Natural convection heat transfer between a fixed array of cylinders and its cubical enclosure

by Gordon Crupper

A thesis submitted in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Mechanical Engineering
Montana State University

Abstract:
Natural convection heat transfer from a fixed array of four isothermal , heated cylinders to an isothermal, cooled cubical enclosure was experimentally investigated for both a horizontal and vertical position of the array. The cylinders were arranged in a square array and four test fluids (air, water, 99% aqueous glycerin, and a silicone oil) yielding Prandtl numbers in the range of 0.7 to 3.1 x 10^4 were used with each geometric position.

The heat transfer results are presented in correlation equations for each fluid and geometry as well as an overall correlation utilizing all data. Several equation forms were used in which the Nusselt number was correlated as a function of combinations of the Rayleigh, Prandtl, and Grashof numbers. The best equation found to correlate all data with a single parameter was NuB = 0.286 RaB 0.275 using the boundary layer length as a characteristic dimension. This equation provided results with a 9.30% average deviation. The results were compared with equations developed for single bodies to cubical and spherical enclosures for the same Prandtl number range. Correlations of the data are provided in tabular form for all fluids and geometries.

Several geometric effects were observed. The vertical configuration convected less heat than the horizontal while a rotation about the vertical axis for each of these configurations had negligible effect. The resulting decrease in heat transfer for the vertical configuration was attributed to a complex interaction between the boundary layer length, the flow patterns which resulted from the geometry, and the cross sectional area exposed to the upward flow. As the Prandtl number of the fluid media increased it tended to damp out the geometric effects. Flow visualization studies and temperature profiles were used to aid in evaluating the geometric effects.

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by

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A thesis submitted in partial fulfillment
of the requirements for the degree

of

MASTER OF SCIENCE

in

Mechanical Engineering

Approved:

Chairperson, Examining Committee

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Bozeman, Montana

May, 1977

ACKNOWLEDGMENT

The author expresses his appreciation to all those who have aided in his work. Special appreciation is extended to Dr. Robert O. Warrington, Jr. for his guidance and counsel during the preparation of this thesis and also to Dr. J. A. Scanlan, Dr. T. C. Reihman, and Dr. R. L. Mussulman for their advice and instruction. The author is especially appreciative of the patience and understanding of his wife and children.

The work in this thesis was supported by the United States Army under the Advance Degree Program for ROTC Instructor Duty.

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ABSTRACT

Natural convection heat transfer from a fixed array of four isothermal, heated cylinders to an isothermal, cooled cubical enclosure was experimentally investigated for both a horizontal and vertical position of the array. The cylinders were arranged in a square array and four test fluids (air, water, 99% aqueous glycerin, and a silicone oil) yielding Prandtl numbers in the range of 0.7 to 3.1 x 10^4 were used with each geometric position.

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$$Nu_B = 0.286\ Ra_B^{0.275}$$

using the boundary layer length as a characteristic dimension. This equation provided results with a 9.30% average deviation. The results were compared with equations developed for single bodies to cubical and spherical enclosures for the same Prandtl number range. Correlations of the data are provided in tabular form for all fluids and geometries.

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NOMENCLATURE

Symbol	Description
A	Total heat exchanger area
A_c	Total area of the cylinders
B	Distance traveled by the boundary layer on the inner body
c_p	Specific heat at constant pressure
f	Denotes function
g	Acceleration of gravity, 9.81 m/sec^2
Gr_x	Grashof number, defined by equation (2.2)
Gr_x^*	Local Grashof number, $g\beta qx^4/k\nu^2$
h	Average heat transfer coefficient, $Q_{CONV}/A_c\Delta T$
k	Thermal conductivity
L	Gap width or hypothetical gap width, R_o-R_i
l	Length of straight cylindrical section
Nu_x	Nusselt number, hx/k, where x is any characteristic dimension
Pr	Prandtl number, $c_p\mu/k$
Q_{COND}	Heat transfer by conduction
Q_{CONV}	Heat transfer by convection
$Q_{RADIATION}$	Heat transfer by radiation
R	Local position of thermocouple probe
Ra_x	Rayleigh number, $\rho^2 g\beta(T_i-T_o)x^3 c_p/\mu k$, where x is any characteristic dimension

Symbol	Description
Ra_x^*	Modified Rayleigh number, $Ra_x(L/R_i)$
$R_{i(o)}$	Inner(outer) body hypothetical radius equal to the radius of a sphere having an equal volume
R_{ND}	Dimensionless radius defined as $R/R(\theta)$
$R(\theta)$	Length of probe for radial position θ
T	Local temperature
T_f	Film temperature, $T_f = (T_w + T_\infty)/2$
T_{ND}	Dimensionless temperature defined as, $(T-T_0)/(T_I-T_0)$
$T_{I(0)}$	Temperature of the inner(outer) body
T_w	Temperature at the outer surface of any body
$TMVL_{I(0)}$	Temperature of the inner(outer) body as measured by the thermocouple in millivolts
T_s	Temperature of the support sphere
T_∞	Temperature of the ambient, far from the inner body
x	Any characteristic dimension
β	Thermal expansion coefficient
μ	Dynamic viscosity
ν	Kinematic viscosity
Π	Ratio of circumference of a circle to the diameter, 3.14159
θ	Temperature probe angular location as defined on page 15
ρ	Density

CHAPTER 1

INTRODUCTION

Investigation in the area of heat transfer by natural convection within enclosures has increased dramatically in the last decade. This has been in response to advances in electrical packaging, solar heating technology, and increased demands for handling nuclear waste. Most recently the demands for energy conservation have increased the importance for a better understanding of natural convection within enclosures.

The purposes of this study are to determine the heat transferred between a set of four isothermal, heated cylinders (multiple bodies) and an isothermal, cooled cubical enclosure, to determine the effect of the position of the tubes within the enclosure, and to compare the results with the findings of previous studies on heat transfer from single bodies to the same form of enclosure. Four fluids and inner body positions are utilized in studying the heat transferred. The body positions include the set of cylinders in both a horizontal and vertical position and include a 45 degree rotation about the vertical axis for each position. The fluids used are air, water, 99 percent glycerin, and a Dow-Corning 20 cs fluid. The fluids provide a Prandtl number range of 0.7 to 31,000.

Although the heat transfer problem is coupled with a fluid-flow problem, the intent of this study is directed primarily toward the heat transfer problem. Flow visualization and temperature profiles within the enclosure are obtained to aid in evaluating the heat transfer.

CHAPTER II

LITERATURE REVIEW

Heat transfer by natural convection is a field in which several subdivisions are evident. The majority of the initial studies dealt solely with heat transfer from common shapes (e.g. plates, cylinders, spheres, etc.) to a sufficiently large surrounding fluid medium as to be termed an infinite atmosphere. Further developments found a need to study the natural convective heat transfer from one body to an enclosing body through a fluid medium that could not be treated as infinite. Both of the major subdivisions mentioned above have experienced emphasis on the separate areas of heat transfer from single bodies, series of like shapes usually symmetrically placed, and other geometric effects as well as specialized fluid properties. For purposes of clarity this discussion will be covered in two phases. These are (1) natural convection to an infinite atmosphere, and (2) natural convection in enclosures.

NATURAL CONVECTION TO AN INFINITE ATMOSPHERE

There have been many major developments in research into the heat transferred from a single geometric shape to an infinite atmosphere. Several texts provide an excellent summary of these advances. McAdams [1] provides a comprehensive review of earlier results of investigations into natural convection heat transfer from single horizontal cylinders to an infinite atmosphere with Rayleigh numbers ranging from 10^{-4} to 10^{9}. Both Gebhart [2] and Holman [3] provide excellent references for the heat transfer from horizontal and vertical cylinders with Rayleigh numbers from 10^{-5} to 10^{12}, well into the turbulent region.

In nearly all cases three dimensionless parameters are utilized to correlate the heat transfer by natural convection. The resulting equation is of the form

$$Nu_x = f(Gr_x , Pr) \tag{2.1}$$

in which the dimensionless parameters are defined as

$$Gr_x = \frac{g\beta(T_w - T_\infty) x^3}{\nu^2} \tag{2.2}$$

$$Nu_x = \frac{hx}{k} \tag{2.3}$$

and

$$Pr = \frac{c_p\mu}{k} \tag{2.4}$$

where x is a characteristic length. Simplification of the correlating equation has been accomplished by combining the Grashof number (Gr_x) and the Prandtl number (Pr) to form a single dimensionless parameter, the Rayleigh number, as follows:

$$Ra_x = Gr_x \cdot Pr \quad . \tag{2.5}$$

The equation form most commonly utilized to correlate heat transfer data is

$$Nu_x = C_1 \ (Gr_x Pr)^{C_2} \qquad (2.6)$$

Fujii and Fujii [4], Roy [5], and Churchill and Ozoe [6] present studies to express C_1 explicitly as a function of the Prandtl number in the equation

$$Nu_x = C_1 \ (Gr_x^* \ Pr)^{1/5} \qquad (2.7)$$

for laminar flow along vertical surfaces with uniform heat flux. The results were satisfactory, but are limited in application to this particular situation.

Lienhard [7] presented theoretical arguments that regardless of shape, the overall heat transfer correlation in the laminar range can be expressed as

$$(Nu_x/Ra_x)^{1/4} \simeq 1/2 \qquad (2.8)$$

for wholly immersed isothermal bodies. He recommended that this relationship be used with the length of travel of the boundary layer as the characteristic length (x) to achieve results varying only by a few percent.

Eckert and Soehngen [8] researched the effects of a vertical series of horizontal cylinders upon each other. Utilizing 2.231 cm diameter, isothermal cylinders, they observed that when two cylinders were utilized, one directly over the other at a distance of four diameters, there was no change in the Nusselt number of the lowest cylinder as opposed to a single cylinder. However, the upper cylinder evidenced a reduction in heat transferred. The resulting Nusselt number of the upper cylinder was 87 percent of the lower cylinder. Similar results were achieved when the series was increased to three cylinders. The Nusselt numbers were 100, 83, and 65 percent, respectively, from bottom to top cylinders. When the middle cylinder was moved laterally out of line by one half a diameter, the Nusselt number of that cylinder rose to 103 percent of the bottom cylinder while that of the top cylinder was 86 percent.

Eckert and Soehngen reasoned that the effect of the warmer wake around the upper tubes reduced the heat transferred since the temperature differential had decreased. Conversely, the staggered tube was not in the natural convection plume and an induced fluid movement of cooler air resulted in a greater capacity to transfer heat. Analysis of the method of heat transfer from the top cylinder in the staggered array was more complex as it was effected by both the induced wake and the convective plume.

Leiberman and Gebhart [9], in 1968, investigated the interactions between the natural convective flows of several closely spaced surfaces by using long, horizontal wires in a parallel array at several spacings and inclinations. Other investigations which include temperature and velocity measurements about a line source have been conducted by Brodowicz and Kierkus [10], and Forstrom and Sparrow [11]. All of the above investigators found that cool air is induced into the plume from the sides and below the source.

The influence of tube spacing and array on natural convection heat transfer coefficients for horizontal tube bundles has been determined experimentally by Tillman [12]. Five pitch to diameter (P/D) ratios were studied for a square array of 16 tubes, and four (P/D) ratios were studied for a staggered array of 14 tubes. The following equations were developed to correlate the data:

$$Nu_f = 0.057\ (Gr\ Pr)_f^{0.5} \tag{2.9}$$

for square arrays, and

$$Nu_f = 0.067\ (Gr\ Pr)_f^{0.5} \tag{2.10}$$

for staggered arrays, where the thermal properties of the fluid were evaluated at the film temperature

$$T_f = \frac{T_2 + T_\infty}{2} \tag{2.11}$$

except β which was evaluated at the ambient temperature. The characteristic dimension was defined as

$$D_h = \frac{4\ A_c x}{A} \tag{2.12}$$

in which A_c is the flow cross section area, A is the total heat exchanger area, and x is the height of the tube bundle. Conclusions drawn by Tillman indicate that tube spacing has more effect on the heat transfer than the type of array. Also, an optimum spacing for each array was determined for separate temperature differentials.

Natural convection from vertical tube bundles has been researched by Davis and Perona [13]. They utilized 42 tubes having an outside diameter of 1.58 cm and a length of 1.23 cm arranged in 7 staggered rows of 6 tubes per row. Experimental results were compared with theoretical results obtained from a finite difference solution of the problem. The results obtained from both experimental data and theoretical calculations correlated extremely well with the exception of the values in the region where the end support system had an apparent influence.

NATURAL CONVECTION IN ENCLOSURES

Natural convection in enclosures, in this discussion, will be restricted to heat transferred from one body or bodies completely enclosed by a second body. With this restriction imposed, one finds an area of investigation that has barely been initiated. The primary emphasis to date has been with single bodies geometrically centered within a second enclosing body.

As discussed previously, several dimensionless parameters were utilized in correlating the heat transfer data for natural convection to an infinite atmosphere. Although these parameters remain valid for enclosures, a third element was found helpful, if not necessary, which alters equation (2.1) to the form

$$Nu_x = f(Gr_x , Pr , N_d) \qquad (2.13)$$

where N_d is a ratio of characteristic dimensions.

There are several sources of information on heat transfer from a single body to its enclosure. Most recently, Warrington [14] developed equations from experimental data which are useful in correlating heat transfer between inner bodies utilizing spheres, cubes, and cylinders of varying sizes to both spherical and cubical enclosures using data obtained by Bishop [18], Scanlan, et al [19] and other major investigations, as well as considerable data of his own. The following correlation:

$$Nu_L = 0.396Ra_L^{0.234} \left(\frac{L}{R_i}\right)^{0.496} Pr^{0.162}$$

was found to adequately describe the heat transferred for 9 different sizes of spherical, 11 cylindrical, and 6 cubical inner bodies when enclosed by a spherical or cubical outer body. Several different fluids were utilized to extend the Prandtl number range from 0.706 to 13,800. The average percent deviation was 13.50%. Correlations are also given for separate combinations of fluids, inner, and outer bodies. Warrington [14] also provides an exhaustive review of the literature pertaining to heat transfer from a single geometric shape to its enclosure.

Kuehn and Goldstein [15] analytically developed a correlating equation for horizontal concentric cylinders. Utilizing the experimental results of eight studies by other authors in which the ratio of outer to inner cylindrical diameters (D_o/D_i) was 2 and 3, and a Prandtl number of 0.7, they found that the analytical model fit the data within a few percent. Included in this study were two major extensions to the concentric cylinder problem. First was an analytical determination of the gap width at which the solution was the same as that for a single cylinder. It was found that when $D_o/D_i > 360$ at $Ra_{Di} = 10^7$ and $D_o/D_i > 700$ at $Ra_{D_i} = 10^{-1}$, the heat transfer coefficient was within 5% of that for a single cylinder. The second extension of their analytical model was the solution to the problem of multiple cylinders contained in a single cylindrical enclosure. For this case the correlating equation became

$$Nu = \frac{2N}{\ln[1+2/Nu_i] - N \ln[1-2/Nu_o]} \tag{2.15}$$

where the Nusselt number of the inner tube boundary conditions is

$$Nu_i = [(0.518Ra_{D_i}^{1/4}[1+(\frac{0.0559}{Pr})^{3/4}]^{-5/12})^{15}+(0.1Ra_{D_i}^{1/3})^{15}]^{1/15} \tag{2.16}$$

and the outer tube Nusselt number is

$$Nu_o = \{([(\frac{2}{1-e^{-0.25}})^{5/3}+(0.587Ra_{D_o}^{1/4})]^{3/5})^{15}+(0.1Ra_{D_o}^{1/3})^{15}\}^{1/15} \quad (2.17)$$

for a set of N inner cylinders. This equation is expected to be valid for small numbers of inner tubes. Although the authors are not specific as to the term small, they leave the impression that more than 4 or 5 cylinders would result in error. No other heat transfer data could be found for this configuration and, therefore, it has not been tested for validity.

The study of vertical tube bundles in enclosures has been mainly within the low Prandtl number range (liquid metals). Dutton and Welty [16] researched the effects of cylinder spacing on the heat transferred from a vertical rod bundle in a vertical cylindrical enclosure utilizing liquid mercury as the fluid medium, Pr = 0.023, with a uniform heat flux applied to the rod bundle. Axial and radial temperature distributions were also measured. Results of this study showed that there is a strong dependence on cylinder spacing and that the equation form for use in studies at low Prandtl numbers should involve the dimensionless parameter product defined as

$$Gr_x^* \cdot Pr \equiv (g\beta q x^4/k\alpha^2) \quad (2.18)$$

where q is a uniform heat flux.

The study of natural convection in enclosures as evidenced by the literature has increased in the last decade. That portion which includes the use of multiple bodies still lacks appreciable knowledge. This study is intended to provide an extension of this area.

CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURE

EXPERIMENTAL APPARATUS

The apparatus for this investigation consisted of a water jacketed cubical outer body, cooling system, power source, and a four cylinder inner body with supporting elements. One outer body system was used to obtain heat transfer data and temperature profiles while a separate system was utilized to photograph flow patterns. The assembled outer body and peripheral components are shown in Figure 3.1.

The outer body in which tests were conducted to determine the heat transfer was constructed from 1.27 cm thick, type 6061 aluminum with an inner 26.67 cm cubical chamber. This was a jacketed design consisting of a separate 3.175 cm wide rectangular channel for each face of the cube. Access to the test chamber was provided through a 25.4 cm removable circular plate on the top inner face and a completely removable outer face. A closed system consisting of a chiller, pump, and storage reservoir provided water to cool the outer body.

The flow rate of cooling water through each channel was controlled by a valve which fed four inlet and outlet ports. This arrangement assured uniform flow along the entire face and also allowed the temperature of each face to be controlled independently in order to achieve an isothermal outer body. The temperature of the inner wall was monitored by 34 copper constantan thermocouples epoxied 0.3175 cm from the inner

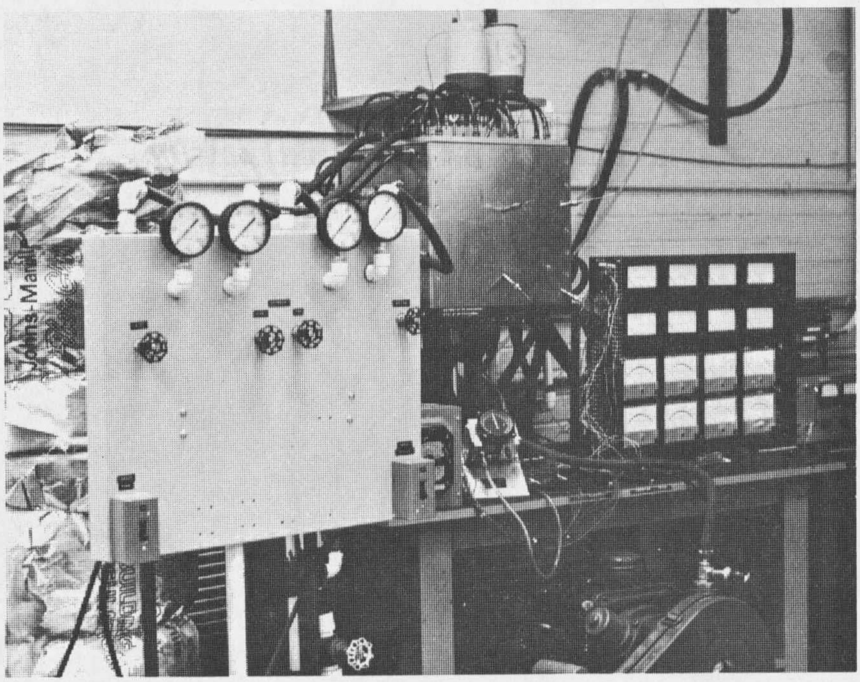

Figure 3.1 Heat Transfer Apparatus

face. The thermocouples for each face of the cube were connected in parallel which provided an average temperature for each face.. By controlling the flow of the cooling water, all faces could be maintained within 2 K.

To obtain temperature profiles within the test chamber, the outer body was designed with nine thermocouple ports. These consisted of one common and four additional ports on each of two separate axis. One axis was on a vertical plane through the center of the cube while the other was the vertical plane through the edge of the cube. The five ports in each axis were at 0°, 34°, 80°, 120°, and 160° measured downward from the top center vertical axis of the body. Each thermocouple port had a center tube which moved through a fixed port tube. The center tube was a 0.1587 cm diameter stainless steel tube which carried the copper-constantan thermocouple epoxied to the inner end. The outer sleeve was a 1.016 cm diameter stainless steel tube threaded into the inner jacket wall, and sealed in both jacket walls with rubber O-rings. The thermocouple lead tube was sealed within the sleeve by a Conax fitting attached to the outer end. A vernier caliper was modified to attach to the Conax fitting while the thermocouple tube was affixed to the sliding scale. This permitted the location of the thermocouple to be established with 0.0025 cm.

The outer body used for flow visualization was nearly identical to the one previously described. The major difference was that the