14), and Sverdrup (20) also discuss the equality of the eddy diffusivities of heat and mass transfer in finding evaporation rates. Marciano and Harbeck (22), Ahmsbrak (1), and Budyko et al (4) have used a ratio derived by Bowen (3) that expresses the ratio of heat loss by convection to that by evaporation. Yamamoto (26) conducted evaporation studies from pans. He compared his experimental results to an approximate solution of the boundary layer equations and found them to be in good agreement.

Convection heat transfer has been treated in a manner similar to that described for evaporation.

THEORETICAL ANALYSIS

In considering the energy transport between a pond of water and the night sky, there will in general be momentum, temperature, and moisture boundary layers on the surface of the water as shown in Figure 1.

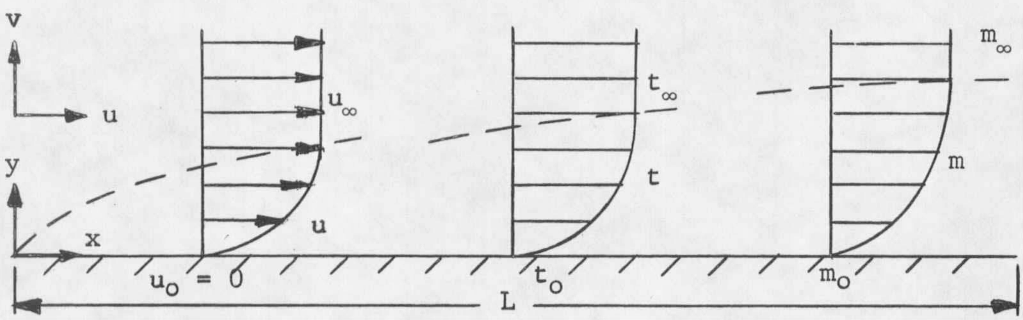

Figure 1. Boundary layer development

The governing conservation of energy equation applied to a moving fluid element may be expressed as (19):

$$\rho \frac{Di}{D\theta} = - \nabla \cdot \vec{\dot{q}}'' + \nabla \cdot \left(\sum_j d_j i_j \nabla m_j \right) + \mu\phi + \frac{Dp}{D\theta} + s \qquad (1)$$

where the left-hand side of the equation represents the rate of change of internal energy of the element. The first term on the right-hand side is the net heat addition to the element where $\vec{q}$ is the heat flux vector. For a fluid that absorbs, emits and scatters radiation, the heat flux vector will be given by

$$\vec{\dot{q}}'' = - k \nabla t + \vec{\dot{q}}''_R$$

where the $k\nabla t$ term represents thermal conduction and $\vec{\dot{q}}''_R$ is the radiant heat flux vector. The second term on the right-hand side of equation (1) is the transport of energy as a result of the enthalpy of each component's undergoing diffusion. The third term, $\mu\phi$, is the energy dissipated by viscous action that is transformed into heat, where ϕ is the dissipation function. The $\frac{Dp}{D\theta}$ term is the reversible compression work done on the element, and the last term represents any source functions such as internal heat generation.

The structure of the energy equation indicates that the transport mechanisms are coupled, i.e., one process may affect the remaining transport processes. By assuming the transport mechanisms to act independently the resulting equations can be solved. The governing differential boundary layer equations from Schlichting (17) and Kays (10) for steady state, incompressible, constant property turbulent flow are:

Conservation of mass,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

Conservation of momentum,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left[(\nu + \varepsilon_M) \frac{\partial u}{\partial y} \right] \qquad (4)$$

Conservation of energy,

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \frac{\partial}{\partial y} \left[(\alpha + \varepsilon_H) \frac{\partial t}{\partial y} \right] \qquad (5)$$

Diffusion,

$$u \frac{\partial m}{\partial x} + v \frac{\partial m}{\partial y} = \frac{\partial}{\partial y} \left[(d + \varepsilon_D) \frac{\partial m}{\partial y} \right] \qquad (6)$$

where ε_M, ε_H and ε_D are respectively the eddy diffusivities of momentum, heat, and diffusion. The boundary conditions for constant free stream velocity, temperature, and vapor concentration are:

at $y = 0$:

$u = 0$

$v = 0$

$t = t_o$

$m = m_o$

as $y \to \infty$:

$u \to u_\infty$

$t \to t_\infty$

$m \to m_\infty$

at $x = 0$:

$t = t_\infty$

$m = m_\infty$.

The solution to (5) for the local heat transfer rate in the Prandtl number range of gases is given by Kays (10) as

$$\dot{q}''_{o_x} = 0.0295\ \rho c u_\infty \Pr^{-0.4} \mathrm{Re}_x^{-0.2} (t_o - t_\infty) \tag{7}$$

Since the differential equations for energy and diffusion are the same their solutions will also be the same. Thus, for the local evaporation rate

$$\dot{E}''_{o_x} = 0.0295\ u_\infty \Pr^{-0.4} \mathrm{Re}_x^{-0.2} (m_o - m_\infty) \tag{8}$$

The corresponding mean rates of heat transfer and evaporation are

$$\dot{q}''_o = 0.0369\ \rho c u_\infty\ Pr^{-0.4}\ Re_L^{-0.2}\ (t_o - t_\infty) \qquad (9)$$

$$\dot{E}''_o = 0.0369\ u_\infty\ Pr^{-0.4}\ Re_L^{-0.2}\ (m_o - m_\infty) \qquad (10)$$

Equation (10) for the mean rate of evaporation is in very good agreement with the work of Yamamoto (26) who correlated data by Millar (13) and presents the following equation:

$$\dot{E}''_o = 0.042\ u_\infty\ Re_L^{-0.2}\ (m_o - m_\infty) \qquad (11)$$

If the Prandtl number for air is taken as 0.72 equation (10) reduces to

$$\dot{E}''_o = 0.0421\ u_\infty\ Re_L^{-0.2}\ (m_o - m_\infty) \qquad (12)$$

Energy transport due to natural convection must also be considered. McAdams (12) recommends the following equations for the mean heat transfer rate from a horizontal plate for natural convection

For $10^5 < Gr\ Pr < 2 \times 10^7$

$$\dot{q}''_o = 0.54(k/L)\ (Gr\ Pr)^{0.25}\ (t_o - t_\infty) \qquad (13)$$

and for $2 \times 10^7 < Gr\ Pr < 10^{10}$

$$\dot{q}''_o = 0.14\ (k/L)\ (Gr\ Pr)^{0.25}\ (t_o - t_\infty) \qquad (14)$$

The mean rate of evaporation due to natural convection is given by Yamamoto (26) as

$$\dot{E}''_o = 0.525\ (d/L)\ (Gr\ Sc)^{0.25}\ (m_o - m_\infty) \qquad (15)$$

The two mechanisms of energy transport, forced and natural convection, are coupled, but again they will be treated independently.

The transport of energy in the absence of wind will be neglected since energy transport by molecular diffusion is several orders of magnitude less than energy transport by other mechanisms (22). During the experimental studies, observations of "zero" wind were rare and never lasted for more than a few minutes. In addition, the threshold velocity of the anemometer used in the experimental tests was about 0.4 ft/sec. so that a recording of "zero" wind indicates only a velocity less than 0.4 ft/sec. and not necessarily truly zero velocity. Therefore, there is sufficient reason to ignore the case of zero wind.

In considering if the boundary layer is turbulent or laminar, Yamamoto's (26) analysis of Millar's (13) data for evaporation indicated that the turbulent solution fit the data for a Reynolds number as low as 10,000. Yamamoto's own data showed a transition Reynolds number from laminar to turbulent flow of 50,000. Yamamoto attributed Millar's results to the fact that the rim of the water container used in Millar's experiment was higher than the water surface whereas Yamamoto used a saturated blotter mounted evenly with the edge of the evaporating pan. Thus, the turbulent solution would appear to fit Millar's data for a lower Reynolds number, because of the disturbance caused by the rim. The ponds used in the experimental tests conducted by the author also had a rim above the water surface as would be expected in a practical design. The lowest Reynolds number observed by the author for a fifteen minute average of wind velocity of 1.37 ft/sec. and a pond length of 5 ft. was 38,000. Thus, only the turbulent solution to the boundary layer equations was used in examining the experimental data.

The net exchange of radiant energy as given by Wiebelt (23) can be found by taking an energy balance on an imaginary plane just above the water surface as shown in Figure 2. The incoming radiation or irradiation is designated by G and the radiation leaving the surface or radiosity is designated as J.

The net rate of radiant energy leaving the surface is then given by

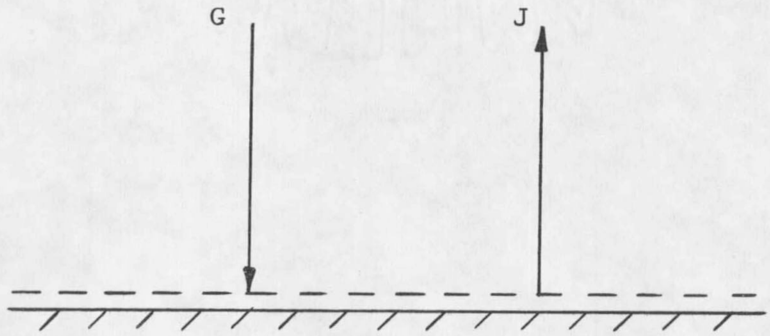

Figure 2

$$\dot{q}''_{o_{net}} = J - G \quad (16)$$

The radiosity is given by

$$J = \rho G + \varepsilon E_b \quad (17)$$

where ρ is the reflectivity, given by one minus the emissivity, ε, and E_b is the black body emissive power of the surface as given by the Stefan-Boltzman law