



The cascode amplifier  
by Glenn B Coughlan

A THESIS Submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering  
Montana State University  
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**Abstract:**

The primary purpose of this thesis was to determine the characteristics of the cascode amplifier and to develop equations for the design of such an amplifier.

This entailed developing equations from the equivalent circuit of the cascode amplifier and then confirming these expressions by making measurements on an amplifier in the laboratory.

Fairly good agreement existed between predicted and experimental results.

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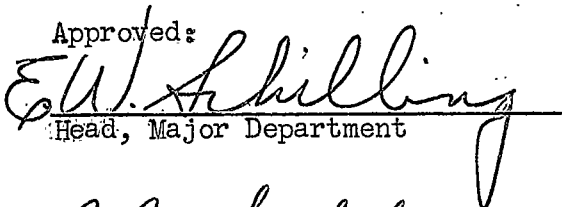
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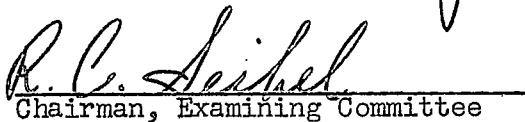
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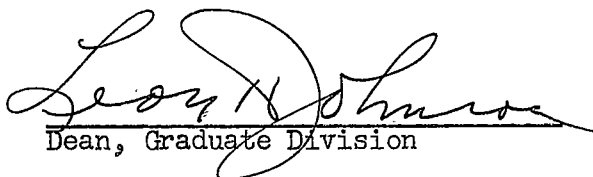
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Montana State College

Approved:

  
Head, Major Department

  
Chairman, Examining Committee

  
Dean, Graduate Division

Bozeman, Montana  
July, 1956

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Bozeman, Montana  
July, 1956

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Glenn B. Coughlan

ABSTRACT

The primary purpose of this thesis was to determine the characteristics of the cascode amplifier and to develop equations for the design of such an amplifier.

This entailed developing equations from the equivalent circuit of the cascode amplifier and then confirming these expressions by making measurements on an amplifier in the laboratory.

Fairly good agreement existed between predicted and experimental results.

### INTRODUCTION

In low frequency radio systems where the greatest sources of noise, such as atmospherics, are external to the radio set itself, the small amount of tube and circuit noise generated within the set is not important. However, in the very-high frequency range, where atmospherics are not so severe, minimum signal discernibility is often determined by noise generated within the receiver itself. Now, the signal entering a vhf receiver is usually very weak and although this signal can be amplified easily, any noise generated in the input stages of the receiver will be amplified along with the signal. Noise generated in subsequent stages is not so important since its magnitude is small compared to the signal which has already been amplified many times. Thus, the maximum attainable signal/noise ratio of a vhf receiver is set by the signal/noise ratio of the input stages. For this reason, low-noise r-f amplifiers are very important in vhf receivers.

It is well known that the random division of cathode current between plate and screen in a pentode makes the partition noise of a pentode three to five times that of the same tube connected as a triode; therefore, it is desirable to use a triode tube in the first stage of any vhf receiver.

Triode amplifiers that must function over a wide frequency range, such as in television receivers, are very difficult to stabilize unless they are heavily loaded. Since one heavily-loaded triode tube has low voltage gain, it would seem desirable to use two triodes at the input of a vhf receiver to improve its noise figure.

Two triode tubes can be cascaded in nine possible ways but the cascode circuit has proved to be the best combination with regard to low noise figure, stability and gain.

The cascode amplifier is a low-noise amplifier and although two triode tubes are employed, the circuit as a whole has the stability, and high gain characterized by the pentode. Much has appeared in the literature concerning the low-noise properties of the circuit but very little has been published about practical design considerations.

This investigation was undertaken to study the gain, bandwidth, input impedance, and stability characteristics of the cascode amplifier and to develop equations and present curves that would be useful in designing a cascode circuit.

### THEORY

The cascode amplifier as shown in Fig. 1(a) and 1(b) consists of a conventional grounded-cathode amplifier  $V_1$  feeding a grounded-grid amplifier  $V_2$ . In both circuits the tubes are series-connected as far as the a-c signal is concerned; however, Fig. 1(a) shows the plate voltage applied to the tubes in series, and Fig. 1(b) shows the tubes paralleled across the plate supply. There are advantages and disadvantages to both circuits.

To secure the same operating point, the plate voltage for the series connection must be twice that for the parallel connection. Fewer components and adjustments are necessary, however, in the series-connected cascode amplifier and it seems to be the most widely used circuit.

Another important consideration is that a-c voltage applied to the grid of  $V_1$  will control both tubes if they are series-connected, whereas a-c voltage must be applied to the grids of  $V_1$  and  $V_2$  separately in the parallel-connected circuit to obtain the same amount of gain control.

In both circuits the grid-bias resistors  $R_{k_1}$  and  $R_{k_2}$  are adjusted so that both triodes are operating at the same point on the linear portion of their characteristic curves. Adjusted this way, both triodes may be assumed to have the same tube coefficients. The by-pass capacitors  $C_b$  are selected so that they have negligible reactance at the operating frequency. The tank circuits consisting of  $L$  and  $C$  are parallel resonant at the operating frequency and the radio-frequency choke, in the parallel-connected circuit, is selected to have a high reactance at the operating frequency. Neglecting interstage shunting effects in the parallel-connected cascode circuit, as they can be if RFC presents a high reactance to the signal



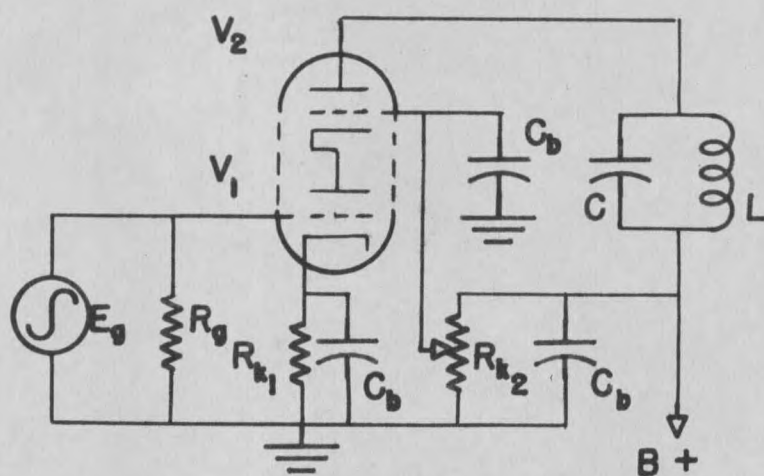


FIG. 1(a). - SERIES D-C CONNECTED CASCODE AMPLIFIER.

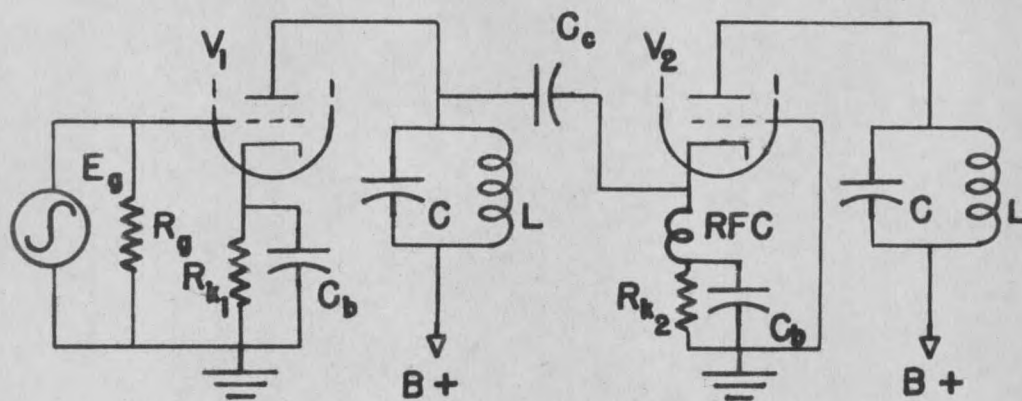


FIG. 1(b). - PARALLEL D-C CONNECTED CASCODE AMPLIFIER.

voltage, the series-connected cascode and the parallel-connected cascode amplifier have the same a-c equivalent circuit; therefore, the equations derived in this investigation will apply equally well to both circuits.

The grounded grid stage  $V_2$  is inherently stable and because of its very low input impedance (approximately equal to the transconductance of  $V_2$ )<sup>1</sup>, it reduces the gain of the first stage by heavy loading. Stability in both circuits is thus achieved. Contrary to the claims of many writers,<sup>2,3</sup> who have failed to specify the degree of loading used on their cascode amplifiers, laboratory tests indicated that the circuit has some tendency to oscillate unless the second stage is loaded. This will be discussed later.

The reasons for the low-noise characteristics of the circuit are quite involved and have been treated thoroughly elsewhere.<sup>3</sup>

As far as the a-c signal is concerned, the cascode circuit is essentially two tubes connected in series. If the polarizing voltages on the tubes are adjusted so that they are operating on a linear portion of their characteristic curve, and if the exciting signal is not allowed to drive the tubes out of their linear region, then we may represent the circuit by the equivalent circuit of Fig. 2. Since both tubes have the same operating point, their tube coefficients  $\mu$ ,  $g_m$ , and  $r_p$ , are the

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<sup>1</sup>Valley, G. E., and Wallman, H., Vacuum Tube Amplifiers, New York; McGraw-Hill, 1948, pp 657.

<sup>2</sup>Reintjes, J. F., and Coate, G. T., Principles of Radar, New York; McGraw-Hill, 1952, pp 429.

<sup>3</sup>Wallman, H., MacNee, A. B., and Gadsden, C. P., "A Low-Noise Amplifier," Proc. IRE., June, 1948, pp 700-708.

same. If a negative-going signal is applied to the input of the amplifier and designated as  $-E_g$ , then the first tube may be considered as an a-c generator of  $\mu E_g$  volts with an internal resistance  $r_p$  equal to the a-c plate resistance of the tube. The same a-c plate current  $I_p$  flows through both tubes, and the signal driving the second tube will be  $\mu E_g$  minus the drop in the plate resistance of  $V_1$  or  $(\mu E_g - I_p r_p)$ . With this driving voltage and an amplification factor  $\mu$ , the second tube  $V_2$  may be represented as a generator of  $\mu(\mu E_g - I_p r_p)$  volts with an internal impedance equal to  $r_p$ . Writing Kirchoff's voltage equation around the circuit in Fig. 2.

$$\mu E_g + \mu(\mu E_g - I_p r_p) = I_p(2r_p + Z_L) \quad (1)$$

where  $Z_L$  represents the combined impedance of the plate tank circuit and any coupled load. Solving Eq. (1) for  $I_p$

$$I_p = \frac{\mu E_g(1+\mu)}{Z_L + r_p(2+\mu)} \quad (2)$$

The output voltage  $E_o$  will be  $I_p Z_L$  and the voltage gain of the stage will be

$$A = \frac{E_o}{-E_g} = - \frac{I_p Z_L}{E_g} = - \frac{\mu(1+\mu)Z_L}{Z_L + r_p(2+\mu)}$$

$$A = - \frac{\mu(\mu+1)}{1 + \frac{r_p}{Z_L}(2+\mu)} \quad (3)$$

where the minus sign indicates that when  $Z_L$  is resistive, there is a 180-degree phase shift between  $E_g$  and  $E_o$ .

To derive the bandwidth equation, we will first convert the constant-voltage equivalent circuit of Fig. 2. to the constant-current equivalent

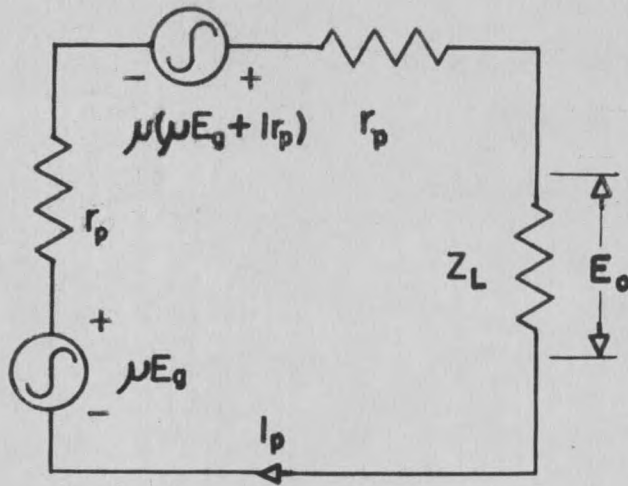


FIG. 2.- CONSTANT-VOLTAGE EQUIVALENT  
PLATE CIRCUIT.

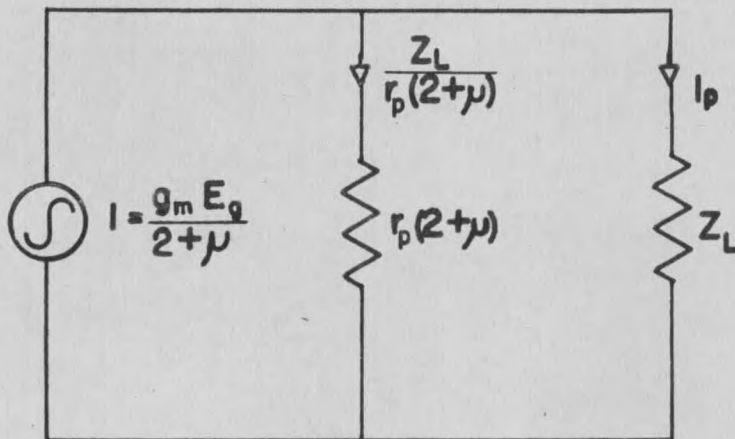


FIG. 3.- CONSTANT-CURRENT EQUIVALENT  
PLATE CIRCUIT.

circuit of Fig. 3. From Eq. (2)

$$I_p = \frac{\mu E_g (1 + \mu)}{Z_L + r_p (2 + \mu)}$$

or

$$\mu E_g = I_p Z_L + I_p r_p (2 + \mu)$$

It is well known that  $\mu = g_m r_p$  and dividing by  $r_p (2 + \mu)$  we get

$$\frac{g_m E_g}{2 + \mu} = I_p \frac{Z_L}{r_p (2 + \mu)} + I_p \quad (4)$$

Both sides of Eq. (4) are expressions for current. In fact this equation describes the circuit of Fig. 4(a), where  $I_p$  is the current flowing through the load  $Z_L$  and  $I_p \frac{Z_L}{r_p (2 + \mu)}$  is the current flowing through the shunting impedance  $r_p (2 + \mu)$ . The sum of these two currents as expressed by Eq. (4) is  $\frac{g_m E_g}{(2 + \mu)}$ , the constant-current source.

The load impedance  $Z_L$  may be a resonant circuit as shown in Fig. 4(a) where  $R$  is the damping resistance due to coil loss and any coupled load. We may simplify this circuit to that of Fig. 4(b) by combining the two resistances into a total damping resistance  $R_T$  given by

$$R_T = \frac{r_p (2 + \mu) R}{r_p (2 + \mu) + R} \quad (5)$$

The quality  $Q$  of this parallel tank is given by<sup>2</sup>

$$Q = \frac{R_T}{\omega L}$$

<sup>1</sup>Cruft Electronics Staff, Electronic Circuits and Tubes, New York: McGraw-Hill, 1947, pp 285.

<sup>2</sup>Martin, T. L., Ultra-high Frequency Engineering, New York; Prentice-Hall, 1950, pp 154.

and the bandwidth BW is

$$BW = \frac{f_r}{Q} = \frac{\omega L}{R_T} f_r \quad (6)$$

where  $f_r$  is the resonant frequency of the tank circuit.

The input impedance of an amplifier is not infinite mainly because of signal currents flowing to ground through the inter-electrode capacitance of the tube. The a-c grid current  $I_g$  consists of two components as shown in Fig. 5; the current  $I_{gk}$  flowing through the grid-to-cathode capacitance  $C_{gk}$ , and the current  $I_{gp}$  flowing through the grid-to-plate capacitance  $C_{gp}$ . These currents may be expressed as

$$I_{gk} = j\omega C_{gk} E_g$$

$$I_{gp} = j\omega C_{gp} E_{gp}$$

Now assuming a resistive  $Z_L$  as would occur at the resonant frequency,  $E_g$  and  $E_{gp}$  are 180 degrees out of phase and their sum is the a-c plate voltage  $E_{p1}$  of  $V_{p1}$ ; therefore, we may write

$$E_{gp} = E_g - E_{p1}$$

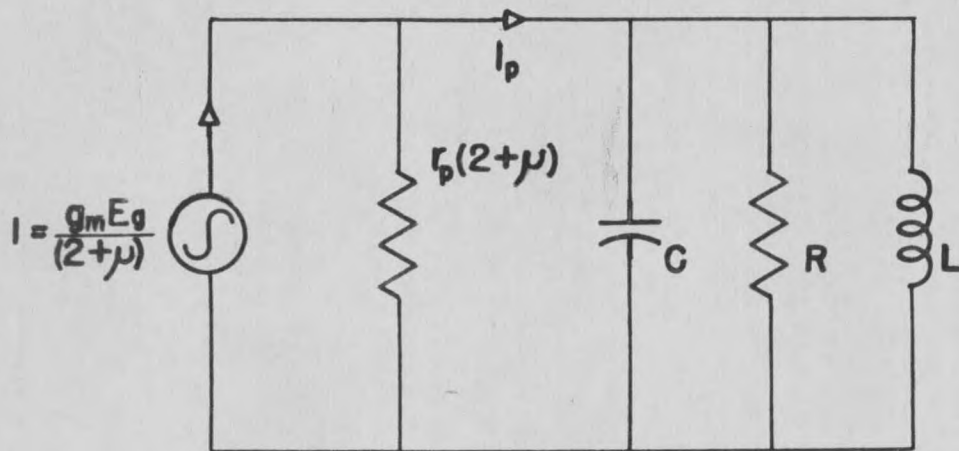
Assuming voltages polarized as shown in Fig. 2,  $E_{p1}$  is

$$E_{p1} = I_p r_p - \mu E_g$$

therefore

$$E_{gp} = E_g - I_p r_p + \mu E_g = E_g(\mu + 1) - I_p r_p$$

Substituting the expression for  $I_p$  given by Eq. (2) we arrive at the



(a)

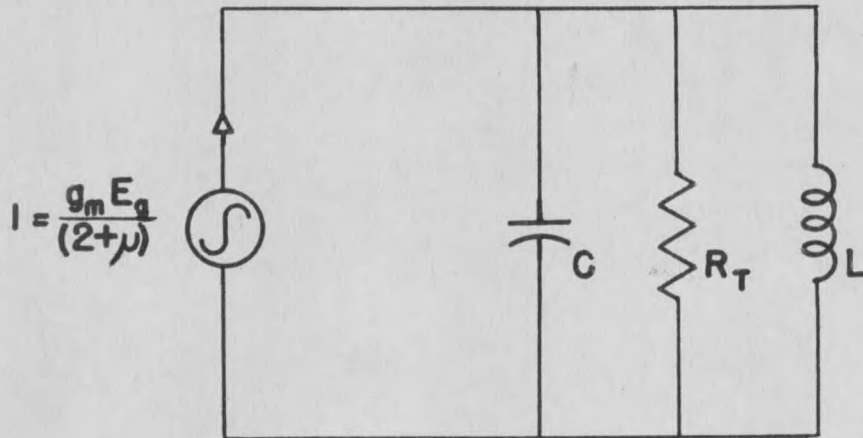


FIG. 4. - CONSTANT-CURRENT EQUIVALENT  
PLATE CIRCUIT.

following

$$E_{gp} = E_g(\mu+1) - \frac{\mu E_g(1+\mu)r_p}{z_L + r_p(2+\mu)}$$

$$= E_g(\mu+1) \left[ 1 + \frac{\mu}{2+\mu + z_L/r_p} \right]$$

Now

$$I_{gp} = j\omega C_{gp} E_{gp} = j\omega C_{gp} E_g(\mu+1) \left[ 1 + \frac{\mu}{2+\mu + z_L/r_p} \right]$$

and

$$I_g = I_{gp} + I_{gk} = j\omega E_g \left[ C_{gk} + C_{gp}(\mu+1) \left( 1 - \frac{\mu}{2+\mu + z_L/r_p} \right) \right]$$

therefore

$$z_g = \frac{E_g}{I_g} = \frac{1}{j\omega \left[ C_{gk} + C_{gp}(\mu+1) \left( 1 - \frac{\mu}{2+\mu + z_L/r_p} \right) \right]} \quad (7)$$

If the load impedance  $z_L$  is a pure resistance, that is if the plate is resonant, then the bracketed expression in the denominator of Eq. (7) is a pure capacitance called the input capacitance.

$$C_{in} = C_{gk} + C_{gp}(\mu+1) \left[ 1 - \frac{\mu}{2+\mu + z_L/r_p} \right]$$

This can be simplified to

$$C_{in} = C_{gk} + C_{gp} \left[ 1 + \frac{\mu r_p + \mu z_L}{r_p(2+\mu) + z_L} \right] \quad (8)$$

This equation for the cascode amplifier input capacitance can be compared with the well-known expression for the input capacitance of a conventional triode amplifier, Eq. (9). The only difference between Eq. (8) and Eq. (9)



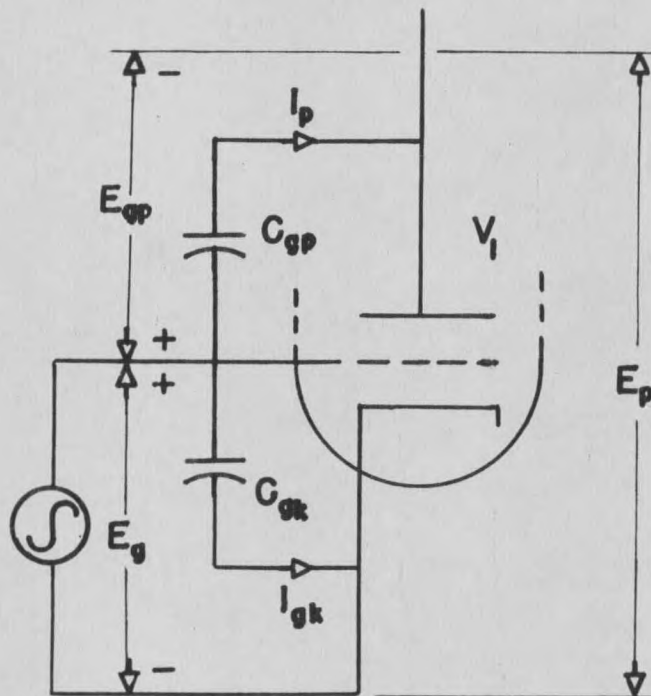


FIG. 5.- EQUIVALENT INPUT CIRCUIT  
SHOWING INTERELECTRODE  
CAPACITANCE.

is in the expression within the brackets.

$$C_{in} = C_{gk} + C_{gp} \left[ 1 + \frac{\mu Z_L}{r_p + Z_L} \right] \quad (9)$$

With the circuit parameters given in Fig. 1 of the Appendix and assuming a  $Z_L$  of 10 K ohms, Eq. (8) gives 6.75  $\mu\text{mf}$  for the input capacitance of the cascode amplifier. Assuming that  $V_1$  in Fig. 1 of the Appendix was operating as a conventional triode amplifier with the operating conditions as given in the figure, the input capacitance from Eq. (9) would be 31.8  $\mu\text{mf}$ .

Thus, it appears that the input capacitance of a cascode amplifier employing a pair of triode tubes is less than the input capacitance of a conventional triode amplifier using one of the same triodes. This gives the cascode amplifier another advantage over the conventional triode amplifier in the vhf range where capacitance shunting effects must be held to a minimum.

If  $Z_L$  in Eq. (7) is not a pure resistance, then  $Z_g$  contains a real and a reactive component. For any  $Z_L$ , the reactive component will be capacitive but the input resistance may be either positive or negative. A positive input resistance results when the load impedance in the plate circuit is capacitive, while a negative resistance is obtained with an inductive load. A negative input resistance indicates that energy is being fed from the plate circuit back to the grid circuit; moreover, for a certain range of inductive plate circuit loads, the magnitude and phase of this feed-back voltage will be such that the amplifier will oscillate. The

magnitude of this feed-back voltage may be reduced below that required for oscillation by loading the amplifier heavily.

METHOD OF INVESTIGATION

During gain measurements, all quiescent voltages and the input voltage  $E_g$  were held constant. The plate circuit was kept tuned to 30 mcps at all times. The General Radio model 1001-A signal generator was tuned above and below this frequency ( $E_g$  being kept constant) to obtain the response curves in Fig. 3 in the Appendix. To study the behavior of the amplifier under various loads, the tank circuit was shunted with several different external resistors  $R_L$ . This  $R_L$  is included in the R of Fig. 4, page 14. Input and output voltages were measured with Hewlett-Packard model 410-B vacuum-tube voltmeters.

Calculation of the bandwidth of the amplifier from Eq. (6) required a value of L and  $R_T$ . The inductance L was removed from the amplifier output tank circuit and measured on a General Radio model 916-A r-f bridge. An average of several measurements was taken as the final value (0.96 micro-henries).

Considerable difficulty was experienced in measuring the effective tank resistance  $R_T$ . This resistance was too high to measure accurately at 30 mcps on the r-f bridge used to measure L. A General Radio model 821-A Twin-T impedance bridge designed to measure conductance was found to give much more consistent results. After much experimentation, it was found that  $R_T$  depended upon the placement of the inductance in relation to near-by metal objects such as the chassis, and the exact position in which the external load resistors  $R_L$  were fastened. Whether the tube filaments were hot or not also made considerable difference. For this reason,  $R_T$  was measured with L and  $R_L$  in the exact positions that they were in during

the corresponding gain measurement and the filament power was supplied. Only in this way could serious discrepancies between experimental data and theoretical calculations be resolved.

Since the input impedance of the amplifier given by Eq. (7) was also very high, the twin-T r-f bridge was found to be more satisfactory than the conventional r-f bridge for input impedance measurements. The tank was kept resonant during these measurements so  $Z_g$  given by Eq. (7) was purely capacitive. This capacitance was so small that it was necessary to take into account the capacity of the leads connecting the r-f bridge to the amplifier input terminals. The capacitance of these leads alone was measured and subtracted from the input capacitance measurement.

DISCUSSION OF RESULTS

The values of components used in the experimental circuit, operating voltages, and tube coefficients are given in Fig. 1 of the appendix. The calculated and measured frequency response, gain, and bandwidth for various plate loads  $R_T$  are graphed in Figs. 2, 3, and 4. These curves can be used to design series- or parallel-connected cascode amplifiers of given bandwidth and gain, providing the same tube and polarizing voltages are used. The data used to plot the graphs is tabulated in Table I of the Appendix. The left column in this table gives the d-c or color-code resistance of the external plate-circuit load. The next column gives the measured value of the total equivalent plate load resistance  $R_T$ . The graphs and tabulated results substantiate the theoretical equations. In all cases the deviation between predicted and measured values is within experimental error. All the calculated values depended upon at least one impedance measurement at 30 mcps, and it was found that the balance of the r-f bridges could be changed markedly by a slight vibration of the instruments or by a small movement of connecting leads. Averages of several measurements had to be taken in all cases.

Nowhere in this work are the d-c resistance values of the external tank-circuit loads mentioned. The reason for this is that the a-c resistance values at 30 mcps could not be predicted from the measured d-c value. For instance, a resistor with a measured d-c resistance and color-code marking of 68 K ohms was found to load the tank circuit much more than a resistor of the same type with a 33 K ohm d-c resistance. Also a 150 K ohm load resistor yielded a gain much lower than a 100 K ohm resistor. The

results tabulated on page v of the Appendix indicate that the loading effect of the resistors was not a predictable function of the d-c resistance. According to theory, for any particular type of resistance of a given size and geometrical configuration, the ratio of the a-c resistance to d-c resistance will depend only upon the product of frequency and d-c resistance and will be independent of the particular value of resistance.

The opinion of the author is that the apparent discrepancies between a-c and d-c resistance in this investigation were results of non-uniform resistance manufacture.

The most important practical application of the cascode circuit is as a very-high frequency amplifier. Used this way, the circuit always has a tuned tank in the grid circuit. The circuit was connected this way at the beginning of this investigation and was found to oscillate unless either loaded or neutralized. Many writers describe the circuit as being very stable, and claim that neutralization is used only to improve the noise figure. Although two different mechanical layouts were tried, and all well-known precautions were observed, the circuit was very unstable unless loaded. With the tuned-plate, tuned-grid arrangement, the circuit was stable for  $R_T$  less than about 8 K ohms. For  $R_T$  between 8 K ohms and 9 K ohms the gain measurements were erratic and for  $R_T$  above 9 K ohms, the circuit broke into oscillation. To eliminate this difficulty, the grid-tank circuit was replaced by a 300-K ohm resistor. Connected this way, there was no sign of instability.

Of course the cascode circuit is normally used well above 30 mcps and at the higher frequencies the maximum attainable tank-circuit Q's

decrease. Hence, it is reasonable to suppose that the circuit would be more stable at frequencies above those used in this investigation.



CONCLUSIONS

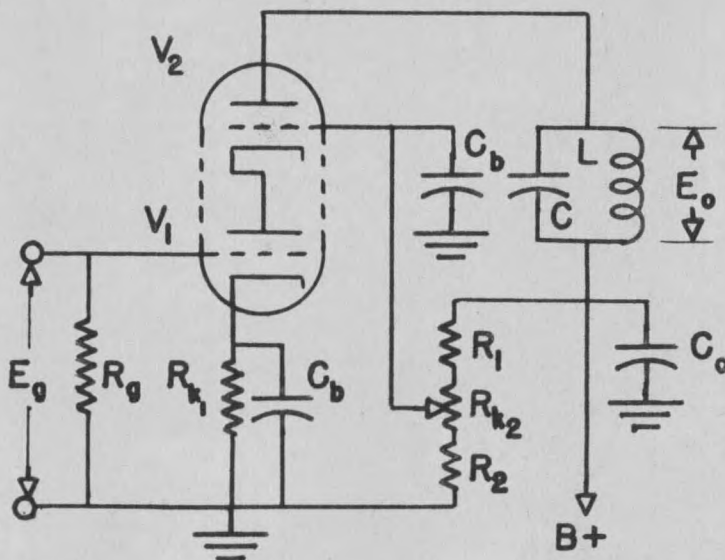
This investigation indicates that with moderate loads, the cascode amplifier has the high-gain and stability characteristic of pentode circuits; moreover, it is well-known that the cascode circuit has a much lower noise figure than amplifiers employing pentodes. At one time the cascode amplifier, because it employed two tubes, was not as economical to build as the one-tube pentode amplifier. This is no longer true, now that duo-diodes have been developed especially for use in the cascode circuit. In fact, the cascode amplifier is now beginning to replace both the high-gain pentode and the low-noise triode in high-fidelity audio-frequency amplifiers. In this application no tank circuits are involved, so instability is no problem.

It is doubtful if the cascode circuit will ever replace the conventional pentode amplifier in r-f applications below 30 mcps where very good selectivity is important and can be obtained only by using high-Q tuned circuits and hence high impedance plate-circuit loads.

In its present stage of development, it appears that the cascode amplifier is limited to use in the vhf range where high-Q tuned circuits can not be constructed practically, and to use at audio frequencies where tuned circuits are not used at all.

APPENDIX

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### CIRCUIT PARAMETERS

$R_g$ - 300 K ohms	$C_b$ 1000 $\mu\text{mf}$
$R_{k1}$ - 220 ohms	$C$ - 50 $\mu\text{mf}$
$R_{k2}$ - 50 K ohms	$L$ - 0.96 $\mu\text{h}$
$R_1, R_2$ - 100 K ohms	$V_1, V_2$ - 6BQ7A

Plate voltage on each section	150 volts
Grid bias voltage on each section	-2 volts
Filament voltage	6.3 volts a-c

$$\mu = 39, \quad g_m = 6400 \mu\text{mhos}, \quad r_p = 6100 \text{ ohms}$$

FIG. 1.- CASCODE AMPLIFIER EXPERIMENTAL CIRCUIT.















