MONTANA STATE UNIVERSITY

Class C amplifier harmonics and efficiency
by Rajendra Dube

A THESIS Submitted to the Graduate Faculty in partial fulfilment of the requirements for the degree of Master of Science in Electrical Engineering
Montana State University
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This thesis presents an investigation into the effects of harmonics on tube efficiency. If is a well-known fact that the presence of appreciable harmonics in the plate circuit results in a loss of efficiency in a tube; but no data on the magnitude' of the effect are published at present. The. reason for the omission of the data, in the opinion of the author, may be attributed to the lack of a suitable practical method of measuring the per-centage of harmonics and tube efficiency with accuracy. This led into, the possibility of devising suitable- methods, of determining harmonics and tube efficiency in the laboratory.

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A THESIS

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ABSTRACT

Recent research in some parts of the vast domain of radio science is characterized by an extensive study of many problems which, as a rule, were formerly either generally considered to be of no great importance or were not considered at all. Frequently, it happens later on that these apparent problems prove to be of fundamental importance.

This thesis presents an investigation into the effects of harmonics on tube efficiency. It is a well-known fact that the presence of appreciable harmonics in the plate circuit results in a loss of efficiency in a tube; but no data on the magnitude of the effect are published at present.[1] The reason for the omission of the data, in the opinion of the author, may be attributed to the lack of a suitable practical method of measuring the percentage of harmonics and tube efficiency with accuracy. This led into the possibility of devising suitable methods of determining harmonics and tube efficiency in the laboratory.

The reader will note that the investigation is not a complete success; but the study has been extensive considering the limitation of the available equipment.

[1]E. E. Staff, M. I. T., "Applied Electronics", John Wiley & Sons, Inc., New York, pp. 590, 1949.

INTRODUCTION

In literature dealing with electric circuits, a number of abbreviations and symbols are used which are common to the technical man. Accepted abbreviations, such as Mcps for megacycles per second, used in radio engineering will be assumed well-known to the reader. It has also been considered desirable to include data for the beam power tube type 807 for ready reference.

A class C amplifier is defined as one in which the tube operates at a bias much greater than cut-off voltage, so that plate power is drawn only on the peaks of the signal voltage. It is not used in audio-amplifiers because distortion is too high but is the most efficient circuit for r-f power amplifiers where harmonics can be reduced by the use of tuned or selective circuits.

Ever since the introduction of class C amplifiers into the field of electronics, high vacuum tubes of the power type have assumed a steadily increasing importance. Improvement in design technique has permitted the construction of tubes giving very large outputs. Owing to the use of large power, any factors which contribute to an increase in efficiency of operation of electron tubes are worthy of serious consideration. Many papers have appeared which deal with technical investigations of the properties of tubes operating as radio-frequency class C amplifiers. These papers are concerned largely with conditions under which optimum conversion of direct current plate supply power into radio-frequency power is obtained, consistent with the demands of the type of service required. One of the factors, the effects of harmonics on tube efficiency, has received little attention.

LIST OF SYMBOLS

Symbol	Meaning
E_{bb}	Plate supply voltage.
E_{cc}	Control grid supply voltage.
E_{co}	Cut-off grid-bias.
E_{sg}	Screen grid-supply voltage.
E_{p1}	Fundamental voltage (rms value) across tank circuit.
E_s	Peak value of grid exciting signal voltage.
e_g	Instantaneous value of grid voltage.
E_{os}	Output voltage of the signal generator.
I_{bb}	D. C. Plate current.
i_p	Instantaneous value of plate current.
I_n	Peak value of the n^{th} harmonic current.
I_{p1}	Fundamental current (rms value).
$I_{R.F.}$	Rms value of r-f current in the load.
Z_T	Impedance of the loaded tank circuit at resonance.
Z_n	Impedance of the loaded tank circuit at the n^{th} harmonic.
R_L	Load resistance in ohms.
R_s	Reflected resistance in the tank circuit.
R_1	Resistance of coil.
R	Equivalent series resistance of coil including effect of connected load.
r	High resistance across tank circuit. This high resistance consists of two resistances, r_1 and r_2 in series, where r_1 is negligibly small in comparision with r_2.
P_1	Power associated with fundamental frequency.

LIST OF SYMBOLS

Symbol	Meaning
P_L	Power dissipated in load R_L.
P_{in}	Power input.
η_1	Fundamental efficiency in percent.
η	Overall efficiency in percent.
C	Capacitance in farads.
L	Inductance in henrys.
X	Reactance of coil.
ω	Angular velocity.
ω_{pr}	Angular velocity at parallel resonance.
θ	Angle of current flow.
Q_o	Quality factor for unloaded coil.
Q_L	Quality factor for loaded coil.
R.F.C.	Radio-frequency choke coil.

DATA ON TRANSMITTING BEAM POWER TUBE[2]
(Type 807)

D. C. Plate Voltage	500 volts
D. C. Grid No. 2 Voltage	250 volts
D. C. Grid No. 1 Voltage	-45 volts
Peak R. F. Grid No. 1 Voltage	65 volts
D. C. Plate Current	100 ma.
D. C. Grid No. 2 Current	6 ma.
D. C. Grid No. 1 Current (Approximately)	3.5 ma.
Driving Power (Approximately)	0.2 watt
Power Output (Approximately)	30 watts
Mu-Factor, Grid No. 2 to Grid No. 1	8

2R. C. A. Tube Handbook HB-3 Vol. 3-4.

Operation Of Beam Power Tube

A beam power tube is characterized by having a plate current that is substantially independent of plate voltage unless the plate voltage is so low that other positive electrodes rob the plate of a disproportionate fraction of the total space current. Since practical class C amplifiers are operated so that the plate potential never becomes as low as this, one can always assume that the plate current is substantially independent of the plate voltage, and hence of load impedance. This makes the analytical analysis of class C amplifiers using beam power tubes entirely possible.

The total space current in any vacuum tube is determined by the electrostatic field in the immediate vicinity of the cathode. In a beam power tube this field is, to a high degree of accuracy, proportional to the quantity $\left(e_g + \frac{E_{sg}}{\mu_{sg}}\right)^n$ where e_g and E_{sg} are the control grid and screen grid potentials respectively; and μ_{sg} is the amplification factor of the control grid against an anode represented by the screen grid.

When a beam power tube is operated so that plate current is substantially independent of plate voltage, the plate current will always be a substantially constant fraction of the total space current, provided the control grid current is negligible. Under the assumption, one can write:

$$i_p = K\left(e_g + \frac{E_{sg}}{\mu_{sg}}\right)^n \qquad (1)$$

where K and n are constants.

In order to determine n, the equation (1) is expressed in logarithmic form as

$$\log i_p = \log K + n \log \left(e_g + \frac{E_{sg}}{\mu_{sg}}\right) \qquad (2)$$

then log i_p and log $\left(e_g + \frac{E_{sg}}{\mu_{sg}}\right)$ are plotted for various values of i_p and e_g taken from the static characteristic curves, keeping E_{sg} constant at operating conditions. The slope of the resulting curve is n. It is better to plot i_p and $\left(e_g + \frac{E_{sg}}{\mu_{sg}}\right)$ on log-log paper, so that the necessity of taking the logarithm may be avoided. When this plot is made, the exponent n for the sake of simplicity in the following analysis may be taken approximately equal to 2. Hence, the expression

$$i_p = K\left(e_g + \frac{E_{sg}}{\mu_{sg}}\right)^2 \quad \ldots \quad (3)$$

is assumed to represent the characteristic of the beam power tube to a sufficient degree of accuracy.

In amplifiers, the voltage applied to the grid of the tube consists of a constant negative bias E_{cc} plus an alternating potential of peak amplitude E_s with angular velocity ω, so that

$$e_g = \left(E_{cc} + E_s \operatorname{Cos} \omega t\right) \quad \ldots \quad (4)$$

From Fig. 1, it can be seen that the angle of current flow θ may be determined by the expression

$$\operatorname{Cos} \frac{\theta}{2} = -\frac{\left(E_{cc} - E_{co}\right)}{E_s} \quad \ldots \quad (5)$$

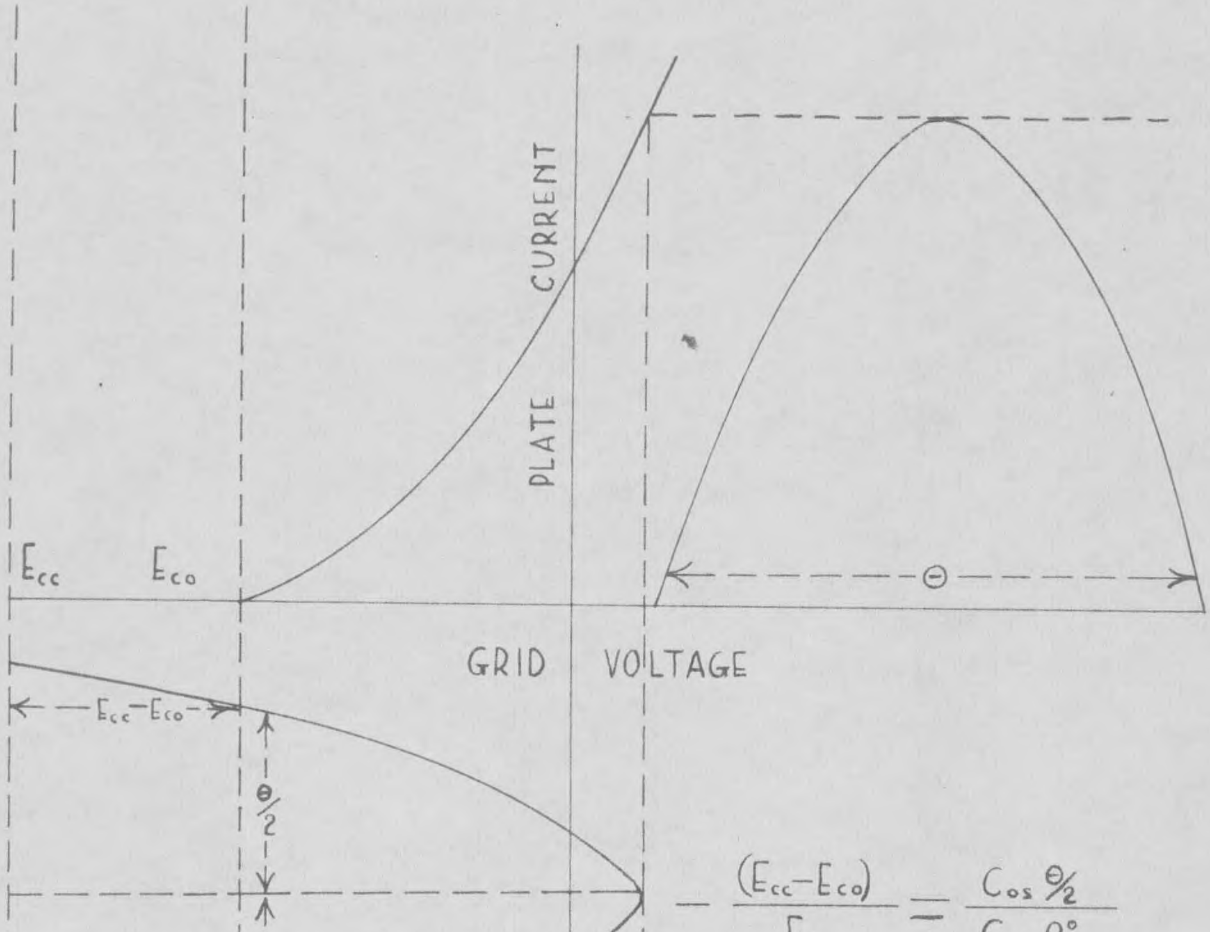
In case of class C amplifiers using tetrodes or pentodes

$$E_{co} = -\frac{E_{sg}}{\mu_{sg}} \quad \ldots \quad (6)$$

Combining equations (3), (4), (5) and (6), one can write

$$i_p = K\, E_s^{\,2}\left(\operatorname{Cos} \omega t - \operatorname{Cos} \frac{\theta}{2}\right)^2 \quad \ldots \quad (7)$$

Since the wave-shape of the current i_p is non-sinusoidal and symmetrical about the vertical axis,



ANGLE OF CURRENT FLOW

FIG. 1

$$i_p = I_o + I_1 \cos \omega t + I_2 \cos 2\omega t + \ldots \qquad (8)$$

By Fourier Analysis,

$$I_o = \frac{KE_s^{\,2}}{\pi} \int_{0}^{\omega t = \theta/2} \left(\cos \omega t - \cos \frac{\theta}{2} \right)^2 d(\omega t) \qquad (9)$$

$$I_n = \frac{2KE_s^{\,2}}{\pi} \int_{0}^{\omega t = \theta/2} \left(\cos \omega t - \cos \frac{\theta}{2} \right)^2 \cos n\omega t \, d(\omega t) \qquad (10)$$

On simplification (see Appendix I), the following important results are obtained:-

$$I_o = \frac{KE_s^{\,2}}{\pi} \left[\frac{\theta}{4} + \frac{\theta}{2} \cos^2 \frac{\theta}{2} - \frac{3}{4} \sin \theta \right] \qquad (11)$$

$$I_1 = \frac{2KE_s^{\,2}}{\pi} \left[\sin \frac{\theta}{2} - \frac{1}{3} \sin^3 \frac{\theta}{2} - \frac{\theta}{2} \cos \frac{\theta}{2} \right] \qquad (12)$$

$$I_2 = \frac{2KE_s^{\,2}}{\pi} \left[\sin \theta \left(\frac{1}{8} + \frac{3}{4} \cos^2 \frac{\theta}{2} \right) - \cos \frac{\theta}{2} \left(\sin \frac{\theta}{2} + \frac{1}{3} \sin \frac{3\theta}{2} \right) + \frac{\theta}{8} \right] \qquad (13)$$

$$I_n = \frac{2KE_s^{\,2}}{\pi} \left[\frac{1}{n(n+2)} \sin \frac{n\theta}{2} \left\{ 1+2(n+1) \cos^2 \frac{\theta}{2} \right\} - \frac{1}{(n-1)} \cos \frac{\theta}{2} \sin (n-1) \frac{\theta}{2} \right.$$
$$\left. - \frac{1}{(n+1)} \cos \frac{\theta}{2} \sin (n+1) \frac{\theta}{2} + \frac{1}{(n^2-4)} \sin (n-2) \frac{\theta}{2} \right] \qquad (14)$$

where $n = 3, 4, \ldots$

Since the beam power tube type 807 was employed for the study, it may be noted that under the operating conditions, $E_{sg} = 250$ volts, $E_{cc} = -45$ volts, $E_s = 65$ volts and $\mu_{sg} = 8$. Therefore taking into account the equations (5) and (6), one obtains

$$\cos \frac{\theta}{2} = 0.212 = \cos 77.75^\circ.$$

$$\therefore \quad \theta = 155.5^\circ = 2.715 \text{ radians.}$$

Substituting this value of θ in equations (12), (13) and (14), the following harmonic components are obtained:-

$$I_1 = \frac{2KE_s^2}{\pi}\left[0.3786\right] \quad \ldots\ldots (15)$$

$$I_2 = \frac{2KE_s^2}{\pi}\left[0.2538\right] \quad \ldots\ldots (16)$$

$$I_3 = \frac{2KE_s^2}{\pi}\left[0.1189\right] \quad \ldots\ldots (17)$$

From equations (15), (16) and (17), one obtains

$$\frac{I_2}{I_1} = 0.668 \quad \ldots\ldots (18)$$

$$\frac{I_3}{I_1} = 0.314 \quad \ldots\ldots (19)$$

Tank Circuit

The plate circuit of the amplifier consists of a tank circuit; therefore the load offers different impedances to different harmonics (the impedance decreasing as the number of harmonics increases). The tank circuit acts as a pure resistance only to the fundamental frequency to which it is tuned.

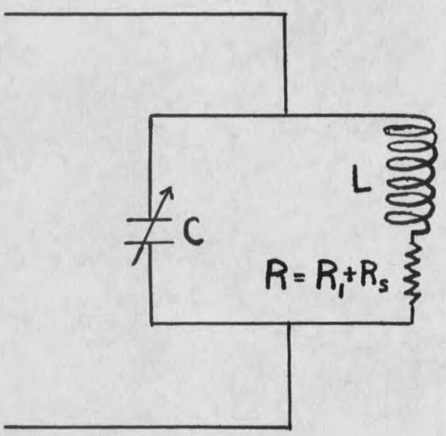


Fig. 2

Let R denote the sum of the resistance R_1 of the inductance and any resistance R_s reflected into the circuit.

Since the inductance L and resistance R in series are in parallel with the capacity C, the impedance[3] of the tank circuit is

$$Z = \frac{L}{CR}\,\frac{1-j\,R/\omega L}{1+j\left(\frac{\omega L}{R}-\frac{1}{\omega CR}\right)} \quad \ldots\ldots (20)$$

[3] Cruft Electronics Staff, "Electronic Circuits and Tubes", McGraw-Hill Company, Inc., New York and London, pp. 43, 1947.

Since the tuning capacity is variable, the condition for maximum impedance is obtained simultaneously with unity power factor[4]. The condition for unity power factor is

$$-\frac{R}{\omega L} = \frac{\omega L}{R} - \frac{1}{\omega CR} \quad \ldots\ldots\ldots\ldots \quad (21)$$

Let ω_{pr} be the parallel resonant frequency and $Q_L = \frac{\omega_{pr}L}{R}$, then the impedance of the tank circuit at parallel resonance is

$$Z_T = \frac{L}{CR} = \omega_{pr}L\left(Q_L + \frac{1}{Q_L}\right) \quad \ldots\ldots\ldots\ldots \quad (22)$$

From equations (20) and (21), the impedance of the tank circuit at the n^{th} harmonic is

$$Z_n = \frac{L}{CR}\,\frac{1-j\,\frac{1}{nQ_L}}{1+j\left[Q_L\left(n-\frac{1}{n}\right)-\frac{1}{nQ_L}\right]} \quad \ldots\ldots\ldots\ldots \quad (23)$$

The percentage of harmonic voltage across the tank circuit is 100 x $\left|\frac{I_n Z_n}{I_1 Z_T}\right|$, where I_n is the n^{th} harmonic component of the plate current I_p.

From equations (18), (19), (22) and (23), one obtains:-

$$\text{Per cent second harmonic voltage} = \left|\frac{66.8\left(1-j\,\frac{1}{2Q_L}\right)}{1+j\left(1.5Q_L-\frac{1}{2Q_L}\right)}\right| \quad \ldots\ldots \quad (24)$$

$$\text{Per cent third harmonic voltage} = \left|\frac{31.4\left(1-j\,\frac{1}{3Q_L}\right)}{1+j\left(2.67Q_L-\frac{1}{3Q_L}\right)}\right| \quad \ldots\ldots \quad (25)$$

For different values of Q_L, the theoretical percentage of second and third harmonic voltages across the tank circuit may be calculated from equations (24) and (25). Some of the values are tabulated in table I and are plotted in Fig. 3.

[4] John D. Ryder, "Networks, Lines, and Fields", Prentice-Hall, Inc., New York, pp. 53, 1949.

Table I

Q_L	Per cent second harmonic	Per cent third harmonic
2	23.6	5.8
3	14.0	3.9
4	11.0	3.06
5	8.84	2.35
6	7.38	1.96
7	6.35	1.68
8	5.56	1.47
9	4.95	1.35
10	4.46	1.175
11	4.05	1.065
12	3.71	1.02
13	3.43	0.9

It is also interesting to note that when the tank circuit is replaced by a pure resistance, the maximum possible percentage of second and third harmonics are:

Table II

Second harmonic voltage across tank circuit	66.8%
Third harmonic voltage across tank circuit	31.4%

Figure 3 indicates that the percentage of harmonics becomes inappreciable when Q_L is large.

The greatest defect with this theoretical analysis is that it is based on a large number of assumptions:-

1. Plate current is independent of the load.
2. Plate current follows a square law.
3. Grid exciting voltage is sinusoidal.
4. No grid current flows.
5. Variation of parameters and insufficient filament emission do not take place.

From these assumptions, it is obvious that the analysis is not rigorous.