Effect of plastic flow on flexural stresses in reinforced concrete beams
by Louis A Divras

A THESIS Submitted to the Graduate Committee in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering
Montana State University

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EFFECT OF PLASTIC FLOW ON FLEXURAL STRESSES IN REINFORCED CONCRETE BEAMS

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A THESIS

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Montana State College

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Bozeman, Montana
September, 1948

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Abstract

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Most designers, while dosigning certain beams for different loads, base their design on the straight line theory, which in turn considers only the instantaneous unit deformation produced by the application of a load and which is knownas elastic strain.

Concrete, however, is a plastic material and additional deformations occur immediately after the application of a load. Even though the load remains constant, these additional strains increase as time passes. These strains, which are usually called plastic strains, vary with the amount of load and with the time. Considering this, it is shown that modulus of elasticity, as far as concrete design is concerned, is meaningless.

In the experimental work, two beams were tested. One beam was designed by the plastic flow method and one was designed by the straight line stress variation method, in which the modulus of elasticity of concrete must be used.

The test results demonstrate that the strains do not vary as a straight line from the neutral axis to the extreme fiber; consequently, the stress does not vary as a straight line from the neutral axis to the extreme fiber.

Symbols Used

A_c = Effective concrete area.

a = Depth of equivalent stress diagram.

A = Overall area of concrete in a column.

A_s = Area of tensile steel.

b = Width of beam.

C = Total compressive force.

T = Total tensile force.

c = Arm of resisting couple.

f_c = Tensile stress in concrete.

f_s = Compression stress in steel.

f^1c = Compressive stress of a concrete cylinder.

fy = Yield point stress of steel.

d = Effective depth of beam.

M = Resisting moment.

n = Modular ratio (which is equal to $\frac{Es}{Ec}$).

E_s = Modulus of elasticity of steel.

E_c = Modulus of elasticity of concrete.

p = Steel ratio (As/bd).

S = Shortening of a non-reinforced concrete block due to shrinkage.

S_a = Shortening of reinforced concrete beam due to shrinkage.

f's = Allowable stress in steel.

f"c = Allowable stress in concrete.

jd = Moment arm of the internal couple.

kd = Position of the neutral axis.

V = Total shear force at the section.

v = Intensity of vertical shearing stress.

ϵ_1 = Total concrete strain represented in the trapezoidal diagram at final rupture.

$\beta\epsilon_1$ = Plastic strain represented by the horizontal portion of the trapezoidal diagram.

β = Plasticity ratio.

ϵ_o = Total concrete strain minus plastic strain.

fs'' = Computed theoretical stress of steel.

fc''' = Computed theoretical stress of concrete.

S_s = Computed actual stress of steel.

S_c = Computed actual stress of concrete.

ϵ_s = Actual strain of steel.

ϵ_c = Actual strain of concrete.

Introduction

Experience has shown that the actual stress distribution in a reinforced concrete beam does not agree with the accepted theory. It is also well known that this usual theory, when used with properly selected unit stresses gives results that are safe in spite of the fact that the actual unit stresses may be entirely different.

It has long been recognized that concrete changes in volume, when subjected to stress (elastic deformation), sustained load (plastic flow), temperature changes, and changes in moisture content (shrinkage), and many investigators have worked upon the determination of the magnitude of these volume changes.

In recent years, the criticisms of the ordinary straight line theory of reinforced concrete have given rise to proposals for the abandonment of the modular ratio $\frac{Es}{Ec}$, in favor of formulas based on various theories of plastic action. In all these theories, emphasis is placed upon the load at final rupture of the beam. While the wisdom of such emphasis is open to question, there can be no doubt that it is desirable to be able to preduct, within reasonable limits, the ultimate capacity of a beam. Since concrete is not truly elastic, the stress variation across the beam section usually is to be assumed a parabola. The writer does not feel satisfied with such an assumption. Since its deformations under load increases with time and because of shrinkage and plastic flow, the elastic or straight line theory has long been recognized as approximate. Since shrinkage and flow are impossible of prediction with any accuracy for a given structure, it appears to be a hopeless task to determine exactly the stresses under a given load. It is possible, however, to derive a simple relationship which will preduct with considerable accuracy the ultimate strength of a reinforced concrete beam, a value little affected by shrinkage and flow since there is a redistribution of stress with the large strains previous to failure. It thus becomes possible to design on the basis of ultimate strength applying a definite factor of safety suitable to the conditions.

The writer, in the experiments conducted in the Civil Engineering Laboratory of Montana State College, attempted to find the actual stress distribution in two full size concrete beams.

The undersigner wants to express his deep gratitude to Professor R. C. DeHart for his kind assistance in carrying on the work. He also wants to thank Mr. J. R. Arboe and Mr. Drummond for their kind assistance in offering the facilities of the Mechanical Department necessary for the completion of the work.

Historical Development

It is probable that a number of observers had noticed the plastic flow phenomenon in concrete before 1907, when Professor W. K. Hatt [1] published the results of a series of deflection tests on beams. This appears to be the first publication on the subject. According to Professor Hatt, these results taken together show a sort of plasticity in concrete by which it yields under the action of a load applied for a long time or applied a number of times.

In 1915, Professor F. R. MacMillan [2] reported tests on beams and slabs in which he measured fiber deformation changes as well as deflections under conditions where the effect of shrinkage due to drying out were separated from the effect of plastic flow. In some of his tests on slabs, though the deflection continued to increase, the fiber stress measurements on the steel indicated compression after one year. Apparently the loading was not great enough to form any cracks in the region of the steel and the total shortening of the beam due to shrinkage was great enough to cause compression in the steel and the combined effect of shrinkage and plastic flow on the compression side were sufficient to allow the increased deflection. The concrete on the tension side was apparently taking the full bending plus the compression, which the shrinkage caused in the steel. These tests illustrated the effect of shrinkage better than that of plastic flow.

In 1916, two other important papers, one by E. B. Smith [3] and another

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1. Superior figures throughout this thesis refer to bibliography.

by A. T. Goldbeck and E. B. Smith [4] and a major discussion by F. R. MacMillan [5] were published. The first of these papers brought out the fact that the plastic recovery after a load that had been sustained for some time was removed with considerably less than the plastic flow under the sustained load. It also showed that concretes under water flow less than those in air. The second paper showed that the plastic flow is comparatively rapid at first and appears to undergo a progressive slowing down as time goes on. Tests on beams and slabs in which fiber deformations were measured brought out the information that the deformations at six months were roughly three times the initial deformations and at two years, four times. MacMillan made calculations and reported the plastic flow in terms of a reduced modulus of elasticity for the concrete or increased modular ratio of (modulus of elasticity of steel to modulus of elasticity of concrete).

In 1917, two other important papers appeared in the Proceedings of the American Concrete Institute. One, a report by A. R. Lord [6] of measurements made during a year on a flat slab panel, confirmed previous reports and emphasized the effect of shrinkage in reducing the tensile stresses in steel. Lord reported that the plastic flow and shrinkage tended to reduce the negative bending resistance, thus producing greater deflections in regions of positive moment. He also found that the "straight line theory" of bending for reinforced concrete did not apply for the conditions of the test even with low stresses in the concrete and the steel.

E. B. Smith [7], in 1917, found that the law of plastic flow of concrete tended toward a limit and that the amount of plastic flow depends somewhat upon the kind of aggregate used. He found that the coarser the aggregate, the greater the plastic flow. He also found that the action of plastic flow is to lower the neutral axis of a beam, which happens when the modulus of elasticity is reduced. It follows then that the compression area is increased, thus reducing the resultant couple arm and consequently, increasing the unit stress in steel.

In 1927, Oscar Faber [8], an authority on mechanics, published in England the results of tests he had made on beams. The effect of shrinkage and plastic flow was brought into the theory of mechanics by way of change of the modular ratio with time.

Professor R. E. Davis of the University of California, was commissioned to make tests so that proper adjustments and corrections for plastic flow could be made. The results of these tests are the most comprehensive that have so far been made and they are published in the Proceedings of the American Concrete Institute, first in 1928 [9], and later in 1931 [10], as well as in the Proceedings of the American Society for Testing Materials in 1930 [11] and 1934 [12].

A number of papers have lately appeared in which theoretical aspects of plastic flow have been discussed. Among them is one by C. S. Whitney [13], in which he shows how the plastic flow property of concrete eliminates many of the secondary stresses in arches, a problem that has concerned designers considerably and has been the cause of very special procedures and devices introduced into design and construction. The effect of plastic flow figures largely in the conclusions drawn from the comprehensive series of tests on columns lately carried out by the University of Illinois.[14]

Lorenz G. Straub [15] made some tests and reports that a repeated application loads produce effects similar to a sustained load because of the difference between the plastic flow and plastic recovery. Results in a series of rigid frames constructed and tested at the University of Illinois [16] serve to check the theoretical calculations.

Plastic Flow of Concrete [17]

Ordinary calculations for stresses and stress distribution are based on the theory that the material is elastic, that the deformation is proportional to stress, and no term is introduced for time. It is now known that some structural materials are affected by time and that strains or deformations continue to change after the stresses have become constant. This property is called "time yield" or "creep" or "plastic flow". None of these terms apply to steel at low temperatures so long as the stresses are within the elastic limit. The designer of steel parts for stress at high temperatures where the elastic properties are not so definite is concerned with the property usually known as "creep" and rather good studies have been made of this property of metals. Other materials such as timber, concrete and resinous substances as celluloid have this plastic property, at ordinary temperatures and at any degree of stress. The nature of this property is different for different materials. The term "plastic flow" as used in this thesis is that property of a material which is evidenced by the long time continuation of a varying increase of deformation or strain under sustained load or decrease of stress during sustained strain.

A plain concrete specimen in compression under a constant load will continue to shorten after loading. This shortening begins immediately after the load is applied. The change in length at constant stress is quite rapid at first and slows down as time goes on. The shape of the curve of time plotted against deformation is that of a power or exponential function not unlike that of a cubic parabola. When steel and concrete are both present, as in a building column, the plastic flow is lessened because of the presence of steel which is not plastic. The steel must take on an increasing proportion of the load as time goes on. In beams, the deflection continues after the load has been applied, and the stress is increased in the tension steel because of the reduction of the resultant internal couple arm. The straight line representing variations of the fiber deformations continues to change and an increasing depth of the beam is brought into compression.

The concept of plastic flow as indicated by the term itself is readily applied to conditions of sustained load or stress but the action of plastic flow under sustained deformation or strain is somewhat more difficult to understand, although it is little, if any, less important. If a plain concrete specimen is placed in a testing machine and a certain deformation is imposed and held, the scale beam will drop. If an attempt is made to keep the beam poised, the indicated load will have to be reduced, rapidly at first and then progressively more slowly. If a beam is loaded to a certain elastic deformation and this deformation is held, the load will have to be reduced in the same way as it would have to be reduced for the specimen on the testing machine. If a beam is subjected to secondary stress, the plastic flow in the concrete will cause the fiber stresses in compression in the concrete to be reduced and as a consequence, the stresses in the steel will have to reduce also. Thus the action of plastic flow on secondary stresses is generally beneficial.

Plastic Flow and Volume Changes of Concrete

The best picture of what goes on inside a piece of concrete when shrinkage or plastic flow takes place seems to be that given by R. E. Davis, H. E. Davis and Hamilton of the University of California.[18] In tests which they made under mass curing conditions, the stresses developed due to thermal changes in large concrete cylinders under complete axial restraint were measured. It was found that the degree to which flow takes place has an important influence upon the stresses developed. They have also found that in a period of 10 years, 95 percent of the plastic flow occurring in 10 years took place in the first 5 years and the remaining 5 percent in the last five years. Tests to determine the effect of water cement ratio and of aggregate cement ratio upon plastic flow indicated that the more the cement used per unit volume of concrete, the less the flow and that this variable was very important. Within the range of normal concretes tested, it was observed that with pastes of the same water cement ratio, the flow was practically proportional to the amount of the paste in the concrete.

In tests to compare the flow of concretes in axial tension and compression, it was observed that at least during the early periods, the flow under tensile stress was greater than that under compressive stress. In the later stages, the rate of flow was less under tensile than under compressive stress.

In technical books for about a decade, there was frequent reference to the strength and elasticity of concrete but only an occasional reference to shrinkage due to drying and almost no reference to the gradual deformation under the action of sustained load, which has been called "creep", "time yield", and "plastic flow". As a result, however, of the research of recent years, there has been developed a general conception of the effect of shrinkage and plastic flow upon the behavior of concrete structures. It is now believed that shrinkage and plastic flow are closely related phenomena, each being primarily due to changes in the amount of absorbed water in the cement gel and being but little directly influenced by the free water occupying the pore spaces within the concrete mass.

Many researches were made to find the magnitude of plastic flow and shrinkage under the conditions surrounding a given concrete structure. However, they were not able to succeed.

The property of shrinkage of concrete due to drying is altogether undesirable. Shrinkage can never be entirely eliminated, yet there seems to be the possibility through the proper selection of materials and methods of reducing shrinkage to a point where it will not be a factor for serious consideration. On the whole, plastic flow does not seem to be an undesirable property. In certain reinforced concrete members, it tends to make possible more efficient use of steel, such as thin structures subjected to drying, as well as in mass structures subjected to thermal changes due to the hydration of cement.

Plastic Flow Under Long Time Loading:

The plastic flow of plain and reinforced concrete cylinders, which have been under load for 10 years, are shown in figure 1. One load of specimens was loaded at the age of 28 days and the other at the age of 3 months. Prior to loading, the concrete was moist cured and after loading, it was stored in air at 70 percent relative humidity and 70° F. The