The response of an airplane to a dynamic load

by Rodney Lee Gilge

A thesis submitted to the Graduate Faculty in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE in Aerospace and Mechanical Engineering
Montana State University

Abstract:
The response of an airplane to a blast load is studied. The effects of rigid body translation and rigid body rotation are taken into account. Linear and non-linear solutions are compared. The non-linearities result from use of a non-linear stress-strain relation and from geometry changes due to large deflection.

It is concluded that the response of an airplane to a dynamic load is definitely influenced by the effects of rigid body translation and rotation. For large loads the non-linear solution predicts a larger wing deflection and a longer period of wing oscillation than does the linear solution.

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Signature Rodney Lee Hilge

Date Nov. 17, 1970

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Approved:

Head, Major Department

Chairman, Examining Committee

Graduate Dean

MONTANA STATE UNIVERSITY
Bozeman, Montana

December, 1970

ACKNOWLEDGMENT

The author is indebted to the National Science Foundation which provided financial aid for this project.

Grateful acknowledgment is also made for the continuing guidance and assistance of Dr. D. O. Blackketter.

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ABSTRACT

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CHAPTER I

INTRODUCTION

Design loads for accelerated airplane structures are frequently based on the assumption that the wing is perfectly rigid. This may lead to failure due to dynamic overstress. For example, a gust load may produce wing bending moments at the fuselage that are 15-20% greater than those calculated on the assumption of a rigid wing. Dynamic loads cause translation and rotation of the airplane as a whole and also cause vibrations of the structure. Dynamic overstress is produced by the additional inertia forces associated with the structure vibrations.

The load distribution on the wing is also affected by the wing deformation and vibration. Determining the load distribution on the basis of a rigid wing may lead to results that are too much in error to be useful. There may also be serious loss of aileron, elevator, and rudder control effectiveness due to deformation of the structure. In this paper only the airplane response to a dynamic load will be considered. The effect of this response on the wing load distribution and control effectiveness will not be analyzed.

The response of an airplane to dynamic loads has been frequently studied. In some studies (11)[1] the airplane was con-

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[1] Numbers in parenthesis refer to literature consulted.

sidered to be perfectly rigid. In other studies (7,10) the wings were considered to be elastic but the effects of rigid body translation and rotation of the airplane were neglected. Bisplinghoff (4) and Houbolt (5) included the effects of rigid body translation in their studies. However, they considered only dynamic loads that were uniform across the airplane so that no rigid body rotation occurred.

Most of the studies that have been made do not include the effects of both rigid body rotation and translation. The possibility of large non-linear wing deflections is considered in only very few studies.

The purpose of this paper is to study the response of an airplane to a dynamic load. The effects of rigid body rotation and translation and wing vibrations will be considered.

A combination of the modified Galerkin method and Hamming's modified predictor-corrector method will be used to solve the governing partial differential equations.

CHAPTER II

FORMULATION OF THE PROBLEM

SYSTEM DESCRIPTION

The system selected for study is shown in figure 1. It is intended to be a simple representation of an airplane. It consists of a free-free beam with a lumped mass at the center. The lumped mass simulates the fuselage of the airplane and the left and right portions of the beam represent the wings. The dynamic load acts across the entire wing cross-section, but only the rectangular portion of the wing is assumed to carry any load. The material properties are assumed to be homogeneous throughout each wing. This system is identical to one used by Bisplinghoff (10) in his study of aeroelasticity.

Three coordinant systems are used to describe the motion of the system. The fixed coordinants X-Y with unit vectors $\bar{i}$ and $\bar{j}$ are used to describe the rigid body translation and rotation of the system. The moving coordinants x_1-y_1 and x_2-y_2 with unit vectors $\bar{i}_1$, $\bar{j}_1$ and $\bar{i}_2$, $\bar{j}_2$, respectively are used to measure the deflection of the wings relative to the fuselage.

EQUATIONS OF RIGID BODY MOTION

The response of the system can be specified by four equations of motion. The equations describe the rigid body translation, the

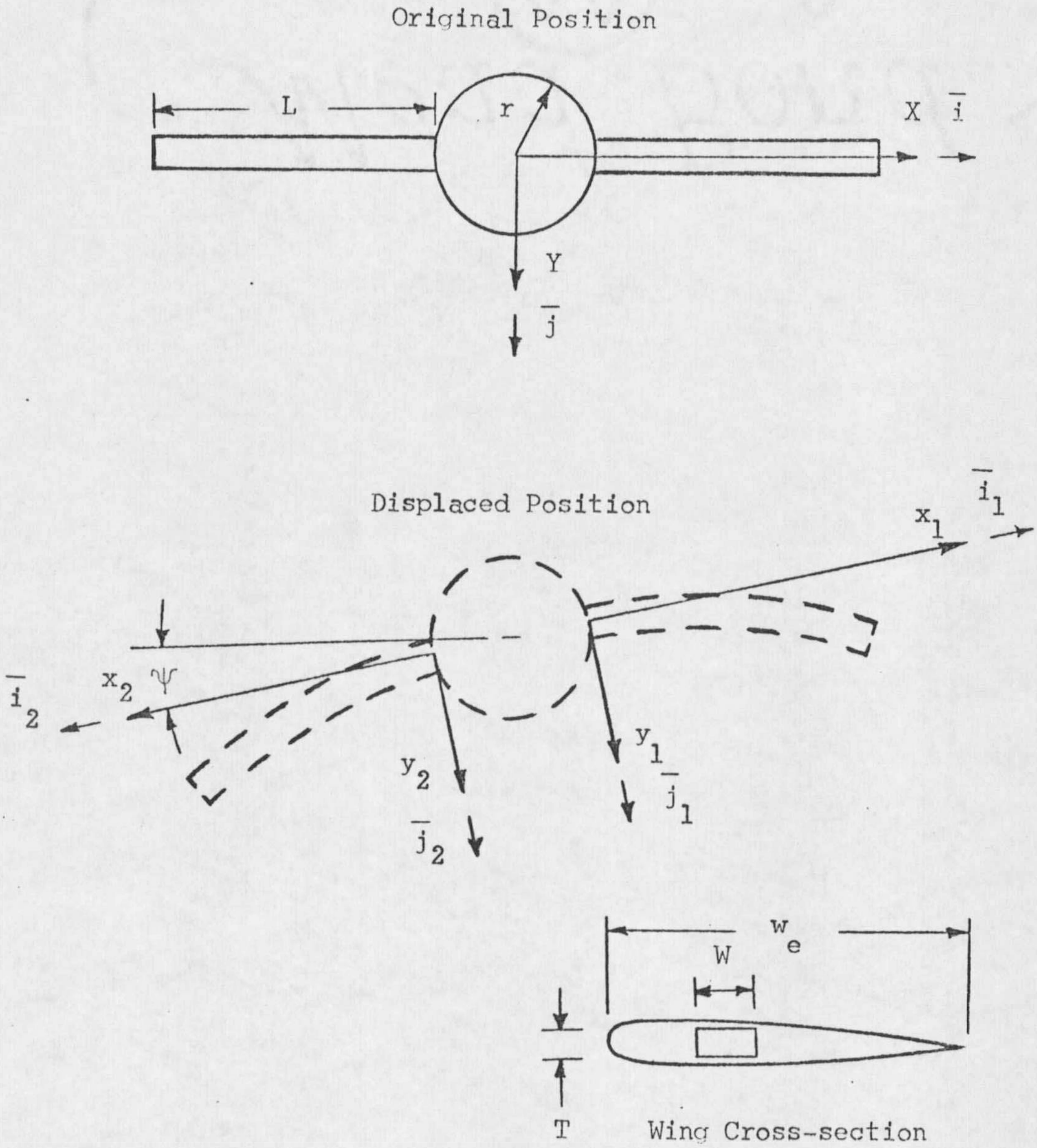

Figure 1. The Airplane Model

rigid body rotation, the relative displacement of the left wing, and the relative displacement of the right wing. The two equations of rigid body motion can be determined by considering the forces acting on the fuselage.

The forces acting on the fuselage are shown in figure 2. It is assumed that the weight of the airplane and the lift on the wings cancel each other. Therefore, these terms are not included in the derivation of the governing equations. Another simplifying approximation is that no forcing function acts on the fuselage.

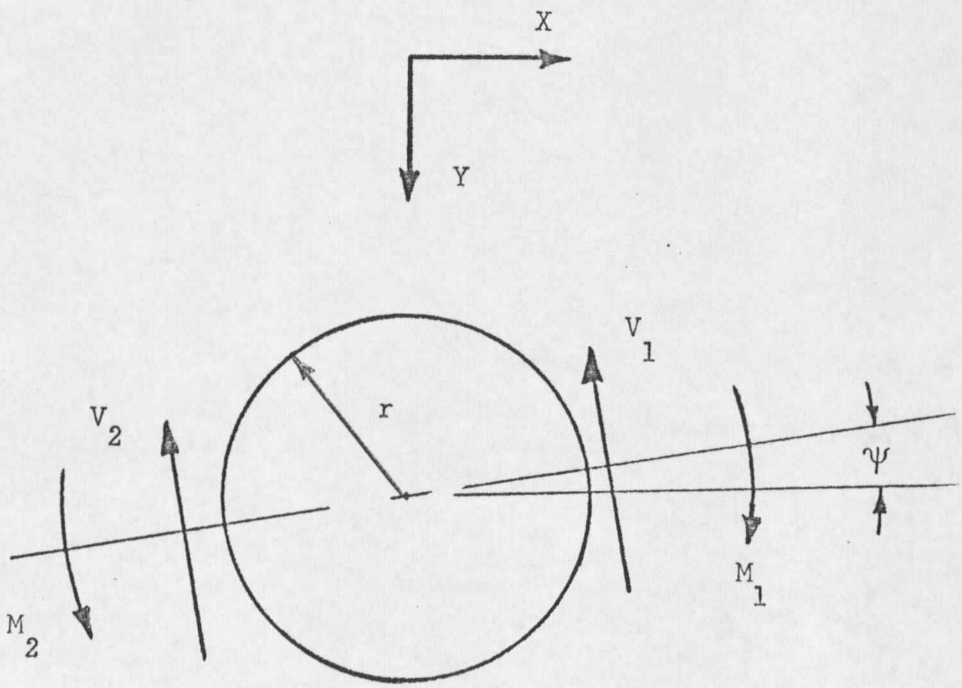

Figure 2. Forces on the Fuselage

The equation for rigid body rotation can be obtained by summing moments about the center of the fuselage. This yields:

$$\ddot{\psi} = \left[r(V_1 - V_2) + M_2 - M_1 \right] / I_b \qquad (1)$$

where ψ is the rigid body rotation, r is the radius of the fuselage, V_1 and V_2 are shears, M_1 and M_2 are internal moments, and I_b is the mass moment of inertia of the fuselage. A dot , $\dot{}$, above a quantity indicates differentiation with respect to time and a prime , $'$, indicates differentiation with respect to position.

By summing forces in the $\bar{j}$ direction the equation for rigid body translation can be shown to be:

$$\ddot{Y} = -(V_1 + V_2) \cos \psi / M_b \qquad (2)$$

where M_b is the mass of the fuselage.

LINEAR EQUATION OF WING MOTION

The fuselage of the airplane can both displace and rotate. Since the wings are rigidly attached to the fuselage, any motion of the fuselage will result in a corresponding motion of the wings. In addition the wings can also move relative to the fuselage.

If it is assumed that cross-sectional planes remain plane and if shear deformation and rotary inertia of bending are neglected, then the linear equations for the relative motion of the wings can be written as: (see appendix B)

$$-EI\,y_1'''' + w_1(x_1,t) = u\left[\ddot{Y}\cos\psi - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] \quad (3)$$

$$-EI\,y_2'''' + w_2(x_2,t) = u\left[\ddot{Y}\cos\psi + r\ddot{\psi} + x_2\ddot{\psi} + \ddot{y}_2 - y_2\dot{\psi}^2\right] \quad (4)$$

where Y is the rigid body translation, ψ is the rigid body rotation, x_1 and x_2 are positions along the wings, y_1 and y_2 are the relative displacements of the wings, w_1 and w_2 are the forcing functions, and u is the mass per unit length of wing.

NON-LINEAR EQUATIONS OF WING MOTION

The non-linear wing equations include the effects of geometry changes due to large deflections and also account for the possibility of strains that are in the non-linear portion of the stress-strain curve.

Consider an element of the right wing as shown in figure 3. Again assume plane sections remain plane and neglect shear deformation and rotary inertia of bending.

The equations of motion of the wings with respect to the fuselage can be shown to be: (see appendix C)

$$M_1'\phi_1'\sin(2\phi_1) - M_1''\cos^2(\phi_1) + u\,T^2\phi_1'\ddot{\psi}\sin(2\phi_1)/12 + w_1(x_1,t) = u\left[\ddot{Y}\cos(\psi) - r\ddot{\psi} - x_1\ddot{\psi} + \ddot{y}_1 - y_1\dot{\psi}^2\right] \quad (5)$$