Performance analysis of the maximum likelihood sequence estimator for known channel impulse responses

by Harold Fred Fisher

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Electrical Engineering
Montana State University

Abstract:
In a pulse amplitude modulation system, intersymbol interference is the primary detriment to reliable high rate digital transmission over narrow bandwidth high signal to noise ratio channels. The maximum likelihood sequence estimator is a receiver structure designed for use under intersymbol interference conditions. The objective of this thesis is to develop and use an analysis technique for evaluating the performance of the maximum likelihood sequence estimator for linear channels with known impulse responses. The performance criterion used is the fractional difference in minimum euclidean weight due to intersymbol interference. Discrete time signal models and flow graph theory are used to develop the analysis technique.

PERMISSION TO COPY

In presenting this thesis in partial fulfillment of the requirements for an advanced degree at Montana State University, I agree that the Library shall make it freely available for inspection. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by my major professor, or, in his absence, by the Director of Libraries. It is understood that any copying or publication of this thesis for financial gain shall not be allowed my written permission.

Signature Harold Fred Fisher

Date June 18, 1973

PERFORMANCE ANALYSIS OF THE MAXIMUM LIKELIHOOD SEQUENCE ESTIMATOR FOR KNOWN CHANNEL IMPULSE RESPONSES

by

HAROLD FRED FISHER

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree

of

MASTER OF SCIENCE

in

Electrical Engineering

Approved:

Head, Major Department

Chairman, Examining Committee

Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

August, 1973

ACKNOWLEDGMENT

The author wishes to thank his committee C.K.Rushforth, D.A.Rudberg and M.A.Faulkner for their guidance in helping prepare this thesis. Also, the author expresses his appreciation to R.E.Leo his research assistantship advisor for his patiences and understanding showed during the entire time spent preparing the thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

In a pulse amplitude modulation system, intersymbol interference is the primary detriment to reliable high rate digital transmission over narrow bandwidth high signal to noise ratio channels. The maximum likelihood sequence estimator is a receiver structure designed for use under intersymbol interference conditions. The objective of this thesis is to develop and use an analysis technique for evaluating the performance of the maximum likelihood sequence estimator for linear channels with known impulse responses. The performance criterion used is the fractional difference in minimum euclidean weight due to intersymbol interference. Discrete time signal models and flow graph theory are used to develop the analysis technique.

I. INTRODUCTION

In a pulse amplitude modulation system whenever the information rate, measured in units of pulses per second (baud), is larger than the bandwidth of the transmission channel, intersymbol inteference may result [1]. In other words, the pulses overlap into adjacent time slots and may cause an erroneous decision at the receiver. This phenomenon is the primary deterrent to reliable, high rate digital transmission over narrow bandwidth, high signal-to-noise ratio channels. Voice-grade telephone channels used for data transmission are typical examples.

The objective of this thesis is to develop and use a technique for analyzing the performance of the maximum likelihood sequence estimator [2] for linear channels whose impulse response is longer than the source symbol separation.

Many receiver structures have been discussed and developed [3,4, 5]. Probably the most talked about structure has been the optimum receiver structure--the one that makes symbol decisions based on the entire received sequence [5]. This structure was not pursued, however, since maximum likelihood calculations increased exponentially with sequence length making hardware development too complex and difficult. As an alternative to this overwhelming complexity problem, structures were developed that made simple symbol-by-symbol decisions [4]. The best forms of these receiver structures were derived using certain criterion of optimality, such as minimum probability of error, minimum probablity of error with intersymbol interference forced to zero, and minimum mean-square error. In every case these receiver structures turned out to be a matched filter in cascade with a tapped delay line (transversal filters) [3].

Nonlinear receivers have also been looked at [6]. Several optimum nonlinear structures have been developed, but these turned out to be very complex. This caused the experts to look at suboptimal nonlinear receivers such as decision feedback. These were far too complex and difficult to analyze to justify their use.

The most recent receiver structure devised for channels introducing gaussian noise and intersymbol interference is a maximum likelihood sequence estimator of the entire received sequence [1]. Unlike past formulations of likelihood sequence estimators its complexity does not increase with sequence length, but is proportional to M^L, where M is the size of the input alphabet and L is the length of channel impulse response in units of symbol separation. In fact this receiver can easily be implemented and analyzed.

Figure 1 is a block diagram of a P.A.M. channel with additive white gaussian noise cascaded with the maximum likelihood sequence receiver. The receiver consists of a linear filter called the whitened matched filter, a sampler taking samples once every T seconds (symbol separation), and a nonlinear recursive algorithm called the Viterbi Algorithm. From Figure 1

$$s(t) = \sum_{i=0}^{i=N} x_i h(t-iT) \tag{1}$$

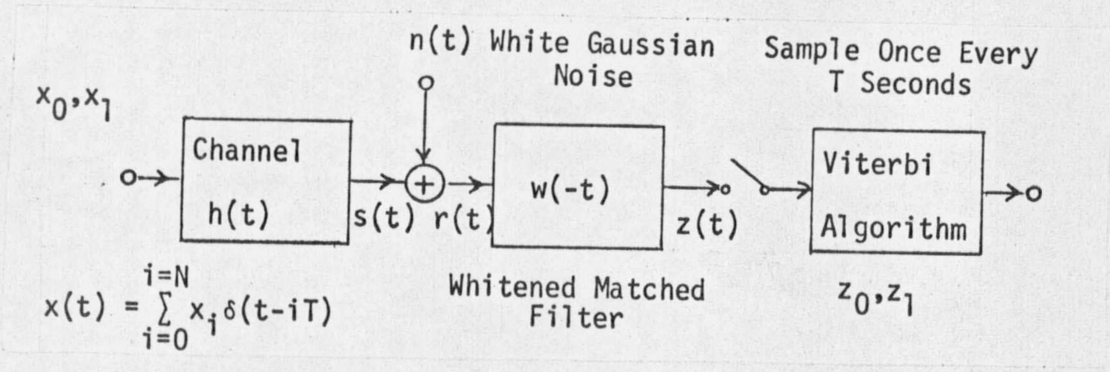

Figure 1. P.A.M. channel with white gaussian noise and the maximum likelihood sequence receiver structure.

where

$s(t)$ = output of channel whose impulse response is $h(t)$

$x(t)$ = input sequence

T = input symbol separation

x_i = $(i + 1)^{th}$ symbol of the input sequence

The implementation of the whitened matched filter is quite simple. Instead of having a bank of matched filters, one for every input symbol sent, only one filter [w(-t)], with samples taken once every symbol instant is necessary. This leaves the output samples with all the necessary information making them a set of sufficient statistics for the estimation of the transmitted sequence. In addition, the output sequence, z_0, z_2, ..., $z_{N + L - 1}$ is a sequence of statistically independent, identically distributed gaussian random variables. This is necessary to insure the simple and recursive property of the Viterbi algorithm.

The Viterbi algorithm was originally used to decode convolutional codes. Because of the similarity between a data transmission channel causing intersymbol interference and a convolutional encoder the Viterbi algorithm can be used for maximum likelihood sequence estimation [1]. As mentioned earlier, the algorithm gives likelihood calculation with complexity proportional to M^L. To implement the algorithm only M^{L-1} comparisons and M^L additions per received symbol are required. Also, M^{L-1} registers for remembering the surviving path and M^{L-1} registers for remembering the appropriate metrics are needed [7].

Much of the literature dealing with detection theory classifies a receiver optimum when the probability of error is minimized [2, 3, 8, 9, 10]. For the maximum likelihood sequence estimator, a bound on the minimum probability of error, can be determined by knowing the minimum euclidean distance or weight of the output signal space. Minimum eucludean distance is the distance between the two closest signals or sequences of the output signal space. If the channel causes intersymbol interference, the minimum distance may become smaller. Some channels cause a larger reduction in distance then others, hence, causing a larger increase in probability of error. This decrease directly reduces the reliability of the system. If it is known which channels yield the smallest differences (percent reduction) then communication systems can be devised that provide maximum reliability in the presence of intersymbol interference and additive white gaussian noise.

The performance criteria used is the fractional difference in minimum euclidean weight. To determine this for each channel, minimum euclidean weight with and without intersymbol interference must be found.

Associated with each channel impulse response there is a set of error states that form nodes of a flow graph. These nodes are connected in such a way that a transition from one node to another determines a single receiver output error event. Further, euclidean weight for each transition can be shown to be the square of a single output error event. This might suggest using the flow graph to solve for minimum euclidean weight by finding the shortest path from some initial node to a final node. Indeed this can be done and is developed and used here in the form of an XDS Extended Fortran IV program as our performance analysis technique.

As a first step toward the flow graph, a more complete analysis of the whitened, matched filter will be given. Also, an equivalent discrete-time model of the P.A.M. channel and maximum likelihood sequence receiver will be presented. This will lead to definitions of input and output error events, state error events, euclidean weight, and then finally to minimum euclidean weight.

II. CHANNEL MODELS AND THE OPTIMUM RECEIVER

Reference to Figure 1 shows the output r(t) of the noisy channel to be a sum of two signals, white gaussian noise n(t) and signal s(t) given by (1). It is well known that detection of signals that are linear combinations of some set of basis signals is accomplished using a bank of filters each matched to a basis signal [8]. Since the signals of (1) are time translations of h(t) (the channel impulse response), only one filter matched to the channel with samples taken once every T seconds is necessary. Letting a(t) represent the output of h(-t) (matched filter), its samples form a sequence of numbers a(0), a(1), ..., a(k), ..., a(N + L - 1).

$$a(k) = \int_{-\infty}^{+\infty} r(t)h(t-kT)dt \qquad k = 0, 1, \ldots, N + L - 1$$

$$= \sum_{i=0}^{i=N} x_i \int_{-\infty}^{+\infty} h(t-iT)h(t-kT)dt + \int_{-\infty}^{+\infty} n(t)h(t-kT)dt \tag{2}$$

Now, let each integral term of (2) be represented as

$$R_{k-i} = \int_{-\infty}^{+\infty} h(t-kT)h(t-iT)dt \tag{3}$$

and

$$n'_k = \int_{-\infty}^{+\infty} n(t)h(t-kT)dt \tag{4}$$

Equation (2) now has a discrete representation given by

$$a(k) = \sum_{i=0}^{i=N} x_i R_{k-i} + n'_k \tag{5}$$

What can be said about the correlation of the noise samples given by (4)? To begin with, the input noise n(t) is white gaussian with autocorrelation function $\sigma^2\delta(\tau)$ where σ^2 is the value of the power over a unit bandwidth. If the duration of h(t) is larger than the input symbol separation, i.e., if L > 1 where L is the smallest integer such that h(t) = 0 for t > LT, then at least adjacent samples are correlated. To get an exact measure of the correlation of the noise samples, their autocorrelation coefficient must be determined. Taking the expected value of the product $n_k'n_i'$, $0 \leq i$ and $k \leq N + L - 1$, gives

$$E(n_k'n_i') = \sigma^2 R_{k-i} \tag{6}$$

Referring to equation (3), it is evident that R_{k-i} is nonzero for k-1<L because of the overlap. Also, R_{k-i} is symmetrical, i.e., $R_{k-i} = R_{i-k}$

It was mentioned earlier that the whitened, matched filter has the property that its output samples are statistically independent. This is necessary to insure the simple and recursive nature of the Viterbi algorithm. But, equations (6) and (3) show that the noise samples n_k' are correlated whenever $L \geq 1$; therefore, the sampled outputs of h(-t) are not statistically independent. Some whitening transformation must be performed. From random variable theory it is known that a linear transformation on a sequence of gaussian random variables is also gaussian. Further, if the sequence is statistically dependent, then there is a linear transformation that will produce a statistically independent sequence.

This is the final process performed by the whitened, matched filter and is accomplished by a transversal filter (tapped delay line). Discussion of the transversal filter is postponed until the discrete time model is introduced.

Recall from Figure 1 that the input to the channel is a sequence of numbers, each separated from its adjacent neighbors by T seconds. Associated with this sequence (or any time sequence of numbers) is a formal power series in D. This series is called the D-transform of x(t) and is given by

$$x(D) = \sum_{i=0}^{i=N} x_i D^i \tag{7}$$

where x_i is a symbol chosen at the transmitter from some predetermined alphabet; e.g., in the binary case, x_i equals 1 or 0. D^i can be thought of as a delay operator representing iT units of delay.

Since the output samples of h(-t) form a sequence with symbol separation T, it too has a D-transform. Equation (5) indicated that each sample a(k) is a function of n'_k and the first k + 1 values for x_i and R_i. This looks as if the D-transform of the sequence a(0), a(1), ..., could be represented in terms of the D-transforms of x_0, x_1, ..., x_N ; n'_0, n'_1, ..., n'_N, and the sequence whose coefficients are given by (3). Now equation (5) has terms representing the convolution of the coefficients x_i and R_i. Whenever two polynomials are multiplied, each term of the product is the convolution of terms from each. Hence, the D-transform of the samples a(k) can be expressed as

$$a(D) = x(D)R(D) + n'(D) \tag{8}$$

where x(D) is the input sequence transform, n'(D) is the correlated noise sample transform and R(D) is the transform of the sequence whose coefficients are given by (3).

Notice, equation (3) is a function of the difference k-1. For example,

$$R_2 = R_{9-7} = \int_{-\infty}^{+\infty} h(t-9T)h(t-7T)dt$$

$$= R_{3-1} = \int_{-\infty}^{+\infty} h(t-3T)h(t-T)dt$$

Also, R_{k-i} equals R_{i-k}, making the corresponding sequence symmetrical about the k-i origin. The number of coefficients in the sequence depends on the amount of overlap, or the value of L. All totaled, there are 2(L-1) + 1 nonzero terms having a D-transform.

$$R(D) = \sum_{i=1-L}^{i=L-1} R_i D^i \tag{9}$$

The D-transform R(D) will be called the autocorrelation function or the pulse autocorrelation function. Since it contains 2L-1 nonzero terms, it has 2L-2 complex roots. Since R(D) is symmetrical [$R(D) = R(D^{-1})$] an inverse of a root is also a root. This makes it possible to factor R(D) into a product of polynomials, f(D) and $f(D^{-1})$.

We are now in a position to talk about the transversal filter. This filter, remember, must transform the correlated noise sequence n'(D) into a white gaussian noise sequence. But the coefficients of (6) are just those of (3) times the noise power per unit bandwidth (σ^2); therefore, the pulse autocorrelation function of n'(D) is

$$R_n'(D) = \sigma^2 R(D) = \sigma^2 f(D) f(D^{-1}) \tag{10}$$

The white noise sequence must have a pulse autocorrelation function that is a constant. Let

$$n'(D) = n(D) f(D^{-1}) \tag{11}$$

where n(D) is our desired white noise sequence. Equation (8) now can be written as

$$a(D) = x(D) f(D) f(D^{-1}) + n(D) f(D^{-1}) \tag{12}$$

Passing the sequence a(D) through a filter with response $f^{-1}(D^{-1})$ (transversal filter) gives

$$z(D) = a(D)/f(D^{-1}) = x(D) f(D) + n(D) \tag{13}$$

with pulse autocorrelation function for n(D) equal to

$$E[n(D)n(D^{-1})] = \frac{E[n'(D)n'(D^{-1})]}{f(D)f(D^{-1})} = \frac{\sigma^2 R(D)}{R(D)} = \sigma^2 \tag{14}$$

where σ^2 is a constant. Equation (13) tells us that the P.A.M. channel and whitened, matched filter can be represented as a discrete time filter with impulse response f(D) plus an additive white gaussian noise sequence (see Figure 2).