Class C amplifier harmonics and efficiency
by Rajendra Dube

A THESIS Submitted to the Graduate Faculty in partial fulfilment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University
© Copyright by Rajendra Dube (1950)

Abstract:
i- Re coat, research in some part of the fast domain of radio science is characterized by an extensive study of many problems which, as a rule, were formerly either generally considered to be of no great importance or were not considered at all. Frequently, it happens later on that these apparent problems prove to be of fundamental importance.

This thesis presents an investigation into the effects of harmonics on tube efficiency. If is a well-known fact that the presence of appreciable harmonics in the plate circuit results in a loss of efficiency in a tube; but no data on the magnitude of the effect are published at present. The reason for the omission of the data, in the opinion of the author, may be attributed to the lack of a suitable practical method of measuring the percentage of harmonics and tube efficiency with accuracy. This led into, the possibility of devising suitable methods, of determining harmonics and tube efficiency in the laboratory.

The reader will note that the investigation is not a complete success; but the study has been extensive considering the limitation of the available equipment.

CLASS C AMPLIFIER HARMONICS
AND EFFICIENCY

by

RAJENDRA DUBE

A THESIS
Submitted to the Graduate Faculty
in
partial fulfilment of the requirements
for the degree of
Master of Science in Electrical Engineering
at
Montana State College

Approved:

[Signatures]

Bozeman, Montana
August, 1950
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Subject</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgment</td>
<td>3</td>
</tr>
<tr>
<td>Abstract</td>
<td>4</td>
</tr>
<tr>
<td>Introduction</td>
<td>5</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>6</td>
</tr>
<tr>
<td>Data on Transmitting Beam Power Tube</td>
<td>8</td>
</tr>
<tr>
<td>Operation of Beam Power Tube</td>
<td>9</td>
</tr>
<tr>
<td>Two Stage Transmitter</td>
<td>17</td>
</tr>
<tr>
<td>Harmonic Analysis by Lissajous Figures</td>
<td>17</td>
</tr>
<tr>
<td>Indirect Measurement of Harmonic Amplitudes</td>
<td>21</td>
</tr>
<tr>
<td>Discussion of Results</td>
<td>31</td>
</tr>
<tr>
<td>Summary and Conclusions</td>
<td>34</td>
</tr>
<tr>
<td>Appendix I</td>
<td>36</td>
</tr>
<tr>
<td>Literature Cited</td>
<td>38</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENT

This thesis work was undertaken at the suggestion and guidance of Professor R. C. Seibel of the Electrical Engineering Department, Montana State College, Bozeman, Montana, U. S. A. The author expresses his deep appreciation to Professor Seibel for his kind help and proper direction.

[R. Seibel]
ABSTRACT

Recent research in some parts of the vast domain of radio science is characterized by an extensive study of many problems which, as a rule, were formerly either generally considered to be of no great importance or were not considered at all. Frequently, it happens later on that these apparent problems prove to be of fundamental importance.

This thesis presents an investigation into the effects of harmonics on tube efficiency. It is a well-known fact that the presence of appreciable harmonics in the plate circuit results in a loss of efficiency in a tube; but no data on the magnitude of the effect are published at present. The reason for the omission of the data, in the opinion of the author, may be attributed to the lack of a suitable practical method of measuring the percentage of harmonics and tube efficiency with accuracy. This led into the possibility of devising suitable methods of determining harmonics and tube efficiency in the laboratory.

The reader will note that the investigation is not a complete success; but the study has been extensive considering the limitation of the available equipment.
INTRODUCTION

In literature dealing with electric circuits, a number of abbreviations and symbols are used which are common to the technical man. Accepted abbreviations, such as Mcps for megacycles per second, used in radio engineering will be assumed well-known to the reader. It has also been considered desirable to include data for the beam power tube type 607 for ready reference.

A class C amplifier is defined as one in which the tube operates at a bias much greater than cut-off voltage, so that plate power is drawn only on the peaks of the signal voltage. It is not used in audio-amplifiers, because distortion is too high, but is the most efficient circuit for r-f power amplifiers where harmonics can be reduced by the use of tuned or selective circuits.

Ever since the introduction of class C amplifiers into the field of electronics, high vacuum tubes of the power type have assumed a steadily increasing importance. Improvement in design technique has permitted the construction of tubes giving very large outputs. Owing to the use of large power, any factors which contribute to an increase in efficiency of operation of electron tubes are worthy of serious consideration. Many papers have appeared which deal with technical investigations of the properties of tubes operating as radio-frequency class C amplifiers. These papers are concerned largely with conditions under which optimum conversion of direct current plate supply power into radio-frequency power is obtained, consistent with the demands of the type of service required. One of the factors, the effects of harmonics on tube efficiency, has received little attention.
# LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{bb}$</td>
<td>Plate supply voltage.</td>
</tr>
<tr>
<td>$E_{cc}$</td>
<td>Control grid supply voltage.</td>
</tr>
<tr>
<td>$E_{co}$</td>
<td>Cut-off grid-bias.</td>
</tr>
<tr>
<td>$E_{sg}$</td>
<td>Screen grid-supply voltage.</td>
</tr>
<tr>
<td>$E_{p1}$</td>
<td>Fundamental voltage (rms value) across tank circuit.</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Peak value of grid exciting signal voltage.</td>
</tr>
<tr>
<td>$e_g$</td>
<td>Instantaneous value of grid voltage.</td>
</tr>
<tr>
<td>$E_{os}$</td>
<td>Output voltage of the signal generator.</td>
</tr>
<tr>
<td>$I_{bb}$</td>
<td>D. C. Plate current.</td>
</tr>
<tr>
<td>$i_p$</td>
<td>Instantaneous value of plate current.</td>
</tr>
<tr>
<td>$I_n$</td>
<td>Peak value of the $n^{th}$ harmonic current.</td>
</tr>
<tr>
<td>$I_{p1}$</td>
<td>Fundamental current (rms value).</td>
</tr>
<tr>
<td>$I_{R,F}$</td>
<td>Rms value of r-f. current in the load.</td>
</tr>
<tr>
<td>$Z_T$</td>
<td>Impedance of the loaded tank circuit at resonance.</td>
</tr>
<tr>
<td>$Z_n$</td>
<td>Impedance of the loaded tank circuit at the $n^{th}$ harmonic.</td>
</tr>
<tr>
<td>$R_L$</td>
<td>Load resistance in ohms.</td>
</tr>
<tr>
<td>$R_s$</td>
<td>Reflected resistance in the tank circuit.</td>
</tr>
<tr>
<td>$R_l$</td>
<td>Resistance of coil.</td>
</tr>
<tr>
<td>$R$</td>
<td>Equivalent series resistance of coil including effect of connected load.</td>
</tr>
<tr>
<td>$r$</td>
<td>High resistance across tank circuit. This high resistance consists of two resistances, $r_1$ and $r_2$, in series, where $r_1$ is negligibly small in comparison with $r_2$.</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Power associated with fundamental frequency.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Power dissipated in load $R_L$</td>
</tr>
<tr>
<td>$P_{in}$</td>
<td>Power input</td>
</tr>
<tr>
<td>$\eta_l$</td>
<td>Fundamental efficiency in percent</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Overall efficiency in percent</td>
</tr>
<tr>
<td>$C$</td>
<td>Capacitance in farads</td>
</tr>
<tr>
<td>$L$</td>
<td>Inductance in henrys</td>
</tr>
<tr>
<td>$X$</td>
<td>Reactance of coil</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular velocity</td>
</tr>
<tr>
<td>$\omega_{pr}$</td>
<td>Angular velocity at parallel resonance</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of current flow</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>Quality factor for unloaded coil</td>
</tr>
<tr>
<td>$Q_l$</td>
<td>Quality factor for loaded coil</td>
</tr>
<tr>
<td>$R_{s}, F, C$</td>
<td>Radio-frequency choke coil</td>
</tr>
</tbody>
</table>
DATA ON TRANSMITTING BEAM POWER TUBE\textsuperscript{2}
(Type 807)

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D, C, Plate Voltage</td>
<td>500 volts</td>
</tr>
<tr>
<td>D, C, Grid No. 2 Voltage</td>
<td>250 volts</td>
</tr>
<tr>
<td>D, C, Grid No. 1 Voltage</td>
<td>45 volts</td>
</tr>
<tr>
<td>Peak R. F. Grid No. 1 Voltage</td>
<td>65 volts</td>
</tr>
<tr>
<td>D, C, Plate Current</td>
<td>100 mA</td>
</tr>
<tr>
<td>D, C, Grid No. 2 Current</td>
<td>6 mA</td>
</tr>
<tr>
<td>D, C, Grid No. 1 Current (Approximately)</td>
<td>3.5 mA</td>
</tr>
<tr>
<td>Driving Power (Approximately)</td>
<td>0.2 watt</td>
</tr>
<tr>
<td>Power Output (Approximately)</td>
<td>30 watts</td>
</tr>
<tr>
<td>Mu-Factor, Grid No. 2 to Grid No. 1</td>
<td>8</td>
</tr>
</tbody>
</table>

\textsuperscript{2}\textsuperscript{R, C, A. Tube Handbook HB-3 Vol. 3-4.}
Operation Of Beam Power Tube

A beam power tube is characterized by having a plate current that is substantially independent of plate voltage unless the plate voltage is so low that other positive electrodes rob the plate of a disproportionate fraction of the total space current. Since practical class C amplifiers are operated so that the plate potential never becomes as low as this, one can always assume that the plate current is substantially independent of the plate voltage, and hence of load impedance. This makes the analytical analysis of class C amplifiers using beam power tubes entirely possible.

The total space current in any vacuum tube is determined by the electrostatic field in the immediate vicinity of the cathode. In a beam power tube this field is, to a high degree of accuracy, proportional to the quantity \( \left( e_g + \frac{E_{sg}}{\mu_{sg}} \right)^n \) where \( e_g \) and \( E_{sg} \) are the control grid and screen grid potentials respectively, and \( \mu_{sg} \) is the amplification factor of the control grid against an anode represented by the screen grid.

When a beam power tube is operated so that plate current is substantially independent of plate voltage, the plate current will always be a substantially constant fraction of the total space current, provided the control grid current is negligible. Under the assumption, one can write:

\[
i_p = K \left( e_g + \frac{E_{sg}}{\mu_{sg}} \right)^n
\]

where \( K \) and \( n \) are constants.

In order to determine \( n \), the equation (1) is expressed in logarithmic form as

\[
\log i_p = \log K + n \log \left( e_g + \frac{E_{sg}}{\mu_{sg}} \right)
\]

(2)
then \( \log i_p \) and \( \log \left( \frac{e_g + \frac{E_{sg}}{m_{sg}}} \right) \) are plotted for various values of \( i_p \) and \( e_g \) taken from the static characteristic curves, keeping \( E_{sg} \) constant at operating conditions. The slope of the resulting curve is \( n \). It is better to plot \( i_p \) and \( \left( \frac{e_g + \frac{E_{sg}}{m_{sg}}} \right) \) on log-log paper, so that the necessity of taking the logarithm may be avoided. When this plot is made, the exponent \( n \) for the sake of simplicity in the following analysis may be taken approximately equal to 2. Hence, the expression

\[
i_p = K \left( \frac{e_g + \frac{E_{sg}}{m_{sg}}} \right)^2
\]

is assumed to represent the characteristic of the beam power tube to a sufficient degree of accuracy.

In amplifiers, the voltage applied to the grid of the tube consists of a constant negative bias \( E_{cc} \) plus an alternating potential of peak amplitude \( E_s \) with angular velocity \( \omega \), so that

\[
e_g = \left( E_{cc} + E_s \cos \omega t \right)
\]

From Fig. 1, it can be seen that the angle of current flow \( \theta \) may be determined by the expression

\[
\cos \frac{\theta}{2} = -\left( \frac{E_{cc} - E_{co}}{E_s} \right)
\]

In case of class C amplifiers using tetrodes or pentodes,

\[
E_{co} = -\frac{E_{sg}}{m_{sg}}
\]

Combining equations (3), (4), (5) and (6), one can write

\[
i_p = K E_s \left( \cos \omega t - \cos \frac{\theta}{2} \right)^2
\]

Since the wave-shape of the current \( i_p \) is non-sinusoidal and symmetrical about the vertical axis,
ANGLE OF CURRENT FLOW

FIG. 1
\[ I_p = I_0 + I_1 \cos \omega t + I_2 \cos 2\omega t + \ldots \]  

By Fourier Analysis,

\[ I_0 = \frac{KE_s^2}{\pi} \int_0^{\omega t = \theta/2} (\cos \omega t - \cos \Theta)^2 d(\omega t) \]  

\[ I_n = \frac{2KE_s^2}{\pi} \int_0^{\omega t = \theta/2} (\cos \omega t - \cos \Theta)^2 \cos n\omega t d(\omega t) \]  

On simplification (see Appendix I), the following important results are obtained:

\[ I_0 = \frac{KE_s^2}{\pi} \left[ \frac{\Theta}{2} + \frac{\Theta}{2} \cos^2 \Theta - \frac{3}{4} \sin \Theta \right] \]  

\[ I_1 = \frac{KE_s^2}{\pi} \left[ \sin \Theta - \frac{1}{2} \sin^2 \Theta - \frac{\Theta}{2} \cos \Theta \right] \]  

\[ I_2 = \frac{KE_s^2}{\pi} \left[ \sin \Theta \left( \frac{1}{8} + \frac{3}{4} \cos^2 \Theta - \cos \Theta \left( \sin \Theta + \frac{1}{3} \sin^2 \Theta \right) \right) \right] \]  

\[ I_n = \frac{2KE_s^2}{\pi} \left[ \frac{1}{n(n+2)} \sin \frac{n\Theta}{2} \left\{ 1 + 2(n+1) \cos \Theta - \frac{1}{2} \cos \Theta \sin \frac{(n-1)\Theta}{2} \right. \right. \]  

\[ \left. - \frac{1}{2} \cos \Theta \sin \frac{(n+1)\Theta}{2} + \frac{1}{(n^2-4)} \sin \frac{(n-2)\Theta}{2} \right] \]  

where \( n = 3, 4, \ldots \)

Since the beam power tube type 807 was employed for the study, it may be noted that under the operating conditions, \( E_{sg} = 250 \) volts, \( E_{oc} = -45 \) volts, \( E_s = 65 \) volts and \( \mu_{sg} = 8 \). Therefore taking into account the equations (5) and (6), one obtains

\[ \cos \frac{\Theta}{2} = 0.212 = \cos 77.75^\circ \]

\[ \Theta = 155.75^\circ = 2.715 \text{ radians} \]

Substituting this value of \( \Theta \) in equations (12), (13) and (14), the fol-
following harmonic components are obtained:

\[
I_1 = \frac{2KE^2}{\pi} \left[ 0.3766 \right] \\
I_2 = \frac{2KE^2}{\pi} \left[ 0.2538 \right] \\
I_3 = \frac{2KE^2}{\pi} \left[ 0.1189 \right]
\]

From equations (15), (16) and (17), one obtains

\[
\frac{I_2}{I_1} = 0.668 \\
\frac{I_3}{I_1} = 0.314
\]

**Tank Circuit**

The plate circuit of the amplifier consists of a tank circuit; therefore the load offers different impedances to different harmonics (the impedance decreasing as the number of harmonics increases). The tank circuit acts as a pure resistance only to the fundamental frequency to which it is tuned.

Let \( R \) denote the sum of the resistance \( R_1 \) of the inductance and any resistance \( R_0 \) reflected into the circuit.

Since the inductance \( L \) and resistance \( R \) in series are in parallel with the capacity \( C \), the impedance \( Z \) of the tank circuit is

\[
Z = \frac{L}{R} \frac{R}{\omega L} \frac{1}{1 + j \frac{\omega L}{R} \left( \frac{1}{R} - \frac{1}{\omega L} \right)} 
\]

---

Since the tuning capacity is variable, the condition for maximum impedance is obtained simultaneously with unity power factor. The condition for unity power factor is

\[- \frac{R}{\omega L} = \frac{\omega L}{R} - \frac{1}{\omega CR} \]

Let \( \omega_{pr} \) be the parallel resonant frequency and \( Q_L = \frac{\omega_{pr} L}{R} \), then the impedance of the tank circuit at parallel resonance is

\[ Z_T = \frac{L}{CR} = \omega_{pr} L \left( Q_L + \frac{L}{Q_L} \right) \]

From equations (20) and (21), the impedance of the tank circuit at the \( n^{th} \) harmonic is

\[ Z_n = \frac{L}{CR} \left( 1 - j \frac{1}{nQ_L} \right) \]

The percentage of harmonic voltage across the tank circuit is 100 \( \% \) \( \left( \frac{I_n Z_n}{I_p Z_T} \right) \), where \( I_n \) is the \( n^{th} \) harmonic component of the plate current \( I_p \).

From equations (18), (19), (22), and (23), one obtains:

Per cent second harmonic voltage = \[ \frac{66.8 \left( 1 - j \frac{1}{2Q_L} \right)}{1 + j \left( 1 + 5Q_L - \frac{1}{2Q_L} \right)} \]

Per cent third harmonic voltage = \[ \frac{31.4 \left( 1 - j \frac{1}{3Q_L} \right)}{1 + j \left( 2 + 67Q_L - \frac{1}{3Q_L} \right)} \]

For different values of \( Q_L \), the theoretical percentage of second and third harmonic voltages across the tank circuit may be calculated from equations (24) and (25). Some of the values are tabulated in table I and are plotted in Fig. 3.

Table I

<table>
<thead>
<tr>
<th>$Q_L$</th>
<th>Per cent second harmonic</th>
<th>Per cent third harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>23.6</td>
<td>5.8</td>
</tr>
<tr>
<td>3</td>
<td>14.0</td>
<td>3.9</td>
</tr>
<tr>
<td>4</td>
<td>11.0</td>
<td>3.06</td>
</tr>
<tr>
<td>5</td>
<td>8.84</td>
<td>2.35</td>
</tr>
<tr>
<td>6</td>
<td>7.38</td>
<td>1.96</td>
</tr>
<tr>
<td>7</td>
<td>6.35</td>
<td>1.68</td>
</tr>
<tr>
<td>8</td>
<td>5.56</td>
<td>1.47</td>
</tr>
<tr>
<td>9</td>
<td>4.95</td>
<td>1.35</td>
</tr>
<tr>
<td>10</td>
<td>4.46</td>
<td>1.175</td>
</tr>
<tr>
<td>11</td>
<td>4.05</td>
<td>1.065</td>
</tr>
<tr>
<td>12</td>
<td>3.71</td>
<td>1.02</td>
</tr>
<tr>
<td>13</td>
<td>3.45</td>
<td>0.9</td>
</tr>
</tbody>
</table>

It is also interesting to note that when the tank circuit is replaced by a pure resistance, the maximum possible percentage of second and third harmonics are:

Table II

| Second harmonic voltage across tank circuit | 66.8% |
| Third harmonic voltage across tank circuit | 31.4% |

Figure 3 indicates that the percentage of harmonics becomes inappreciable when $Q_L$ is large.

The greatest defect with this theoretical analysis is that it is based on a large number of assumptions:

1. Plate current is independent of the load.
2. Plate current follows a square law.
3. Grid exciting voltage is sinusoidal.
4. No grid current flows.
5. Variation of parameters and insufficient filament emission do not take place.

From these assumptions, it is obvious that the analysis is not rigorous.
Two Stage Transmitter

In order to determine the percentage of harmonics in a class C amplifier, a two stage transmitter was built. The wiring diagram, Fig. 4, shows the circuit of the simple transmitter built for the experimental analysis.

A crystal oscillator (employing 6V6G tube) drives an amplifier. For the amplifier, a type 807 tube was selected in order to avoid the trouble of neutralization. Tetrodes and pentodes usually do not require neutralization, because the screen grid reduces the grid to plate capacitance sufficiently to eliminate practically all the interchange of power between input and output circuits.

The oscillator circuit operates with a 4.35 Mcps crystal. The oscillator output is obtained at the crystal fundamental frequency. The diagram indicates that only the oscillator and amplifier tank circuits need adjustment. The load (at the output stage of the amplifier) was connected across part of the coil, so that \( Q \) could be changed easily.

Harmonic Analysis By Lissajous Figures

In some fields of investigation, Lissajous figures are studied. These figures are produced by the motion of a point whose plane Cartesian coordinates vary. The shape of the figure depends upon the relative frequencies producing the horizontal and vertical motion.

In particular, the simultaneous application of sinusoidal voltages to the two pairs of deflector plates in a cathode ray tube produces the figures on the screen. Consequently, observation of Lissajous patterns in this manner provides a very convenient method of performing certain studies in connection with the behaviour of electric circuits.
TWO STAGE TRANSMITTER

FIG. 4

C₁ = 150 μf VARIABLE
C₂ = 400 μf VARIABLE
L₁ = 40 μh
L₂ = 15 μh
The author attempted to use these figures to determine harmonic distortion in the class C amplifier. The experimental set-up is shown in Fig. 5.

**Theory**

Let it be assumed that the grid exciting voltage is

\[ y = E_s \cos \omega t \]  

As the output voltage of the amplifier is non-sinusoidal, it can be represented by

\[ x = E_1 \cos \omega t + E_2 \cos 2 \omega t + E_3 \cos 3 \omega t + \ldots \]  

Eliminating \( \cos \omega t \) from equation (27) by using equation (26), one obtains

\[ x = \left( \frac{y}{E_s} \right) E_1 + \left( \frac{2y}{E_s} - 1 \right) E_2 + \left( \frac{4y^2}{E_s^2} - \frac{2y}{E_s} \right) E_3 + \ldots \]  

Since the amplitude of the exciting voltage can be determined experimentally, and \((x,y)\) co-ordinates can be determined from the Lissajous figure, one can obtain the values of the unknowns \( E_1, E_2, E_3 \) etc. by solving simultaneous equations.

**Causes of Failure**

From the theory it seems that the experiment should be successful; but it was found that the success of the attempt was far below expectation.

If the causes of failure are analysed, it becomes apparent that one of the most serious assumptions that the analysis makes is that the grid exciting voltage is sinusoidal. The presence of harmonics in the grid exciting voltage causes so much trouble that although \( Q_s \) of the tank circuit of the amplifier was kept very high, still a straight line figure was not obtained.

This was not the only source of difficulty. The crystal, which was
TWO STAGE TRANSMITTER

CRYSTAL OSCILLATOR

CLASS C AMPLIFIER

0.002 μf

OSCILLOSCOPE

G - GROUND
V - VERTICAL PLATES
H - HORIZONTAL PLATES

HARMONIC ANALYSIS BY LISSAJOU FIGURES

FIG. 5
expected to have one particular mode of oscillation, was found to have many modes of oscillations. Many of these modes of vibration were excited simultaneously when the crystal was operating, with the result that the harmonics of these modes of oscillation were found quite close to the principle mode of oscillation. These could not be removed by a filter circuit because of their proximity to the desired signal. In the laboratory there was not a single crystal available which was free from all modes of oscillation other than the principle mode.

There is, however, another difficulty which is no less important than the other two mentioned above. A close examination of the theory reveals that the phase of the input and output frequency components should be the same. If there is a phase difference on account of stray inductance and capacitance, the formula determined becomes so complicated that one is unable to solve the equation from the Lissajous figure obtained from the oscilloscope.

One more source of error may be pointed out, although that did not come in the picture; because of the failure of the method. For the reader, it may be stated that the width of the moving spot is great enough to obscure smaller percentages of harmonic voltages.

As freedom from these troubles could not be achieved, the method was abandoned.

**Indirect Measurement of Harmonic Amplitudes**

When the first method was unsuccessful, another simple arrangement was devised in the laboratory for the measurement of the amplitudes of harmonics. The block diagram is shown in Fig. 6.
**CLASS C AMPLIFIER TANK CIRCUIT**

---

**STANDARD SIGNAL GENERATOR**

**TYPE No. 1001-A**
**SERIAL No. 224**
**GENERAL RADIO Co. MASS, U.S.A.**

---

**SUPER SKYRIDER RECEIVER**

**MODEL Sx-28**
**THE HALLICRAFTERS INC. CHICAGO, U.S.A.**

---

**HICKOK VTVM**

**MODEL No. 202**

---

\[ r_1 = 12 \, \Omega \]
\[ r_2 = 5.6 \times 10^6 \, \Omega \]

---

**INDIRECT MEASUREMENT OF HARMONIC AMPLITUDES**

---

**FIG. 6**
A very large non-inductive resistance \((r = r_1 + r_2 \text{ where } r_1 \ll r_2)\) was connected in parallel with the tank circuit of the class C amplifier. The high resistance has a negligible effect on the characteristic of the tank circuit. The voltage developed across resistance \(r_1\) was injected into the receiver which was tuned to the frequency whose amplitude was to be measured. The gain control of the receiver was suitably adjusted in order to get reasonable output voltage at the second detector. The voltmeter reading was recorded. The receiver was then connected to the output of the signal generator. The signal generator was then tuned to the frequency to which the receiver was already tuned. The output voltage of the signal generator was adjusted to get the same output voltage at the second detector of the receiver. It is needless to point out that the gain control and frequency dial should not be disturbed, while connecting the receiver to the signal generator; otherwise conditions will change and the output voltmeter readings will be of no value. The calibrated output voltage of the signal generator then equals the voltage across the resistance \(r_1\).

From this data, one can calculate the voltage developed across the tank circuit.

Let the output voltage of the signal generator be denoted by \(E_{0s}\). Then the fundamental voltage developed across the tank circuit is

\[
E_{f1} = \frac{E_{0s} \times r}{r_1}
\]

Since \(r_1\) is a very small resistance, \(r = r_2\)

Note: The voltmeter was connected across a resistor at the second detector of the receiver. Its choice was prompted by the fact that distortion was very little when a reasonably large input signal was fed into the receiver.
Percentage of the \( n \)-th harmonic voltage across the tank circuit is

\[
\% \text{\( n \)-th harmonic voltage} = \frac{\text{Harmonic voltage across } R_1}{\text{Fundamental voltage across } R_1} \times 100
\]

Power \( P_1 \) associated with the fundamental frequency is given by

\[
P_1 = \frac{E_1^2}{R_1} \quad (32)
\]

The \( Q_L \) of the loaded coil was determined by measurement with the \( R, F \) Bridge, so that \( Z_T \) could be calculated from equation (22),

\[
\text{Power input } P_{\text{in}} = E_{bb} \cdot I_{bb} \quad (33)
\]

Fundamental efficiency \( \eta_1 = \frac{P_1}{P_{\text{in}}} \quad (34) \)

Power dissipated in the load \( R_L \) is

\[
P_L = I_{R_L}^2 \times R_L \quad (35)
\]

Overall-efficiency

\[
\eta = \frac{P_L}{P_{\text{in}}} \quad (36)
\]

The experimental data are given in tables III to VI; and the results of these measurements are given in tables VII and VIII. The important curves are shown by Figs. 7 to 9.
# MEASUREMENT OF HARMONIC AMPLITUDES

### Table III

<table>
<thead>
<tr>
<th>R, F, Bridge Readings</th>
<th>E&lt;sub&gt;bb&lt;/sub&gt; = 500 v.</th>
<th>I&lt;sub&gt;R, F&lt;/sub&gt; = 0.79 amp.</th>
<th>x = \frac{1800}{4.35} ohms.</th>
<th>R = 87 ohms.</th>
</tr>
</thead>
</table>

| I<sub>bb</sub> = 100 ma. | R<sub>L</sub> = 56 ohms. |

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Voltmeter Readings</th>
<th>Signal Generator Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.35 Mops</td>
<td>110 V</td>
<td>520 µV</td>
</tr>
<tr>
<td>8.70</td>
<td>96</td>
<td>330</td>
</tr>
<tr>
<td>13.05</td>
<td>85</td>
<td>130</td>
</tr>
<tr>
<td>17.40</td>
<td>65</td>
<td>80</td>
</tr>
</tbody>
</table>

### Table IV

<table>
<thead>
<tr>
<th>R, F, Bridge Readings</th>
<th>E&lt;sub&gt;bb&lt;/sub&gt; = 500 v.</th>
<th>I&lt;sub&gt;R, F&lt;/sub&gt; = 0.68 amp.</th>
<th>x = \frac{1700}{4.35} ohms.</th>
<th>R = 59 ohms.</th>
</tr>
</thead>
</table>

| I<sub>bb</sub> = 95 ma. | R<sub>L</sub> = 71 ohms. |

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Voltmeter Readings</th>
<th>Signal Generator Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.35 Mops</td>
<td>105 V</td>
<td>580 µV</td>
</tr>
<tr>
<td>8.70</td>
<td>75</td>
<td>230</td>
</tr>
<tr>
<td>13.05</td>
<td>55</td>
<td>115</td>
</tr>
<tr>
<td>17.40</td>
<td>42</td>
<td>35</td>
</tr>
</tbody>
</table>
Table V

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Voltmeter Readings</th>
<th>Signal Generator Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.35 Mcps</td>
<td>100 V</td>
<td>660 $\mu$V</td>
</tr>
<tr>
<td>8.70</td>
<td>65</td>
<td>258</td>
</tr>
<tr>
<td>13.05</td>
<td>25</td>
<td>58</td>
</tr>
<tr>
<td>17.40</td>
<td>10</td>
<td>19</td>
</tr>
</tbody>
</table>

Table VI

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Voltmeter Readings</th>
<th>Signal Generator Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.35 Mcps</td>
<td>90 V</td>
<td>750 $\mu$V</td>
</tr>
<tr>
<td>8.70</td>
<td>85</td>
<td>166</td>
</tr>
<tr>
<td>13.05</td>
<td>40</td>
<td>28</td>
</tr>
<tr>
<td>17.40</td>
<td>30</td>
<td>21</td>
</tr>
</tbody>
</table>
CALCULATED RESULTS

Table VII

<table>
<thead>
<tr>
<th>$Q_L$</th>
<th>$P_{in}$</th>
<th>$P_1$</th>
<th>$\eta_1$</th>
<th>$P_L$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.76</td>
<td>50.0 watts</td>
<td>28.70 watts</td>
<td>57.4%</td>
<td>35.0 watts</td>
<td>70.0%</td>
</tr>
<tr>
<td>6.62</td>
<td>47.5</td>
<td>27.80</td>
<td>58.5</td>
<td>32.8</td>
<td>69.0</td>
</tr>
<tr>
<td>9.26</td>
<td>45.0</td>
<td>26.75</td>
<td>59.5</td>
<td>29.9</td>
<td>66.5</td>
</tr>
<tr>
<td>12.68</td>
<td>43.0</td>
<td>26.10</td>
<td>60.6</td>
<td>28.2</td>
<td>65.5</td>
</tr>
</tbody>
</table>

Table VIII

<table>
<thead>
<tr>
<th>$Q_L$</th>
<th>Second Harmonic</th>
<th>Third Harmonic</th>
<th>Fourth Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.76</td>
<td>63.5%</td>
<td>25.00%</td>
<td>6.510%</td>
</tr>
<tr>
<td>6.62</td>
<td>51.6%</td>
<td>19.85%</td>
<td>5.600%</td>
</tr>
<tr>
<td>9.26</td>
<td>39.6%</td>
<td>5.78%</td>
<td>2.880%</td>
</tr>
<tr>
<td>12.68</td>
<td>22.1%</td>
<td>3.73%</td>
<td>1.465%</td>
</tr>
</tbody>
</table>
EXPERIMENTAL PERCENTAGE OF HARMONICS VERSUS $Q_L$

FIG. 9
31

DISCUSSION OF RESULTS

As the discussion centers around the efficiency in the tube, the enunciation of the definition will not be considered out of place. The fundamental efficiency is defined as the efficiency with which the tube converts the direct current power input into the fundamental a.c. power output in the load. Mathematically,

\[ \eta_i = \frac{E_{p1}I_{p1}}{E_{bb}I_{bb}} = \frac{P_i}{P_{in}} \]

Since the characteristic curves of a tube are never straight, no amplifier is linear in the strict sense of the word. The effect of non-linearity of the tube characteristics is to generate new frequency components, so that the wave-forms of the plate voltage and plate current variations are not the same as the wave-form of the grid signal voltage. The result is that the a.c. power associated with these new frequency components do not contribute anything to the useful a.c. power output. This should, therefore, be considered as a loss. The usual plate dissipation is also present and contributes to the losses.

It is obvious that if a tube converts a large d.c. power input into a.c. power with considerable harmonics, there results a loss of efficiency. The greater the percentage of harmonics, the smaller the fundamental efficiency. The effects of harmonics on the fundamental efficiency is shown in Fig. 7.

The overall efficiency is defined as the ratio of the power \( P_L \) delivered to the load \( R_L \) to the power input \( P_{in} \). The power \( P_L \) includes the power associated with harmonics. The table VII reveals that \( P_L \) increases as \( Q_L \) decreases. It is important to note that \( Q_L \) of a coil is a valuable index of
its quality. The circuit efficiency is high if $Q_L$ of the loaded coil is low. This is the reason why the overall efficiency increases as $Q_L$ decreases. Moreover, when $Q_L$ is small, the percentage of harmonics is large, and the contribution of the power associated with harmonics makes the overall efficiency large.

The overall efficiency should not be confused with the fundamental efficiency. In order to remove the confusion, Fig. 8 is drawn to show the effects of $Q_L$ on the overall efficiency.

The table VII also reveals that the power $P_1$ associated with the fundamental frequency decreases, and the fundamental efficiency increases with the increase of $Q_L$. The reason is obvious when it is observed that the power input $P_{in}$ decreases as $Q_L$ increases in such a way that the efficiency $\eta_1$ goes higher.

On comparing Figs. 3 and 9, it is observed that both indicate that the percentage of harmonics decreases with the increase in $Q_L$, but the theoretical and experimental curves do not agree so far as the magnitudes of the harmonics are concerned. The reason for this abnormal behaviour lies in the sources of errors behind the two ways in which the problem has been approached. It is not necessary to repeat that the theoretical analysis has been derived on making a number of assumptions; which are not at all met in practice. Nevertheless, the theoretical analysis describes the operation of the beam power tube and the interpretation of the results are in agreement to the expected behaviour.

The experimental approach is not free from defects. The advantage of the practical method is that no assumption is made to determine the percentage of harmonics. But the sources of errors involved are too many, and they may be enumerated below:

1. In the laboratory it is not possible to measure \( r_f \) voltages of the order of micro-volts with accuracy.
2. The signal generator output voltage scale cannot be taken as accurate. There is no means available in the laboratory to check its accuracy. The accuracy claimed by the manufacturer is \( \pm (6\% + 0.1 \mu V) \). In the mid-scale region, the error may be greater or smaller by \( 4\% \).
3. Due to large pickup from stray fields, the output of the receiver is adversely affected. The shielding of the individual parts of the entire set-up is a big problem which cannot be solved readily in the laboratory.
4. The \( r_f \) signals pass through the power lines and are picked up by the receiver. As suitable \( r_f \) chokes were not available in the laboratory, this source of error could not be eliminated. In the opinion of the author, this caused large errors in the experimental data.
5. Although the indirect measurement of harmonic amplitudes is not based on the assumption that the grid exciting signal voltage is sinusoidal, one reason for disagreement between the theoretical and practical results may be attributed to the grid exciting signal voltage which was definitely non-sinusoidal.
SUMMARY AND CONCLUSIONS

Of the many branches of non-linear electric circuit theory, perhaps none is of greater theoretical or practical interest than that relating to the operation of class C radio frequency power amplifiers. Unfortunately, the large number of variables present in such an amplifier renders a precise theoretical treatment of the various circuits employed quite impracticable. The result is that the majority of treatments of these circuits have been confined to analysis of idealized systems in which mathematically troublesome tube characteristics are replaced by simpler linear or square law characteristics. The theoretical analysis, in a general way, attempts to present a method of analysing the characteristics of class C amplifiers using beam power tubes. The analysis is not rigorous, being derived from a number of assumptions. Still, in the opinion of the author, the analysis has its own importance for studying the subject in the light of mathematics.

The practical method of analysis based on Lissajous figures, in the opinion of the author, can be successful if the grid-exciting signal voltage is sinusoidal and the effects of stray inductance and capacitance are negligibly small. The effects of the stray inductance and capacitance could be reduced by using a low frequency. The author did not use a low frequency, as the coils and capacitors required for building the two stage transmitter are quite large. Large variable capacitors are not available in the laboratory and the winding of large coils was no easy task.

Another method for analysing the harmonics based on the Fourier analysis of the actual wave-shape of the voltage across the tank circuit of the amplifier could not be attempted, as the oscilloscopes available in the laboratory
have sweep frequencies which are too low compared to the signal frequency.

The indirect measurement of harmonic amplitudes is the best if one could remove all the sources of errors involved. If the experiment is performed on a well-built and shielded transmitter with $f_0$ choke coils in the power line of both transmitter and receiver, there seems to be no reason why the results obtained should be unsatisfactory.
APPENDIX I

1. Derivation of \( I_0 \), component of current:

\[
I_0 = \frac{KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} (\cos \omega t - \cos \frac{\omega}{2})^2 d(\omega t)
\]

\[
= \frac{KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} (\cos^2 \omega t - 2\cos \omega t \cos \frac{\omega}{2} + \cos^2 \frac{\omega}{2}) d(\omega t)
\]

\[
= \frac{KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} (\frac{1}{2} + \cos 2\omega t - 2\cos \omega t \cos \frac{\omega}{2} + \cos^2 \frac{\omega}{2}) d(\omega t)
\]

\[
I_0 = \frac{KE_s^2}{\pi} \left[ \frac{1}{4} + \frac{1}{2} \cos^2 \frac{\omega}{2} - \frac{3}{4} \sin^2 \theta \right]
\]

\[\text{(11)}\]

2. Derivation of fundamental component of current:

\[
I_1 = \frac{2KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} (\cos \omega t - \cos \frac{\omega}{2})^2 \cos \omega t d(\omega t)
\]

\[
= \frac{2KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} \left[ \frac{1}{4} \cos^2 \omega t + \frac{1}{4} \cos^2 \frac{\omega}{2} + \cos \omega t \cos \frac{\omega}{2} \right] d(\omega t)
\]

\[
= \frac{2KE_s^2}{\pi} \left[ \frac{1}{2} \sin^2 \theta + \frac{1}{4} \sin^2 \theta + \frac{1}{4} \cos^2 \theta + \frac{1}{2} \sin \theta \cos \theta + \frac{1}{2} \cos \theta \right]
\]

\[
I_1 = \frac{2KE_s^2}{\pi} \left[ \frac{1}{2} \sin^2 \theta + \frac{1}{4} \sin^2 \theta + \frac{1}{4} \cos^2 \theta - \frac{1}{4} \sin \theta \cos \theta \right]
\]

\[\text{(12)}\]

3. Derivation of second harmonic component of current:

\[
I_2 = \frac{2KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} (\cos \omega t - \cos \frac{\omega}{2})^2 \cos 2\omega t d(\omega t)
\]

\[
= \frac{2KE_s^2}{\pi} \int_{\omega t = 0}^{\omega t = 2/2} \left[ \frac{1}{4} \cos^2 \omega t + \frac{1}{4} \cos^2 \frac{\omega}{2} - \frac{1}{4} \cos \omega t \cos \frac{\omega}{2} + \frac{1}{2} \cos^2 \frac{\omega}{2} \right] d(\omega t)
\]
\[ I_2 = \frac{2KE_a^2}{\pi} \left[ \sin \left( \frac{1 + \cos^2 \theta}{2} \right) \cos \left( \frac{\sin \theta + \sin 3\theta}{2} \right) + \frac{e}{\theta} \right] \]  

\text{Derivation of } n\text{-th harmonic component of current:}

\[ I_n = \frac{2KE_a^2}{\pi} \int_{-\pi/2}^{\pi/2} \left( \cos \omega t - \cos \frac{\theta}{2} \right)^2 \cos n\omega t \, d(\omega t) \]

\[ = \frac{2KE_a^2}{\pi} \int_{-\pi/2}^{\pi/2} \left( \frac{1 + \cos n\omega t + 1 \cos (n+2)\omega t + 1 \cos (n-2)\omega t - 1 \cos (n+1) \omega t - \cos (n-1) \omega t}{4} \right) \cos \frac{\theta}{2} \cos n\omega t \, d(\omega t) \]

\[ = \frac{2KE_a^2}{\pi} \left[ \frac{1}{2n} \frac{\sin \frac{\theta}{2}}{2} \frac{\sin \frac{n\theta}{2} + 1 \sin \frac{(n+2)\theta}{2} + 1 \sin \frac{(n-2)\theta}{2} - 1 \cos \frac{\theta}{2} \sin (n+1) \theta}{2n+2} \right] \]

\[ = \frac{2KE_a^2}{\pi} \left[ \frac{1}{n(n+2)} \frac{\sin \frac{\theta}{2}}{2} \frac{1 + 2(n+1) \cos^2 \theta}{2} - \frac{1}{n(n-1)} \frac{\cos \frac{\theta}{2} \sin (n+1) \theta}{2} \right] \]

\[ = \frac{2KE_a^2}{\pi} \left[ \frac{1}{n(n+2)} \frac{\sin \frac{\theta}{2}}{2} \frac{1 + 2(n+1) \cos^2 \theta}{2} - \frac{1}{n(n-1)} \frac{\cos \frac{\theta}{2} \sin (n+1) \theta}{2} \right] \]
LITERATURE CITED


Dube: Class C amplifier 1950 harmonics and efficiency.